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"Outlier Robust Specification of Multiplicative Time-Varying Volatility Models"

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Outlier Robust Specification of Multiplicative Time-Varying Volatility Models*

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Abstract

Nonstationarity and outlying observations are commonly encountered in financial time series. It is thus expected that models are able to accommodate these stylized facts and the techniques used are suitable to specify such models. In this paper we relax the assumption of stationarity and consider the problem of detecting smooth changes in the unconditional variance in the presence of outliers. It is found by simulation that the misspecification test for constancy of the unconditional variance in GARCH models can be severely adversely affected in the presence of additive outliers. An outlier robust specification procedure is also proposed to mitigate the effects of outliers for building multiplicative time-varying volatility models. The outlier robust variant of the test is shown to perform better than the conventional test in terms of size and power. An application to commodity returns illustrates the usefulness of the robust specification procedure.

JEL Classification Codes: C12; C32; C51; C52.

Keywords. Conditional heteroskedasticity; Testing parameter constancy; Model specification; Time-varying unconditional variance; Outliers.

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1 Introduction

Empirical evidence suggests that when fitting a sufficiently long return series with a stationary GARCH model the estimated parameters are very close to those of an integrated GARCH model. Diebold (1986) and Lamoureux and Lastrapes (1990) advocate that the phenomena of high persistence in volatility can be well described by unmodelled deterministic shifts in the unconditional variance and therefore the assumption of weak stationarity may be inappropriate under the evidence of changes in long series of returns. One solution to deal with extreme persistence in volatility is to extend the GARCH model by explicitly assuming the unconditional variance to be time-varying. Examples of this line of research include models with multiplicative decomposition of the variance into a stochastic component and a deterministic component. The parametric model introduced by Amado and Teräsvirta (2008) and further discussed in Amado and Teräsvirta (2013, 2014, 2017) belongs to this class of models. Pioneered examples of this view include Feng (2004) and van Bellegem and von Sachs (2004), and succeeding approaches by Engle and Rangel (2008), Brownlees and Gallo (2010) and Mazur and Pipień (2012).

In this paper, we investigate the properties of the data-driven strategy outlined by Amado and Teräsvirta (2017) in the presence of outliers. In their approach the nonstationary component of volatility is represented by a linear combination of generalised logistic functions of rescaled time and it relies on statistical inference for specifying the deterministic component. The modelling strategy for building the parametric structure of the time-varying component consists of sequential hypotheses testing by means of Lagrange multiplier (LM) tests. If the statistical test suggests inadequacy of the GARCH model, the specification of the model must be modified accordingly. Since neglected outliers can suggest model misspecification, a careful specification and estimation of the nonstationary component is needed for fitting an adequate model.

Outliers are known to severely affect the asymptotic properties of the test statistics for nonlinearity; see for example van Dijk, Franses and Lucas (1999b) for LM test for smooth transition autoregressive nonlinearity or van Dijk, Franses and Lucas (1999a) and Franses, van Dijk and Lucas (2004) for test for autoregressive conditional heteroskedasticity (ARCH) in the presence of additive outliers. For effects of different types of outliers on the LM tests for ARCH and bilinearity see Tolvi (2000). Balke and Fomby (1994) find that after controlling for outliers much of the evidence of nonlinearity in major macroeconomic time series is weakened and hence suggesting that extreme observations are linked with nonlinear data structures. Therefore, when outliers are neglected, one expects the test for constancy of the unconditional variance to be biased and thus becoming difficult to discriminate between nonstationarity in the variance and outlying observations. Motivated by these issues, this paper intends to fill the gap in the literature by examining the effects of additive outliers on the misspecification test for stability of the unconditional variance. Our simulation results substantiate the findings that additive outliers lead to spurious GARCH-type misspecification by rejecting the null hypothesis of constant unconditional variance too often when it is true. We find that additive outliers distort the distributional properties of the test statistics and thus incorrectly pointing toward nonstationarity. To overcome this limitation this paper proposes a modified specification procedure for building multiplicative time-varying GARCH models which is robust to the presence of additive outliers. A simple modification to the maximization by parts described in Amado and Teräsvirta (2013) is proposed by modifying the stochastic component with the bounding mechanism of Muler and Yohai (2008) and thereby having more desirable properties in the presence of outliers. Other ways of dealing with additive outliers in GARCH models have been proposed by Franses and Ghijsels (1999) and Park (2002) for improving the quality of volatility forecasts. It is also found by simulation that neglected additive outliers bias the estimated parameters of the stochastic and deterministic components in finite samples. Similar conclusions have been drawn for the GARCH model in Carnero, Peña and Ruiz (2007, 2012) and Muler and Yohai (2008), among others. An empirical application to daily commodity returns data shows that the robust data-based modelling technique of the multiplicative decomposition of the variance supports the simulation findings and a careful and thorough analysis must be carried out in the presence of outliers.

This paper is organised as follows. In Section 2 we briefly review the multiplicative time-varying GARCH model of Amado and Teräsvirta (2008, 2013) and the available data-driven specification procedure. Section 3 attempts to provide further insight into specifying the time-varying GARCH model in the presence of outliers and furthermore discuss the outlier robust estimation of parameters. The effects of outliers on the misspecification test of constancy unconditional variance are investigated in Section 4. An empirical illustration is provided in Section 5 in which the robustified version of the specification procedure is applied to a couple of commodity returns. Conclusions can be found in Section 6.

2 Multiplicative time-varying GARCH

2.1 The model

In this paper the tool for modelling return series is the multiplicative time-varying GARCH model of Amado and Teräsvirta (2008,2013) in which the unconditional variance is assumed to evolve smoothly over time. To define the model, consider the sequence of returns $\{y_t\}$

$$y_t = \mathsf{E}(y_t|\mathcal{F}_t) + \varepsilon_t \tag{1}$$

where $\mathcal{F}_t$ contains the historical information available at time $t-1$ and the conditional mean of the returns is assumed to have a time-varying structure, i.e., $E(y_t|\mathcal{F}_t) = \mu_t$. Let $\{\varepsilon_t\}$ be an innovation sequence with conditional mean $\mathsf{E}(\varepsilon_t|\mathcal{F}_t) = 0$ and time-varying conditional variance $\mathsf{E}(\varepsilon_t^2|\mathcal{F}_t) = \sigma_t^2$. The error term ε_t is further parameterized as

$$\varepsilon_t = \xi_t h_t^{1/2} \tag{2}$$

where h_t describes conditional heteroskedasticity in the observed process y_t and ξ_t is a time-varying random variable satisfying

$$\xi_t = \zeta_t g_t^{1/2} \tag{3}$$

where $\{\zeta_t\}$ is a sequence of independent and identically random variables with $\mathsf{E}\zeta_t = 0$, $\mathsf{E}\zeta_t^2 = 1$, $\mathsf{E}\zeta_t^3 = 0$ and $\mathsf{E}|\zeta_t^2|^{2+\phi} = 0 < \infty$, $\phi > 0$, and g_t is a positive-valued deterministic component which allows the unconditional variance of ξ_t to change smoothly over time. The time-varying conditional variance is thus modelled using the following multiplicative decomposition

$$\sigma_t^2 = h_t g_t \tag{4}$$

where the function h_t describes the short-run dynamics of the variance of the returns and follows the standard GARCH(p,q) model of Bollerslev (1986)

$$h_t = h_t(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2) = \alpha_0 + \sum_{i=1}^{q} \alpha_i \phi_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \tag{5}$$

where $\phi_t = \varepsilon_t / g_t^{1/2}$, $\boldsymbol{\theta}_2 = (\alpha_0, \boldsymbol{\alpha}', \boldsymbol{\beta}')' \in \Theta_2 = (\alpha_0 \times A \times B)$ with $\boldsymbol{\alpha} = (\alpha_1, ..., \alpha_q)' \in A$, and $\boldsymbol{\beta} = (\beta_1, ...\beta_p)' \in B$ and the parameter restrictions for positivity and stationarity of the conditional variance of ϕ_t are satisfied; see Bollerslev (1986) and Nelson and Cao (1992). This implies $\alpha_0 > 0$, $\alpha_i \geq 0$, $i = 1, ..., q-1$, $\beta_j \geq 0$, $j = 1, ..., p$, $\alpha_q > 0$ and $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1$. In order to generate smooth changes in the unconditional variance and introducing nonstationarity into σ_t^2 the function g_t has the following representation

$$g_t = g_t(\boldsymbol{\theta}_1, t/T) = \delta_0 + \sum_{l=1}^{r} \delta_l G_l(t/T; \gamma_l, \mathbf{c}_l) \tag{6}$$

where $\boldsymbol{\theta}_1 = (\boldsymbol{\delta}', \boldsymbol{\gamma}', \mathbf{c}_1', ..., \mathbf{c}_r')' \in \Theta_1 = (\Delta \times \Gamma \times C)$ with $\boldsymbol{\delta} = (\delta_0, \delta_1, ..., \delta_r)' \in \Delta$, $\boldsymbol{\gamma} = (\gamma_1, ...\gamma_r)' \in \Gamma$, $\mathbf{c}_l = (c_{l1}, ..., c_{lK_l})' \in C$, $l = 1, ..., r$, and $G_l(t/T; \gamma_l, \mathbf{c}_l)$ is the so-called transition function continuous and bounded between zero and one. A suitable choice for $G_l(t/T;\gamma_l,\mathbf{c}_l)$ is the general logistic transition function defined as

$$G_l(t/T;\gamma_l,\mathbf{c}_l) = \left(1+\exp\left\{-\gamma_l\prod_{k=1}^{K_l}(t/T-c_{lk})\right\}\right)^{-1}. \qquad (7)$$

Following Amado and Teräsvirta (2017) the following assumptions are made about (6) and (7):

AG1. The elements of $\boldsymbol{\delta}\in\Delta$ are restricted such that $\delta_0>0$ is fixed, $\max_{j=1,\ldots,q}|\delta_j|\leq M_\delta<\infty$ and $\inf_{\delta_1\in\Theta_1} g_t(\boldsymbol{\theta}_1,t/T)\geq g_{\min}>0$.

AG2. The slope parameter $\gamma_l>0$, $l=1,...,r$, and the location parameters $c_{1k}<c_{2k}<...<c_{rk}$.

The assumptions δ_0 fixed in AG1 and AG2 are identifying restrictions. The former is needed due to the multiplicative decomposition and because both h_t and g_t contain a positive constant. One of these constants must be fixed to achieve identification and we set $\delta_0=\delta_0^*$ (known constant).

