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Mobile Math Trails: An Experience in Teacher Training with Mathcitymap

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ABSTRACT

Background: Studies show that the outdoors can be a privileged context to promote positive attitudes towards mathematics. **Objectives**: We aim to address the following questions: 1) Which pros and cons are pointed out by pre-service teachers of elementary education to the use of MathCityMap (MCM)? 2) How can we characterise the participants' engagement in a math trail performed with MCM? **Design**: We followed a qualitative, interpretative methodology. The paradigm choice was because the main goal was to understand the perspective and reactions of the participants to a particular situation. **Setting and Participants**: The participants were 48 pre-service teachers of elementary education attending the first semester of the 3rd year of an undergraduate course at a public higher education institution in Portugal. **Data collection and analysis**: Data were collected during the classes of a unit course on Didactics of Mathematics. The pre-service teachers answered a questionnaire, followed by the implementation of a mobile math trail. At the end of this experience, the participants filled out another questionnaire. Participant observation and audio-visual records were also applied. Thus, the analysis involved a qualitative and inductive approach, resorting to content analysis. **Results**: The pre-service teachers were actively engaged during the math trail, showing interest in solving the tasks, using MCM, and focused on finishing the activity, evidencing persistence and will to be involved in the discussions. We also identified anxiety and frustration related to the outcome of a particular task. Strategies of different nature were used, mainly associated to the level of cognitive demand of the tasks. As pros, they considered the app intuitive, promoting autonomy, spatial orientation and collaborative work and highlighted as cons the inaccessibility to Wi-Fi, younger students' not having smartphones/tablets, and the limitation of answer formats in task design. **Conclusions**: Results encourage the use of MCM as a valuable tool in outdoor mathematics education.

Keywords: authentic tasks; math trail; mobile learning; teacher education.

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Trilhos matemáticos digitais: uma experiência na formação de professores com o MathCityMap

RESUMO

Contexto: Vários estudos mostram que o meio envolvente pode ser um contexto privilegiado para promover atitudes positivas em relação à matemática. Neste âmbito, destacam-se os trilhos matemáticos. **Objetivos**: Pretendemos responder às seguintes questões de investigação: 1) Quais os prós e contras apontados por futuros professores do ensino básico à utilização do MathCityMap (MCM)? 2) Como podemos caracterizar o envolvimento dos participantes ao longo de um trilho matemático com o MCM? **Design**: Seguiu-se uma metodologia qualitativa e interpretativa, sendo a escolha do paradigma sustentada pelo facto de o objetivo principal ser entender a perspectiva e as reações dos participantes a uma determinada situação. **Ambiente e participantes**: Participaram neste estudo 48 futuros professores do ensino básico a frequentar o primeiro semestre do 3.º ano de uma Licenciatura em Educação Básica de uma instituição de ensino superior pública em Portugal. **Coleta e análise de dados**: Os dados foram recolhidos nas aulas de uma unidade curricular de Didática da Matemática. Inicialmente, os futuros professores responderam a um questionário, seguindo-se a implementação de um trilho matemático digital. No final desta experiência os participantes preencheram outro questionário. A observação participante e os registos audiovisuais foram também aplicados. A análise dos dados envolveu uma abordagem qualitativa e indutiva, recorrendo à análise de conteúdo. **Resultados**: Os futuros professores estiveram ativamente envolvidos ao longo do trilho matemático, demonstrando interesse na resolução das tarefas, utilizando o MCM, e focados em finalizar a atividade, evidenciando persistência e vontade de participar nas discussões. Também identificamos ansiedade e frustração relacionadas com o resultado de determinada tarefa. Foram utilizadas estratégias de natureza diversa, principalmente associadas ao nível de exigência cognitiva das tarefas. Como prós, consideraram a app intuitiva, promotora de autonomia, orientação espacial e trabalho colaborativo, e destacaram como contras a inacessibilidade a Wi-Fi; alunos mais novos não possuírem smartphones/tablets; e a limitação dos formatos de resposta na criação de tarefas. **Conclusões**: Os resultados encorajam a utilização do MCM como uma ferramenta com potencial no âmbito da educação matemática fora da sala de aula.

Palavras-chave: tarefas autênticas; trilho matemático; aprendizagem móvel; formação de professores.

INTRODUCTION

The outdoors has been frequently referenced as a privileged educational context that promotes positive attitudes and additional (cognitive, affective, behavioural) engagement in the learning of mathematics, acting as a complement to the formal learning environment of the classroom (e.g., Barbosa

& Vale, 2016; English et al., 2010; Richardson, 2004; Vale et al., 2019). Outdoor learning can take many forms, but our focus has been set on a specific strategy: the math trail. This non-formal context has great potential to evidence the connections between mathematics and reality, particularly the surrounding environment, giving new meaning to previously learnt concepts and ideas. Our work concerning math trails has been focused on two dimensions: teaching and learning aspects related to problem solving, underlying the trail dynamics, and features of task design, in the scope of teacher education, addressing problem posing. In this paper, we add the dimension of mobile learning due to its undeniable evolution, particularly access to mobile devices and educational apps. MathCityMap (MCM) is an app intentionally designed for mobile devices to create and solve tasks, organised as a math trail that articulates the real context with a digital environment. This tool has had a positive impact on supporting teachers and students in the teaching and learning of mathematics outside the classroom, acting as a valuable resource to explore the outdoors through a mathematical lens (e.g., Cahyono & Ludwig, 2019; Ludwig & Jablonski, 2019).

