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Using a picture to challenge creativity in mathematics class with 1st and 6th graders

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Abstract:

This paper focuses on first- and sixth-graders' performance on posing problems. It addresses three questions: 1) How do these students understand problem-posing tasks? 2) What sort of intentions do these students have? 3) What difficulties do students have when posing problems? Qualitative methods with a case study approach were used to describe students' reactions when challenged to formulate a problem using a picture. Results revealed that students were able to pose interesting problems, suggesting some knowledge systematisation, and opportunities to building new learnings had emerged, in spite of having some difficulties in terms of the complexity and variety of their formulations.

Keywords: mathematics education, creativity in mathematics, problem posing.

Introduction

In the XXI century, creativity has been seen as one of the most critical issues in education. We live in a dynamic world, where inventiveness and originality play a key role in our society. Like many other skills, proficiency can only be obtained by practice and, as a result, our schools must be able to develop and expand creativity, in as many ways as possible. Therefore, it is crucial that the issue of creativity must be a focal point in education nowadays.

One of the main goals of mathematical education in today's view is to light a spark in the creative thinking process (Ayllón, Gómez, & Ballesta-Claver, 2016). Gone are, or should be, the days of a passive and monotonous mathematics education. This is no longer suitable for XXI century students, not only because it does not promote key aspects in education, but also because students' profiles have changed. In a society where everything is dynamic, we cannot have a traditional mathematics class, which would not be able to promote an effective knowledge construction by students. Such a class would become "boring to them" (Agić & Rešić, 2015, p.11). The main purposes of mathematics education are to promote mathematical creativity (Aktaş, 2016), to arouse interest around new learnings, and to make a significant connection between the structured curriculum and the lives and interests of students. The concept of creative mathematics should not be only in seminars, but mostly in schoolrooms, in everyday mathematics tasks, and in mathematical reasoning throughout the learning process.

Problem solving, and specifically problem posing activities, can be regarded as a path to explore creativity in mathematics classes. In the past few decades, the attention of mathematics educators, teachers, and researchers, has been caught by the relevance of mathematical problem posing as a way for the development of students' understanding and creation of mathematical knowledge (Harpen & Presmeg, 2013). The action of posing a problem, links to a number of relevant cognitive processes such as creating an appropriate question, analysing the formulation critically, examining the data used in such formulation, and processing possible solving strategies that allow obtaining success in the resolution of such problems (Ayllón, Gómez, & Ballesta-Claver, 2016). All of these are considered crucial to strengthening mathematical knowledge and building the bridge between theory and practice.

Theoretical framework

Despite the recent emphasis toward the importance of creativity in education, the scientific community has not yet reached a consensual definition of creativity (Agić & Rešić, 2015; Akgul & Kahveci, 2016; Aktaş, 2016), and some even suggest that there is no "unique and authoritative point of view or definition of creativity" (Agić & Rešić, 2015, p.12). However, there are some attempts and some guidelines that can help trace a profile to identify it.

The National Advisory Committee on Creative and Cultural Education (NACCCE, 1999) coined one of the most commonly used definitions of creativity, stating creativity as "imaginative activity fashioned so as to produce outcomes that are both original and of value" (NACCCE, 1999, p.30). Other attempts have been made, such as Kandemir and Gür (2007), that defined creativity as an idea or product that "should be new, original and appropriate" (p.108), suggesting that is a "thinking process that results in new, unusual, and insightful solutions to a given problem that can branch from any difficulty, by thinking divergently and looking at problem solving using new perspectives" (Kandemir & Gür, 2007).

Historically, creativity in mathematics was not seen as a skill to develop in students; it was reserved for academics (Silver, 1997; El-Sahili, Al-Sharif & Khanafer, 2015). Nowadays, however, the point of view has changed and, therefore, mathematical creativity is seen as a skill that can be promoted and developed in schools (Silver, 1997; Leikin, 2009; El-Sahili, Al-Sharif & Khanafer, 2015). Other goals have merged with the traditional purpose of transmitting knowledge, like learning how to learn, to collaborate, and to be autonomous (Haddad, 2012). Creativity in education goes a step even further and requires us to "learn how to unlearn, to doubt, to call ourselves into question, to rebel against established paradigms, and to seize opportunities, so that we can clear new pathways, taking us into uncharted territories" (Haddad, 2012, p. 2).

Problem-solving, and specifically problem-posing tasks, has emerged, over the years, as an important strategy for the development students' understanding of mathematics (Kilpatrick, 1987; Silver, 1997; Pólya 1995; Vale & Pimentel, 2004; Boavida *et al.*, 2008; Palhares 1997; Dante, 2009; Harpen & Presmeg, 2013), stimulating the enhancement of creative skills. The National Council of Teachers of Mathematics (2007) states that opportunities should be given to students to solve mathematical problems using multiple solution strategies, and to formulate and create problems from given situations.

Silver (1997) considers that the formulation of problems refers either to the conception of new problems or to the reformulation of a given problem. For Palhares (1997), the formulation of problems occurs when an individual invents or discovers a problem, which can arise in conjunction with problem solving. According to Pólya (1995), Kilpatrick (1987), Ernest (1991), and Dante (2009), articulation enhances the success of the mathematical learning process, contributing positively to the development of problem-solving skills, while promoting deepening of the mathematical concepts involved, stimulating thinking and reasoning. Silver goes along the same line, stating that “the connection to creativity lies not so much in problem posing itself, but rather in the interplay between problem posing and problem solving. It is in this interplay of formulating, attempting to solve, reformulating, and eventually solving a problem that one sees creative activity” (1997, p. 76).

