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# **COMPARISON OF DIFFERENT MODELLING APPROACHES FOR THE ASSESSMENT OF THE OUT-OF-PLANE BEHAVIOUR OF TWO-LEAF STONE MASONRY WALLS**



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#### **ABSTRACT**

This work addresses the comparison of different modelling approaches to simulate the out-ofplane behaviour of two-leaf stone masonry walls. Two modelling strategies are selected and compared in this the study, namely finite element (FE) by considering micro and macro modelling approach, and the distinct element method (DEM).

The study intends to: i) provide an insight regarding parameter estimation and calibration procedure for each modelling approaches; ii) compare different modelling strategies highlighting their pros and cons in terms of computational effort and results' accuracy.

**Keywords**: Two-leaf stone masonry walls; FE micro modelling; FE macro modelling, distinct element method; comparison of modelling strategies.

#### **1. INTRODUCTION**

Masonry is a composite material made of units (natural or manmade) arranged in space usually following a regular pattern characterized by a succession of horizontal overlapping layers which are staggered to avoid the formation of continuous vertical joints. Ideally, the presence of mortar (made of fine aggregate, sand, water, and air or hydraulic lime binders) should ensure masonry structures with monolithic behaviour. In the case of dry-stone masonry, no mortar is present, and the units are stacked on top of each other trying to achieve a good interlock such as to provide effective structural stability.

A significant part of the existing worldwide building stock consists of ordinary and historical masonry buildings (churches, temples, fortresses, etc.) [1].

Indeed, historical masonry buildings are often the result of a non-engineered building practice rooted in the workmanship's practical expertise developed over the centuries and successively codified as rules of thumb, which are essentially an array of techniques consistently detected in historical constructions.

Therefore, when it comes to assess the structural performance of historical masonry buildings, qualitative features (e.g. geometrical configuration of the masonry bond) and quantitative features (e.g. mechanical properties) need to be examined in order to provide an accurate estimation of the overall structural behaviour [2].

Among the recurring collapse mechanisms, the overturning of the buildings' external walls (first damage mode or out-of-plane failure) represents the most severe condition of vulnerability in masonry buildings [3].

With regard to a structural analysis of masonry buildings, several modelling approaches, ranging from highly simplified to highly advanced, have been developed to simulate the numerical behaviour of masonry structures throughout the decades [4]. Each of these approaches has pros and cons in terms of results' accuracy and computational effort. Moreover, the estimation of input parameters (from empirical formulas or experimental data) has a great influence on the final output affecting the reliability of the numerical simulations.

Therefore, this study presents the comparison among different modelling strategies with the aim of providing a deeper insight regarding (1) input parameters estimation and (2) reliability and effectiveness in simulating the out-of-plane behaviour of two-leaf stone masonry walls.

This study will focus on a comparison between a macro-model and a simplified micro-model built and analysed by means of a finite element software (DIANA FEA) [12]. Moreover, an additional application of the simplified micro-modelling approach will be carried out using a distinct element software (3DEC) [13]. The experimental data adopted for the calibration of the models and the comparison of the results refers to an out-of-plane test carried out by means of an airbag on a reduced scale (1:2) U-shaped dry-stone masonry walls (DS).

#### **2. MAIN FEATURES OF THE SELECTED MODELLING STRATEGIES**

Even though the array of numerical strategies used for both academic purposes and professional practice is extremely wide, a categorization of these procedures can be provided considering the level of refinement achieved once the numerical model is built [4] [5]. In macro-modelling approaches (1), masonry can be modelled as a one-phase material, where units, mortar and units-mortar interfaces are smeared out in a homogeneous continuum (Figure 1 – a). Simplified micro-modelling approaches (2) rely on the definition of "expanded" masonry units combined with zero-thickness interface elements to simulate mortar joint behaviour (Figure 1 – b). Conversely, in a detailed micro-modelling approach (3) masonry is represented as a three-phase material (Figure  $1 - c$ ) implying that masonry units and mortar joints are represented by continuum elements, whereas the unit–mortar interface is represented by discontinuous elements [5].