Mathematics does not have to take place exclusively in the classroom. The possibility of solving authentic tasks connected to situations in the real world can potentially increase students' learning opportunities, giving meaning to abstract concepts. Also, the extension of the classroom to the surrounding environment, enhanced by the use of mobile devices, may provide more significant, personalised and autonomous experiences. Assuming the relevance of these principles and the fact that teacher education programs should include experiences that stimulate pre-service teachers' knowledge, using the same teaching and learning principles that they are expected to use with their own students, we focused our study on understanding the perspective of pre-service teachers of elementary education about mobile learning in outdoor mathematics. To address this problem, we considered the following research questions: 1) Which pros and cons are pointed out by pre-service teachers of elementary education to the use of MCM? 2) How can we characterise the participants' engagement in a math trail performed with MCM?

THEORETICAL BACKGROUND

Current trends in the teaching and learning of mathematics

There is a common agreement that, to be successful in mathematics, students must learn it with understanding, actively building new knowledge from their own experiences and prior knowledge (NCTM, 2000, 2014). Adding these requirements to the need to prepare students to face the challenges of the 21st century effectively, we must recognise that the development of procedural efficiency is not enough; students should also be incited to use rigorous ways of thinking/reasoning and evidence deeper levels of conceptual understanding (Stein et al., 2017). Mastering higher-order skills, like problem solving, reasoning, communication, establishing connections or creativity, is also a concern in this scope (NCTM, 2000, 2014; WEF, 2016).

The teaching practices must align with these goals, abandoning traditional approaches where the teacher introduces new concepts or procedures through stereotyped tasks, and students are merely asked to apply this knowledge to similar tasks. The teaching and learning process should be student-centred, supported by multiple-solution tasks that promote conceptual understanding and the development of problem solving, reasoning, and communication skills (e.g., Doyle, 2010; Stein et al., 2017). This type of instruction is challenging for the teachers; going beyond selecting appropriate tasks, they must be prepared to orchestrate productive mathematical discussions emerging from students' work (Smith & Stein, 2011). This is certainly more demanding as students may use different solution strategies and pose unforeseen questions but allowing them to construct their own understandings and share their methods in a whole-class environment, building on personal and collective sensemaking, certainly supports learning in a more effective way (NCTM 2014; Smith & Stein, 2011).

Students' role must be more than listening and applying. They must read, write, discuss and, above all, be engaged in their own mathematical activity, following the principles of active learning (Prince, 2004), which simultaneously demands cognitive (attaining deeper understanding), social (communication through classroom interactions) and physical (movement) engagement (Edwards, 2015). The interaction between these dimensions contributes to the most lasting learning outcomes, leading students to gather information, think and solve problems collaboratively, showing increased levels of attention (Vale & Barbosa, 2020). However, the analysis of students' engagement is a complex endeavour as it is a multi-faceted concept that contemplates multiple variables. One of the most common perspectives involves considering three levels of engagement: cognitive, affective and behavioural (Attard, 2012; Fredricks et al., 2004). These levels should be regarded as part of the learning process in the context of students' activity related to a particular task. Cognitive engagement relates to investment, recognition of the value of learning, and a willingness to go beyond the minimum requirements. It refers to aspects such as concentration, motivation, effort, ability to self-regulate, mastering knowledge and skills and using strategies to overcome challenges (Attard, 2012; Fredricks et al., 2004). Affective engagement includes students' reactions to school, teachers, peers and academics, influencing their willingness to become involved in schoolwork. This level of engagement is associated with positive or negative emotional reactions (affections), such as acceptance, interest, enthusiasm, sense of belonging and attitude towards learning, influencing the predisposition to carry out a task (Attard, 2012; Fredricks et al., 2004). Behavioural engagement refers to the active participation in academic, social, and extracurricular activities and is crucial for achieving positive educational outcomes. This level of engagement includes compliance with rules and classroom aspects related to the duration of a task, persistence, attention, posing questions and participation in collective discussions. To delimit and characterise these levels of engagement in the scope of mathematics, Kong et al. (2003) developed a set of descriptors that facilitate the analysis of the relationship between students' engagement and the learning outcomes (Table 1).

Table 1

Engagement levels and descriptors (Kong et al., 2003)

Engagement levels	Descriptors
Cognitive	Surface strategy (memorisation; procedural knowledge; handling tests) Deep strategy (understanding; establish connections; justification) Reliance (pose questions; follow the teacher's instructions)
Affective	Interest (joy; pleasure; sense of satisfaction; curiosity; excitement) Achievement orientation (effort to get good results; focus on finishing a task) Anxiety (nervous, worried; afraid of poor results) Frustration (uncomfortable; tired; dislike for the task)
Behavioural	Attentiveness (listen; take an active part in the discussion; make an effort; concentration) Diligence (effort to understand; try again; persistence) Time spent (on out-of-class learning)

Engagement may directly connect to students' achievement and performance (in mathematics). Due to its importance for learning, there is a growing interest in researching the effects of using new approaches and practices in parallel or as an alternative to more traditional ways of teaching.

