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A SIMPLE AND COMPREHENSIVE APPROACH TO FORMULATE AND SOLVE DYNAMICS PROBLEMS IN A NON-TRADITIONAL ENGINEERING COURSE

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1. INTRODUCTION

Learning and teaching dynamics is often considered by students and instructors to be a difficult and demanding subject, in the measure that it involves a good number of ingredients, chiefly those related to mechanics, physics, algebra, calculus, numerical methods and computational aspects [1]. This is particularly true when dealing with non-traditional courses. In this context, the present study is carried out using a simple and comprehensive approach to formulate and solve any dynamics problem. For that purpose, a seven-step methodology is presented together with a demonstrative example of application, namely a robotic arm. The proposed approach has been implemented in the "Virtual Product Development" course" (4 hours per week) that is included in the first year of the curriculum of the master's degree in mechanical engineering offered at the University of Minho. Preliminary findings indicate that, overall, students enjoyed the learning activities and content of the dynamics module.

2. DESCRIPTION OF THE PRESENTED APPROACH

A comprehensive description of a general methodology to handle dynamics problems is briefly described in this section. In order to better understand the sequence of steps of the proposed approach, an example of application is utilized, namely a robotic arm (Fig. 1a). The formulation of a dynamics problem can be condensed in the following steps:

i) ***Discretization of the system***. The first step consists of modeling the system under analysis, which should be conducted with an appropriate level of detail, bearing in mind the aim of the problem to be solved. The mathematical model should be as simple as possible without compromising the primary objectives of the analysis or the results to be obtained. For the robotic arm considered here, it is evident that the system can be modeled as a double pendulum (Fig. 1b).

ii) ***Selection of coordinates***. In this step, the type of coordinates utilized to define the position and orientation of the bodies that compose the system must be chosen. Among the various types of coordinates commonly utilized in dynamics, absolute coordinates are selected for this study. Thus, the pose of each body of the robotic arm under analysis can be defined by using the x and y coordinates of the center of mass, together with a rotational coordinate, θ, associated with the orientation of the body (Fig. 1b). One of the key aspects when dealing with dynamics problems is the establishment of the inertial frame of reference. The x-y Cartesian system of coordinates is a good choice for the inertial frame (Fig. 1b).

iii) ***Drawing free-body diagrams***. The third step deals with the construction of the free-body diagrams of the parts that compose the system, which provide a simple and clear picture of the forces that act on each body of the mechanical system. Figure 1c shows the free-body diagrams of the arms 1 and 2 that constitute the robotic manipulator considered as a double pendulum in the present study. With the purpose of keeping the analysis simple, the gravitational force is the only external force that acts on the mechanical system. A separate free-body diagram is drawn for each body in the system.

iv) ***Development of the equations of motion***. Based on the free-body diagrams constructed in the previous step, the Newton-Euler equations of motion must be written. For planar cases, Newton's second law states that the sum of the applied forces equals the mass times acceleration. In turn, Euler established that the sum of applied moments with respect to the center of mass equals the mass moment of inertia at the center of mass times the angular acceleration. Thus, with regard to Fig. 1c, the Newton-Euler equations of motion for arms 1 and 2 can be written as

$$m_1\ddot{x}_1 = -\lambda_1 - \lambda_3 \tag{1}$$

$$m_1\ddot{y}_1 = -\lambda_2 - \lambda_4 - m_1 g \tag{2}$$

$$I_1\ddot{\theta}_1 = -\lambda_1 \frac{l_1}{2}\sin\theta_1 + \lambda_2 \frac{l_1}{2}\cos\theta_1 + \lambda_3 \frac{l_1}{2}\sin\theta_1 - \lambda_4 \frac{l_1}{2}\cos\theta_1 \tag{3}$$

$$m_2\ddot{x}_2 = \lambda_3 \tag{4}$$

$$m_2\ddot{y}_2 = \lambda_4 - m_2 g \tag{5}$$

$$I_2\ddot{\theta}_2 = \lambda_3 \frac{l_2}{2}\sin\theta_2 - \lambda_4 \frac{l_2}{2}\cos\theta_2 \tag{6}$$

