Statistical Dependence Investigation Related to Dowel-Type Timber Joints

Caroline D. Aquino, Leonardo G. Rodrigues, Wellison S. Gomes, and Jorge M. Branco

Abstract The design of timber connections with dowel-type fasteners is dependent on the knowledge of their mechanical behavior and failure modes. Concerning the main design parameters, the timber embedment strength and the dowel bending moment capacity are the parameters that govern the load-carrying capacity. The correlation between the timber embedment strength and the dowel bending moment capacity has not been sufficiently addressed in the literature yet. However, since they both share a common dependency to the timber density, they are probably correlated. To investigate this, traditional distribution fitting procedures, as well as copula functions, are implemented to consider the correlation between them. By doing so, it is aimed to evaluate the effectiveness of the different approaches in describing the dependence structure of the variables and their influence on the structural reliability. It was found that, for single dowel-type connections, the impact of the copulas on the results is small. It is indicated that, unless significantly nonlinear correlations exist among the data, the results obtained by applying different copula functions will probably be very close.

Keywords Timber joints · Reliability · Joint behavior · Copula theory

1 Introduction

An adequate design of a timber structure is very dependent on the efficiency and safety of its connections. In fact, it has been suggested that they can govern the overall structural strength, serviceability, and fire resistance [1]. In addition, assessments of severely damaged timber structures, after extreme events, often point to an inadequacy in the connections as the primary cause of failure [2]. Thus, it is crucial

C. D. Aquino (✉) · W. S. Gomes
CORE, Department of Civil Engineering, Federal University of Santa Catarina, Florianópolis, Brazil

L. G. Rodrigues · J. M. Branco
ISISE, Department of Civil Engineering, University of Minho, Guimarães, Portugal

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to study the behavior of connections in order to achieve a safe design of timber structures.

The variability of materials in structural timber members is relatively well understood and has been addressed in the literature (e.g., [3–5]). However, there is only few researches performed regarding the reliability assessment of timber connections with dowel-type fasteners as well as data characterization of the parameters involved in the design of timber connections.

The existing studies on the reliability assessment of timber connections with dowel-type fasteners are scarce. Köhler [3] proposed a probabilistic framework for the reliability assessment of connections with dowel-type fasteners, considering the resistance of the connection in terms of the timber embedment strength (f_h) and the effective bending capacity of the fastener ($M_{y,eff}$). However, the correlation between these parameters is not addressed. Later, Jockwer et al. [6] addressed the assessment of the failure behavior and reliability of timber connections with multiple dowel-type fasteners. The distribution characteristics of the parameters are based on [3], and regarding the dependence modeling, linear pair-wise correlation coefficients are considered. However, the statistical dependence structure between f_h and $M_{y,eff}$ is also not addressed.

The embedding strength of the timber members (f_h) and the effective bending moment capacity of the dowel ($M_{y,eff}$) are the parameters that govern the structural design. These parameters are ultimately related to the timber density (ρ) and, therefore, are correlated. Based on that assumption, correlation models that represent the statistical dependence between these two variables are developed. Independent and Gaussian copula functions are considered, and these dependence structures are compared with the inferred non-linear copula function based on theoretical data. Such comparison is done in terms of the reliability results achieved with the different dependence structures.

2 Dowel-Type Timber Joints

The load carrying capacity of single dowel-type fasteners loaded perpendicular to the fastener axis can be described by different failure modes according to the Johansen's yield theory [7]. In this study, we are particularly interested in the failure mode where one plastic hinge occurs per shear plane (see Fig. 1). The expression for the load carrying capacity according to Johanses's yield theory for this failure mode is given in Eq. (1).

