STRATEGIES USED BY 11TH GRADE PORTUGUESE STUDENTS IN SOLVING MATHEMATICAL PROBLEMS

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Abstract

Problem-solving, organised as individual or group activity, should be highlighted in the practice of Mathematics. This is emphasized by the Portuguese Mathematics Working Group (GTM) in the report entitled "Recomendações para a melhoria das aprendizagens dos alunos em Matemática" [Recommendations for improving student learning in Mathematics] (2019). In addition to problem-solving, the authors of this document also mention the importance of communication and of resorting to different representations, among other mathematical skills. It was to highlight these capacities that the research presented in this paper was carried out, as part of a PhD project. The project's main objective is to understand the how problem-solving and written communication skill mutually reinforce each other. Within this broad context, one of the research questions enquires "What are the strategies used by students in problem-solving?". Problem-solving strategies is precisely the focus of this paper, where following strategies are considered: trial and error, search for a pattern, generalization, deduction, end-to-start resolution, construction of diagrams or figures, construction of tables, construction of a model, resolution by parts, application of formulas, exhaustion, and particularization.

Twenty-nine 11th grade students from two classes of the same school participated in this study. Throughout the academic year, students were invited to participate in 16 problem-solving sessions, lasting about 90 minutes each, conducted virtually over an online platform. These students were divided into six working groups which remained unchanged until the end of the project. In each session, a mathematical problem was proposed to be addressed and solved by the groups. At the end of each session, all groups delivered a single resolution to the proposed problem, which resulted in 92 resolutions at the end of the project. This data collection was carried out by the researcher and first author of this communication, who was present at all times of data collection, using a qualitative methodology within an interpretative paradigm. After collecting all the resolutions over the 16 sessions, the same author analysed the resolutions and identified which strategies the students used.

After analysing the resolutions of all groups in all sessions, we concluded that all strategies listed were used by at least one group throughout the 16 sessions. Furthermore, it was found that all groups resorted to almost all the strategies listed, which means that they felt the need to address the problems with different processes. It was also possible to detect that the most used strategy by the students was the "construction of diagrams or figures" – 35 resolutions resort to it – while the least used were "search for a pattern", "generalization", "end-to-start resolution" and "particularization" – all with 4 resolutions only.

Keywords: Problem-solving, strategies, high school, math.

1 INTRODUCTION

A challenging learning environment can be the key to improve students' mathematical reasoning [1]. To promote this environment, we can propose different problem-solving activities to students, which are an essential part of mathematics education [2]. According to [3], problem-solving gives to students the opportunity to practice and interpret the concepts previously learned. This is one reason why problem-solving is so important. To solve the problem, students need to comprehend what is given and what is asked, to think about the way to do that, and to plan and organize the answer [3]. This involves the application of some strategies and the development of written communication. The focus of this paper is on the strategies students' use while solving the problems proposed. To attend this focus, it's important to know which strategies could be used by the students.

In reference [4] a list of different strategies is presented: make a model; make a table; try, check and review; simplify; eliminate; find patterns. Another list is presented in [5] which has a number of similarities with the previous one. This list includes guess and check, work backwards, look for a pattern, use simpler numbers, draw a picture, write an equation, make a table, and organize the data. But [5] warns that we can't wait that one specific strategy or method will work well to every problem: no strategy fits

all. To these lists we can add two other strategies, for example, creating a simpler problem, and using logic [6].

The list of strategies adopted in this research was based on the previous ones. So, we considered the following eleven different strategies: trial and error, search for a pattern, generalization, deduction, end-to-start resolution, construction of diagrams or figures, construction of tables, construction of a model, resolution by parts, application of formulas, and exhaustion.

2 METHODOLOGY

This research is based on a project called *ProbleMath.Com*, which was developed along the academic year 2021/2022. The project involved twenty-nine 11th grade students of the Sciences and Technologies' course. These kids were from two different classes of a school in the North of Portugal, located in an urban center. All of them had one thing in common: the same Math's teacher. Besides that, all students were in this project on a voluntary basis. Throughout the academic year, students were invited to participate in 16 sessions of problem-solving, lasting 90 minutes each, on an online format. They were organized in six groups, and each group was the same in every session. The students come to the sessions through a link and they were separated in different rooms (one group per room), to solve the problem proposed for the session. In the end, each group sent to the first author of this paper one photo of the group's resolution. After this, the students had the possibility of sharing different resolutions and ideas in a moment of collective discussion. For this paper, we analyzed the work of every group in all 16 sessions of the project to identify and understand what strategies students used.