**Figure 1. Modelling strategies for masonry structures: (a) macro-modelling; (b) simplified micro-modelling; (c) detailed micro-modelling (adapted from [5])** 

Finite element macro-modelling approximates masonry as a homogeneous isotropic continuum material. The practical advantage of this approach relies on the use of simpler finite element meshes since there is no need of accurately simulating masonry components [10]. Damages are described as a smeared property spreading over a large volume of the structure, which is an approximation that may lead to some inaccuracy because actual cracks in masonry structures usually arise in concentrated or isolated locations [14]. The detailed micro-modelling approach considers independently masonry units, mortar joints and mortar-unit interfaces. It is an extremely accurate method although it is highly time-consuming.

Simplified micro-models overcome the computational drawbacks of the standard micromodelling technique. In this approach, a sort of "average interface" merges together each mortar joint and two adjacent unit-mortar interfaces, whereas the units are expanded to keep constant the overall geometrical configuration. Thus, expanded units represented by continuum elements are used to model both units and mortar material, whereas the behaviour of the mortar joints and unit-mortar interfaces is lumped to the discontinuous elements [15].

The "distinct element method" (DEM) was proposed by Cundall in 1971. DEM solution procedure is based on the integration of the equations of motion of the rigid blocks, which allows the possibility of considering large displacements and to update the block positions [22]. In DEM, masonry is represented as an assemblage of distinct blocks, where masonry joints are modelled as contact surfaces among different blocks; it can be classified as a simplified micro-model approach. The basic assumptions related to a computer-based distinct element modelling approach are [22]: (1) development of finite displacements and rotations of distinct bodies (blocks), including the complete detachment; (2) automatic recognition of new contacts between blocks as the calculation progresses.

Macro- and micro-models have been extensively used to analyse the seismic response of a wide range of masonry structures characterized by different boundary and load conditions [16] [17] [18]. Both approaches proved their reliability in capturing the out-of-plane behaviour of masonry structures showing a good agreement with the experimental results presented in research works available in the literature, even though further work to address the behaviour of two-leaf masonry walls can still be done [19] [20] [21].

# **3. PARAMETERS ESTIMATION**

As aforementioned, the modelling approaches considered in this study are: (1) macromodelling and (2) simplified micro-modelling. The macro-modelling analyses will be carried out using FE-based commercial software (DIANA [6]), whereas in the simplified micro-modelling, a comparison between DIANA [6] and the DEM-based software 3DEC by Itasca [7], will be presented.

The input parameters required to carry out the numerical analysis may vary depending on the constitutive model selected to simulate the behaviour of a certain structural system and on the selected modelling approach.

To this end, it must be underlined that macro-modelling approaches consider masonry as an isotropic continuum material having linear and non-linear properties; conversely, in a simplified micro-model it is assumed that the non-linear properties of masonry are concentrated on the interface connecting the unit.

The estimation of the mechanical properties related to the macro-modelling approach is based on the recommendations provided by Lourenço [5]. The equations used to estimate macromodel mechanical properties in this study are:

$$
f_c = \frac{E}{\alpha}
$$
  
\n
$$
f_t = \frac{1}{10} f_c
$$
  
\n
$$
G_t = d_t \cdot f
$$
\n(3)

$$
G_{fc} = d_{u,c} \cdot f_c \tag{3}
$$
  
\n
$$
G_{f1} = 12 \, N/m \tag{4}
$$

Where *E* is the Young Modulus, *f<sup>c</sup>* is the compressive strength, *α* is a coefficient assumed to be equal to 1000,  $f_t$  is the tensile strength,  $G_t$  is the compressive fracture energy,  $d_{u,c}$  is the ductility index in compression assumed equal to 1.60 mm and  $G_{f1}$  is the Mode I fracture energy. It must be noted that the values of *α*, *du,c* and *Gf1* have been set based on the same set of recommendations [5].