Outdoor mathematics: the case of math trails

Nowadays, educational contexts are more and more diversified, going beyond the formal classroom environment. Rich and meaningful learning in mathematics or other curricular areas must be based on the selection of diverse spaces, allowing contextualised, complementary and creative approaches. Non-formal education, which usually takes place outside the classroom in a planned and organised way, contributes to challenging contexts, prone to the application of previously acquired knowledge while positively influencing students' learning and engagement (OECD, 2016). So, besides diversifying strategies, tasks and other resources, teachers should also provide different learning environments, enhancing the usefulness of mathematics and simultaneously aiming at the development of higher-order abilities (e.g., Vale & Barbosa, 2021). The importance of the outdoors has been gaining momentum to complement the formal work developed inside the classroom, hence assuming that learning can occur in different environments and contexts (e.g., Kenderov et al., 2009; Vale et al., 2019). Learning beyond the traditional four walls of the classroom empowers and helps students discover and interpret the world around them, giving meaning to mathematical concepts and tools, applying their knowledge to real-world problems (English et al., 2010). Teachers can provide a diversity of experiences that meet these requirements; however, there is strong evidence that well-organised activities outside the classroom contribute significantly to the quality and depth of students' learning, including their personal, social, and emotional development (e.g., English et al., 2010; Vale et al., 2019). Among these possibilities, math trails are known to support such enhanced learning.

We define math trail as a sequence of tasks along a pre-planned route (with beginning and end) composed of a set of stops in which students solve mathematical tasks in the surrounding environment (Vale et al., 2019, adapted from Cross, 1997). Exploring mathematical concepts outdoors provides students with rich examples of where mathematics can be observed and discovered, and, particularly, a math trail challenges them to solve authentic mathematical tasks contextualised in the environment and its elements. It constitutes a vehicle for applying the knowledge acquired in the classroom and developing transversal skills, such as problem solving, reasoning, communicating, and establishing connections (Richardson, 2004). Math trails facilitate group work through collaboration in the multiple requirements of the route (e.g., writing, measuring, discussing); involve movement, walking around a particular site, observing specific features of elements in the surroundings; and engage students with the tasks, thinking about adequate strategies to reach a solution (Barbosa & Vale, 2016; Vale et al., 2019); which are traits of active learning. These aspects, along with the implicit atmosphere of discovery, can be crucial in inducing positive attitudes towards mathematics, with special emphasis on curiosity, motivation and interest (Bonotto, 2001; Kenderov et al., 2009).

As expected, tasks are a nuclear component of a math trail, as they are one of the main tools of the teaching practice, so they should be carefully designed, having some principles in mind. The surrounding environment can be a source of inspiring elements that may trigger mathematical activity through the formulation of questions that result from the observation of the natural and architectural heritage (e.g., buildings, gardens, stained glass, trees, traffic signs). It is also important to note that students can easily become bored with routine tasks and even show learning difficulties unless challenged, which is why this aspect is so important in mathematics. To clarify, the idea of a challenging task is usually used to describe an interesting, perhaps pleasant task –but not always easy to solve– that must actively engage students, adapting to a diversity of thinking and learning styles (Vale & Barbosa, 2023). Adding to this idea, tasks should have different levels of cognitive demand (Smith & Stein, 1998) and admit multiple solutions. Another aspect that characterises the tasks in a math trail is authenticity. There is some convergence in the literature in considering that authentic tasks are based on situations that, although sometimes fictitious, represent types of problems that we could find in a real context (e.g., Jurdak, 2006; Tan & Nie, 2015), which facilitates the establishment of connections between school mathematics and reality.

In addition to task design, planning and implementing a math trail must have some guiding principles in mind. According to Richardson (2004), we must begin by choosing the location, which can be anywhere as long as it’s rich in mathematical elements. The teacher starts by observing those elements and looking for patterns, shapes, and objects to measure, count or represent. After this step, the author suggests taking photos at each chosen stop to later select and use in the task design. In addition to formulating the tasks and the instructions to reach each stop, a map should also be created, identifying the places where the tasks are supposed to be solved. Complementing Richardson’s (2004) ideas, Shoaf et al. (2004) argue that mathematical trails should also be for everyone, regardless of age and experience of solvers, with the intention that they discuss and compare their reasoning and strategies, promote collaboration rather than competition, be time manageable, be voluntary, be presented in a safe public place, as mathematics is everywhere, and temporary, as locations/elements may be subject to change over time.

Mobile math trails: the MathCityMap app

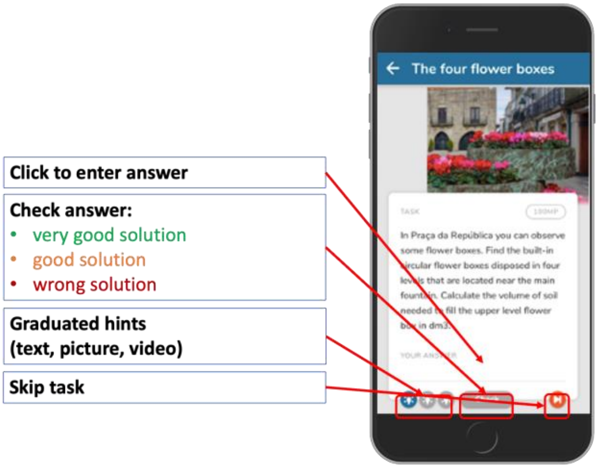
Using digital technologies is an added value in acquiring knowledge because they promote new opportunities to improve and guide the teaching and learning process. The National Council of Teachers of Mathematics (2014) reinforces that, for students to understand mathematical concepts and ideas and learn to reason and communicate their mathematical thinking, a mathematics program combined with using and integrating resources like digital technologies is essential. Given the technological world we live in and the fact that students are digital natives from very young ages, it is unavoidable that mathematics education includes technology and the resource to a diversity of digital resources. Furthermore, it is essential to state that mobile learning is also becoming a trend, particularly in non-formal learning contexts, such as the outdoors (Barbosa et al., 2022; Sung et al., 2016). This reality is due to the rapid development of mobile devices and the diversity of educational apps that have been created, increasing the opportunities for teachers to use tools of this nature with their students. Undeniably, there are many advantages to using mobile devices in teaching and learning, as is the case with smartphones or tablets, which allow students to work individually, in small groups or with the whole classroom, reducing the barriers between formal and non-formal education. This type of technology improves social interactions, provides a more personalised experience for each student and encourages the emergence of creative processes (Shuler, 2009). In a non-formal learning context, these resources allow students to learn as they move, interacting with each other and their surrounding environment (Fessakis et al., 2018).