The transition function $G_l(t/T;\gamma_l,\mathbf{c}_l)$ allows the unconditional variance to change smoothly as a function of the calendar time t/T. The parameters c_{lk} and γ_l determine the location and the speed of the transition between different regimes. The slope parameter γ_l in (7) controls the degree of smoothness of the transition. When $\gamma_l\longrightarrow\infty$, g_t collapses into a step function and the process contains structural breaks at $c_{l1}<c_{l2}<...<c_{lK_l}$ and it switches instantaneously over time from one regime to another. The order $K_l\in\mathbb{Z}_+$ determines the shape of the transition function. Typical choices for the transition function in practice are $K=1$ and $K=2$. When $r=1$ and $K=1$ the model is suitable for describing monotonic changes in the unconditional variance from δ_0 to $\delta_0+\delta_1$ with the location centred at $t=c_{11}T$ for return processes whose dynamics is different before and after the smooth structural change. When $r>1$ and $K=2$ the parameterization is capable of describing nonmonotonic deterministic changes in the unconditional variance. Under $\delta_1=...=\delta_r=0$, the unconditional variance $\mathsf{E}(\varepsilon_t^2)=\mathsf{E}(\zeta_t^2h_tg_t)=g_t\mathsf{E}(h_t)$ becomes constant and equals $\mathsf{E}(\varepsilon_t^2)=\delta_0\mathsf{E}(h_t)$.

This parameterization can explain systematic movements of the conditional variance as in the GARCH process, but relaxing the assumption of constancy of the unconditional volatility and therefore introducing nonstationarity in σ_t^2. This formulation allows the standard GARCH(p,q) to be nested in (4) when $g_t\equiv 1$.

2.2 Specification of the unconditional variance

The model-building cycle for specifying the multiplicative time-varying GARCH model is identical to the specific-to-general strategy for nonlinear models recommended by

Granger (1993). We first begin by modelling the conditional variance h_t under the assumption that $g_t \equiv 1$. The selection of the number of transitions r in (6) is determined thereafter by sequential testing. The problem of testing constancy in the unconditional variance by misspecification test was considered by Amado and Teräsvirta (2008, 2017) and Silvennoinen and Teräsvirta (2016). For the purpose of discussing the test statistic consider $p = q = 1$ in (5) and $r = 1$ in (6) and rewrite the conditional variance as

$$h_t g_t = (\alpha_0 + \alpha_1 \phi_{t-1}^2 + \beta_1 h_{t-1})(\delta_0 + \delta_1 G_1(t/T; \gamma_1, \mathbf{c}_1)) \tag{8}$$

We start by testing the null hypothesis of constant unconditional variance $H_0 : \gamma_1 = 0$ against $H_1 : \gamma_1 > 0$ in (8) at a predetermined significance level $\alpha^{(1)}$. When the null hypothesis holds,

$$\delta_1(G_1(t/T; \gamma_1, \mathbf{c}_1) - 1/2) = 0 \tag{9}$$

$(\delta_1, \mathbf{c}_1')'$ is a vector of nuisance parameters (subtracting 1/2 from $G_1(t/T; \gamma_1, \mathbf{c}_1)$ for notational convenience does not affect the conclusion). This makes the standard asymptotic inference invalid as the test statistic has a nonstandard asymptotic null distribution. We circumvent this identification problem as in Luukkonen, Saikkonen and Teräsvirta (1988) and approximate the transition function by its first-order Taylor expansion around $\gamma_1 = 0$. This approach facilitates the derivation of a simple applicable misspecification test. After merging terms we obtain

$$h_t g_t = (\alpha_0 + \alpha_1 \phi_{t-1}^2 + \beta_1 h_{t-1})(\varphi_0 + \varphi_1(t/T) + \varphi_2(t/T)^2 + ... + \varphi_K(t/T)^K + R_{1t}) \tag{10}$$

where $\varphi_0 = \delta_0 + \gamma_1\delta_1\widetilde{c}_0$, $\varphi_k = \gamma_1\delta_1\widetilde{c}_k$, $k = 1, ..., K$, such that the parameters $\widetilde{c}_k$ are functions of the location parameters c_k and R_{1t} is the remainder. Using the reparameterization (10) it follows that the null hypothesis of parameter constancy of the unconditional variance becomes: $H_0' : \varphi_1 = ... = \varphi_K = 0$. Under H_0', $R_{1t} = 0$, thus the remainder does not affect the asymptotic null distribution of the test statistic. Constructing a LM test for testing parameter constancy in the unconditional variance has the advantage that the model is only estimated under the null hypothesis. To introduce the test statistic, let the "hats" denote the maximum likelihood estimates, $\widehat{\mathbf{v}}_t = (1, \widehat{\varepsilon}_t, \widehat{h}_t)'$ and denote $\widehat{h}_t$ the estimated h_t evaluated under H_0. Furthermore, let $\boldsymbol{\varphi} = (\varphi_1, ..., \varphi_K)'$, $\widehat{\mathbf{x}}_{1t} = \widehat{h}_t^{-1}(\partial\widehat{h}_t/\partial\boldsymbol{\theta}_2)|_{H_0}$ and $\widehat{\mathbf{x}}_{2t} = \widehat{g}_t^{-1}(\partial\widehat{g}_t/\partial\boldsymbol{\varphi})|_{H_0}$ where $\partial\widehat{h}_t/\partial\boldsymbol{\theta}_2 = \widehat{\mathbf{v}}_{t-1} + \widehat{\beta}_1\partial\widehat{h}_{t-1}/\partial\boldsymbol{\theta}_2|_{H_0}$ and $\partial\widehat{g}_t/\partial\boldsymbol{\varphi}|_{H_0} = (t/T, (t/T)^2, ..., (t/T)^K)'$ with $\widehat{g}_t$ equal to g_t estimated under H_0. The standard LM test statistic derived by Amado and Teräsvirta (2017) is given by

$$T\mathbf{s}(\widehat{\boldsymbol{\theta}})'(\widehat{\boldsymbol{\Sigma}}_{22}(\widehat{\boldsymbol{\theta}}) - \widehat{\boldsymbol{\Sigma}}_{12}(\widehat{\boldsymbol{\theta}})\{\widehat{\boldsymbol{\Sigma}}_{11}(\widehat{\boldsymbol{\theta}})\}^{-1}\widehat{\boldsymbol{\Sigma}}_{21}(\widehat{\boldsymbol{\theta}}))^{-1}\mathbf{s}(\widehat{\boldsymbol{\theta}}) \xrightarrow{d} \chi^2_{(K)} \tag{11}$$

where $\mathbf{s}(\widehat{\boldsymbol{\theta}}) = (1/2T)\sum_{t=1}^{T}(\widehat{\varepsilon}_t^2/\widehat{h}_t - 1)\widehat{\mathbf{x}}_{2t}'$ and $\widehat{\boldsymbol{\Sigma}}_{ij} = (1/2T)\sum_{t=1}^{T}\widehat{\mathbf{x}}_{it}\widehat{\mathbf{x}}_{jt}'$, $i, j, = 1, 2$, is a consistent estimator of $\boldsymbol{\Sigma}_{ij}$ assuming normality for ζ_t. The LM test statistic can be computed in a fairly straightforward way by constructing an auxiliary regression version of the test as $T(SSR_0 - SSR_1)/SSR_0$, where $SSR_0 = \sum_{t=1}^{T}(\widehat{\varepsilon}_t^2/\widehat{h}_t - 1)^2$ and SSR_1 is the sum of squared residuals from a regression of $\widehat{\varepsilon}_t^2/\widehat{h}_t - 1$ on $\widehat{\mathbf{x}}_{1t}$ and $\widehat{\mathbf{x}}_{2t}$. After the number of transitions r has been determined, one needs to specify the order K of the polynomial of the transition function. For choosing K we use a model selection rule based on a sequence of nested tests as in Amado and Teräsvirta (2017). Assume $K = 3$ to ensure a parameterization sufficiently flexible and test the sequence of hypotheses:

$$H_{03} \quad : \varphi_3 = 0 \tag{12}$$

$$H_{02} \quad : \varphi_2 = 0|\varphi_3 = 0 \tag{13}$$

$$H_{01} \quad : \varphi_1 = 0|\varphi_3 = \varphi_2 = 0 \tag{14}$$

One of the assumptions underlying the LM test is that the conditional fourth moment is constant, but this condition will be violated in the presence of additive outliers. When the errors are not normal, a robust version of the test statistic to certain departures from normality can be derived to fit this situation. One can construct a robust version of the LM-type statistic using the procedure by Wooldridge (1990, 1991). In practice the test can be carried out in a straightforward way using an auxiliary regression as follows:

1. Estimate the GARCH(1,1) model by quasi maximum likelihood and compute the squared standardised residuals $\widehat{\eta}_t^2 = \widehat{\varepsilon}_t^2/\widehat{h}_t$, $\widehat{\mathbf{x}}_{1t}$ and $\widehat{\mathbf{x}}_{2t}$, $t = 1, ..., T$.

2. Regress $\widehat{\mathbf{x}}_{1t}$ on $\widehat{\mathbf{x}}_{2t}$ and save the residual vectors $\mathbf{w}_t$, $t = 1, ..., T$.

3. Regress 1 on $(\widehat{\eta}_t^2 - 1)\mathbf{w}_t$ and compute the SSR_0 from this regression. Under the null hypothesis, the test statistic $\xi_{LM_R} = T - SSR_0$ has an asymptotic χ^2 distribution with K degrees of freedom.

Next, estimate g_t with a single transition function and test against another transition at the significance level $\alpha^{(2)} = \tau\alpha^{(1)}$, where $\tau \in (0, 1)$. In our application we set $\tau = 0.5$. The significance level is reduced at each stage by a factor τ in order to favour parsimony. More generally, when g_t has been estimated with $r-1$ transition functions one tests for another transition in g_t at the significance level $\alpha\tau^{r-1}$ and proceed sequentially until the first non-rejection of the null hypothesis.

3 Model specification in the presence of outliers

We start by briefly reviewing the generating mechanism of an outlier in time series. Let the observed series x_t, $t = 1, ..., T$, with finite fourth-order moment be contaminated by outliers of magnitude ω described by

$$x_t = y_t + \omega v(L)\xi_t^{(s)} \tag{15}$$

where the outlier-free time series y_t follows an autoregressive (AR) process (for ARMA models see Tsay (1986, 1988)) and $\xi_t^{(s)}$ is an indicator variable such that $\xi_t^{(s)} = 1$ if $t = s$, and $\xi_t^{(s)} = 0$ otherwise. Outliers are incorporated through the lag polynomial function $v(L)$ and its form depends on the types of outliers. Two types of contamination on time series are considered in the standard outlier literature. They are the additive outlier (AO) and innovational outlier (IO). For an AO only the disturbance of magnitude ω affects the sth observation, and therefore $v(L) = 1$. In what follows, we denote the additive outlier by ω_{AO}. An IO is a disturbance ω affecting the innovation series ε_t in (2) and future observations x_{s+1}, $x_{s+2}, ...$ through the autoregressive dynamic pattern $v(L)$. In order to simplify the exposition, assume that y_t follows

$$y_t = \phi_0 + \phi_1 y_{t-1} + \varepsilon_t, \qquad |\phi_1| < 1 \tag{16}$$

where

$$\varepsilon_t = \zeta_t h_t^{1/2} g_t^{1/2} \tag{17}$$

with h_t and g_t as in (5) and (6), respectively, and $\varepsilon_t|\mathcal{F}_t \sim iid(0, \sigma_t^2)$. In this work, we focus on additive outliers because the analysis lies on financial returns which are often characterized by being weakly uncorrelated and thus the distinction between additive outliers and innovative outliers becomes trivial. The additive outliers shall be assumed as level outliers since they merely affect the level of the series but not the dynamics of the underlying volatility and they can be interpreted as a deviation from conditional normality and some type of misspecification in the conditional mean (see van Dijk et al. (1999a) for details).