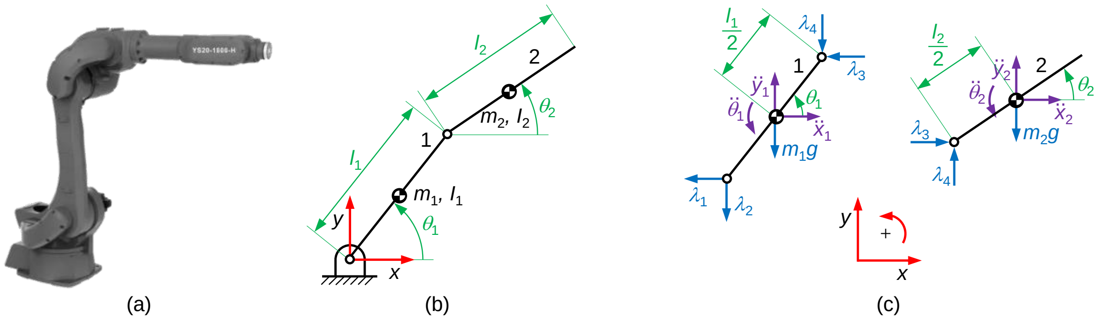

Figure 1. (a) Robotic arm; (b) Model of the robotic arm as a double pendulum; (c) Free-body diagrams.

v) ***Establishment of the kinematic constraints***. The equations of motion (1)-(6) represent a system of six equations with 10 unknowns, precisely six accelerations and four joint reaction forces. Thus, kinematic constraint equations associated with the system restrictions, namely the constraints imposed by the revolute joints, must be established. For this purpose, the position of the center of mass of each body, expressed in terms of the chosen coordinates, must be generated first. With regard to Fig. 1, the robotic arm can be constrained by four kinematic conditions expressed as

$$\Phi_1 = x_1 - \frac{l_1}{2}\cos\theta_1 = 0 \tag{7}$$

$$\Phi_2 = y_1 - \frac{l_1}{2}\sin\theta_1 = 0 \tag{8}$$

$$\Phi_3 = x_1 + \frac{l_1}{2}\cos\theta_1 - x_2 + \frac{l_2}{2}\cos\theta_2 = 0 \tag{9}$$

$$\Phi_4 = y_1 + \frac{l_1}{2}\sin\theta_1 - y_2 + \frac{l_2}{2}\sin\theta_2 = 0 \tag{10}$$

vi) ***Assembly and resolution of the equations of motion and kinematic constraints***. In this step, the equations of motion and the kinematic constraint equations must be combined and solved numerically. It should be noticed that Eqs. (7)-(10) are formulated at the position level and, therefore, they cannot be directly incorporated with Eqs. (1)-(6), since the last ones are expressed at the acceleration level. Thus, in order to overcome this situation, the constraint equations must be differentiated twice with respect to time. Finally, the combination of the equations of motion (1)-(6) and the constraint equations at the acceleration level leads to the set of differential-algebraic equations, written in matrix form as

$$\begin{bmatrix} \mathbf{M} & \mathbf{\Phi}_{\mathbf{q}}^{\mathrm{T}} \\ \mathbf{\Phi}_{\mathbf{q}} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} \mathbf{g} \\ \boldsymbol{\gamma} \end{Bmatrix} \tag{15}$$

where $\mathbf{M}$ is the system's mass matrix, $\mathbf{\Phi}_{\mathbf{q}}$ represents the Jacobian matrix of the constraint equations, $\ddot{\mathbf{q}}$ denotes the vector containing the system accelerations, $\boldsymbol{\lambda}$ is the Lagrange multipliers vector that includes the joint reaction forces, $\mathbf{g}$ is the generalized vector of applied forces, and $\boldsymbol{\gamma}$ denotes the right-hand side vector of the acceleration equations. Equation (15) represents a differential-algebraic system of equations (DAE) of index-1 that is solved for the accelerations and Lagrange multipliers. Then, the accelerations and velocities are integrated in time to determine the new velocities and positions. This numerical procedure is repeated until the final time of simulation is reached.

vii) ***Visualization of the outcomes.*** The last step deals with the verification and visualization of the results produced, which can be presented in the form of tables, plots and animations. This step is crucial to evaluate the accuracy and effectiveness of the developed model, as it allows for the physical interpretation of the obtained data in the analysis.

3. CONCLUDING REMARKS

A simple and systematic approach to deal with dynamics problems is presented in this work, which is included in a non-traditional dynamics course. The proposed formulation comprises seven steps, which are described together with an example of application, chiefly a robotic manipulator. The presented methodology is also utilized to analyze and discuss a variety of mechanical systems, including the academic slider-crank mechanism, a formula student car model, and a biomechanical human system. Additionally, the use of commercial codes, as an alternative for modeling and analyzing complex systems, is considered within the scope of this work, particularly in their application to industrial problems. Overall, the proposed approach has been recognized as effective by students and instructors. These and other issues will be object of discussion at the symposium.

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