Mobile technology can also facilitate authentic learning, helping students contact with and solve problems in a real context, whether allowing access to virtual contexts, favouring interactions, or collecting/processing/presenting data. In particular, it can help develop a deeper understanding of mathematics, acting as a thinking tool for exploration, decision-making, reflection, reasoning, problem solving, and collaboration (Fessakis et al., 2018). The extension of the classroom to the outdoors is easily accomplished by the portability and wireless functionalities of mobile devices, which present students with a more meaningful and appropriate context and experience in the exploration of the surrounding environment (Cahyono & Ludwig, 2019; Shuler, 2009). Mobile devices, and consequently apps, help establish a bridge between educational contexts designed with a specific pedagogical intention and learning situated in a real context (Shuler, 2009), making it more authentic for the students.

There are several apps created specifically for outdoor education. The MCM system was designed to articulate the dimensions of outdoor mathematics and mobile learning. This system has two components: an app, designed for smartphones and tablets, that allows the user to make math trails; and a web portal that works as an international database, in which tasks and trails are created and published, being linked to GPS coordinates. The main goal of MCM is to facilitate and maximise the experience of outdoor mathematics for both teachers and students, applying previously learned concepts while solving authentic tasks (Ludwig & Jablonski, 2019). The tasks created in the web portal must comply with a set of criteria to be properly implemented in the context of a math trail, namely: uniqueness, each task must have an associated photo that helps identify the object on which the task focuses; presence, the task can only be solved on-site, which prevents the statement and the photo from having revealing information; activity, highlighting the idea that mathematics can only be fully understood through an active experience (Jablonski et al., 2018). On the mobile device, the user has access to the trail and the tasks to be solved, which are spatially located with the help of the GPS functionality, and submits the solution, receiving immediate feedback about its validity. The app has other features, such as the possibility of consulting up to three hints on demand that aim to help the user reach the solution when they have doubts about solving a particular problem. The hints are formulated as guiding suggestions without the intention of directly presenting the solution, alerting, for example, to the conditions of the problem that are essential in its resolution or remembering formulas that may be forgotten. The users can also skip tasks and go back to solving them if they wish, for example, because they feel that they are investing too much time to find a solution or because they reached a stop on the trail that currently has too many users at the same time, making data collection difficult. Figure 1 shows the layout of a task seen through MCM, summarising the main features.

Figure 1

Layout of a task seen through the app. (Milicic et al., 2020)



This digital technology has proved to be helpful in exploring mathematics in the surrounding environment, supporting teachers and students in the teaching and learning process, including in the affective dimension, reflected in positive attitudes shown by its users (e.g., Barbosa et al., 2022; Cahyono & Ludwig, 2019).

METHODOLOGY

This paper presents a study with elementary education pre-service teachers to understand their perspective on mobile learning in outdoor mathematics, namely resorting to MCM. Based on this problem, we formulated the following research questions: 1) Which pros and cons are pointed out by pre-service teachers of elementary education to the use of MCM? 2) How can we characterise the participants' engagement in a math trail performed with MCM? Following the main goal of this study, we adopted a qualitative and interpretative approach (Erickson, 1993). This choice of paradigm was sustained by the purpose of understanding the participants' perspectives and reactions to a particular situation. The study was developed as a part of a broader research project about math trails, curriculum and educational environments in European countries. It was conducted according to the guidelines of the Declaration of Helsinki and approved by the Institutional Review Board of Instituto Politécnico de Viana do Castelo.

We conducted this research with 48 future teachers who were attending the second semester of the 3rd year of an undergraduate course in basic education lasting three years (six semesters) and preceding a master's degree course that qualifies them for teaching preschool and primary education (6-12 years old students). This undergraduate course comprises subjects related to didactics, general education, content knowledge, and practice in formal and non-formal educational contexts. This group had 45 women and three men, with an average age of 23 and, among other subjects, they were enrolled in a curricular unit of Didactics of Mathematics that served as the context for the data collection, which included teaching modules focusing on tasks, instructional strategies and resources that meet the current trends of mathematics teaching and learning. Particularly, this curricular unit allowed the pre-service teachers to contact with and experience some active learning strategies they could use in their practice, namely a math trail.

The researchers were also the teachers responsible for this curricular unit, an aspect that facilitated participant observation. To contextualise the work carried out, it is first important to emphasise that these pre-service teachers had little or no experience with outdoor mathematics, which led us to apply a first questionnaire to characterise their experiences and perspectives about the teaching and learning of mathematics outside the classroom, as well as the possibility of using digital technology/mobile learning in this type of context. After completing this questionnaire, the participants carried out a math trail in the city centre, using MCM, divided into small groups. As resources, they used a smartphone to access the tasks and submit the answers, an articulated meter, a calculator, and paper and pencil to make the necessary records. After this experience, we applied a second questionnaire to analyse the changes in the participants' perspectives and to know their opinions about the MCM use.