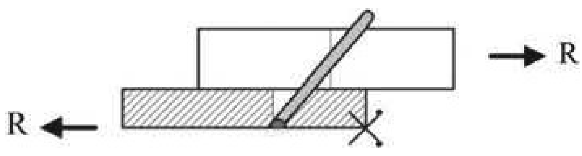

Fig. 1 Failure mode where one plastic hinge occurs per shear plane for dowel-type timber joints

$$R = \frac{f_{h,1} \cdot t_1 \cdot d}{2 + \beta_{f_h}} \left[2\beta_{f_h}\left(1 + \beta_{f_h}\right) + \frac{4\beta_{f_h}\left(4 + \beta_{f_h}\right) \cdot M_y}{f_{h,1} \cdot t_1^2 \cdot d} - \beta_{f_h} \right] \tag{1}$$

where t_1 and t_2 are the thickness of member 1 (side members) and member 2 (middle member); $f_{h,1}$ and $f_{h,2}$ are the embedding strength corresponding to members 1 and 2; β_{f_h} is the ratio between embedding strengths $f_{h,1}$ and $f_{h,2}$; d is the fastener diameter; and M_y is the fastener yield moment.

2.1 Embedment Strength f_h

Associated with the resistance of the timber element against lateral penetration of a stiff fastener, the embedding strength is mostly influenced by the timber density (f_h increases linearly with density), the diameter of the fastener (f_h decreases with increasing fastener diameter), and moisture content (f_h decreases with increasing moisture content) [8, 9]. The experimental procedures for measuring the embedding characteristics of wood are discussed in [10]. Additional discussions considering implications in the timber embedment strength, such as different wood species and grain direction are referred to [11, 12].

There are empirical equations available in the literature to determine f_h. The present study bases on the normative expression given by Eurocode 5 [13] for the wood embedding strength parallel to grain $f_{h,0}$, presented in Eq. (2), dependent on the wood density ρ (kg/m^3) and on the diameter of the dowel d (mm).

$$f_{h,0} = 0.082(1 - 0.01d)\rho \tag{2}$$

2.2 Bending Moment M_y

Related to the dowel's strength against bending, it is mainly influenced by the dowel diameter and the yield strength of the dowel material. In the scope of this paper, the smooth dowels are considered made of mild steel. In the failure modes associated to plastic deformations of dowels, where their bending capacity is mobilized, the effective bending moment directly influences the load-carrying capacity of the connection. In terms of experimental evaluation, the characterization of the bending moment can be performed through a four-point bending test, according to the guidelines of EN 409 [14]. In the experiment, the yield moment of a fastener is determined at a bending angle of 45°. In this configuration, the whole cross-section of the dowel is assumed to be under plastic strain [15]. However, it is argued that when bolted connections are tested, and the failure modes achieved are characterized by the bending of dowels, the bending angles often lie below 45° (see [16]). In this scenario, the plastic capacity of the dowels is partially used. The effective bending moment resides between the elastic ($M_{y,el} = 0.8 \cdot f_u \cdot \pi \cdot d^3/32$) and plastic ($M_{y,pl} = 0.8 \cdot f_u \cdot d^3/6$) bending capacity of the dowels' cross-section, considerably lower than the results achieved through EN 409 [14]. Here, f_u is the fastener tensile strength. Blass et al. [15] proposed a correction factor $\overline{M}(\alpha)$, given in Eq. (3), to address this partially mobilized plastic moment. The effective bending moment $M_y(\alpha)$ is given by the product between the plastic bending capacity $M_{y,pl} = M_y(\alpha = 45°)$ and the factor $\overline{M}(\alpha)$ (see Eq. 4).

$$\overline{M}(\alpha) = (0.866 + 0.00295\alpha)\left(1 - \exp\left(\frac{-0.248\alpha}{0.866}\right)\right) \tag{3}$$

$$M_{y,eff}(\alpha) = \overline{M}(\alpha) \cdot M_y(\alpha = 45°) \tag{4}$$

Therefore, in order to calculate the effective bending moment, one must know α. The bending angle can be measured directly from the load-carrying experiments or obtained by theoretical approaches such as the one presented by Blass et al. [15]. The expressions to assess α are derived based on equilibrium conditions. A scheme of the connection acting forces is given in Fig. 2.