An important aspect related to the comparison of the modelling strategies presented in this study involves the use of suitable equations enabling the transition from one method to another depending on the input data available (e.g. estimation of Young Modulus starting from joints' normal stiffness and vice versa). In fact, the calculation of interface stiffness is a crucial aspect for the application of simplified micro-modelling approaches both for FE-based software and DE-based software. Therefore, providing reference values for interface elements can represent a good starting point in the calibration procedure characterizing any numerical simulation, especially when initial information is limited and/or available experimental data is scarce. To this end, Table 1 presents a brief review of mechanical properties available in the literature addressing the analysis of the behaviour of masonry structural systems by means of DEM.





The ratio between normal and shear stiffness ranges from 2 to 2.4 (*k<sup>n</sup>* = 2 - 2.24 *ks*). The cohesion to tensile strength ratio ranges from 1.25 and 1.5 (c = 1.25 – 1.50  $f_t$ ). The friction angle values range from 30° to 40°.

In the present study a normal stiffness was assumed as  $k_n = 2k_s$ , the cohesion was estimated as 1.25 *ft*. and any variation characterizing the calibration procedure has been carried out keeping it constant. In the case of dry joint masonry structures, the tensile strength and cohesion are assumed to be equal to zero.

### **4. DESCRIPTION OF THE CASE STUDY**

The double-leaf stone masonry walls analysed in this work replicate the main characteristic of stone masonry walls commonly found in vernacular buildings in the northern region of Portugal [22] but can be found in other regions in Portugal, namely the south, and other Mediterranean countries. In order to study experimentally its out-of-plane behaviour, reduced scale (1:2) masonry walls specimens with U-shaped plan configuration were adopted.

The final wall specimens present a span of 2.25 m, a height of 1.35 m and a thickness of 0.30 m. In the dry-stone masonry wall specimen (DS), through-stones (headers) were used to ensure an adequate connection between the wall leaves (realized using roughly cut stone units) and distributed throughout the area of the walls, as shown in Figure 2, where headers have been highlighted in grey colour. Further details about the geometrical configuration of the reference stone masonry walls can be found in Martins [23] (drystone masonry wall).



**Figure 2. Plan configuration tested specimens (a) and (b) front view drystone masonry wall (DS)**

The experimental setup simulated out-of-plane the seismic action by means of an airbag (area of 1.65×1.35 m<sup>2</sup>), exerting a uniformly distributed load on the rear surface of the façade wall. Additionally, a vertical load was also applied to the lateral walls to simulate the self-weight of a timber roof. A supporting steel frame was placed between the reinforced concrete reaction wall of the laboratory and the airbag. Four load cells, placed between the steel profiles and the reaction wall, allowed to record the load applied by the airbag to the wall. The out-of-plane test was carried out under displacement control, being the control point located at the top of the façade wall at mid-span, where the highest displacement was expected (Figure 3). The monitoring of the displacements of the façade wall during the out-of-plane test was carried out using linear variable differential transformers (LVDTs), further details about the testing setup can be found in Martins[23], Maccarini [22] and Murano [24].



**Figure 3. Load configuration and test setup configuration adopted for the OOP test**

The numerical macro-model of the wall was defined with DIANA software [6] using twentynode tetrahedron solid 3D elements (CHX60). Since the model is intended to simulate the experimental test, the reinforced concrete base was also included in the numerical model using the same solid 3D elements.

The reinforced concrete base of the wall is considered fully constrained to the floor. Indeed, some metallic elements have been used to avoid any uplift or sliding movements of the base itself (Figure 3). Moreover, a linear elastic behaviour is assumed for the reinforced concrete base since it did not experience any sort of damage during the testing procedure.

The material model adopted to represent the non-linear behaviour of the stone masonry is a standard isotropic Total Strain Rotating Crack Model (TSRM). The model describes the tensile and compressive behaviour of the material with one stress-strain relationship and assumes that the crack direction rotates with the principal strain axes [6]. An exponential softening function simulates the non-linear behaviour of the material in tension, whereas a parabolic function was adopted to describe the crushing behaviour in compression [6].

