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Magic squares

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Abstract: Magic squares are an important part of recreational mathematics. And they can be used in mathematics education, through problem solving, explorations, investigations, even with algorithmic procedures. Magic squares, on the other hand have not been invented in western mathematics, they were known in China and certainly were passed on from Africa to Europe (Gervais, 1997).
We will explain what a magic square is, which types exist, give some examples of methods to build them, and suggestions to their historical roots. Then we will solve some tasks based on magic squares and discuss the possibilities for mathematics education. Our claim is that magic squares are a rich environment to reflect on mathematics education.

Résumé: Les carrés magiques sont une partie importante des mathématiques récréatives. Et ils peuvent être utilisés dans l'enseignement des mathématiques, pour la résolution de problèmes, des explorations, des enquêtes, même avec des procédures algorithmiques. Les carrés magiques, en revanche, n'ont pas été inventés dans les mathématiques occidentales, ils étaient connus en Chine et ont certainement été transmis d'Afrique à Europe (Gervais, 1997). Nous allons expliquer ce qu'est un carré magique, quels types existent, donner quelques exemples de méthodes pour les construire et des suggestions pour leurs racines historiques. Nous allons ensuite résoudre certaines tâches basées sur des carrés magiques et discuter des possibilités d'enseignement des mathématiques. Nous affirmons que les carrés magiques constituent un environnement riche pour réfléchir à l'enseignement des mathématiques.

Magic squares

If all the lines, columns and diagonals have the same sum, it's a magic square. Missing the diagonals, it's called half magic. A magic square is said to be pandiagonal when the sum of all diagonals, including broken ones, is equal to rows and columns (Rouse Ball, 2010). With only these definitions, we have already a lot to think about.
Ponte (2005) has developed a classification of mathematical tasks that is very useful. According to him, we can analyze mathematical tasks following two criteria, their degree of challenge and their degree of openness. Then, if a task is challenging and closed it is a problem. If it is closed but contain little challenge, it is an exercise. If it is open with reduced challenge, then it is an exploratory task. Finally, when it is open and challenging then it is an investigation task. Examples from each type of task can be constructed in the context of the magic squares.

Exploratory tasks

We will start with exploratory tasks. An exploratory task has reduced challenge and it is open. We could state one like this:
This magic square was invented by Benjamin Franklin (Fonseca, 2005). The magic sum is 260.

There are many combinations of numbers that make sum 260. Also, there are many combinations of four numbers that have sum 130. Can you find other 8 number combinations of sum 260? And of 4 numbers to sum 130?

52	61	4	13	20	29	36	45
14	3	62	51	46	35	30	19
53	60	5	12	21	28	37	44
11	6	59	54	43	38	27	22
55	58	7	10	23	26	39	42
9	8	57	56	41	40	25	24
50	63	2	15	18	31	34	47
16	1	64	49	48	33	32	17

None of the questions above are much difficult. Since it is an open task, when students start looking for the number combinations they will probably follow different paths, with different answers.

EXERCISES

8	1	6
3	5	7
4	9	2

A method to construct magic squares of odd order is to place the number 1 in the central square of the top line, then always placing the next number diagonally to the top and right. If it falls outside the square, we have to imagine the rectangle bending to become a cylinder. If it's already occupied, or in the continuation of one main diagonal, we place the number just under the number already placed (Rouse Ball, 2010). Try to follow the placement of the numbers in the order 3 magic square given above to understand the method. Then construct an order 5 magic square.
This task is not very difficult since it is about following rules. That is clearly an exercise. The answer has to be the same, the procedure is known and is the same for every student.

Problems

This magic square below is a good example of a pandiagonal magic square. Not only all horizontal, vertical and diagonal line all add up to the same value, the broken diagonals too:

20	8	21	14	2
11	4	17	10	23
7	25	13	1	19
3	16	9	22	15
24	12	5	18	6

$20+12+9+1+23 = 8+11+5+22+19 = 7+4+21+18+15 = 3+25+17+14+6 = 65$

In addition to the rows, columns, and diagonals, a 5×5 pandiagonal magic square also shows its magic sum in four types of "quincunx" patterns:

20+2+6+24+13 = 20+21+13+7+4 = 21+19+5+7+13 = 8+17+25+11+4 = 65
Using these properties, can you finish the pandiagonal below (Fonseca, 2005)?

		7		
			15	
	12	4		
1	8	20	22	
	24			

This task contains a series of sometimes very difficult situations, sometimes not so difficult, but reasoning and decision making are necessary at all times. The path to the solution does not have to be unique, but the solution is unique.

INVESTIGATION TASKS

I have a friend whose birthday will be on the 17 of July, 2018. I want to offer her a magic square whose first line contains in sequence, 17, 7, 18. Is it possible?

17	7	18

In this kind of task, we start with a problem that is not so difficult in order to students get a sense of it. Then after they answer it, we introduce new questions and later encourage them to pursue their own interest. One possible follow up that will inevitably send students in different paths is "Can you use this idea to construct your own birthday magic square?" Many students will have lines whose sum is not a multiple of three. This means that they will leave the comfort zone of natural numbers and several lines of inquiry will be possible from there, expanding the notion of magic square and the discovery of possible properties.

Discussion

We think there are advantages of using magic squares to illustrate the differences between different kind of tasks. First, it's something many people heard about but probably does not know much about. So, there is something new and interesting to be learnt. Second, it is not so hard to understand, therefore the content is not an obstacle.

From the experience of solving different kind of tasks emerges reflection about their potentialities and benefits that can enrich a final discussion. In my experience that discussion is always rich and built upon the recent common experience students had.

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