From Fig. 2, α can be written in the format of $\alpha = \arctan(\delta/l)$, where δ is the maximum deformation ($\delta = 15$ mm) according to the guidelines of EN 26891 [17], and l is the length where the embedding strength is reached. Obtaining the geometry variables x_1, x_2, y_1 and y_2 from equilibrium conditions (see [16]), α can be determined from Eq. (5).

$$\alpha = \arctan\left(\frac{\delta}{\frac{t_1}{2} + \frac{t_1}{2+\beta_{f_h}}[a]\left(\frac{1}{2} + \beta_{f_h}\right)}\right)$$

where,

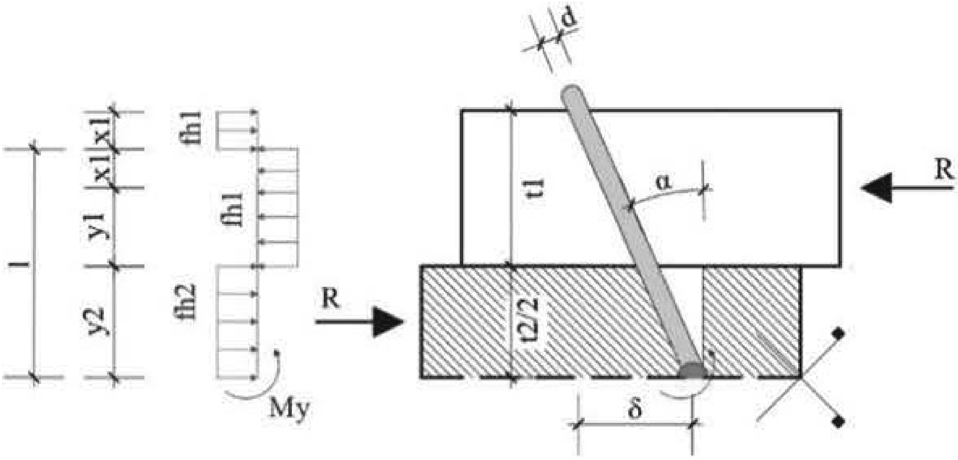

Fig. 2 Dowels' bent in double shear

$$a = \sqrt{2\beta_{f_h}\left(1+\beta_{f_h}\right) + \frac{4\beta_{f_h}\left(2+\beta_{f_h}\right)\cdot M_{y,eff}}{f_{h,1}\cdot d\cdot t_1^2}} - \beta_{f_h} \tag{5}$$

According to Eq. (5) α depends on M_y. The dependence between these variables can be considered by an iterative procedure using Eq. (6). A first estimation is taken as $\alpha = 45°$. Subsequently, the $\overline{M}(\alpha)$ factor in Eq. (3) is inserted in Eq. (5). The theoretical angle α usually converges after three iteration steps [15], considering a tolerance of 1°. Once α is determined, the effective bending moment $M_{y,eff}$ can be calculated through Eq. (4).

$$\alpha_{i+1} = \arctan\left(\frac{\delta}{\frac{t_1}{2} + \frac{t_1}{2+\beta_{f_h}}\left[a_{i+1}\right]\left(\frac{1}{2}+\beta_{f_h}\right)}\right)$$

where,

$$a_{i+1} = \sqrt{2\beta_{f_h}\left(1+\beta_{f_h}\right) + \frac{4\beta_{f_h}\left(2+\beta_{f_h}\right)\cdot \overline{M}(\alpha)\cdot M_y(\alpha = 45°)}{f_{h,1}\cdot d\cdot t_1^2}} - \beta_{f_h} \tag{6}$$

2.3 Dependence Structure Investigation

When $M_{y,eff}$ is determined following the approach presented by Blass et al. [15], the parameter is related to the wood embedment strength f_h. In addition, from Eq. (2), it can be seen that f_h is mainly related to the timber density ρ. Therefore, one can state that both f_h and $M_{y,eff}$ are related to the timber density ρ. Since both mechanical properties share a dependency of timber density, their statistical correlation can be addressed.