The specification problem for building the model consists first of estimating the short-run component h_t and thereafter specifying the long-run component g_t. The idea is to begin with a parsimonious model and proceed to more complicated ones sequentially with LM tests until an adequate model has been obtained. This may be preceded by employing an outlier detection technique to distinguish an AO from an IO using the testing criteria of Chang, Tiao and Chen (1988). In what follows, we summarize the different stages involved in the specification of the multiplicative time-varying GARCH model in the presence of outliers.

3.1 Modelling the short-run component of volatility

Instead of proceeding with an iterative procedure characterized by an outlier-detection stage, correction and estimation to handle situations with an unknown number of outliers as in Tsay (1986, 1988) and Chang et al. (1988), among others, we shall use robust estimation techniques to modify the specification procedure for building multiplicative GARCH models in the presence of outliers. van Dijk et al. (1999b) show that the LM test for testing linearity in the conditional mean can be severely distorted by additive outliers and that neglected outliers in a linear time series may incorrectly suggest some type of nonlinearity. The same authors derive a modified test statistic in the presence of outliers when the model under the null hypothesis is estimated using a robust estimation technique. The same idea of estimating the model under the null using an outlier robust estimator can also be used to robustify the LM test for testing constancy in the unconditional variance in the presence of outliers.

We begin the model specification problem by first modelling the conditional variance component h_t as in (5) with $p = q = 1$. In this work the short-run component of volatility is replaced by the robust estimator of Muler and Yohai (2008) who proposed a robust estimation of the GARCH by limiting the propagation of the outlier effect on the estimated volatility. These estimators belonging to the class of generalized M-estimators downweight influential observations and thus are less sensitive to outliers than the quasi-maximum likelihood (QML) estimators. These robust estimators are called Bounded-M (BM) estimators. In this setting parameters are estimated maximizing a modified log-likelihood with the following specification for the stochastic component

$$h_t^{BM} = \alpha_0 + \alpha_1 h_{t-1}^{BM} r_c\left(\frac{\phi_{t-1}^2}{h_{t-1}^{BM}}\right) + \beta_1 h_{t-1}^{BM} \tag{18}$$

where $\phi_t = \varepsilon_t / g_t^{1/2}$, and $r_c(k) = k$ if $k \leq c$, and $r_c(k) = c$ if otherwise. Carnero, Peña and Ruiz (2012) show using Monte Carlo experiments that the robust method of Muler and Yohai (2008) outperform maximum likelihood techniques to estimate volatility with a GARCH model in the presence of outliers.

3.2 Specification of the unconditional variance in the presence of outliers

The specification of the unconditional variance component involves two sets of decision problems. First, one has to determine the number of transitions r in (6) and second, K_l

for each transition function in (7) has to be selected; see Amado and Teräsvirta (2017). The choice of the number of transitions has been discussed in Section 2.2. Similarly to the results of van Dijk et al. (1999a) for testing conditional heteroskedasticity in the presence of additive outliers, one expects that the LM-type test discussed in Section 2.2 can also be severely affected by additive outliers. Monte Carlo experiments in Section 4 show that this is indeed the case. The outlier robust estimator discussed above for estimating GARCH models can be used to construct outlier robust versions of the LM-type test statistic presented in Section 2.2 where the model needs to be estimated under the null employing the robust technique in Section 3.1. If the null hypothesis is rejected, one proceeds to estimating the model with multiplicative decomposition of the variance by robust maximization by parts as discussed in Section 3.3 as the tentative specification for the model.

3.3 Outlier robust maximization by parts

In order to consider the maximum likelihood estimation of the model, write

$$h_t = h_t(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2) \quad \text{and} \quad g_t = g_t(\boldsymbol{\theta}_1, t/T)$$

and for notational simplicity let $p = q = 1$. Then the conditional (quasi) log-likelihood function of the model for observation t has the form:

$$\ell_t(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\varepsilon}) = -(1/2)ln2\pi - (1/2)\{\ln h_t(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2) + \ln g_t(\boldsymbol{\theta}_1, t/T)\} - (1/2)\frac{\varepsilon_t^2}{h_t(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2) g_t(\boldsymbol{\theta}_1, t/T)} \tag{19}$$

Since maximization of (19) is numerically very difficult, a solution lies in estimating the unconditional and the conditional variance components separately using maximization by parts; see Song, Fan and Kalbfleisch (2005) and Amado and Teräsvirta (2013) for details. The algorithm proceeds as follows:

Iteration 1: Maximize

$$\sum_{t=1}^{T} \ell_t^U(\boldsymbol{\theta}_1) = -(1/2) \sum_{t=1}^{T} \{\ln g_t(\boldsymbol{\theta}_1, t/T) + \tilde{\varepsilon}_t^2 / g_t(\boldsymbol{\theta}_1, t/T)\}$$

with respect to $\boldsymbol{\theta}_1$ by constraining $\boldsymbol{\theta}_2 = (\alpha_0, 0, 0)'$ and setting $\tilde{\varepsilon}_t = \varepsilon_t / \{h_t(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2)\}^{1/2}$. At this stage, let $h_t(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2) \equiv 1$ and define

$$g_t(\boldsymbol{\theta}_1, t/T) = \delta_0^* + \sum_{l=1}^{r} \delta_l^* G_l(t/T; \gamma_l, \mathbf{c}_l) \tag{20}$$

where $\widehat{\delta}_l^{*(1)} = \widehat{\delta}_l^{(1)}/\widehat{\alpha}_0^{(0)}$, $j = 0, ..., r$, and $\widehat{\alpha}_0^{(0)}$ is the estimate of $\alpha_0 > 0$ obtained in the 0th iteration. Denote the estimator as $\widehat{\boldsymbol{\theta}}_1^{(1)}$.

Conditioning on $\widehat{\boldsymbol{\theta}}_1^{(1)}$, maximize

$$\sum_{t=1}^{T} \ell_t^C(\widehat{\boldsymbol{\theta}}_1^{(1)}, \boldsymbol{\theta}_2) = -(1/2) \sum_{t=1}^{T} \{\ln h_t(\widehat{\boldsymbol{\theta}}_1^{(1)}, \boldsymbol{\theta}_2) + \varepsilon_t^{*2}/h_t(\widehat{\boldsymbol{\theta}}_1^{(1)}, \boldsymbol{\theta}_2)\}$$

with respect to $\boldsymbol{\theta}_2$, where $\varepsilon_t^* = \varepsilon_t/\{g_t(\widehat{\boldsymbol{\theta}}_1^{(1)}, t/T)\}^{1/2}$. Call the resulting estimator as $\widehat{\boldsymbol{\theta}}_2^{(1)}$.

Iteration 2: Maximize

$$\sum_{t=1}^{T} \ell_t^U(\boldsymbol{\theta}_1) = -(1/2) \sum_{t=1}^{T} \{\ln g_t(\boldsymbol{\theta}_1, t/T) + \tilde{\varepsilon}_t^2/g_t(\boldsymbol{\theta}_1, t/T)\}$$

with respect to $\boldsymbol{\theta}_1$, where $\tilde{\varepsilon}_t = \varepsilon_t/\{h_t(\widehat{\boldsymbol{\theta}}_1^{(1)}, \widehat{\boldsymbol{\theta}}_2^{(1)})\}^{1/2}$. This yields the estimator $\widehat{\boldsymbol{\theta}}_1^{(2)}$. Next, making use of $\widehat{\boldsymbol{\theta}}_1^{(2)}$, maximize

$$\sum_{t=1}^{T} \ell_t^C(\widehat{\boldsymbol{\theta}}_1^{(2)}, \boldsymbol{\theta}_2) = -(1/2) \sum_{t=1}^{T} \{\ln h_t(\widehat{\boldsymbol{\theta}}_1^{(2)}, \boldsymbol{\theta}_2) + \varepsilon_t^{*2}/h_t(\widehat{\boldsymbol{\theta}}_1^{(2)}, \boldsymbol{\theta}_2)\}$$

with respect to $\boldsymbol{\theta}_2$, where $\varepsilon_t^* = \varepsilon_t/\{g_t(\widehat{\boldsymbol{\theta}}_1^{(2)}, t/T)\}^{1/2}$. Call the resulting estimator as $\widehat{\boldsymbol{\theta}}_1^{(2)}$.

Iterate until convergence.

Iteration n: More generally, maximization by parts is carried out by solving the score equations:

$$(1/2) \sum_{t=1}^{T} \left(\frac{\tilde{\varepsilon}_t^2}{g_t(\widehat{\boldsymbol{\theta}}_1^{(n)}, t/T)} - 1 \right) \frac{1}{g_t(\widehat{\boldsymbol{\theta}}_1^{(n)}, t/T)} \frac{\partial g_t(\widehat{\boldsymbol{\theta}}_1^{(n)}, t/T)}{\partial \boldsymbol{\theta}_1} = 0$$

for $\boldsymbol{\theta}_1$ assuming $\tilde{\varepsilon}_t = \varepsilon_t/\{h_t(\widehat{\boldsymbol{\theta}}_1^{(n-1)}, \widehat{\boldsymbol{\theta}}_2^{(n-1)})\}^{1/2}$, and

$$(1/2) \sum_{t=1}^{T} \left(\frac{\varepsilon_t^{*2}}{h_t(\boldsymbol{\theta}_1, \widehat{\boldsymbol{\theta}}_2^{(n)})} - 1 \right) \frac{1}{h_t(\boldsymbol{\theta}_1, \widehat{\boldsymbol{\theta}}_2^{(n)})} \frac{\partial h_t(\boldsymbol{\theta}_1, \widehat{\boldsymbol{\theta}}_2^{(n)})}{\partial \boldsymbol{\theta}_1} = 0$$

for $\boldsymbol{\theta}_2$, where $\varepsilon_t^* = \varepsilon_t/\{g_t(\widehat{\boldsymbol{\theta}}_1^{(n)}, t/T)\}^{1/2}$. The resulting estimators are denoted as $\widehat{\boldsymbol{\theta}}_1^{(n)}$ and $\widehat{\boldsymbol{\theta}}_2^{(n)}$. For further details see Amado and Teräsvirta (2013).

An important feature of the modelling strategy is that parameters of the short-run component are estimated robustly using maximum likelihood in the presence of outliers. In this work we modify the estimation of the multiplicative GARCH model by maximization by parts where equation (5) with $p = q = 1$ is replaced by (18). This mechanism bounds the propagation of the effect of outliers on the estimated conditional variance and thus becoming robust to the presence of outliers. Provided that some regularity conditions hold, the parameter vector $\boldsymbol{\theta}_2$ can be consistently estimated, and estimators of $\boldsymbol{\theta}_2$ are asymptotically normal; for more details see Muler and Yohai (2008) who prove consistency and asymptotic normality for BM estimators. Using the asymptotic results from Amado and Teräsvirta (2013) and Muler and Yohai (2008) one can conclude that parameters of the model (1)-(7) estimated by outlier robust maximization by parts are also consistent and asymptotic normal.

