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HETEROGENEOUS AND HOMOGENIZED FE MODELS FOR THE LIMIT ANALYSIS OF OUT-OF-PLANE LOADED MASONRY WALLS

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Summary. In the present contribution, a full 3D heterogeneous and a 2.5D homogenized kinematic FE limit analysis approach are employed for the evaluation of collapse loads and failure mechanisms of both running and English bond masonry slabs simply supported at the edges and out-of-plane loaded. Information at failure given by the full 3D approach underlines that particular care should be used in the evaluation of collapse loads with 2.5D approaches in case of multi-wythes panels.

1 INTRODUCTION

The prediction of the ultimate load bearing capacity of masonry walls out-of-plane loaded is technically very interesting. Out-of-plane failures are mostly related to seismic and wind loads and earthquake surveys have demonstrated that the lack of out-of-plane strength is a primary cause of failure in the most traditional forms of masonry. Up to now, limit analysis and the yield line method seem the only methods suitable to be applied in practice for the evaluation of the ultimate load bearing capacity of masonry out-of-plane loaded. Furthermore, limit analysis concepts have been adopted by many codes, as for instance BS 5628 [1]. In this paper, two FE limit analysis models based respectively on a heterogeneous approach [2] and on homogenization [3] are presented. For the heterogeneous model, a possible jump of the velocity field is assumed at the interface between adjoining elements (i.e. between bricks). The collapse load of a rectangular masonry slabs simply supported at the edges and out-of-plane loaded is evaluated with both models, considering different thickness and bricks arrangements. Results underline that particular care should be used in the evaluation of collapse loads with 2D approaches for multi-wythes panels.

2 THE HETEROGENEOUS AND THE HOMOGENIZED NUMERICAL MODELS

In this section, the numerical bases of a full 3D FE kinematic limit analysis approach for the analysis of masonry walls with irregular multi-wythes thickness texture is briefly recalled [2]. Let a multi-wythes thick stone/brick masonry wall with irregular texture be considered. As commonly accepted in the framework of a heterogeneous approach, joints are reduced to interfaces with zero thickness and possible jumps of displacements, whereas bricks are modeled as infinitely resistant parallelepipeds, except for a dissipation on internal bricks interfaces. Since each 3D element E (parallelepiped) is supposed infinitely resistant, thus 3 velocities unknowns $\mathbf{w}_{\mathbf{g}}^{E} = \left[w_{1-g}^{E} \quad w_{2-g}^{E} \quad w_{3-g}^{E} \right]^{T}$ and 3 rotation rates $\mathbf{\Omega}^{E} = \left[\Omega_{1}^{E} \quad \Omega_{2}^{E} \quad \Omega_{3}^{E} \right]^{T}$ per element are assumed as kinematic variables, corresponding respectively to $\mathbf{g}$ centroid velocity and rigid rotation rates around $\mathbf{g}$. Being velocities interpolation inside each parallelepiped linear, jumps of velocities field on interfaces varies linearly. Hence, for each interface, nine unknowns are introduced ($\Delta\mathbf{u}^{I} = \left[\Delta u^{1} \quad \Delta v_{1}^{1} \quad \Delta v_{2}^{1} \quad \ldots \quad \Delta u^{3} \quad \Delta v_{1}^{3} \quad \Delta v_{2}^{3} \right]^{T}$), representing the normal (Δu^{i}) and tangential ($\Delta v_{1}^{i} \quad \Delta v_{2}^{i}$) jumps of velocities (with respect to a suitable interface frame of reference). For any pair of nodes on the interface between two adjacent parallelepipeds $M-N$, the tangential and normal velocity jumps can be written in terms of the Cartesian nodal velocities of elements $M-N$. Consequently, for each interface, a set of equations in the form $\mathbf{A}_{11}^{eq}\mathbf{w}^{Mp} + \mathbf{A}_{12}^{eq}\mathbf{w}^{Ns} + \mathbf{A}_{13}^{eq}\Delta\mathbf{u}^{I} = \mathbf{0}$ can be written, where $\mathbf{w}^{Mp}$ and $\mathbf{w}^{Ns}$ are $6x1$ vectors that collect centroid velocities and rotation rates of elements M and N respectively and $\mathbf{A}_{11}^{eq}$, $\mathbf{A}_{12}^{eq}$, $\mathbf{A}_{13}^{eq}$ are matrices which depend only on the geometry of the elements M and N. Since jump of velocities field varies linearly at each interface, it is necessary to impose also plastic flow constraints on three vertices n of the rectangular interface, see [2] for details.