Copula functions may be employed to represent the statistical dependence structure, since they are capable of modeling non-linear correlated behavior [18]. Long-established reliability methods commonly adopt the Nataf transformation (see [19, 20]) to consider linear correlation coefficients on the data, which is equivalent to a Gaussian copula correlation construction, according to Lebrun and Dutfoy [21]. However, Wang and Li [18] highlights the fact that the implicit Gaussian dependence structure assumed is not necessarily true and may bias the reliability results.

Therefore, based on theoretically generated data, copula functions are inferred to consider the correlation between f_h and $M_{y,eff}$. Independent and Gaussian copula functions are fitted to data, and these dependence structures are compared with the inferred non-linear copula function. Such comparison is done in terms of the reliability indexes obtained by using different dependence structures.

3 Reliability Analysis and the Copula Theory

The main objective of a structural design is to fulfil the requirements for which it is being designed. Thus, the capacity of the system must exceed the demand. There are always uncertainties involved in the representation of the main variables considered in the structural design and other sources of uncertainty. Thus, unfavorable combinations of these random variables may lead the structure to reach ultimate and/or service limit states. This event can be described by using a limit state function, given as follows:

$$g(\boldsymbol{X}) = R(\boldsymbol{X}) - S(\boldsymbol{X}) \tag{7}$$

where $\boldsymbol{X}$ is vector of random variables; $R(\boldsymbol{X})$ is the random variable regarding to resistance; and $S(\boldsymbol{X})$ is the random variable representing the load effect.

Failure occurs if $g(\boldsymbol{X}) \leq 0$. The probability of failure is obtained by integrating the joint probability density function of the random variables $f_X(\boldsymbol{x})$ over the failure domain:

$$P_f = P[g(\boldsymbol{X}) \leq 0] = \int\limits_{g(\boldsymbol{X}) \leq 0} f_X(\boldsymbol{x}) d\boldsymbol{x} \tag{8}$$

The probability of failure (P_f) of a given structural system may be difficult to determine because it depends on the joint probability density function of the random variables in the failure domain and the problem may involve lots of random variables. In this context, approximate methods, such as Monte Carlo Simulation, are usually employed for reliability analysis.

The Monte Carlo simulation method consists of generating samples of the random variables according to their joint probabilities, and evaluating, through a numerical or analytical model, the structure response for these samples. After several simulations, a statistical analysis is performed to determine the probability of failure and the corresponding reliability index [22, 23]. It should be emphasized that the lower the probability of failure, the greater the number of simulations (n_s) required for convergence. In order to accelerate the convergence of the simulation, one can use a number of techniques presented in the literature, such as the so-called importance sampling. In this case, there is an attempt to generate samples closer to the limit state boundary, which accelerates convergence. Adopting an importance sampling distribution $h_X(x)$, and using an indicator of failure $I[\boldsymbol{X}]$, where $I[\boldsymbol{x}] = 1$ if $g(\boldsymbol{x}) \leq 0$, and $I[\boldsymbol{x}] = 0$ if $g(\boldsymbol{x}) > 0$, P_f can be estimated as in Eq. (9), where the ratio $f_X(\boldsymbol{x}_i)/h_X(\boldsymbol{x}_i)$ is the weight of each simulation [23].

$$P_f \cong \sum_{i=1}^{i=n_s} I[\boldsymbol{x}_i] \frac{f_X(\boldsymbol{x}_i)}{h_X(\boldsymbol{x}_i)} \tag{9}$$

The importance sampling distribution may be obtained by centering the original distribution at the most probable failure point (MPP) over the failure surface, since this is the point that contributes the most to the P_f. The reliability problem can be transformed to the standard normal space, where all random variables are converted to equivalent variables with standard normal distribution. In this space, the MPP correspond to the closest point between the origin and the fault surface $g(\boldsymbol{X}) \leq 0$, such distance is defined as the reliability index β, which refers to the safety level of a structure. This study employees the Finite-Step-Length algorithm (FSL) to search for the MPP [24].