4 Monte Carlo experiments

4.1 Simulation design

In this section, we conduct Monte Carlo simulations to investigate the finite sample properties of the test for testing constancy in the unconditional variance in the presence of additive outliers. In the experiments, 2000 artificial series are generated each of lengths of $T = 1000$ and $T = 3000$ observations. To avoid the dependence of the results on starting values, the first 1000 observations of each series have been discarded. Following the suggestion of Bollerslev (1986), recursive computation of h_t is initialized using the estimated constant unconditional variance for the pre-sample values $t \leq 0$. Contaminated series x_t are obtained by adding additive outliers to y_t according to (15). Following Tolvi (2000), we consider contamination with outliers of magnitude $\omega_{AO} = 3, 5, 7, 10$. Outliers of large magnitudes are very likely to appear in financial returns whereas smaller magnitudes are likely to appear in any type of data. Since outliers are view as a rare events, in the experiments we consider the cases of a single outlier and two consecutive outliers. Here we opted by assuming a more controlled experiment instead of generating artificially outliers with the occurrence of a certain probability. The standard robust test of Wooldridge and the outlier robust parameter constancy test are applied to both the clean and contaminated series to obtain estimates of their size and power. Results of the standard test are not presented as they perform rather poorly compared to the robust tests, but they are available from the author upon request. All tests are evaluated at the 1%, 5% and 10% and the asymptotic χ^2 critical values are used.

In our experiments, since the focus lies on modelling the dynamics of volatility of weakly autocorrelated returns, we consider the effects of a small AR parameter in the specification of the conditional mean. In practice, the order of the linear AR model needs to be decided. In our simulation study, we fix this order at $p = 1$. The behaviour of size of the test statistic is examined for three data generating processes (DGP's) that can be nested in the following specification:

$$\begin{aligned} y_t &= \phi_0 + \phi_1 y_{t-1} + \varepsilon_t, \quad t = 1, ..., T \\ \varepsilon_t &= \zeta_t \sigma_t, \quad \zeta_t \sim iid(0,1) \\ \sigma_t^2 &= (\alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2)(\delta_0 + \delta_1 (1 + \exp\{-\gamma(t/T - c)\})^{-1}) \end{aligned} \tag{21}$$

where $\phi_0 = 0.05$, $\phi_1 = 0.10$, $\gamma = \{5, 10\}$, $\delta_0 = 1$, $\delta_1 = \{-0.5, 0, 1.5\}$, and $c = 0.5$. The data generating processes for the short-run component are as following:

DGP1: $\sigma_t^2 = 0.05 + 0.15\varepsilon_{t-1}^2 + 0.75\sigma_{t-1}^2$

DGP2: $\sigma_t^2 = 0.05 + 0.10\varepsilon_{t-1}^2 + 0.85\sigma_{t-1}^2$

DGP3: $\sigma_t^2 = 0.05 + 0.05\varepsilon_{t-1}^2 + 0.90\sigma_{t-1}^2$

The effects of isolated outliers on the size of the tests are analysed by generating the process y_t as the model (21) with $\delta_1 = 0$ with conditionally heteroskedastic GARCH(1,1) errors given by DGP1-DGP3. To consider the power of the tests, the GARCH(1,1) model is replaced by model (21) with $\delta_1 = \{-0.5, 1.5\}$ with a gradual transition between the two regimes at the threshold value $c_1 = 0.5$. The power properties of the tests are examined for three different specifications with the multiplicative decomposition of the variance. The specifications differ in their degree of volatility persistence which varies between moderate to high persistence. In practice, this is done by letting the short-run component of the variance changing across the artificial series generated from DGP1-DGP3.

4.2 Discussion of results

In this section we shall look at the small-sample properties for the two variants of the LM test statistic and the empirical densities of the parameter estimators. First we examine the rejection frequencies of the parameter constancy tests. Then we turn to the empirical distributions of the estimators.

4.2.1 Size and power simulations

The effects of additive outliers on the size simulations are studied for series generated from a GARCH(1,1) whose parameter values are defined in DGP1-DGP3. Rejection frequencies of the null hypothesis for the robust variant of the LM test at 1%, 5% and 10% nominal significance levels for normally distributed ζ_t's are reported in Table 1.

The series are generated according to a weakly autocorrelated process with conditional heteroskedasticity and constant unconditional variance with parameters $\delta_0 = 1$ and $\delta_1 = 0$ in (21). It is assumed that the true autoregressive AR(1) in (21) is known. In our simulations we conduct the experiments with parameter values of the conditional mean set equal to $(\phi_0, \phi_1) = (0.05, 0.10)$ which are typically found for financial series and focus on modelling volatility clustering. Since isolated outliers are known to bias the estimation of the autoregressive coefficient, the conditionally heteroskedastic residuals of the model are also expected to be affected by their presence. The column headed $\omega_{AO} = 0$ shows the empirical size when the test is applied to the series without outliers. The size of the test in the case of no outliers is quite reasonable, but the test becomes somewhat size-distorted for artificial data with higher volatility persistence for smaller samples. The test applied to the "clean" series becomes reasonably well-sized for larger samples in all data generating processes which corroborates the findings of Amado and Teräsvirta (2008,2017). The effects of isolated outliers on the size are investigated by adding outliers of magnitude $\omega_{AO} = 3, 5, 7, 10$ to the model at $s = T/2$. In the presence of outliers, the actual size of the test at conventional significance levels is always above the nominal size for all DGPs, albeit to a less extent for $T = 3000$. It is seen that the size distortion is an increasing function of the magnitude of the outlier. Outliers of larger magnitude ($\omega_{AO} = 7, 10$) lead to more frequent rejection of the null, but the test results become more accurate for larger samples. Moreover, the behaviour of the test tends to overreject the null hypothesis more in the presence of two consecutive outliers than for a single outlier. Overrejection of the null is even more pronounced in the presence of consecutive outliers of larger magnitude. These results signal that the presence of very large outliers tend to dominate the pattern of the data. As an attempt to correct the size distortions we proceed with computation of the test based on the BM-estimator discussed in 3.1. Table 2 shows the results of the rejection frequencies based on the BM-estimator for the contaminated series generated by model (21) with $\delta_0 = 1$ and $\delta_1 = 0$. The distortions in the level of the BM-based test are weaker for outliers of small magnitude. For outliers of large magnitude, it is seen that the size of the BM-based test becomes much smaller and very close to the nominal at the three significance levels even for shorter samples. For longer series, the size of the test based on the BM estimator becomes even closer to the nominal size. These findings are uniform across all the DGPs. Our conclusion is that the outlier robust version of the test statistic is a good approximation to the finite-sample distributions for all samples.

The effects on the power of the LM test are examined by generating series according to the multiplicative time-varying GARCH model (21) for various combinations of δ_1 and γ. Table 3 shows the rejection frequencies for these experiments when ζ_t is normally distributed for the 5% nominal significance level. We allow the change to occur in the middle of the sample. Silvennoinen and Teräsvirta (2016) conclude that shifts occurring early are easier to detect than if similar shifts occur late in the sample. The location of the shift is chosen halfway through the sample, but other simulations are available upon request for shifts located elsewhere in the sample. The column $\omega_{AO} = 0$ shows estimates of the power of the LM-type test statistic applied to the uncontaminated series. The test statistic turns out to be very powerful for short time series when there is moderate persistence in volatility. For large sample sizes, the selection frequencies of the true model become quite high even for smooth changes. It is seen that the correct model is selected more often for quicker changes in the unconditional variance than for slow changes when δ_1 is positive for uncontaminated series. Conversely, for a negative δ_1, the test is considerably more powerful when the transition is fairly smooth with $\gamma = 5$ compared to the case of a higher change with $\gamma = 10$. We further observe for $\delta_1 < 0$ that the estimated power is an increasing function of the magnitude and number of outliers. It thus becomes easier to identify a single transition when two consecutive outliers are present in the data compared to a single outlier for $\delta_1 < 0$. Furthermore, when $\delta_1 > 0$ for DGP3 the test statistic has rather low if any power at small samples when the smoothness parameter is small and approaches its size for larger values of ω_{AO}. However, the test has a considerable increase in power when the smoothness parameter shifts from 5 to 10. If δ_1 is positive, the power of the test in the presence of two consecutive outliers is approximately equal to that of a single outlier. Table 4 shows the rejection frequencies for the robust outlier test statistic when applied to series generated by model (21) for different combinations of δ_1 and γ. Monte Carlo simulations suggest that the empirical performance of the estimated power of the test based on the robust outlier estimation method is quite satisfactory. The BM-estimator based test has reasonable power for smaller samples, but it has a loss of power compared to the robust misspecification test when δ_1 is negative. The results show that the test is able to distinguish quite easily between stationary and nonstationary conditional heteroskedastic processes. For $\delta_1 > 0$ the power loss of the test applied to data with high persistence in volatility is corrected already for shorter samples.

4.2.2 Effects of outliers on the estimation of multiplicative GARCH models

In this section we compare the robustness properties of the quasi-maximum likelihood estimator and the robust bounded-M estimator for estimating the time-varying unconditional volatility model from a series contaminated by outliers. Since estimation by full maximum likelihood is computationally demanding we apply the iterative algorithm maximization by parts for estimating the multiplicative GARCH model using the non-robust and robust procedures. Simulation evidence is presented by means of

