Risk assessment in industry using expected utility: an application to accidents' risk analysis

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Abstract. Expected utility theory can be relevant for decision-making under risk, when different preferences should be taken into account. The goal of this paper is to present a quantitative risk analysis methodology, depending on expected utility, where the risk consequences are determined quantitatively, the risk is modelled using a loss random variable and the expected utility loss is used to classify and rank the risks. Considering the relevance of risk management to reduce workers' exposure to occupational risks, the methodology is applied to the analysis of accidents in industry, where six different injury categories are distinguished. The ranking of the injury categories is determined for three different utility functions. The results indicate that the slope of the utility function influences the ranking of the injury categories. The choice of the utility function may thus be relevant for the risk classification in order to prioritize different aspects of risk consequences.

Keywords: risk assessment, risk analysis, expected utility, utility function, accidents, industry

1 Introduction

In the theory of decision making under risk the expected utility theory developed by von Neumann & Morgenstern (1947) describes the representation of preference relations on risky alternatives using expected utility. The expected utility model is used to model how decision makers choose between uncertain or risky prospects [1]. According to that model, there exists a utility function, which depends on the individual's preferences, to appraise different risky outcomes and a decision maker chooses the outcome which maximizes expected utility. Expected utility theory has applications in the context of economic and actuarial sciences, however it can also be useful for applications in industrial settings, as it will be shown in the present paper. There exist applications of the theory to risk analysis in industry (e.g. in the examples presented below), where utility functions are introduced, however we could not find applications of expected utility, where the utility functions are applied to random risk consequences.

Considering the risk analysis in industry, one important aim is to order risks quantitatively, so that higher ranked risks can be identified and risk-handling approaches can be applied, such as risk-controlling, risk-avoiding and risk-mitigating actions [2].

One common method used in industries, e.g. in many applications of safety analysis or in systems development, is to evaluate risks based on the determination of the level of risk (see [3]) or of the risk factor (see e.g. [4]), which are both expressed in terms of combinations of the occurrence probability of a certain risk event and of the size of its consequence. For example in the risk analysis of accidents, the risk matrix is a popular and common approach to evaluate occupational risks, where the probability of the occurence of accidents and the consequences of accidents are categorised and each cell of the matrix is associated with a level of risk. Several risk matrices have been used and proposed. In the end, risk can be classified in relation to its acceptance or ranked, as e.g. [3] intolerable, undesirable, tolerable and negligible; or high, medium high, medium and low.

Another approach, for example in the failure analysis in systems development (see [4]), is to determine a risk factor RF, more used for complex systems, which depends on the probability of failure P and on the consequence of failure or measure of the consequence of failure C. A failure can be generalized to a broader sense of risk and therefore the risk factor RF can be defined as the product of the probability of risk occurence and the consequence of risk, which can be interpreted as a loss. A simple formulation of the risk factor is to define it as the product of both factors [4]

$$RF = PC. \tag{1}$$

The values for C are determined based on the classification of different risk categories by assigning a value between 0 (negligible impact) and 1 (high or catastrophic impact) to C. This classification can be performed by experts in the various technological areas and several tables with decision criteria have been developed to facilitate the assessment and to evaluate the consequences of risk (see e.g. [5],[6],[7],[8],[9]). Some tables, for example, consist of 4 risk categories: catastrophic, critical, marginal, negligible and the risk factor RF ranges then between 0 and 1, where 0 means that there is no risk and 1 means that there is a high or maximal risk [9].

Based on this risk assessment formulation, Ben-Asher [9] developed a risk assessment method using utility theory, which improves the previous model (1). Due to symmetries, drawbacks were identified in the formula for the risk factor, e.g. high probabilities with negligible consequences and low probabilities with high (catastrophic) consequences can be ranked equally. However, as remarked in [9] most people emphasize risks with higher catastrophic consequences, so that more attention should be payed to the latter case and it should therefore be ranked differently from the first case. Using utility theory the new proposed model was formulated by [9]

$$RF = Pu(C), \tag{2}$$

where u is a utility function, called utility-based loss function.

The loss value for the worst possible outcome is defined by $u(C_{\text{worst}}) = 1$ and, in the absence of risk, the value is given by $u(C_{\text{best}}) = 0$. For other intermediate consequences C_i, the values are determined by asking the agent (or risk management board) for which value of $p \in (0, 1)$ he would be indifferent between getting the outcome C_i with certainty and a lottery yielding the outcome 1 (worst outcome) with probability p and 0 with probability $1 - p$. The utility based loss values are then defined by $u(C_i) = p$, so that C_i is the certainty equivalent of the lottery. In this context, a typical utility function is convex, which increases more for higher values of consequences. This resolves the mentioned drawback.