In reliability assessments of structures, the problem usually involves more than one random variable. Consequently, it is necessary to characterize their joint behaviour. With respect to the dependence structure, copula functions allow the modeling of non-linear correlated behaviour, which cannot be achieve through traditional approaches (e.g. the Nataf transformation [20]). Copulas are defined as functions that join or "couple" multivariate distribution functions to their one-dimensional marginal distribution functions. An M-copula is defined as an M-variate joint cumulative distribution function CDF $C : [0, 1] \rightarrow [0, 1]$ with standard uniform marginals [25]. Sklar's theorem [26] allows to express joint CDFs in terms of their marginal distribution and a copula that represents the multivariate dependence structure. Considering a random vector $\boldsymbol{X} = (X_1, X_2, \ldots, X_M)$, the theorem states that for its M-variate CDF, referred as $F_{\boldsymbol{X}}$, with marginals CDFs, referred as $F_1, \ldots, F_M$, an M-copula $C_{\boldsymbol{X}}$ exists, such that for all $\boldsymbol{x} \in \mathbb{R}^M$ [25]:

$$F_{\boldsymbol{X}}(\boldsymbol{x}) = C_{\boldsymbol{X}}(F_1(x_1), \ldots, F_M(x_M)) \tag{10}$$

Given $U_i = F_i(x_i), i = 1, \ldots, M$, the copula from (10) is unique and has the expression:

$$C_{\boldsymbol{X}}(\boldsymbol{U}) = F_{\boldsymbol{X}}\left(F_1^{-1}(U_1), \ldots, F_M^{-1}(U_M)\right), \quad u \in [0, 1]^M \tag{11}$$

where $F_i^{-1}(U_i)$'s are the marginals inverse CDF's.

Thus, the construction of the joint distribution consists in two separate problems. First, it is required to model the marginals F_i, this can be done through an inference process based on data (see [22]). Then, it is necessary to transform the original components of X_i into uniform random variables $u_i = F_i(X_i)$. Given that the joint probability density function PDF $f_{\boldsymbol{X}}(\boldsymbol{x}) = dF_{\boldsymbol{X}}(\boldsymbol{x})/d\boldsymbol{x}$, and generalizing it to multiple variables, $f_{\boldsymbol{X}}$ can be derived using the chain rule as:

$$f_{\boldsymbol{X}}(x_1, \ldots, x_M) = c_{1 \ldots M}\{F_1(x_1), \ldots, F_M(x_M)\} \prod_{i=1}^{M} f_i(x_i) \tag{12}$$

where $c_{1 \ldots M}(\cdot)$ is an M-variate copula density function. Therefore, one can look at a copula function as what remains of the joint cumulative distribution once the effect of the marginal distribution function has been removed [21]. To fully determine the joint distribution behaviour, a copula family needs to be assigned to the data available. In this study, the copula inference process is based on theoretical data and done by using the open source package Vine Copula Matlab [27].

4 Results and Discussion

The analysis is based on the limit state function g which is given as follows:

$$g = 2z_d R - S_G - S_Q \tag{13}$$

where z_d is the design variable given in Eq. (14), R is the load-carrying capacity given in Eq. (1), and S_G and S_Q are the permanent and variable loads, respectively.

$$z_d = \frac{\gamma_G S_{G,k} + \gamma_Q S_{Q,k}}{2R_k}\gamma_M \tag{14}$$

The subscript k refers to the characteristic value of the loads, and γ_G, γ_Q, and γ_M are the partial safety factors related to the permanent and variable loads and resistance, respectively. Table 1 presents the statistical information of the input variables involved in the problem. The random variables f_h and $M_{y,eff}$ are determined from ρ and f_u through the approaches presented in Sects. 2.1 and 2.2.