Monte Carlo experiments to analyse the biases caused by isolated and consecutive additive outliers on the QML and BM estimators of the parameters of the multiplicative GARCH model. The simulations are carried out by generating 2000 artificial series of sizes $T = 1000$ and 3000 from a multiplicative GARCH representation (21) with the short-run component generated from DGP1-DGP3 while the long-run component has been generated with $\delta_1 = 1$, $\gamma_1 = 5$ and $c_1 = 0.5$. The series have been contaminated at $s = T/2$ first by an isolated outlier and second by two consecutive outliers of size $\omega_{AO} = 10$. Table 5 reports the Monte Carlo medians and standard deviations for the QML and BM estimates when $T = 1000$. We can observe that the QML estimators with the exception of $\widehat{\delta}_1$ have small bias when there are no outliers. The same conclusions can be obtained from Figure 1 which plots the corresponding kernel densities of the QML estimators of the parameters of the short-run component and volatility persistence in the case of no outliers. For the sake of saving space only the estimated densities for the parameters of the stochastic component h_t are plotted. The figure shows that the bias for samples of smaller size becomes almost negligible for larger samples. On the other hand, QML estimators are not robust to the presence of outliers. This is in line with the simulations by Sakata and White (1998) and Carnero, Peña and Ruiz (2007) who found that QML estimators of the parameters of GARCH models based on non-leptokurtic distributions are not robust to outliers. We further observe that the outlier robust maximization by parts has either the same or very competitive efficiency as the QML maximization by parts for finite samples. We note that the autoregressive parameter ϕ_1 is severely overestimated in the presence of isolated outliers in small samples, but the bias is reduced for larger samples as reported in Table 6. Similar observation can be drawn for the QML and BM estimators of α_0 whose sample distributions have large positive bias when the data has been contaminated by two consecutive outliers. It is also seen that the bias for the BM estimator of α_0 is lower than that of the QML estimator in the presence of a single outlier for DGP1-DGP2. As expected, the accuracy in the parameter estimation improves with the sample size. In the presence of isolated outliers, the sample distribution of QML and BM estimators of β_1 shows large negative bias for DGP1 and DGP3 when $T = 1000$, but the bias is noticeably reduced when the sample increases. It is such that standard inference becomes unreliable in small samples for α_0 and β_1 for series contaminated with large outliers. A visual inspection of Figure 2 with the kernel estimates of the densities of the QML and BM estimators of α_0, α_1 and β_1 validate these statements. We observe that, in the presence of isolated outliers, both estimators have similar sample distributions with positive biases in smaller samples and that the dispersion reduces for larger samples. With respect to the parameters of the deterministic component, the QML and BM estimators of the smoothness parameter are overestimated for DGP1 and DGP2, but the bias reduces with the sample size. On the other hand, the estimator of the location parameter c_1 is unbiased for all studied cases. It is also seen that the negative bias of both QML and BM estimators of δ_1 persists even for large samples and its accuracy in estimation does not improve with the sample size. Next, we compare the robustness of QML and BM estimators in the presence of two consecutive outliers. We observe from Figure 3 that the effects caused by two consecutive outliers on the QML and BM estimators resemble those caused by the presence of a single outlier, but the bias is further aggravated for the estimators of ϕ_1 and parameters of the stochastic component h_t. The QML and BM estimators of α_1 are characterized by large positive biases for series contaminated with two consecutive outliers in smaller samples, but the bias for QML estimator is slightly weaker than that of the BM estimator. Monte Carlo densities of the QML and BM estimators plotted in Figure 3 corroborate these findings.

5 Empirical illustration

In this section the above data-based modelling technique of the multiplicative decomposition of the return variance is illustrated in practice by examining daily commodity market futures prices of corn and sugar. Corn futures are traded in The Chicago Board of Trade (CBOT) and sugar futures are available on the Intercontinental Exchange (ICE). The sample covers the period from 08 January 1997 until 14 March 2022 which amounts to 6272 observations. The daily prices data have been transformed into percentage logarithmic returns and the series are graphed in Figure 4. Because of the long observation period it is unlikely that the series are stationary. Figure 4(a) displays the corn returns and one can distinguish two different regimes in volatility for the series apart from very large (absolute values) returns. It contains a period of higher volatility following the 2008 financial crisis lasting until 2015 and thereafter descends to a lower level of volatility. Sugar returns plotted in Figure 4(b) show a fairly high amplitude of the clusters in volatility in the beginning and middle of the sample to decrease around 2014 to a smaller level of volatility. The exposition of the modelling cycle follows Amado and Teräsvirta (2017). We begin the model specification problem by first modelling the short-run volatility component h_t as in (5) or (18) with $g_t = 1$. Thereafter, the specification of the deterministic function g_t is determined by sequential testing. Parameter constancy of the unconditional variance is tested using the robust and the outlier robust versions of the Lagrange multiplier tests. For comparison purposes we also provide the results of the standard LM test. The initial significance level of the sequence of tests is $\alpha^{(1)} = 0.05$. At each stage of the sequence we halve the significance level of the test, i.e. $\tau = 0.5$. The test results appear in Table 7. For the sugar returns, constancy of the deterministic component is strongly rejected using the robust version of the test, but the outlier robust version of the test fails to reject the null hypothesis of constancy. This leads to the selection of a GARCH model whose parameter estimates are reported in Table 8. Evidence in favour of the smoothly time-varying unconditional variance is found for corn returns as the null hypothesis of constant unconditional variance is rejected for the robust and outlier robust versions of the tests. The shape of the transition is determined as described in Section 2.2. It is seen that for both versions of the tests the strongest rejection occurs for $K = 2$ with a p-value equal to 0.010 for the outlier robust test and therefore $K = 2$ is selected as the exponent of the transition for the corn returns. Fitting the model with one transition and testing for another transition yields a p-value equal to 0.579 which is remarkably higher than the significance level $\alpha^{(2)} = 0.025$. Thus, the LM test does not provide evidence of yet another transition, so that the sequential testing leads to the specification of a multiplicative time-varying GARCH model with a single transition as the final parameterization for corn returns.

The estimates along with their standard errors for the short-run component of volatility can be found in Table 8. For comparison we also provide the estimates of the GARCH(1,1) model. The estimation of the standard multiplicative time-varying GARCH (MTV-GARCH) model and its robust BM counterpart (BM-MTV-GARCH) has been carried out by maximization by parts to avoid convergence problems. For estimating the model with time-varying unconditional variance we use the parameter estimates of the time-varying variance model with $h_t = 1$ as starting-values. A general finding is that in-sample fit of the robust and standard multiplicative GARCH(1,1) models are superior to the fit obtained by the GARCH(1,1) model. The results suggest that models with deterministic nonstationary component outperforms the constant unconditional variance model with the standard MTV-GARCH providing the best in-sample fit. It is seen that the persistence measured by $\widehat{\alpha}_1 + \widehat{\beta}_1$ reduces considerably when the model accounts for slow movements in volatility by introducing the deterministic component g_t into the model. The deterministic component may not have removed all the long run dependence since the level of persistence is still high after the long-run movements in the series have been taken into account. The final estimated deterministic components from the standard MTV-GARCH model equals (standard errors in parentheses)

$$g_t = \underset{(-)}{9.517} - \underset{(0.0439)}{7.6481} G(t/T; \widehat{\gamma}_1, \widehat{c}_1) \tag{22}$$

where

$$G(t/T; \widehat{\gamma}_1, \widehat{c}_1) = \left(1 + \exp\left\{ -\underset{(2.7660)}{31.396}(t/T - \underset{(0.0723)}{0.4963})(t/T - \underset{(0.0723)}{0.4963}) \right\}\right)^{-1} \tag{23}$$

and the estimated time-varying component from the robust MTV-GARCH model has the form:

$$g_t = \underset{(-)}{9.517} - \underset{(0.0688)}{5.8101} G(t/T; \widehat{\gamma}_1, \widehat{c}_1) \tag{24}$$

where

$$G(t/T; \widehat{\gamma}_1, \widehat{c}_1) = \left(1 + \exp\left\{ -\exp(\underset{(7.1348)}{17.079})(t/T - \underset{(0.0012)}{0.4026})(t/T - \underset{(0.0003)}{0.5958}) \right\}\right)^{-1} \tag{25}$$

The shape of the fitted deterministic time-varying components from the MTV-GARCH model appear in Figure 5. Note that since the intercept δ_0 is kept fixed to avoid identification problems it does not have a standard error. We observe rather smooth slow movements in volatility for the standard multiplicative GARCH whereas for its robust counterpart the transition function is very much close to a step function. The estimates of the location parameters lie within the range 0.40 - 0.60. This range contains the turbulent period of the global financial crisis indicating that the changing unconditional variance is associated with the largest economic recession during the observation period. This is line with previous studies; see for example Amado and Teräsvirta (2014).

Figure 6 shows the estimated conditional standard deviation obtained from the GARCH model. The series looks clearly nonstationary. There is a systematic increased in the baseline volatility from 2005 until 2015 and also at the beginning and end of the sample period. On the other hand, conditional standard deviations from the MTV-GARCH models do not show any signs of nonstationarity. The figure shows that the deterministic time-varying component removes long-run movements in conditional standard deviation between the years 2005 and 2015 and both ends. It is seen that the spikes in the conditional standard deviations from the robust multiplicative GARCH model are of smaller magnitude than those of its standard counterpart. This effect may be explained to the bounding mechanism of propagation of the outlier effects used in the estimation of the short run dynamics of volatility in the robust estimator of the BM-MTV-GARCH model.

6 Conclusions

In this paper, we propose an outlier robust LM-type misspecification test for testing smooth changes in the unconditional variance. Monte Carlo evidence suggests that the robust variant of the misspecification test suffers from size distortions and loss of power in the presence of outliers. It is further seen that the poor performance of the robust misspecification test depends on the number and magnitude of outliers.

On the other hand, the outlier robust version of the test statistic constructed from an outlier-robust estimation technique performs quite satisfactorily in finite samples in the presence of additive outliers. A modified data-driven strategy for building the parametric deterministic time-varying component of the multiplicative GARCH model based on the outlier-robust testing procedure can be designed and carried out which is robust to the presence of additive outliers. An application to a couple of commodity returns demonstrate the usefulness of the robust specification procedure. It is seen that one should carefully interpret the evidence of nonstationarity from conventional robust tests because the presence of a few outlying observations may cause spurious nonstationarity. Exploring the forecasting properties of the out-of-sample volatility for the multiplicative time-varying GARCH model in the presence of outliers warrants further investigation.

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A Tables

Table 1: Rejection frequencies for the robust LM misspecification test for constancy of the unconditional variance based on 2000 replications

				$1\omega_{AO}$				$2\omega_{AO}$			
T	DGP	α	$\omega_{AO}=0$	3	5	7	10	$3,3$	$5,5$	$7,7$	$10,10$
1000	DGP1	0.01	0.012	0.015	0.017	0.022	0.038	0.018	0.021	0.024	0.028
		0.05	0.063	0.070	0.077	0.094	0.119	0.083	0.091	0.101	0.105
		0.10	0.118	0.134	0.147	0.178	0.192	0.146	0.167	0.185	0.188
	DGP2	0.01	0.017	0.021	0.023	0.034	0.058	0.023	0.032	0.045	0.067
		0.05	0.093	0.100	0.106	0.118	0.165	0.106	0.124	0.153	0.176
		0.10	0.160	0.172	0.183	0.205	0.255	0.184	0.217	0.246	0.279
	DGP3	0.01	0.023	0.025	0.031	0.047	0.071	0.027	0.043	0.056	0.073
		0.05	0.096	0.105	0.111	0.139	0.179	0.117	0.138	0.176	0.194
		0.10	0.170	0.175	0.195	0.230	0.285	0.197	0.237	0.276	0.295
3000	DGP1	0.01	0.009	0.011	0.010	0.012	0.015	0.012	0.013	0.016	0.018
		0.05	0.059	0.061	0.056	0.050	0.055	0.064	0.065	0.058	0.063
		0.10	0.107	0.114	0.108	0.101	0.113	0.115	0.123	0.116	0.126
	DGP2	0.01	0.014	0.014	0.016	0.015	0.017	0.014	0.016	0.017	0.024
		0.05	0.060	0.067	0.071	0.065	0.068	0.067	0.077	0.080	0.082
		0.10	0.118	0.122	0.127	0.122	0.128	0.126	0.136	0.145	0.157
	DGP3	0.01	0.014	0.016	0.016	0.019	0.021	0.017	0.021	0.028	0.032
		0.05	0.061	0.063	0.068	0.077	0.084	0.067	0.090	0.104	0.122
		0.10	0.125	0.123	0.131	0.136	0.156	0.128	0.158	0.175	0.208