Other applications of utility theory to the risk analysis in the industrial settings were proposed in [10], where risk matrices were established that integrate risk attitudes quantified by utility functions, in [11], where the assessment of safety risks in oil and gas industry was considered, or in [12], where in the context of ports vulnerability analysis in shipping and port industries the ranking of vulnerable levels was determined by utility values.

In this work we present a methodology for the analysis of risks in industry, where the risk evaluation and classification is based on expected utility theory. The main difference between this methodology and the methodologies based on utility theory mentioned above is that this is a quantitative methodology, where the risk consequences are determined numerically and modelled as random variables and expected utility is used to assess the risks, whereas the other methodologies are semi-quantitative, the consequences being determined qualitatively and ranked subjectively through the utilities. Here, a loss random variable is defined and the expected utility loss is used to rank the different risks. Different utility functions are introduced presenting more or less agressive decision-makers, so that a given risk can be classified differently, more or less severe, according to the different properties related with the first and second derivative of the utility function, modelling the decision-maker's risk attitude. The risk analysis based on expected utility is applied to a case study of risk analysis of occupational accidents in the furniture industry, considered in [13], and the results are compared with the results of that previous study. In [13], the risks, associated with lost days due to different injury categories, were modelled by loss random variables and expected loss, loss variance and risk measures, such as Value-at Risk and Tail-Value-at-Risk (or Expected Shortfall), were used to analyse the risk levels of different accident categories. The data used in the case study corresponds to accidents in the furniture industry in Portugal in the year 2010, this industry being one of the most relevant activity sectors in Portugal [14], which consists predominantly of small and medium-sized enterprises [15].

2 Risk assessment based on expected utility

Consider the following decision problem. Suppose that a decision maker with wealth w must decide between two random losses X and Y. A simple decision model that one can apply in order to choose between X or Y is the expected value model. According to the expected value principle, a decision maker compares $\mathbb{E}[w - X]$ with $\mathbb{E}[w - Y]$ and chooses the random amount which maximizes the expected value, so that he would prefer X to Y if $\mathbb{E}[w - X] > \mathbb{E}[w - Y]$, he would choose Y if $\mathbb{E}[w - X] < \mathbb{E}[w - Y]$ and in the case $\mathbb{E}[w - X] = \mathbb{E}[w - Y]$ the decision maker would be indifferent between X and Y.

In the expected utility theory developed by von Neumann and Morgenstern (1947) a utility function $u(\cdot) : \mathbb{R} \to \mathbb{R}$ is introduced that represents the preference ordering of the decision. The decision maker judges the utility of a given quantity instead of the simple value of that quantity, so that he takes into account the utility of the wealth $u(w)$ instead of w. The utility function is an increasing function, $u'(\cdot) > 0$, since utility increases with wealth.

According to the expected utility model, if the decision maker must decide between two random losses X and Y, he compares the expected utilities $\mathbb{E}[u(w-X)]$ with $\mathbb{E}[u(w-Y)]$ and opts for the quantity having the higher expected utility, so that he chooses X if $\mathbb{E}[u(w - X)] > \mathbb{E}[u(w - Y)]$, or Y if $\mathbb{E}[u(w - X)] < \mathbb{E}[u(w - Y)]$ and if $\mathbb{E}[u(w - X)] = \mathbb{E}[u(w - Y)]$ the decision maker is indifferent between X and Y.

The decision that an individual takes depends on his attitude towards risk, which is characterized by the second derivative of the utility function, by its shape. A decision maker can be classified into three risk attitude categories: risk-seeking, risk-neutral and risk-avoiding (or risk-averse). For a risk-seeking individual the utility function is convex, $u''(\cdot) > 0$, for a risk-avoiding individual the utility function is concave, $u''(\cdot) < 0$, and for a risk-neutral individual the utility function is linear, $u''(\cdot) = 0$. The following example illustrates this characterization.