The data collection was carried out by the first author, who was present in every moment of data collection. She had access to every group during the sessions, and the possibility to listen their ideas and to answer students' questions. After data collection, the same author analyzed the resolutions and identified the strategies used by the students. So, this research followed a qualitative methodology within an interpretative paradigm. This is a research centered in discovering and comprehension of reality [7]. Besides that, there is a strong interaction between the researcher and the investigated [8]. The investigator is actively involved on the process of data collection and analysis, and the data are based on words and images [9] – in this specific case, through images of the students' resolutions and recorded dialogues during the sessions.

3 RESULTS

When planning the problems to propose for all the sessions, we tried to give to students the opportunity to explore and use different strategies. This objective was achieved, as shown in Table 1, since all strategies were used for at least one group throughout the project. Another thing we can observe on Table 1 is the existence of one more strategy that wasn't in our previous list: particularization. This strategy was used by the students who resorted to a particular case to solve a more generic problem.

Table 1. Group strategies throughout the 16 sessions.

Strategies used	*Group 1*	*Group 2*	*Group 3*	*Group 4*	*Group 5*	*Group 6*	*Total*
Trial and error	4	3	2	2	4	8	23
Search for a pattern	1	-	1	1	-	1	4
Generalization	1	1	1	-	1	-	4
Deduction	6	5	5	4	4	1	25
End-to-start resolution	-	-	1	1	-	2	4
Construction of diagrams or figures	4	6	8	5	6	6	35
Construction of tables	2	-	2	3	1	1	9
Construction of a model	3	3	3	7	3	1	20
Resolution by parts	4	4	5	5	3	3	24
Application of formulas	4	6	6	3	6	6	31
Exhaustion	2	3	4	3	3	2	17
Particularization	-	1	-	1	1	1	4

As we can see on Table 1, all groups used almost every strategy on the list, which means they felt the necessity of solve the problems with different processes. We can also notice that *construction of diagrams or figures* is the most used strategy. On the contrary, strategies such as *search for a pattern*, *generalization*, *end-to-start resolution*, and *particularization* were the least used ones. The strategies least used appear in four resolutions each. In the case of *search for a pattern* and *particularization* strategies, all four resolutions applying each of these strategies referred to the same problem (session 14 and session 4, respectively). *Generalization* can be seen in two different problems (three resolutions on session 3 and one on session 4). Finally, *end-to-start resolution* was used in two problems too, three on session 6 and one on session 12. It is good to notice one session – session 4 – resorted to two of these strategies: session 4. On the other hand, the only group which used *generalization* was not any of the groups which resorted to *particularization*.

We can add other curiosities found on resolutions with the less used strategies. *Search for a pattern* was not used alone, since every resolution resorting to it also used *construction of diagrams or figures* and *construction of a model* (in three of the cases), and *construction of tables* and *construction of a model* (in one case). The strategy *particularization* was a little different, because one of the groups used this strategy without any other one – one group used *particularization* with *application of formulas* and two other groups combined it with *deduction*. *Generalization* has a similar story, since one group used this strategy in an isolated way and two other groups used it with *resolution by parts* and *application of formulas* (the three cases of session 3). The group who used *generalization* on another session used this strategy with *application of formulas* too. Lastly, *end-to-start resolution* was never used in isolation. On session 6, the three groups used the strategy with *resolution by parts*, and two of them also used *construction of tables*. The group which resorted to *end-to-start resolution* on session 12 conjugated this strategy with *trial and error* and *application of formulas.* It is important to note that in sessions we referred for the least used strategies, other groups used different strategies and were successful, so there was no problems requiring specific strategies to be solved.

4 CONCLUSIONS

Students need to work in small groups involved in problem-solving activities or in establishing a strategy to follow [6]. According to these authors, it is relevant that students write down their resolutions and discuss their ideas with others. That was what we tried to do throughout of this project. Due to all these dynamics of resolution in small groups and posterior collective discussion, the students were able to confront themselves with different ways to solve problems and with different strategies which they were challenged to put in practice throughout the project. Thus, it was possible to all participants to be in contact with every strategy listed, plus another one that we called *particularization*. We also noticed that the least used strategies were proposed by the students and chosen based on their previous experiences.

Students are more comfortable using strategies used more often in school, strategies which make them draw or make schemes or apply formulas they previously knew. It is easy to understand why *construction a diagram or figure* was the most used strategy. This happened because this is a more versatile and flexible strategy that can be adapted and used in almost every type of problem. The least used strategies, like *search for a pattern*, *generalization*, *end-to-start resolution*, and *particularization*, seems to fit to quite specific problems and their application is probably less flexible.

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