Table 2: Rejection frequencies for the outlier robust LM misspecification test for constancy of the unconditional variance based on 2000 replications

			$1\omega_{AO}$				$2\omega_{AO}$			
T	DGP	α	3	5	7	10	3, 3	5, 5	7, 7	10, 10
1000	DGP1	0.01	0.014	0.010	0.011	0.024	0.013	0.009	0.011	0.015
		0.05	0.074	0.065	0.063	0.092	0.068	0.056	0.055	0.060
		0.10	0.136	0.126	0.127	0.152	0.137	0.112	0.114	0.132
	DGP2	0.01	0.016	0.015	0.014	0.016	0.020	0.015	0.012	0.012
		0.05	0.086	0.081	0.072	0.067	0.094	0.080	0.068	0.063
		0.10	0.162	0.153	0.134	0.122	0.169	0.151	0.139	0.129
	DGP3	0.01	0.012	0.011	0.012	0.014	0.019	0.019	0.016	0.012
		0.05	0.079	0.076	0.065	0.061	0.091	0.081	0.059	0.064
		0.10	0.150	0.141	0.122	0.109	0.158	0.157	0.130	0.125
3000	DGP1	0.01	0.015	0.014	0.014	0.017	0.015	0.013	0.015	0.017
		0.05	0.079	0.064	0.056	0.072	0.074	0.061	0.058	0.070
		0.10	0.144	0.126	0.116	0.134	0.137	0.119	0.112	0.126
	DGP2	0.01	0.019	0.016	0.016	0.016	0.019	0.016	0.015	0.013
		0.05	0.076	0.075	0.063	0.056	0.078	0.071	0.063	0.063
		0.10	0.141	0.133	0.126	0.117	0.140	0.136	0.123	0.119
	DGP3	0.01	0.015	0.015	0.012	0.010	0.016	0.016	0.014	0.011
		0.05	0.064	0.066	0.058	0.047	0.070	0.079	0.068	0.057
		0.10	0.130	0.127	0.117	0.099	0.134	0.139	0.134	0.115

Table 3: Estimated power for the robust LM misspecification test for constancy of the unconditional variance based on 2000 replications

				$1\omega_{AO}$			$2\omega_{AO}$		
T	DGP	γ	$\omega_{AO}=0$	3	5	7	$3,3$	$5,5$	$7,7$
$\delta_1=-0.5$									
1000	DGP1	5	0.787	0.843	0.901	0.930	0.937	0.958	0.949
		10	0.775	0.782	0.856	0.899	0.935	0.964	0.955
	DGP2	5	0.458	0.547	0.695	0.821	0.701	0.875	0.920
		10	0.373	0.410	0.552	0.709	0.598	0.817	0.884
	DGP3	5	0.242	0.312	0.522	0.765	0.510	0.841	0.928
		10	0.130	0.155	0.311	0.577	0.320	0.698	0.886
3000	DGP1	5	0.999	0.996	0.901	0.930	0.937	0.958	0.949
		10	0.993	0.989	0.975	0.976	0.998	0.999	1.000
	DGP2	5	0.992	0.988	0.986	0.975	0.996	0.998	0.998
		10	0.952	0.953	0.934	0.923	0.972	0.989	0.993
	DGP3	5	0.677	0.689	0.713	0.765	0.510	0.841	0.928
		10	0.384	0.376	0.409	0.511	0.500	0.752	0.911
$\delta_1=1.5$									
1000	DGP1	5	0.687	0.688	0.693	0.697	0.688	0.695	0.698
		10	0.927	0.926	0.927	0.925	0.927	0.925	0.924
	DGP2	5	0.960	0.960	0.960	0.957	0.960	0.958	0.958
		10	0.999	0.999	0.999	0.999	0.999	0.999	0.999
	DGP3	5	0.045	0.046	0.048	0.054	0.047	0.049	0.060
		10	0.239	0.237	0.241	0.249	0.238	0.240	0.246
3000	DGP1	5	0.989	0.990	0.989	0.989	0.990	0.990	0.989
		10	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	DGP2	5	0.999	1.000	1.000	1.000	1.000	1.000	1.000
		10	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	DGP3	5	0.484	0.485	0.500	0.491	0.499	0.497	0.497
		10	0.891	0.890	0.882	0.883	0.885	0.887	0.889

Table 4: Estimated power for the outlier robust LM misspecification test for constancy of the unconditional variance based on 2000 replications

				$1\omega_{AO}$			$2\omega_{AO}$		
T	DGP	γ	$\omega_{AO}=0$	3	5	7	$3,3$	$5,5$	$7,7$
$\delta_1=-0.5$									
1000	DGP1	5	0.778	0.789	0.789	0.859	0.881	0.917	0.953
		10	0.752	0.703	0.694	0.820	0.848	0.918	0.962
	DGP2	5	0.436	0.488	0.555	0.627	0.614	0.741	0.836
		10	0.344	0.353	0.385	0.481	0.489	0.630	0.813
	DGP3	5	0.219	0.254	0.340	0.480	0.398	0.621	0.748
		10	0.118	0.123	0.153	0.278	0.207	0.424	0.676
3000	DGP1	5	0.998	0.997	0.988	0.968	0.998	0.998	0.998
		10	0.995	0.986	0.951	0.937	0.993	0.987	0.998
	DGP2	5	0.990	0.987	0.975	0.939	0.991	0.986	0.974
		10	0.952	0.942	0.904	0.853	0.958	0.944	0.951
	DGP3	5	0.627	0.620	0.613	0.607	0.680	0.756	0.841
		10	0.361	0.345	0.327	0.326	0.419	0.518	0.643
$\delta_1=1.5$									
1000	DGP1	5	0.998	0.998	0.997	0.997	0.998	0.997	0.997
		10	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	DGP2	5	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		10	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	DGP3	5	0.729	0.731	0.733	0.738	0.733	0.738	0.746
		10	0.952	0.951	0.953	0.956	0.951	0.955	0.959
3000	DGP1	5	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		10	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	DGP2	5	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		10	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	DGP3	5	0.999	0.999	0.999	0.999	0.999	0.999	0.999
		10	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 5: Monte Carlo medians and standard deviations of the multiplicative GARCH estimates without outliers, an isolated outlier and two consecutive outliers of size $\omega_{AO} = 10$ for $T = 1000$ based on 2000 replications.

			$1\omega_{AO}$		$2\omega_{AO}$	
		$\omega_{AO} = 0$	QML	BM	QML	BM
DGP1	ϕ_0	0.051 (0.028)	0.061 (0.028)	0.061 (0.028)	0.063 (0.025)	0.063 (0.025)
	ϕ_1	0.097 (0.041)	0.096 (0.039)	0.096 (0.039)	0.202 (0.037)	0.202 (0.037)
	α_0	0.068 (0.040)	0.093 (0.104)	0.059 (0.088)	0.119 (0.114)	0.084 (0.116)
	α_1	0.138 (0.040)	0.138 (0.087)	0.128 (0.082)	0.182 (0.081)	0.229 (0.091)
	β_1	0.755 (0.080)	0.718 (0.181)	0.778 (0.162)	0.640 (0.198)	0.661 (0.213)
	δ_1	0.397 (0.808)	0.455 (0.465)	0.458 (0.312)	0.526 (0.203)	0.558 (0.234)
	γ_1	9.678 (4.744)	9.775 (4.169)	9.779 (3.947)	9.746 (2.118)	9.748 (1.579)
	c_1	0.576 (0.172)	0.496 (0.030)	0.496 (0.029)	0.497 (0.023)	0.498 (0.019)
DGP2	ϕ_0	0.051 (0.039)	0.061 (0.039)	0.061 (0.039)	0.067 (0.037)	0.067 (0.037)
	ϕ_1	0.097 (0.039)	0.096 (0.038)	0.096 (0.038)	0.156 (0.037)	0.156 (0.037)
	α_0	0.069 (0.067)	0.081 (0.129)	0.061 (0.094)	0.112 (0.173)	0.079 (0.161)
	α_1	0.091 (0.035)	0.088 (0.050)	0.088 (0.048)	0.117 (0.053)	0.132 (0.062)
	β_1	0.850 (0.071)	0.842 (0.127)	0.861 (0.099)	0.791 (0.165)	0.815 (0.158)
	δ_1	0.417 (0.792)	0.382 (0.576)	0.381 (0.640)	0.424 (0.563)	0.438 (0.608)
	γ_1	5.473 (3.500)	7.156 (2.494)	7.044 (2.546)	7.732 (2.451)	7.869 (3.221)
	c_1	0.547 (0.195)	0.493 (0.062)	0.493 (0.058)	0.495 (0.047)	0.495 (0.044)
DGP3	ϕ_0	0.052 (0.039)	0.061 (0.039)	0.061 (0.039)	0.067 (0.037)	0.067 (0.037)
	ϕ_1	0.097 (0.035)	0.097 (0.035)	0.097 (0.034)	0.155 (0.034)	0.155 (0.034)
	α_0	0.066 (0.122)	0.089 (0.245)	0.092 (0.334)	0.193 (0.358)	0.216 (0.346)
	α_1	0.049 (0.025)	0.041 (0.042)	0.056 (0.040)	0.086 (0.044)	0.120 (0.050)
	β_1	0.894 (0.111)	0.879 (0.206)	0.864 (0.257)	0.758 (0.299)	0.719 (0.296)
	δ_1	0.356 (0.647)	0.387 (0.798)	0.365 (0.158)	0.508 (0.621)	0.430 (0.180)
	γ_1	4.999 (2.586)	5.018 (13.72)	5.018 (0.710)	5.049 (2.032)	5.053 (0.977)
	c_1	0.513 (0.159)	0.482 (0.038)	0.482 (0.029)	0.482 (0.029)	0.487 (0.027)

Table 6: Monte Carlo medians and standard deviations of the multiplicative GARCH estimates without outliers, an isolated outlier and two consecutive outliers of size $\omega_{AO} = 10$ for $T = 3000$ based on 2000 replications.