Example 1. Consider a lottery, where the decision maker can win 2 monetary units with probability 0.5 or 0 with the same probability, and let X denote the random variable representing the corresponding monetary outcome. The expected utilities for the three different utility functions: $u_1(x) = x^2$ – risk-seeking, $u_2(x) = x$ – risk-neutral, $u_3(x) = \sqrt{x}$ – risk-avoiding, are given by:

$$\mathbb{E}[u_1(X)] = 2, \;\; \mathbb{E}[u_2(X)] = 1, \;\; \mathbb{E}[u_3(X)] = \frac{\sqrt{2}}{2} \approx 0.71. \tag{3}$$

Comparing the expected value of the monetary outcome of the lottery, given by $\mathbb{E}[X]$=1, with the certainty equivalents corresponding to the different utilities (a certainty equivalent is the amount of cash that an individual would accept with certainty instead of facing the lottery), which are determined through $CE(X; u_i) = u_i^{-1}(\mathbb{E}[u_i(X)])$, $i = 1, 2, 3$:

$$CE(X; u_1) = \sqrt{2} \approx 1.41, \;\; CE(X; u_2) = 1, \;\; CE(X; u_3) = \frac{1}{2}, \tag{4}$$

one can observe that the lottery for a risk-seeking decision maker is valued higher, $CE(X; u_1) > \mathbb{E}[X]$, than for a risk-avoiding decision maker, $CE(X; u_3) < \mathbb{E}[X]$. For the risk-neutral utility, the certainty equivalent coincides with the expected value.

In risk theory, a risk can in certain situations be modeled by a loss random variable X, a random risk, of the form (see for example [1]):

$$X = IB, \tag{5}$$

where I is the indicator random variable, which can take the values $I = 1$, indicating that a risk event or a loss has occurred, or $I = 0$, indicating that no risk event or no loss has occurred, and B represents the random amount of loss.

The indicator random variable I can be described by a Bernoulli(p) distribution, $0 \leq p \leq 1$. The occurrence probability of the risk event is $p = P(I = 1)$ and $1 - p = P(I = 0)$ corresponds to the probability of no risk occurrence. If $I = 1$, then the loss X is drawn from the distribution of the random variable B and if $I = 0$, then $X = 0$, meaning that no loss has occurred.

The moments of X can be calculated using the iterative formula of conditional expectations. The expected loss is determined as follows

$$\mathbb{E}[X] = \mathbb{E}[\mathbb{E}[X|I]] = p\mathbb{E}[B]. \tag{6}$$

In the particular case, where the amount of loss is fixed $B = b$, then the formula for the expected loss simplifies to

$$\mathbb{E}[X] = pb. \tag{7}$$

The expected utility loss of $X = IB$ is given by

$$\mathbb{E}[u(X)] = p\mathbb{E}[u(B)] + (1 - p)u(0) \tag{8}$$

and the expected utility for $X = Ib$, considering a fixed loss $B = b$, by

$$\mathbb{E}[u(X)] = pu(b) + (1 - p)u(0). \tag{9}$$

Using a normalization condition $u(0) = 0$, the formulas (8) and (9) become, respectively,

$$\mathbb{E}[u(X)] = p\mathbb{E}[u(B)] \tag{10}$$

and

$$\mathbb{E}[u(X)] = pu(b). \tag{11}$$

Comparing the previous formulas for the expected loss and expected utility loss of a random risk with the definitions for the risk factor used in the industry context (1) and (2), the latter depending on the utility function, then the expected loss defined in (7) resembles formula (1) and (11) resembles formula (2), where

p plays the role of P of the risk occurrence probability and b the role of C, since a risk consequence can be perceived and represented as a loss in a certain sense. In the expected utility framework, that we will adopt here, the consequences or losses will also considered to be random variables, so that the definitions (6) and (10) would represent the analogous risk factors for random consequences.

In the risk assessment approach based on expected utility we will use the utility function to characterize the outcome of the risk representing a loss (or a consequence), so that as in [9] the utility function u is a utility loss function (or utility-based loss function). Thus, $u(\cdot)$ is an increasing function of loss or consequence, $u'(\cdot) > 0$. We will apply the definitions (11) and (10), where the losses are modelled as fixed or random variables, respectively. In order to assess and classify the risks in industry, we will use the expected utility losses to rank the risks. We will say that a risk X is higher or more severe than the risk Y, or simply that X is riskier than Y, if the expected utility loss of X is higher than those of Y and we will represent this risk relation order by $X \succ Y$. Thus, we have that

$$X \succ Y \Leftrightarrow \mathbb{E}[u(X)] > \mathbb{E}[u(Y)], \tag{12}$$

meaning that X is riskier than Y.