The simplified micro-model built using 3DEC software [7] envisages the use of rigid blocks to simulate masonry stone units, whereas the nonlinear behaviour is simulated by means of interface elements based on a Mohr-Coulomb constitutive model [7].

Similarly, the micro-model of the wall in DIANA software also considers interface elements to which the same constitutive model (Mohr-Coulomb) is assigned. In this case, stone units are assumed to have linear elastic behaviour, implying that cracks occur at the masonry joints, whether they are dry or mortar joints.

The geometrical models of the tested specimens have been built using a 3D CAD software and later imported into DIANA and 3DEC to characterize the geometry based on the input parameters required for each modelling approach and constitutive model considered (Figure 4).



**Figure 4. Geometrical 3D model for macro-modelling application (a); Drystone masonry wall (DS) geometrical 3D model for simplified micro-modelling applications (b); Drystone masonry wall (DS) macro-model (c), DIANA micro-model (d) and 3DEC distinct elements model (e)**

# **5. CALIBRATION PROCEDURE AND RESULTS COMPARISON**

The calibration procedure, applied for the drystone wall (DS), consists in the following stages: (1) preliminary analysis carried out using the simplified micro-model built with 3DEC software (DEM-based software). The starting mechanical properties used to characterize the behaviour of the interface elements have been set equal to the average values proposed in Table 1; (2) adjustment of the preliminary numerical load-displacement pushover curve to fit the experimental results; (3) the stiffness values (normal stiffness - *kn*) obtained at the end of the tuning process carried out in the previous step, have been used to estimate a corresponding value of Young Modulus in order to define the corresponding properties to be used in the macro-model, based on equations (1, (2, (3 and (4; (4) once macro-model's mechanical properties have been defined, a modal analysis was carried out to compare numerical mode shapes and natural frequencies of vibration for all the considered modelling approaches, to have a first insight regarding the effectiveness of the parameters estimation procedure and the consistency of the behaviour of the models.

As mentioned above, the tuning process of the interface mechanical properties has been carried out by means of 3DEC (DE-based software) since it allows to get faster results compared to DIANA (FE-based software).

The load-displacements curves of dry stone wall obtained in the early stage of the calibration procedure (DS\_Step 1) and after the iterative fitting process carried out in order to match numerical and experimental data (DS Step 2) are presented below (see respectively Figure 5 – a and Figure  $5 - b$ ).



**Figure 5. Drystone masonry wall (DS) calibration procedure step 1 (a) and step 2 (b)**

The use of average values proposed in Table 1 resulted in a preliminary analysis which significantly overestimates the experimental results both in terms of initial stiffness and peak load attained. The required adjustment of the interface mechanical parameters (see Table 2) led to a better approximation of the experimental evidence provided by the out-of-plane test  $(Figure 5 - b)$ .

The numerical curve shows a hardening behaviour characterizing the post peak branch. In order to better capture this trend, a more refined constitutive model could have been selected among those available in 3DEC software [7] (e.g. Softening-Healing Mohr-Coulomb Joint Model) or, as an alternative, it could have been possible to specify additional parameters such as residual friction angle, residual cohesion and residual tensile strength in the constitutive model selected to carry out the analyses presented (Mohr-Coulomb). However, since the experimental data for the drystone wall examined is scarce it was decided to reduce the number of assumptions related to the interface mechanical parameters as much as possible, hence only considering the minimum requirements in terms of input parameters needed to carry out the numerical analysesThe same mechanical properties have been used for the FEbased simplified micro-model (DIANA) and the DE-based simplified micro-model (3DEC), since in both cases the Mohr-Coulomb constitutive model was considered.