			$1\omega_{AO}$		$2\omega_{AO}$	
		$\omega_{AO} = 0$	QML	BM	QML	BM
DGP1	ϕ_0	0.051 (0.016)	0.054 (0.016)	0.054 (0.016)	0.055 (0.015)	0.055 (0.015)
	ϕ_1	0.099 (0.024)	0.099 (0.023)	0.099 (0.023)	0.139 (0.023)	0.139 (0.023)
	α_0	0.064 (0.017)	0.072 (0.045)	0.062 (0.029)	0.081 (0.034)	0.073 (0.036)
	α_1	0.142 (0.028)	0.144 (0.039)	0.142 (0.039)	0.155 (0.039)	0.178 (0.042)
	β_1	0.761 (0.045)	0.750 (0.079)	0.770 (0.067)	0.726 (0.073)	0.722 (0.081)
	δ_1	0.330 (0.431)	0.323 (0.139)	0.323 (0.142)	0.349 (0.124)	0.367 (0.154)
	γ_1	7.628 (4.497)	7.307 (2.121)	7.300 (2.145)	7.530 (2.229)	7.744 (2.198)
	c_1	0.565 (0.163)	0.492 (0.042)	0.493 (0.040)	0.494 (0.025)	0.495 (0.025)
DGP2	ϕ_0	0.051 (0.023)	0.054 (0.023)	0.054 (0.023)	0.056 (0.022)	0.056 (0.022)
	ϕ_1	0.099 (0.022)	0.099 (0.022)	0.099 (0.022)	0.121 (0.022)	0.121 (0.022)
	α_0	0.063 (0.049)	0.066 (0.054)	0.058 (0.035)	0.075 (0.066)	0.064 (0.054)
	α_1	0.087 (0.030)	0.085 (0.032)	0.088 (0.033)	0.096 (0.034)	0.101 (0.039)
	β_1	0.860 (0.054)	0.860 (0.060)	0.866 (0.046)	0.843 (0.071)	0.850 (0.065)
	δ_1	0.358 (0.626)	0.303 (0.685)	0.306 (0.643)	0.334 (0.604)	0.347 (0.630)
	γ_1	7.688 (8.018)	7.052 (5.894)	7.022 (2.942)	5.290 (11.29)	5.311 (25.05)
	c_1	0.548 (0.190)	0.491 (0.080)	0.491 (0.080)	0.491 (0.061)	0.491 (0.062)
DGP3	ϕ_0	0.051 (0.023)	0.054 (0.023)	0.054 (0.023)	0.056 (0.022)	0.056 (0.022)
	ϕ_1	0.099 (0.020)	0.099 (0.020)	0.099 (0.020)	0.121 (0.019)	0.121 (0.019)
	α_0	0.050 (0.049)	0.056 (0.081)	0.064 (0.067)	0.081 (0.121)	0.089 (0.112)
	α_1	0.048 (0.022)	0.045 (0.023)	0.052 (0.015)	0.059 (0.024)	0.071 (0.020)
	β_1	0.912 (0.052)	0.911 (0.071)	0.899 (0.060)	0.876 (0.100)	0.863 (0.100)
	δ_1	0.212 (0.930)	0.214 (0.875)	0.293 (0.104)	0.258 (0.883)	0.311 (0.095)
	γ_1	4.999 (0.902)	5.002 (0.683)	5.014 (0.450)	5.010 (0.955)	5.125 (0.891)
	c_1	0.500 (0.090)	0.499 (0.059)	0.487 (0.047)	0.499 (0.066)	0.489 (0.034)

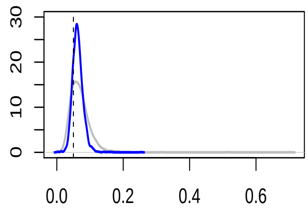
Table 7: Results of the misspecification tests of constant unconditional variance

	Standard test		Robust test		Outlier robust test	
	Corn	Sugar	Corn	Sugar	Corn	Sugar
$H_0 : \varphi_1 = \varphi_2 = \varphi_3 = 0$	0.052	0.054	0.015	2×10^{-5}	0.017	0.126
$H_{03} : \varphi_3 = 0$	0.410	0.568	0.383	0.585	0.390	0.301
$H_{02} : \varphi_2 = 0 \mid \varphi_3 = 0$	0.018	0.564	0.018	0.555	0.010	0.076
$H_{01} : \varphi_1 = 0 \mid \varphi_2 = \varphi_3 = 0$	0.222	0.008	0.051	1×10^{-6}	0.301	0.187

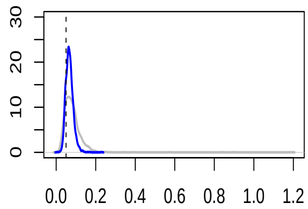
Table 8: Estimation results of the h_t component for the GARCH and MTV-GARCH models (Standard errors in parentheses)

	α_0	α_1	β_1	Log-Lik
	Sugar returns			
GARCH	0.0186 (0.0054)	0.0338 (0.0039)	0.9621 (0.0044)	-12994.9
	Corn returns			
GARCH	0.0425 (0.0076)	0.0602 (0.0058)	0.9251 (0.0073)	-11659.9
MTV-GARCH	0.0443 (0.0083)	0.0670 (0.0078)	0.8913 (0.0139)	-11630.0
BM-MTV-GARCH	0.0195 (0.0039)	0.0692 (0.0073)	0.8987 (0.0128)	-11635.0

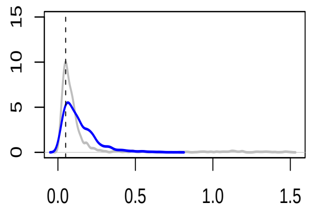
B Figures



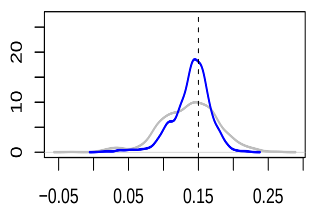
(a) DGP1: α_0=0.05



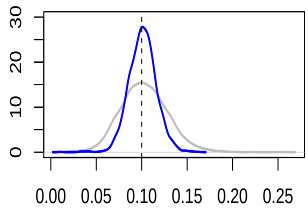
(b) DGP2: α_0=0.05



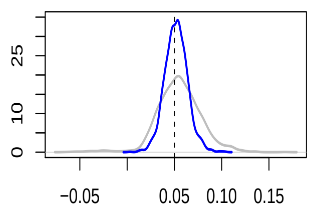
(c) DGP3: α_0=0.05



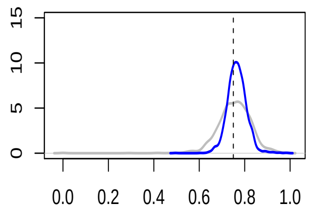
(d) DGP1: α_1=0.15



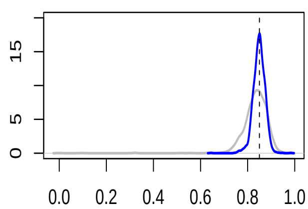
(e) DGP2: α_1=0.10



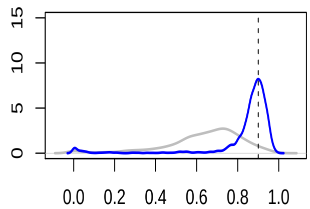
(f) DGP3: α_1=0.05



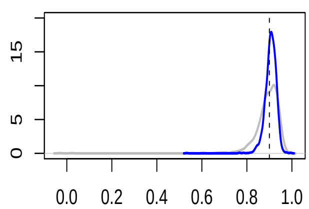
(g) DGP1: β_1=0.75



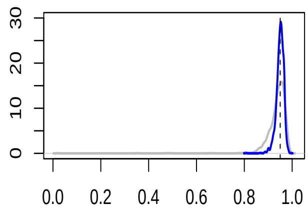
(h) DGP2: β_1=0.85



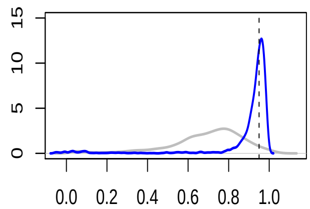
(i) DGP3: β_1=0.90



(j) DGP1: $\alpha_1 + \beta_1$=0.90

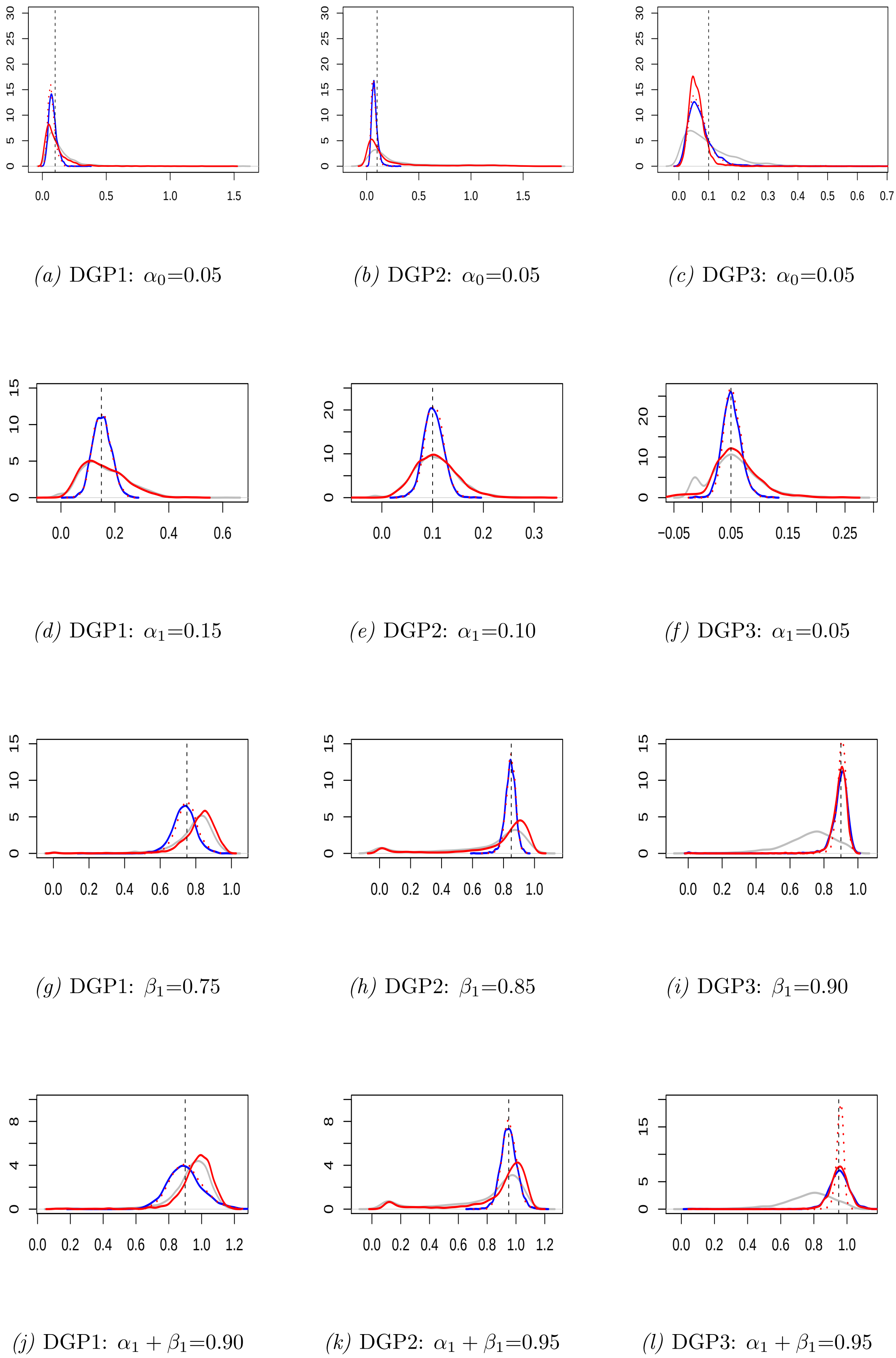


(k) DGP2: $\alpha_1 + \beta_1$=0.95



(l) DGP3: $\alpha_1 + \beta_1$=0.95

Figure 1: Kernel densities of the QML estimators of the GARCH parameters without outliers for $T = 1000$ (grey curve) and $T = 3000$ (blue curve).