With respect to the dependence structures, Figs. 3 and 4 show the scatter plot and the respective inferred copula function from theoretically generated data for the side and middle members, respectively. The generated data consists of 1000 samples obtained via Monte Carlo method, without importance sampling. For the side members, $M_{y,eff}$ tends to decrease with the increase of $f_{h,1}$, therefore, the variables are negatively correlated. The copula inferred for the side members is the Gumbel one with $\theta = 1.8592$ rotated 270°. When the Gaussian copula is considered, the copula parameter is $\theta = -0.6554$. Regarding the middle member dependence structure, the correlation behavior is reversed, that is, $M_{y,eff}$ tends to increase with the increase of $f_{h,2}$. The copula function inferred is the Gaussian one with $\theta = 0.7333$. Therefore, since the Gaussian is the copula inferred from the theoretical data, it is

Table 1 Statistical information of the input variables (based on [3])

	ρ (kg/m^3)	f_u (MPa)	S_G (N)	S_Q (N)
Distribution	Normal	Lognormal	Normal	Gumbel
Mean value	450	427	1000	1200
St. Dev.	45	17	100	480
Fractile	5%	5%	50%	0.4
Char. value	376	400	1000	2444
	$\gamma_M = 1.3$		$\gamma_G = 1.35$	$\gamma_Q = 1.5$

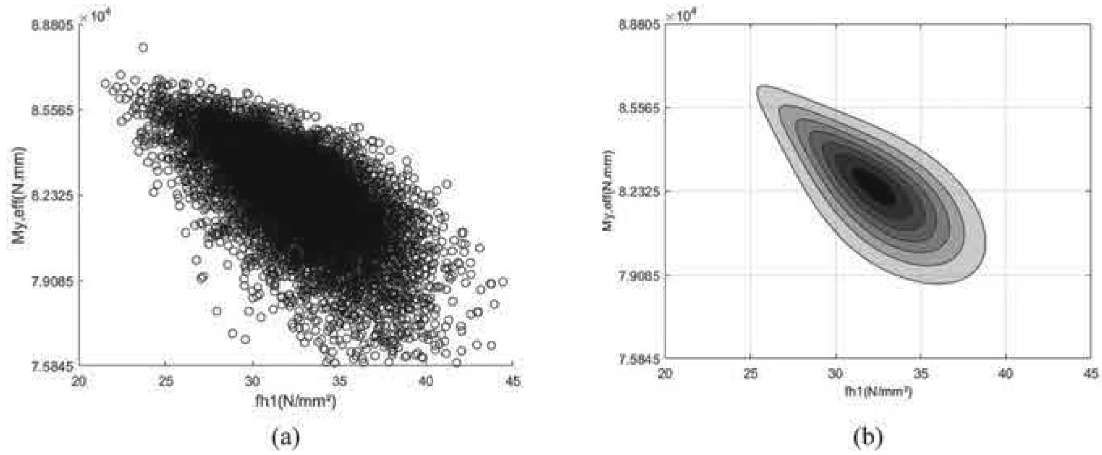

Fig. 3 Joint behavior of the generated data for side members **a** scatter plot, and **b** joint distribution