Data were collected in a holistic, descriptive and interpretive way through naturalistic participant observation (reactions/interactions/discussions along the math trail, recorded through free-flowing notes), two online questionnaires (with open-ended questions), written productions (tasks' solutions) and photographic records. The researchers accompanied the participants during the trail, observing in loco the work developed by each group and taking notes about their performance. Then, we analysed the data following a qualitative and inductive approach using content analysis (Miles & Huberman, 1994). After a repeated reading and consultation of the information collected through the different methods, we categorised the data to systematise and cross-reference the evidence referring to the participants' perspectives with those that emerged from the observation. Through this process, it was possible to generate categories of analysis, influenced mainly by the research questions, complemented by the theoretical framework and the data collected: pros of MCM; cons of MCM; and participants' engagement (Table 1) during the math trail, focusing on cognitive, affective and behavioural engagement.

RESULTS AND ANALYSIS

To report the main findings, we used the information from the questionnaires, the observational notes of the pre-service teachers' conversations, reactions and interactions, and photographic records. We start with a brief characterisation of the participants in terms of their initial perspectives and experiences on outdoor mathematics, moving to the analysis of their engagement along the math trail, and ending with the identification of the pros and cons of using the MCM.

The participants before the math trail

The first questionnaire was applied before the trail implementation, at the beginning of the semester, so that the researchers could have timely access to the participants' answers, having the possibility to carefully prepare the math trail and adapt it based on the results. We mainly intended to access and characterise the initial perspectives and previous experiences of these pre-service elementary education teachers regarding teaching and learning mathematics outside the classroom and using mobile learning in this specific non-formal context. The questionnaire was filled out online and was composed mainly of open-ended questions.

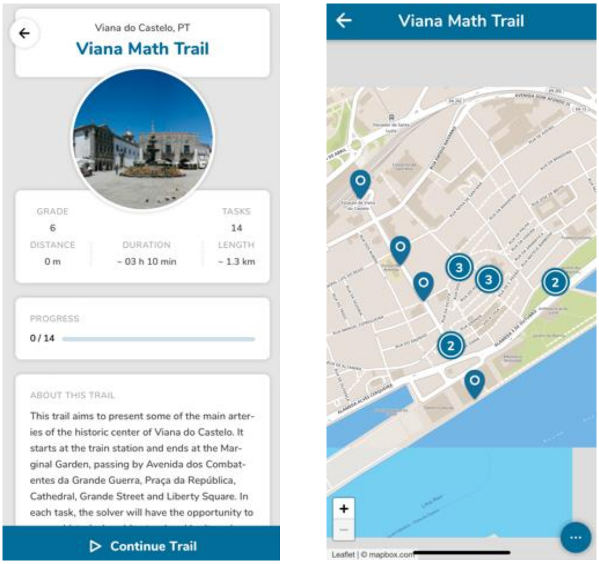
The analysis of the results drawn from the pre-service teachers' answers allowed us to identify, in general, a vague awareness of outdoor mathematics. Although the majority of the participants considered teaching and learning mathematics outside the classroom as a possibility and a relevant practice, they mainly referred to poorly supported examples based on isolated experiences, such as daily life tasks and routines, counting tasks, money-related tasks, games, competitions, math clubs, field trips, observation of architecture/shapes in the surrounding environment, mathematics in nature and peddy paper. Regarding their experiences as students, most participants said they never had the opportunity to experience mathematics outdoors, which may explain their generic ideas about how to implement this approach from the teacher's perspective. This information was necessary for the researchers to conclude that pre-service teachers must experience specific methodologies and strategies first-hand before incorporating them into their future practices, and this unit course (Didactics of Mathematics) was the proper context to do so. As for their knowledge of digital technology and mobile learning as a resource for outdoor mathematics, little more than half of the participants admitted that they did not know any resources for this purpose. This fact was not surprising, as many of these pre-service teachers assumed they had no previous experiences with mathematics teaching and learning outside the classroom. The rest, who recognised knowing resources of this nature, mentioned examples such as digital games, apps, and robots, but none allowed the meaningful exploration of the surrounding environment. Globally, they were not aware of resources in the scope of mobile learning adequate for outdoor mathematics.

Participants' engagement in the math trail

The trail implementation was preceded by a session in the problem-solving module of the curricular unit dedicated to the principles of active learning and particularly the specifics of a math trail. MCM was also presented as a digital tool to support the teacher as the trail designer and the student as the user. However, the focus was on highlighting the main features of the MCM app (e.g., layout, location of the tasks, answer submission, hints, gamification). The pre-service teachers also downloaded the app and the trail to their smartphones to analyse the route they would have to walk, and organised themselves in groups. In the following class, the participants, accompanied by the researchers, went to the city centre to experiment with the trail, and each group decided which would be their first task from the fourteen available, as seen on the map (Figure 2).

Figure 2

Trail map with the location of the tasks.