In order to evaluate power dissipation π^{I} on interfaces, for each interface I a 3D Lourenço and Rots [4] strength domain is adopted. Following [2], within each interface I of area A, the power dissipated is: $\pi^{I} = A/4\sum_{n=1}^{4}\sum_{i=1}^{N_{I}^{pl}}\dot{\lambda}_{i}^{I,n}B_{i}^{I}$, where node 4 results linearly dependent with respect to previous nodes. After some elementary assemblage operations, a simple linear programming problem is obtained [2], where the objective function consists in the minimization of the total internal power dissipated:

$$\left\{ \min\left(\sum_{I=1}^{N^{I}} \pi^{I} - \boldsymbol{\pi}_{0}^{T}\mathbf{w} \right); \; \mathbf{A}^{eq}\mathbf{U} = \mathbf{b}^{eq}, \; \boldsymbol{\pi}_{1}^{T}\mathbf{U} = 1, \; \dot{\boldsymbol{\lambda}}^{I,ass} \geq \mathbf{0} \right\} \tag{1}$$

where N^{I} is the total number of interfaces in which plastic dissipation occurs, $\mathbf{U} = \left[\mathbf{w} \quad \Delta\mathbf{u}^{I,ass} \quad \dot{\boldsymbol{\lambda}}^{I,ass} \right]^{T}$ is the vector of global unknowns, which collects the vector of assembled centroid velocities ($\mathbf{w}_{\mathbf{g}}^{E}$), the vector of assembled parallelepipeds rotations rates ($\mathbf{\Omega}_{\mathbf{g}}^{E}$), the vector of assembled jump of velocities on interfaces ($\Delta\mathbf{u}^{I,ass}$) and the vector of assembled interface plastic multiplier rates ($\dot{\boldsymbol{\lambda}}^{I,ass}$). Finally, $\mathbf{A}^{eq}$ ($\mathbf{b}^{eq}$) is the overall constraints matrix (right hands vector) and collects velocity boundary conditions, relations between velocity jumps on interfaces and elements velocities and velocity constraints for plastic flow in discontinuities.

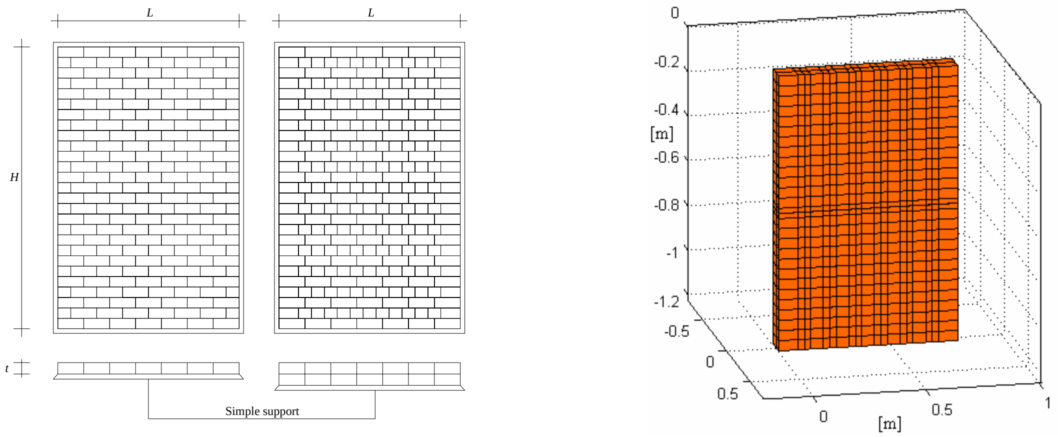


Figure 1: -a: Single and two-wythes thick brick simply supported masonry slab arranged in running or English bond pattern. –b: Mesh used for the heterogeneous model.