Normal Stiffness (k <sub>n</sub> )	$2.00E + 9$ N/m <sup>3</sup>	
Shear Stiffness (ks)	1.00E+9 $N/m3$	
Tensile Strength (f <sub>t</sub> )	0 N/m <sup>2</sup>	
Cohesion (c)	0 N/m <sup>2</sup>	
Dilatancy (dl)	0 N/m <sup>2</sup>	
<b>Friction Angle (fr)</b>	$9^{\circ}$	

**Table 2. Mechanical parameters considered in FE-based (DIANA) and DE-based (3DEC) simplified micro-model drystone wall**

Once the final mechanical properties have been adjusted in the DE-based simplified micromodel, the following step involved the estimation of a Young Modulus to be used for the calculation of the remaining input parameters needed to the macro-model developed for the drystone wall.

To this end, starting from the value of normal stiffness *k<sup>n</sup>* presented in Table 2, a first estimation of the Young Modulus has been carried out based on the expressions provided in the literature. The main assumptions adopted to estimate an equivalent Young Modulus of masonry for the drystone wall were: (1) the average height of stone units has been set equal to 0,20 m (20 cm); (2) mortar Young Modulus and joints' thickness have been considered to be close to zero. The results obtained using different formulations proposed in the literature are summarized in Table 3. The average value for the Young Modulus of the drystone wall stands at  $4.00E+8$  N/m<sup>2</sup> (400 MPa).

**Table 3. Estimation of equivalent Young Modulus for drystone masonry wall (DS)**

<b>Itasca Manual [7]</b>	<b>Bui</b> [18]	<b>Gonen</b> [25]
4.00E+8 $N/m2$	3.93E+8 $N/m2$	4.00E+8 $N/m2$

The empirical expression, proposed by different literature sources and used to estimate an equivalent Young Modulus for the macro-model, yielded similar results, confirming their consistency regardless of the initial assumptions applied to comply to the characteristics of the structural system analysed (drystone masonry wall).

The average equivalent Young Modulus for drystone masonry (400 MPa), estimated according to the expressions provided in the literature, has been used to derive the input parameters for the macro-model analysis (by means of equations 1 to 4).

The preliminary simulation carried out resulted in an overestimation of both initial stiffness and peak load when compared to the experimental results and to the calibrated DE model outcomes (Figure  $6 - a$ ), whereas after some additional adjustments of the mechanical properties, the macro-model numerical simulation showed a good agreement with the outcomes related to the other numerical approaches and to the experimental data as well  $(Figure 6 - b)$ .



**Figure 6. Drystone masonry wall preliminary analyses macro model vs DE-based simplified micro-model (a); Experimental and numerical Load vs Displacement curves drystone masonry wall (DS) (b)**

In the DE analyses, the zero cohesion and zero tension assumptions result in a significant decrease in terms of stiffness and peak load as soon as the contact between two stone units weakens. This physical condition cannot be replicated in the macro-model because, from the geometrical point of view, the model itself assumes a uniform material and no contact between stone units is simulated.

Furthermore, in the macro-model analyses performed in DIANA, convergence problems occurred when tensile strength, compressive strength, compressive fracture energy and mode I fracture energy are set equal to zero.

For this reason, in order to obtain a macro-model able to replicate the same assumptions characterizing the DE model, namely no tension and no cohesion at the level of dry joints, the values of the input mechanical parameters have been set equal and with extremely low values, ignoring the dependency of parameters such as tensile/compressive strength and compressive fracture energy from the masonry Young Modulus (see Table 4).



**Table 4. Initial (Step 1) and final (Step 2) properties for the macro-model drystone masonry wall**

Once the mechanical parameters have been modified, the numerical natural frequencies of vibration obtained in the macro-model approached the values of the micro-model analysis with an overall percentage variation lower than 10% in the first three modes.

A good agreement has been also obtained between DIANA and 3DEC simplified micro models. The mode shapes were essentially the same for all the models analysed. It should be stressed that the same mechanical properties have been used for the interfaces considering the Mohr-Coulomb interface model.