The tasks were mainly problems involving diversified contents (e.g., symmetry, patterns, estimates, area, volume) and levels of cognitive demand. We tried to provide pre-service teachers with examples of authentic and challenging tasks, highlighting the possibility of working with contents from the main domains of school mathematics, establishing connections with reality and, particularly, with the historical and cultural heritage of the city. After reading each task, the groups' participants decided which would be the most effective strategies and procedures to apply to reach the submitted answer. This trail had the gamification function active, attributing points depending on the correction of the answer, i.e., if the answer was correct, they scored 100; otherwise, they could try again and again but with a decreasing score.

To analyse participants' engagement along the trail, we mainly relied on data related to observation, observational notes, written productions, and photographs, focusing on the cognitive, affective, and behavioural dimensions.

Cognitive engagement

The trail was composed of tasks with different levels of cognitive demand (Smith & Stein, 1998), low and high, as seen by the examples in figures 3a and 3b, to balance the level of challenge of the math trail globally.

Figure 3a

Example of a low-level task.

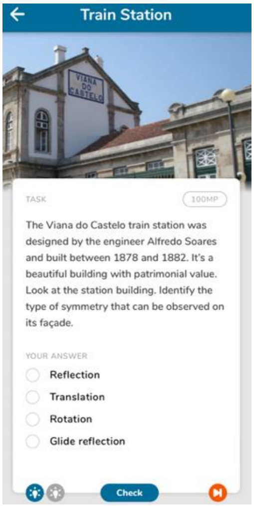
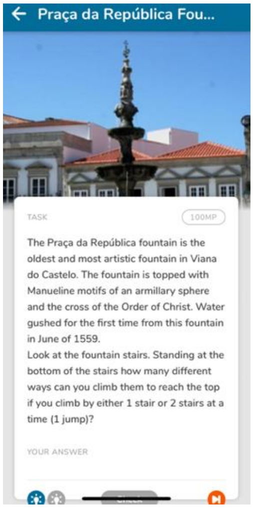


Figure 3b

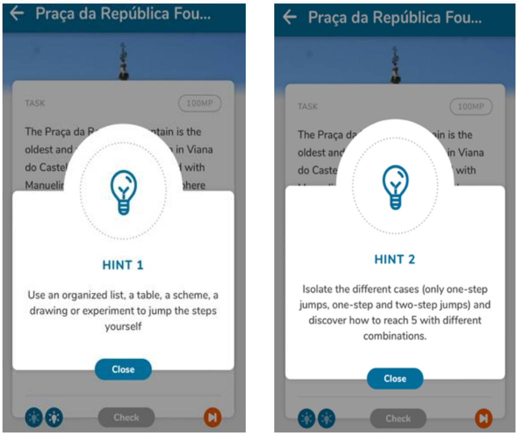
Example of a high-level task.



Incorporating active learning principles in the trail dynamics enhances students' engagement and promotes deeper learning. But, at the same time, the nature of the posed tasks also influences the knowledge and abilities required to reach the solution. Therefore, as expected, the participants used surface and deep strategies, depending on the level of cognitive demand of the tasks. The first was applied to low-level tasks, resorting to memorisation (e.g., task in Figure 3) or procedural knowledge (e.g., using measurements and formulas to determine the volume of a flowerpot). The latter was applied to high-level tasks, using non-routine strategies that demanded understanding or the establishment of connections, like finding a pattern (e.g., estimating the length of an avenue) or using visualisation (e.g., counting the number of different rectangles in a glass window). In some cases, due to initial difficulties in understanding the task or coming up with a strategy, some groups resorted to the available hints (Figure 4).

Figure 4

Hints for the task presented in Figure 3.



The hints were formulated to guide the user without revealing the (re)solution. However, after students accessed a hint, we observed that the level of cognitive demand was reduced, which did not always imply that the nature of the strategies automatically changed from deep to surface; in most cases, it just helped the pre-service teachers unlock an idea to attack the problem. Hence, we can assume that the math trail prompted surface and deep strategies.

In general, participants were successful in solving the proposed tasks. The possibility of having hints on demand and the fact that the tasks were posed in a real context gave them clearer insights into the problems. This experience constituted an opportunity to favour the establishment of connections between mathematics and the local environment, triggering the application of formally learned concepts to an authentic context.

As for reliance, we can say that, in general, participants sought validation from the teachers only in occasional situations, posing questions about the tasks' conditions. They frequently discussed their ideas and doubts within each group and resorted to hints whenever needed. Despite having the possibility to consult the researchers, the groups were generally extremely autonomous in managing their work, considering the app easy to use, which may explain why they did not experience any noteworthy difficulties at this level. As we were able to observe, they managed to recognise their spatial position on the map with the help of the GPS, and, at each stop, represented by a pin in the map, they accessed the respective task, read it out loud, observed the targeted object, and collected the necessary data to solve it and reach a solution.

Affective engagement

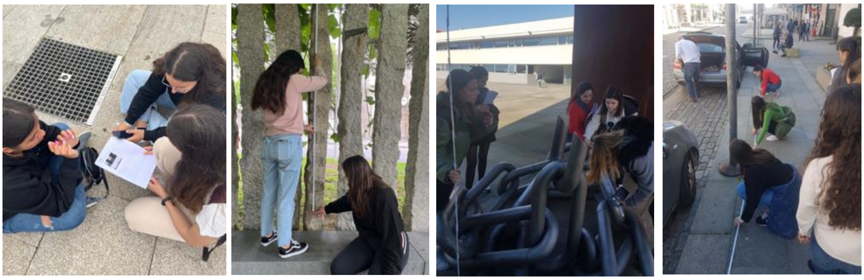
Most participants expressed interest during the math trail, which included solving the tasks collaboratively to achieve the solution that "would not be so efficient and rich if done individually". They were compelled by the opportunity of "working outside the classroom", "exploring the local environment with a mathematical lens", and "resorting to technology", valuing the potential of the app. The gamification feature acted as a triggering factor for extra motivation: on the one hand, the participants were excited when the solution was correct; on the other hand, it implied certification of the validity of the solution before the introduction of the answer, which was naturally discussed within each group. The pre-service teachers valued the experience, stating that they "would like to have more experiences like this", recognising that "a math trail makes it possible to apply different knowledge from different domains".