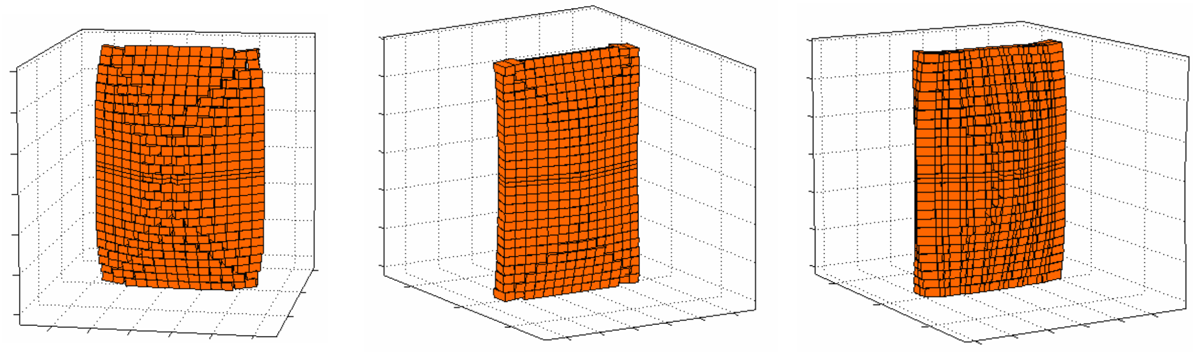
For what concerns the homogenized limit analysis approach, the reader is referred to [3] for a full theoretical description of the model. Here, we only underline that the model proposed is a Kirchhoff-Love approach, therefore suitable for thin plates.

3 NUMERICAL EXAMPLES

The simply supported English bond slab of Figure 1 is considered. A similar wall arranged in running bond was experimentally tested by Chee Liang in [5]. Half scale common English bricks ($112 \times 53 \times 36$ mm^3) were used, with approximately 10 mm thick mortar joints. In order to prevent any rotational restraint at the supports, frictionless materials were placed along each support. A total of 15 walls with different Length/Heigth ratios were tested, but here only two panels (labeled as Wall 8 and Wall 12) of dimensions $LxHxt = 795 \times 1190 \times 53$ mm^3 are analyzed for the sake of conciseness. The overall height of the wall was generally maintained between 1140 mm and 1200 mm, depending on the actual thickness of the joints arranged in the laboratory. The walls were subjected to an increasing uniformly distributed out-of-plane pressure.

In order to show the influence of both masonry texture and panel thickness on the ultimate load, three different FE models, here denoted as Model A, Model B and Model C, are compared for the evaluation of the ultimate out-of-plane pressure of the wall. Model A and Model B are based on the heterogeneous 3D limit analysis formulation presented in the previous section. Nevertheless, Model A and Model B differ for the texture. In particular, for Model A, a running bond texture is assumed for the wall (Model A1: thickness t, Model A2 thickness $2t$), whereas for Model B an English bond is adopted. The 3D mesh used for Model

B is shown in Figure 1-b. The third model (Model C) is the homogenized Kirchhoff-Love approach proposed in [3]. In Figure 2, a comparison among the deformed shapes at collapse obtained with Model A1, A2 and B is shown. Furthermore, in Table I, a comparison among failure loads obtained numerically with all the models at three different values of mortar tensile strength is summarized. For joints, a Lourenço-Rots failure criterion is chosen with $f_t = c/\tan(\Phi)$, $\Phi = 30°$, $f_c = 10 f_t$ $\Phi_2 = 45°$. From the comparison reported in Table I and from Figure 2, it is possible to notice that the formation of well defined yield lines is not possible for English bond and that a relevant difference on the collapse load occurs for different bricks dispositions.



-a -b -c

Figure 2: Masonry panel out-of-plane loaded. –a: running bond deformed shape at collapse, thickness t. –b: running bond deformed shape at collapse, thickness $2t$. –c: English bond deformed shape at collapse.

Table I: Out-of-plane loaded masonry slabs. Comparison among Model A1, Model A2, Model B, Model C at two different values of mortar tensile strength and for wall thickness equal to t and $2t$.

Collapse load [kN/m^2]					
wall thickness t		f_t [N/mm2]	wall thickness $2t$		
Model A1	Model C		Model A2	Model B	Model C
6.01	6.56	0.10	19.01	13.80	26.30
21.10	22.97	0.35	65.81	43.68	91.89

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