Looking at the damage pattern corresponding to a displacement of 40 mm measured at the top mid span of the wall, it is possible to highlight a significant agreement between the simplified micro-model results (DIANA and 3DEC) and the experimental damage pattern (Figure 7 i-l-m, Figure 7 d-e-f, Figure 7 a-b-c).

During the experimental out-of-plane test, the upper left façade of the wall (see Figure 2) experienced a significant crack opening (Figure 7 a-b-c). The opening of this crack occurs at end of the linear behaviour of the wall, and it determines a progressive detachment of the façade from the lateral wall resulting in a translation movement of the façade itself for the highest level of displacements. This phenomenon has been captured in the micro-models (3DEC - Figure 7 d-e-f and DIANA - Figure 7 i-l-m).

It is also interesting to notice that micro-models are very sensitive to the critical points related to the lack of interlocking of vertical joints, as in both micro-models the localized crack at the main façade is clearly captured by them. Due to its intrinsic features, the macro-model has not proven to be able to capture this crack pattern. Despite this expected limitation, the macromodel captured the detachment phenomenon of the façade regarding to the lateral walls (Figure 7 g-h), revealed by the strain concentration at the connection between the main façade and transversal walls.

In both micro-model analyses, damage on the lateral walls consisted in small crack opening and sliding of stone units in the out-of-plane direction. At the same time, sliding phenomena occurred at the base of both micro-models as well.

Even if these mechanisms could not be properly captured by the macro-model analyses, the strain distribution can be considered reasonably compatible with the experimental damage pattern and the numerical damage pattern resulted from both micro-model analyses.

In the macro model (Figure 7 g-h), the highest strain concentration characterizes the upper part of the wall in the areas connecting façade and lateral walls. Moreover, even if the detachment of the façade from the lateral wall showed in the macro-model does not appear to be consistent to the damage presented in both micro-models, it is possible to say that the overturing phenomenon in the macro-model is compatible with the outcomes of the micromodels analyses in their later stage, which consists in the complete collapse of the façade.



**Figure 7. Numerical and experimental damage pattern drystone masonry wall (Displacement level 40 mm)**

#### **6. FINAL REMARKS**

This study presents a comparison among different modelling approaches applied in the simulation of the out-of-plane behaviour of a two-leaf stone masonry wall and a comparison with experimental results from out-of-plane tests.

After an overview regarding the main features of the selected modelling approaches (FE/DE simplified micro-model and FE-based macro-modelling), attention has been paid to the expressions available in the literature to estimate the input parameters describing the constitutive material behaviour in the numerical model.

Moreover, a literature review was conducted to estimate the properties of masonry interfaces, which is a key aspect in case of micro-models. The collected mechanical properties have been used as a starting point for the calibration procedure of the models simulating the wall.

Overall, it is possible to conclude that both micro-models approaches proved to be reliable in simulating the out-of-plane behaviour of the drystone masonry wall, capturing the damage/collapse mechanisms occurred during the experimental tests.

On the other hand, macro-model numerical simulation, despite its limitations, namely the inability to capture asymmetrical or irregular damage mechanisms, due to the basic assumption of the method (masonry considered as homogeneous isotropic material), proved to be reliable in estimating the areas of the masonry wall most prone to failure.

The main drawback related to the FE-based micro model is the computational effort characterizing this type of analysis. Conversely the simplified DE-based micro-model showed a significant efficiency from a computational point of view, representing a good compromise between accuracy of results and duration of the analysis. It is possible to say the same for the macro-model simulation, but in this case, the lack of accuracy of the results must not be overlooked.

A FE-based micro-model pushover analysis approximately required a run time higher than 7 days, whereas the run time for a DE-based pushover analysis roughly required less than 30 minutes; a is similar computational effort (average of 30 minutes) characterised the FE-based macro-model analyses carried out.

Finally, a key aspect highlighted in this work refers to the consistency between the input mechanical parameters estimated with different empirical expressions; attention should be paid to the characterization of the behaviour of the models in order to have good agreement between numerical and experimental curves.

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