Along the math trail, we observed different reactions depending on the outcome. We noticed some anxiety, reflected on the expectation of the result when they were about to submit the answer to find out its' validity, and frustration, especially when they took too much time to solve a task or when the result was wrong, leading to the loss of points, as the gamification feature of the app was activated. However, the moment they reached the solution – validated by the app– was translated into a generalised sense of satisfaction. The inevitable comparison between the groups' outcomes (e.g., the number of tasks solved) made some participants worried about the possibility of poor results: "We made a mistake in the measurement", "Our calculations were wrong", "We could have done better".

Although this group of pre-service teachers revealed different performance levels in mathematics curriculum units, there was no significant impact on their affective engagement. Globally analysing the math trail dynamics and the participants' reactions, it was evident that it impelled collaborative work, both within and between groups, leading participants to share responsibilities (e.g., operating the app on the mobile device; carrying out measurements/data collection; recording data and calculations on paper), but we have also observed examples of cooperation between different groups with the same goal in mind (e.g., joining several articulated meters to measure a certain length) (Figure 5).

Figure 5

Moments of the trail implementation.



Behavioural engagement

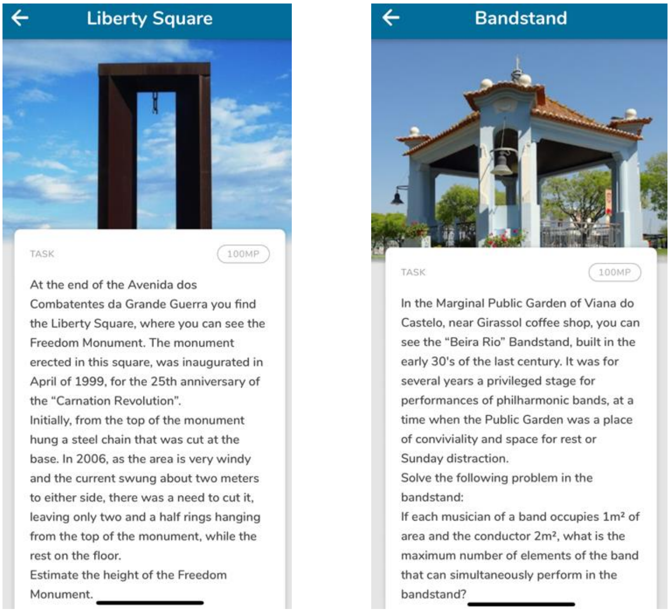
Behavioural engagement is closely related to the nature of student participation; in this case, given the type of experience, it can be analysed through aspects of attentiveness and diligence.

The interest revealed by these pre-service teachers in experiencing a mobile math trail led them to attentively listen to the teachers' instructions prior to and during the trail. They also posed some questions before the implementation, mostly regarding the app's functions (e.g., "Do we need Wi-Fi?" "do we lose points if the answer is wrong?"). During the trail, most groups shared ideas, trying to decide the best strategy to solve each task. We mostly observed intense discussions within the groups, with the participation of the majority of the respective elements (none thought individually), trying to listen to each other and reach an agreement about the best decision to proceed to the next step. At specific moments, we also observed group discussions when solving the same task.

The participants also reflected on the tasks solved during the trail. Most enjoyed solving the trail tasks, but showed having their most and least favourites. The ones highlighted as their favourite had two main features: either being the most challenging or those that added to their knowledge of historical and cultural aspects associated with the elements of the city involved in the trail (Figure 6). The tasks the participants liked the least involved exhaustive calculations or too many steps to reach the solution, which caused some demotivation (Figure 6).

Figure 6

One of the favourite tasks (left) and one of the least liked tasks (right).



Most groups were very careful, checking the validity of the solution before submitting it in the MCM app. Everyone made an effort, revisiting the conditions of the tasks or checking aspects like measurements, calculations, counting or formulas to ensure the correction of these processes. They showed persistence, either when they got a wrong answer or were having difficulties in a certain task, channelling their efforts to find an adequate strategy. Although not all students were completely satisfied with the result, mentioning, for example, that they "did not achieve the maximum score", they also recognised that “the most important thing was doing the trail and understanding its’ purpose”.

Pros and cons of MCM from participants’ perspective

After the trail implementation with MCM, we applied the second online questionnaire. The aim was to understand possible changes in the participants’ perspectives and know their opinions about the math trail as a strategy and the app, particularly the pros and cons of its use. The questions were mainly open-ended and addressed aspects such as the importance of teaching and learning mathematics outside the classroom, the impact of the math trail experience, opinions about the tasks of the trail, and the pros and cons of MCM from the perspective of the teacher and the user/student.