3 A case study – risk assessment of industrial accidents

We will apply the expected utility based risk analysis method to a case study that was considered in [13]. The aim is to classify and assess injury categories of occupational accidents that occurred in the furniture industry in Portugal in 2010. Official accident reports data were provided by the Portuguese Office of Strategy and Planning (GEP), which are aligned with European Statistics on Accidents at Work (ESAW III). The six categories of contact-modes of injuries, denoted by C_i, $i = 1, \dots, 6$, are presented in Table 1.

Table 1. Contact mode of injury categories.

C_i	injury category
C_1	Contact with electrical voltage, temperatures, hazardous substances
C_2	Horizontal or vertical impact with or against a stationary object (victim in motion)
C_3	Struck by object in motion, collision with
C_4	Contact with sharp, pointed, rough, coarse Material Agent
C_5	Trapped, crushed, etc.
C_6	Physical or mental stress

The occurrence of a safety risk, which in the present case is a occupational accident belonging to one of the six injury categories, is accompanied by a consequence and its severity can be measured by the number of lost work days. Therefore, here the loss (or consequence, or severity) will be measured in terms of number of lost days implied by an accident. Different injuries will lead to different numbers of lost work days, where the case of zero lost work days is also possible to occur. The risk of accident can then be characterized by its occurrence probability, estimated based on the past accidents' frequencies, and by its loss, measured in terms of lost work days.

The risk corresponding to the injury category C_i will be modeled using the loss random variable

$$X_i = I_i B_i, \tag{13}$$

where $I_i \sim \mathrm{Ber}(p_i)$, with p_i being the accidents' occurrence probability in category C_i, and the random variable B_i represents the number of lost work days in category C_i.

We will consider two cases, where in the first case a fixed estimated number of lost work days will be used, $B_i = b_i$, and in the second case a random variable B_i will be used, where further the number of zero and non-zero lost work days will be taken into account. The estimated number of lost work days due to an accident of category C_i, $i = 1, \ldots, 6$, will be defined by

$$b_i = \frac{b_{Ti}}{n_i}, \tag{14}$$

where b_{T_i} represents the total number of lost days associated with accidents of category Ci and n_i is the number of accidents in category C_i. The occurrence probability of an accident in category C_i is given by

$$p_i = \frac{n_i}{n}, \tag{15}$$

where n_i stands for the number of accidents in category C_i and n is the total number of accidents that occurred in the furniture industry. The probability that an accident belonging to category C_i will lead to at least one lost work day can be estimated taking into account the number of accidents that had as consequence one or more than one lost work day as follows

$$q_i = \frac{n_{i,\geq 1}}{n_i}, \tag{16}$$

where $n_{i,\geq 1}$ denotes the number of accidents in category C_i leading to at least one lost work day. Table 2 contains the results for accidents of each category C_i.

Table 2. Results for the contact modes of injury categories.

C_i	n_i	$n_{i,\geq 1}$	q_i	b_{Ti}	b_i	p_i
C_1	97	77	0.79	1135	11.70	0.02
C_2	523	361	0.69	17457	33.38	0.12
C_3	958	585	0.61	18082	18.87	0.22
C_4	1406	1188	0.84	53661	38.17	0.33
C_5	331	270	0.82	13594	41.07	0.08
C_6	998	809	0.81	27062	27.12	0.23

Case 1: Risk model with a fixed number of lost work days

Considering the risk model $X_i = I_i b_i$, we will apply the expected utility loss (11) $\mathbb{E}[u(X_i)] = p_i u(b_i)$ using three different utility functions exemplifying the different risk attitudes: the linear utility $u_1(x) = x$, a quadratic utility $u_2(x) = x^2$ and the exponential utility $u_3(x) = e^x$. Note that for the linear utility the expected utility corresponds to the expected value. The other two utility functions are convex, where the exponential utility increases more for higher losses than the quadratic utility. Table 3 contains the calculated expected utility losses for each injury category.

Table 3. Expected utility losses of injury categories.

C_i	$\mathbb{E}[u_1(X_i)]$	$\mathbb{E}[u_2(X_i)]$	$\mathbb{E}[u_3(X_i)]$
C_1	0.23	2.74	2411.43
C_2	4.01	133.71	3.77×10^3
C_3	4.15	78.34	3.45×10^7
C_4	12.60	480.79	1.25×10^{16}
C_5	3.29	134.94	5.49×10^{16}
C_6	6.24	169.16	1.38×10^{11}

The injury categories can be ordered with respect to their risks using the results of Table 3 and the representation (12) for each utility function (see Table 4).