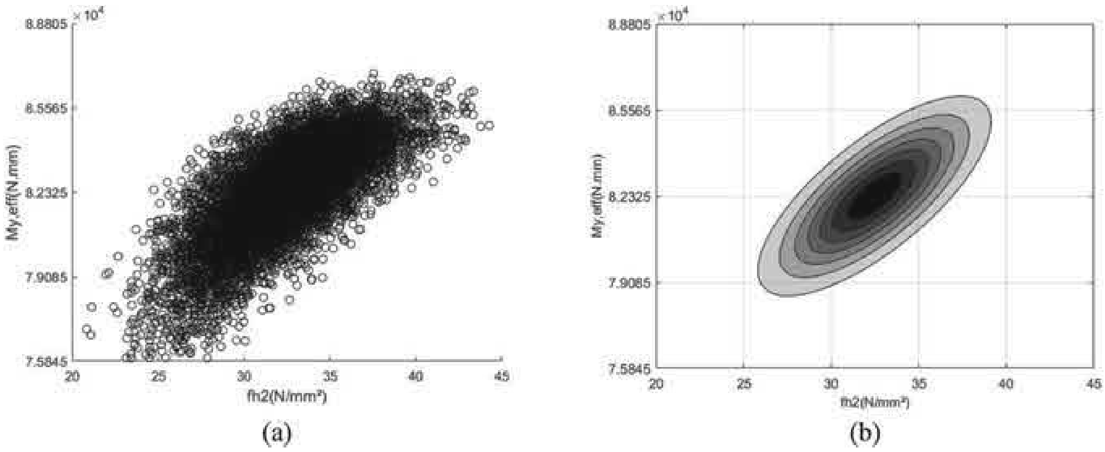

Fig. 4 Joint behavior of the generated data for middle members **a** scatter plot, and **b** joint distribution.

adopted either when the Gaussian and the Gumbel copula is considered for the side members.

The results in terms of the reliability index are given in Fig. 5 and are referred according to the side members dependence structures considered: Independent, Gaussian, and Gumbel.

It is found that the correlation model does not present a great influence on β in this case. If sensitivity indexes are computed by using the First Order Reliability Method, it is noted that the parameter with greater influence on the probability of failure is S_Q (variable load). By reducing its coefficient of variation in an attempt to increase the impact of the copulas on the results, it is seen that the reliability indexes found are still very similar. This indicates that, unless significantly nonlinear correlations exist among the data, the results obtained by applying the different copulas will probably be very close. However, it is difficult to establish beforehand if the correlation is nonlinear enough, since the reliability indexes are also very dependent on the limit state functions and on the other variables. Nevertheless, the results shown herein

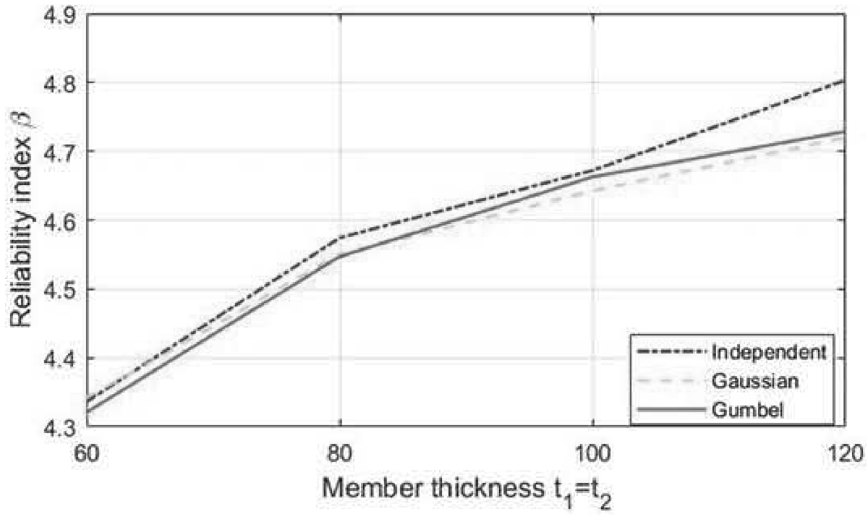

Fig. 5 Reliability results

indicate that for this particular case study a simpler approach, *e.g.* using the Nataf transformation [19], would be enough to deal with the correlation.

5 Conclusions

This study investigated the correlation behavior between the timber embedment strength and the dowel bending moment capacity, which are two parameters of significant impact on the design of dowel-type timber joints. Since, to the best of the authors knowledge, the correlation between them had not been addressed in the literature yet, traditional distribution fitting as well as copula functions were implemented to describe their statistical dependence structure. The results showed that the impact of correlation modeling on the results is small. In this context, it is indicated that for the study of single dowel- type connections, especially for the failure mode where one plastic hinge occur per shear plane, the copula functions are not necessary, and the correlation can be dealt with a simpler approach. Nevertheless, it is crucial to conduct further investigations related to distinct failure modes, specially the one that comprises two plastic-hinges per shear plane. Moreover, further studies shall include connections with multiple fasteners.

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