Analysing the results, all the pre-service teachers recognised the importance of outdoor mathematics to complement what is learned in the classroom. Contrary to the answers obtained in the first questionnaire, everyone was convinced of the possibility of teaching and learning mathematics outside the classroom, which means that some of these pre-service teachers changed their opinion after experiencing the math trail. To support their perspectives, the pre-service teachers argued that a math trail: follows the principles of active learning, promoting intellectual, social and physical engagement; makes learning more meaningful for students because they are directly involved in the problem-solving process, collecting the needed data; evokes affective aspects such as motivation, enthusiasm and curiosity; helps students understand the usefulness and applicability of mathematics; makes more visible the connections between mathematics and cultural and historical heritage; facilitates collaborative work and helps develop communication skills, as well as critical thinking; may lead to the use of technology, such as the MCM. As for MCM use and the assessment of its features, the participants could identify the pros and cons from the user’s and the teacher’s perspective. As the pros they highlighted, we have: the possibility of applying the knowledge learned in school mathematics; being friendly and intuitive; promoting the user’s autonomy, facilitating collaborative work, assuming different roles and responsibilities; promoting the development of a mathematical lens to observe the surrounding environment; being more interactive than a paper trail, given the app’s features; getting immediate feedback about the solution; and the use of gamification with the attribution of points. From the participants’ point of view as future teachers, the pros mentioned were: the possibility of designing tasks adapted to the local environment, being creative and meeting the students’ interests; approaching different mathematical contents and formulating interdisciplinary tasks; diversifying educational contexts in and out of the classroom; and allowing the teacher to supervise and monitor the work developed by the groups, due to the autonomy it provides to the user. Some cons were also pointed out to MCM from the participants' perspective: the possible lack of access to the Wi-Fi at the moment of the trail download; the fact that younger students usually do not have mobile devices (smartphones or tablets), which may lead to the lack of resources in these age groups; and, in terms of tasks, lack of diversity, having only as answer formats the exact value, interval, or multiple-choice types.

CONCLUSIONS

This study, conducted with pre-service elementary education teachers, aimed to understand their perspective about mobile learning in outdoor mathematics, in this case, MathCityMap.

Overall, the participants valued the math trail experience as a meaningful strategy to engage students in solving authentic tasks (Jurdak, 2006; Richardson, 2004; Tan & Nie, 2015) and as a means of creating opportunities to promote the establishment of connections between mathematics and other content areas and even cultural/historical heritage, hence real-life (Bonotto, 2001). The participants also pointed out active learning as a fundamental attribute in a math trail, as it facilitates intellectual (problem solving), physical (movement) and social (collaboration) engagement, whose interaction tends to generate positive attitudes (Vale et al., 2019). MathCityMap was used to present and solve the trail and its tasks, seeking to understand the participants' opinions about its use and engagement along the math trail. These pre-service teachers positively highlighted the adequacy of the app, finding it user-friendly, interactive and motivating, mainly due to the gamification feature, the possibility of immediate feedback about the validity of the solution and having hints on demand in case of difficulties (e.g., Cahyono & Ludwig, 2019). They also mentioned its positive contribution to the development of spatial orientation (associated with the GPS locator), collaboration (intra and inter-groups), student autonomy and being more practical and interactive than a paper trail. The only cons mentioned by the participants were related to possible constraints, such as lack of Wi-Fi or resources (smartphones/tablets) and the limited possibilities regarding answer formats. However, we consider it relevant to clarify two aspects: MCM allows the creation of trails without needing to have an active Wi-Fi network, being only used when the trail is downloaded to the mobile device; the MCM system is constantly being improved since its creation, taking into account, whenever possible, users' needs, and at the moment the system, through the web portal where the tasks are submitted, has a broader range of answer formats in addition to those used by these participants at the time of the trail implementation (e.g., fill in the blanks, vector exact value, vector interval, fraction, set). Concerning the second research question, we concluded that this experience allowed us to identify different types of engagement: cognitive, affective, and behavioural (Kong et al., 2003). Considering the different levels of cognitive engagement, we noticed that, to solve the different tasks, the participants had to apply surface and deep strategies, depending on the low or high level of cognitive demand (Smith & Stein, 1998) of the tasks. The reliance on the teacher in specific moments of the trail was occasional since they also relied on the app's available hints and the elements in their group. In the affective domain, the participants expressed interest in the authenticity of the tasks and the connections to the surrounding environment, but also in exploring the features of MCM. Depending on the outcomes of the tasks, we also observed some anxiety (expectation of the result validity), frustration (obtaining a wrong answer), concern (comparison with other groups), and satisfaction (achieving a correct solution). In general, the elements of the several groups revealed achievement orientation, working together to reach a common goal, sometimes interacting with other groups, and making an effort to solve the tasks. The behavioural engagement was translated through observable indicators like attentiveness and diligence. The participants actively took part in solving the tasks, listening to the teachers and each other, and considering the instructions in the app, making collective decisions when needed (deciding the best strategy and procedures). The need to review a solution showed the participants' persistence, who automatically discussed and analysed possible adjustments to the processes used.

To conclude, with this experience, the participating pre-service teachers generally recognised the potential of a math trail and also of the use of digital technology, specifically MathCityMap, to support teachers in their practices and students in the learning process, actively applying mathematical knowledge and abilities and also promoting positive attitudes.

AUTHORS' CONTRIBUTION STATEMENT

All authors actively participated in the conception of the presented idea, development of the theory, adaptation of the methodology to this context, performance of the activities, data collection, discussion of the results, revision and approval of the final version of the work.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the authors upon reasonable request.

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