Table 4. Risk ordering of injury categories using expected utility losses.

$\mathbb{E}[u_1(X_i)]$	$C_4 \succ C_6 \succ C_3 \succ C_2 \succ C_5 \succ C_1$
$\mathbb{E}[u_2(X_i)]$	$C_4 \succ C_6 \succ C_5 \succ C_2 \succ C_3 \succ C_1$
$\mathbb{E}[u_3(X_i)]$	$C_5 \succ C_4 \succ C_6 \succ C_3 \succ C_2 \succ C_1$

In this case, the utility function is applied to the loss model depending on the accident ocurrence probability and on the estimated number of lost work days, which is fixed, with the effect that only the number of lost work days, b_i, is influenced by the utility function.

From the results in Table 4 one concludes that the injury category C_1 is classified as the lowest risk category with all three utility functions. The linear and the quadratic utility classifiy C_4 as the higher risk category, followed by C_6. However the exponential utility classifies C_5 as the higher risk category, followed by C_4. The reason for this difference can be explained as follows. As one can see from Table 2, C_5 has the highest estimated number of lost days $b_5 = 41.07$ and a low occurrence probability $p_5 = 0.08$, whereas C_4 has the highest occurence probability $p_4 = 0.33$ and the number of lost days is also considerable large $b_4 = 38.17$. With an increasing slope of the utility function the impact on b_i also increases. Thus, the increasing slope penalizes the risk and the penalization is more accentuated for higher values of b_i. This has the influence that C_5 is considered with u_1 the fifth risk category, with u_2 the third risk and with u_3 (the exponential utility having the highest slope), the highest risk category. For low values of b_i, as in C_1, the influence of the slope has low impact, leaving C_1 at the same risk level. In general, one can conclude that attention should first be paid to the injury category C_4 and then to the category C_6 which occupy higher risk positions in the different orderings. There is a further evidence to consider C_5 as the third risk category. If one wants to weight more the number of lost days and the aim is to reduce accidents with higher number of lost days, although the occurence probability being low, then more attention should be paid to injury category C_5.

With the given model the results of Table 4 suggest the following ranking:

$$C_4 \succ C_6 \succ C_5 \succ C_2, C_3 \succ C_1, \tag{17}$$

where the position of C_2 and C_3 varies more with the utility function and it is not clear, in general, which of both should be prioritized. This depends effectively on the decision-makers' attitude reflected by the utility function.

One can conclude that the utility function influences the risk ordering of the injury categories, where different utility functions weight the number of lost days differently. A utility function with a higher slope, as the exponential utility, could be employed if one wants to prioritize risks with higher number of lost work days.

Case 2: Risk model with a random number of lost work days

Now, we will consider the risk model $X_i = I_i B_i$, where B_i is a random variable. In this case, we will further take into account the occurrence probability of accidents with non-zero lost work days, given by q_i (see (16)). The distribution of B_i can be defined by: $P(B_i = b_i) = q_i$, $P(B_i = 0) = 1 - q_i$. The expected utility loss for category C_i, using (10), is then given by

$$\mathbb{E}[u(X_i)] = p_i q_i u(b_i). \tag{18}$$

Calculating the expected utility losses for the utility functions $u_1(x) = x$, $u_2(x) = x^2$ and $u_3(x) = e^x$, one obtains the results in Table 5.

Table 5. Expected utility losses of injury categories.

C_i	$\mathbb{E}[u_1(X_i)]$	$\mathbb{E}[u_2(X_i)]$	$\mathbb{E}[u_3(X_i)]$
C_1	0.18	2.16	1905.03
C_2	2.76	92.26	2.60×10^{13}
C_3	2.53	47.79	2.10×10^{7}
C_4	10.58	403.87	1.05×10^{16}
C_5	2.69	110.65	4.50×10^{16}
C_6	5.05	137.02	1.12×10^{11}

The injury categories can be ordered with respect to their risks using the results of Table 5 and the representation (12) for each utility function (see Table 6).

Table 6. Risk ordering of injury categories using expected utility losses.