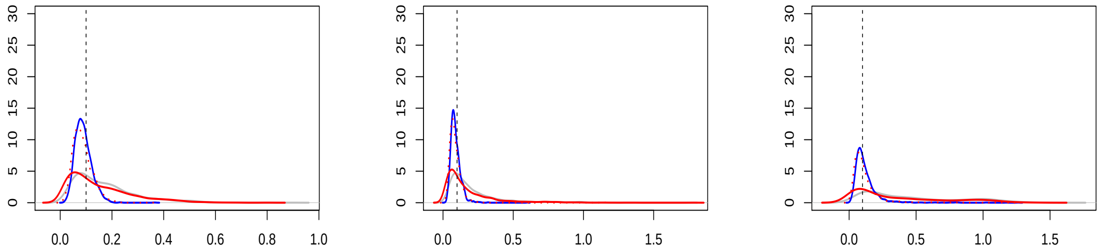
(a) DGP1: α_0=0.05 *(b)* DGP2: α_0=0.05 *(c)* DGP3: α_0=0.05

(d) DGP1: α_1=0.15 *(e)* DGP2: α_1=0.10 *(f)* DGP3: α_1=0.05

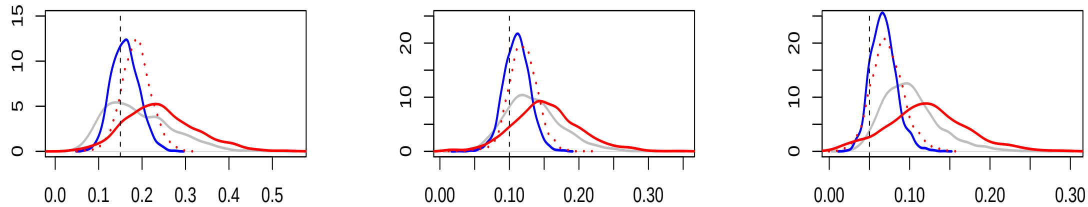
(g) DGP1: β_1=0.75 *(h)* DGP2: β_1=0.85 *(i)* DGP3: β_1=0.90

(j) DGP1: $\alpha_1 + \beta_1$=0.90 *(k)* DGP2: $\alpha_1 + \beta_1$=0.95 *(l)* DGP3: $\alpha_1 + \beta_1$=0.95

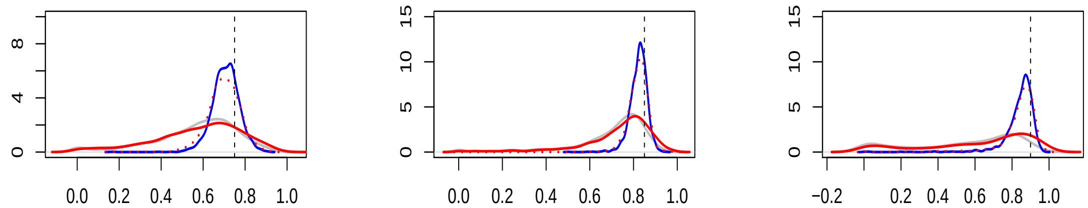
Figure 2: Kernel densities of the QML and BM estimators of the GARCH parameters with a single outlier of size $\omega_{AO} = 10$. Kernel densities are plotted for the QML estimator when $T = 1000$ (grey line) and $T = 3000$ (blue line) and BM estimator when $T = 1000$ (red line) and $T = 3000$ (dotted line).



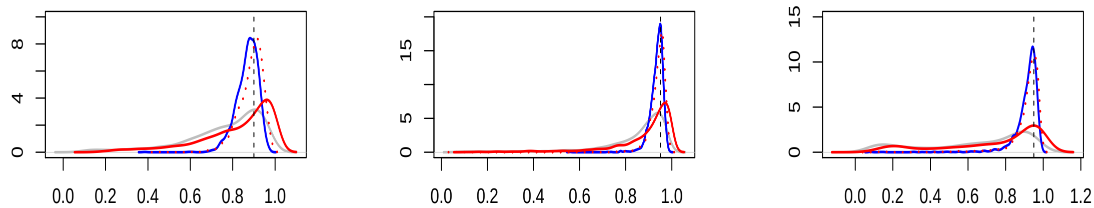
(a) DGP1: α_0=0.05 *(b)* DGP2: α_0=0.05 *(c)* DGP3: α_0=0.05



(d) DGP1: α_1=0.15 *(e)* DGP2: α_1=0.10 *(f)* DGP3: α_1=0.05

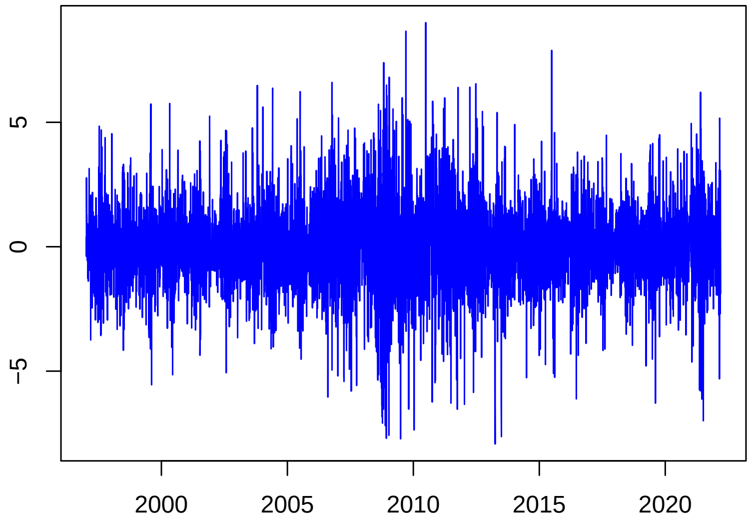


(g) DGP1: β_1=0.75 *(h)* DGP2: β_1=0.85 *(i)* DGP3: β_1=0.90

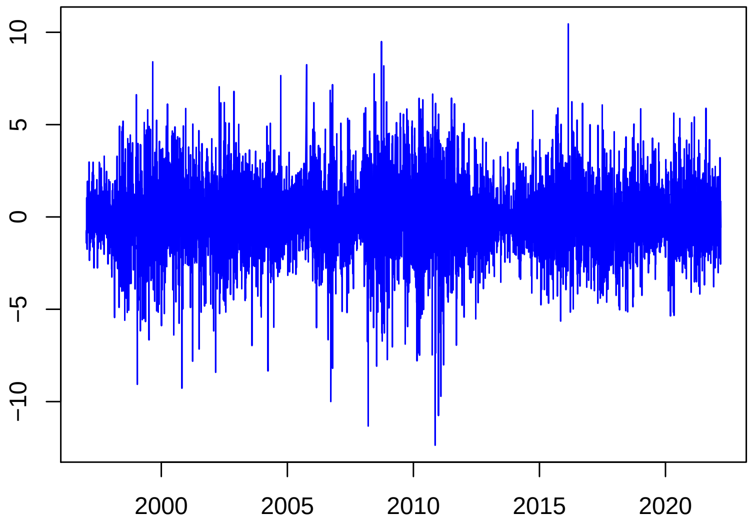


(j) DGP1: $\alpha_1+\beta_1$=0.90 *(k)* DGP2: $\alpha_1+\beta_1$=0.95 *(l)* DGP3: $\alpha_1+\beta_1$=0.95

Figure 3: Kernel densities of the QML and BM estimators of the GARCH parameters with two consecutive outliers of size $\omega_{AO}=10$. Kernel densities are plotted for the QML estimator when $T=1000$ (grey line) and $T=3000$ (blue line) and BM estimator when $T=1000$ (red line) and $T=3000$ (dotted line).



(a) Corn



(b) Sugar

Figure 4: Daily logarithmic commodity returns for corn and sugar from 08 January 1997 until 14 March 2022 ($T = 6272$ observations).

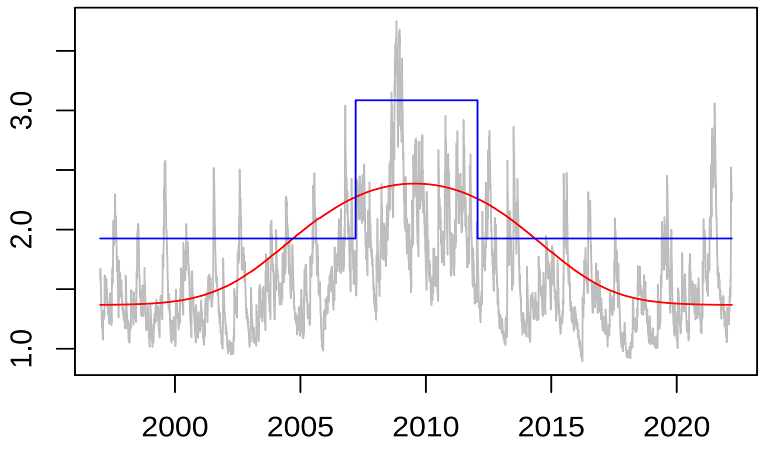


Figure 5: Estimated g_t functions from the standard MTV-GARCH model (red curve) and the robust MTV-GARCH model (blue curve), and estimated conditional standard deviation for the commodity daily returns for corn from the GARCH model (grey curve).

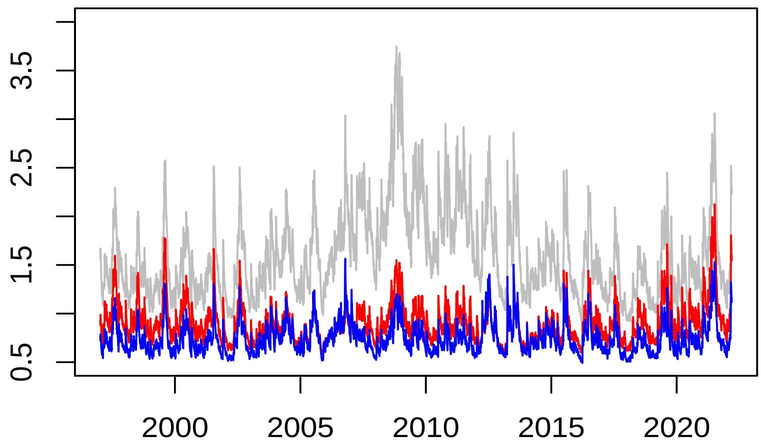


Figure 6: Estimated conditional standard deviations for the commodity daily returns for corn from the GARCH model (grey curve), the MTV-GARCH model (red line) and the robust MTV-GARCH model (blue curve).

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