$\mathbb{E}[u_1(X_i)]$	$C_4 \succ C_6 \succ C_2 \succ C_5 \succ C_3 \succ C_1$
$\mathbb{E}[u_2(X_i)]$	$C_4 \succ C_6 \succ C_5 \succ C_2 \succ C_3 \succ C_1$
$\mathbb{E}[u_3(X_i)]$	$C_5 \succ C_4 \succ C_2 \succ C_6 \succ C_3 \succ C_1$

Analysing the results presented in Table 6, one can observe that, as with the application of the previous model, C_4 and C_6 occupy the first and second risk position with u_1 and u_2, respectively, and using u_3, C_5 occupies the first and C_4 the second risk position. The values for the new introduced quantity q_i are in fact also higher for C_4, C_5 and C_6 (see Table 3), so that these categories remain unchanged in the ranking, and this also due to the fact that in formula (18), the utility function continues influencing only the quantity b_i. Considering the low risk injury category, C_1 is also classified with this model as having the lowest risk with all three utility functions. The difference is now that, furthermore, the category C_3 is classified with the three utilities as second lowest risk category, whereas in the other model C_3 appeared in the third, fourth and fifth risk position, depending on the utility function. The reason for C_3 appearing now in the fifth risk position with all three utilities is that C_3 has the lowest probability of accidents with non-zero lost work days: $q_3 = 0.61$ (see Table 2). In general, if one should rank the injury categories with the proposed model and the three utilities, the results of Table 6 suggest the following ranking:

$$C_4 \succ C_6 \succ C_5 \succ C_2 \succ C_3 \succ C_1. \tag{19}$$

With this second model, the aspect of the probability of non-zero lost work days was further taken into account, so that, with the low probability q_3, the category

C_3 was classified consistently as second lowest risk category. Considering the impact of the number of lost work days on the risk classification, the exponential utility with its higher slope is more sensitive to this aspect than the other utilities and also in this case the expected utility loss with u_3 prioritizes the risk of injury category C_5. On the contrary, the expected utility loss with u_1 and u_2 ranks C_4 as the highest risk category, which has the highest occurence probability and the highest probability of non-zero lost work days.

Comparing the ranking of the six injury categories of industrial accidents obtained with the expected utility loss models with the classification obtained with other risk measures, such as the Value-at Risk (VaR), in the study conducted in [13], where the proposed ranking was

$$C_4 \succ C_6 \succ C_2 \succ C_5 \succ C_3 \succ C_1,$$

one can conclude the following. The ranking based on VaR coincides with the expected loss (expected utility loss with linear utility u_1) with random number of lost days (cf. Table 6). In general, the expected utility loss models (cf. (17) and (19)) and VaR prioritize the risk of category C_4 followed by C_6 and all classify C_1 as the category with minimum risk. The main difference lies in the classification of the intermediate risk levels. For example, considering the third risk position in the ranking, the expected utility loss models select C_5, whereas VaR selects C_2. Here, one can observe the role of the utility function, which penalizes C_5, that in fact has the highest estimated number of lost work days.

4 Conclusions

In this work we proposed a risk analysis approach based on expected utility. The industry risk was modelled by a loss random variable and the expected utility loss was used to classify and rank industry risks. The role of the utility function is to weight differently certain aspects of risk and the utility function can represent the risk attitude of a decision-maker.

The methodology was applied to the risk analysis of accidents in industry, which can be categorized into six classes of injuries. A loss random variable, depending on the number of lost work days, which in a first model was considered fixed and in a second model was considered random, and on the accident occurence probability was defined. Then, the expected utility loss was calculated for each injury category. In the second model, the number of non-zero lost work days was further distinguished for each injury category. Different utility functions, a linear utility, a quadratic utility and an exponential utility, were used to determine the expected utility loss. The results showed that different utility functions provide different rankings of the categories, where the higher the slope of the utility function was, the higher was the penalization of categories with high accident numbers. The introduction of the utility function in the risk assessment can therefore be useful to prioritize certain aspects of risk, penalizing more or less a given risk event (or risk consequence). The utility function serves therefore also to model the preferences and the risk attitude of the decision-maker. This analysis method, besides being a quantitative method, considers a qualitative or subjective evaluation through the utility function. The selection of the utility function, based e.g. on empirical studies in risk judgement in industry, is a topic that can further be studied, also a possible relation of the utility function with existing risk matrices can be investigated.

The risk analysis approach can be extended to other domains, in situations where it is possible to quantify the risk consequence, having an associated occurence probability, and represent it by a loss random variable. The case study serves as an example of how to apply the approach and how to model the risk. One of the main and new achievements is the introduction of the utility function to take into account the decision-maker's risk attitude and the possiblity of ranking the risks accordingly using expected utility. In the future we will use more actual data, apply the different risk methodologies, based on expected utility, on VaR and on the risk matrix and analyse and compare the results. We also plan to apply the methodologies to different industry sectors.

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