Stoyanova (1998) identifies three categories of problem-posing situations: free situations - students formulate problems without any restrictions; Semi-structured situations - students formulate problems similar to others they know or based on figures or diagrams; Structured situations - students create problems by reformulating problems already solved or by changing conditions or issues in a known problematic situation. In this way, it is through tasks in the different spheres that the development of new learning becomes possible (Stoyanova, 1998), making it necessary to adapt the tasks to the contexts, as well as enhance the exploration of different problem formulation strategies.

Regarding the problem-posing analyses, it is important to emphasize that “there is no standardized technique for measuring creativity” (Agić & Rešić, 2015, p.12). Therefore, the criteria of analyses must always take that into consideration. Analysing in the light of the criteria of mathematical creativity, present in Silver (1997), three categories can be used to analyse the students' productions: fluency, flexibility, and novelty. Pinheiro and Vale (2013) interpret that fluency corresponds to the number of problems raised that fit the requirements of the task; flexibility corresponds to the number of different types of problems posed; novelty corresponds to the number of problems posed that are unique or rare.

Vale and Pimentel (2004) emphasize the relevance of problem posing for the development of students is unquestionable, as it is a fundamental activity that contributes considerably to the understanding of mathematical concepts by providing a review of both the process needed to solve a problem and the content developed. Vale and Pimentel also warn that teachers have to be attentive to the situations they are confronted with, which can be provoked or occasional, to recognize their mathematical potential and from them be able to transform them into challenging and mathematically rich situations for students (2004).

The official curriculum guiding documents in Portugal highlight the importance that creativity assumes in the educational process (Martins et al., 2017). This should arise in the most varied areas, with explicit references to problem posing as having a determining role. In the documents called ‘Aprendizagens Essenciais’ [Essential Learnings] (DGE, 2018), approved by Normative Order No. 6944-A / 2018, of 19 July, it is suggested that students must develop the ability to solve and pose problems (DGE, 2018, p. 2), while in the document called ‘Perfil dos Alunos à Saída da Escolaridade Obrigatória’ [Profile of Students Leaving Mandatory Schooling] (Martins et al., 2017) the posing of new questions is also mentioned (Martins et al., 2017, p. 23) as a relevant teaching vector. However, the practical application of these guidelines has been taking a long time to materialize, being still crucial to promote awareness of the importance of these types of tasks for the development of students’ creativity.

There are few studies developed in this field, and a relevant opportunity arises to explore the ideas that students have on these subjects. These studies are hardly found in the 1st Cycle of Basic Education (1st to 4th- grades), and even in the 2nd Cycle of Basic Education (5th – 6th – grades) they are not common in Portugal. Therefore, this study aims to have an insight of 1st and 6th - graders when posing problems. Three questions were addressed: 1) How do these students understand problem-posing tasks? 2) What sort of intentions do these students use? 3) What difficulties do students have when posing problems?

Methods

Qualitative methods were used (Bogdan & Bicklen, 2006) in a case study approach (Yin, 2014) to have an insight of students' performance when formulating problems. The participants were a group of 22 first-graders (6- to 7-years-old) and 30 sixth-graders (10-11-years-old) who attended public schools, in Braga, Portugal.

Student's reactions were observed when a task of formulating problems was presented to them. In this task, the students were given a picture (see Figure 1) and were challenged to formulate a problem. This task is a semi-structured situation of problem-posing, in agreement with Stoyanova (1998).

Figure 1: Picture presented to students of the 1st and 6th – grades.
(Credits: Dreamstime image bank)

In the 1st-grade, the students were challenged to solve the task individually. The task resolution lasted approximately 20 minutes. In the 2nd –grade the students were organised into pairs and were challenged to use the picture to formulate a problem regarding one of the 17 global Goals of the Sustainable Developmental of the United Nations Organization. This task lasted 10 minutes for the 6th - graders.

During the task resolution, students were supported by the researcher, giving them the opportunity to learn by action, challenging them to formulate the problems by their own. In all the moments, students were free to interact with each other and with the researcher, one of the authors of this paper.

Data collection was carried out using photos, audio and video records of student's performance when solving the task. Field notes were also used by the researcher.

Results

The 1st - graders

When solving their first task of formulating problems, students felt a bit lost in the beginning, as it was something absolutely new for them, but quickly realised that they could create a situation as they wish, feeling motivated to formulate their problems.

Students resolutions of the tasks were analysed attending to their written productions and to their justifications. As 1st graders were not yet able to write long sentences, oral explanations of their resolutions were asked for each task. It is interesting to realise that students' poor preparation to accomplish written justifications did not limit their ability to pose problems, with a great diversity of creations, especially at the level of fluency and novelty. Figure 2 presents an example of a problem formulation given by Alberto, based on one of the elements presented in the picture provided in the task.

Figure 2: Problem statement created by Alberto.

Alberto explained that "*O Alberto tinha três casas. Desapareceu uma. Fiquei com duas*" "Alberto had three houses. One disappeared. I got two". In this example, Alberto focused only on one element in the image and extrapolated a hypothetical situation in which that element appears as the only protagonist. The use of subtraction in the formulation stands out, realizing that the student intends to use a more complex algorithm, compared to addition. This is a relevant fact since most students prefer to use simpler content in their formulations, as a systematization process, instead of risking the use of more complex algorithms.

It is also possible to emphasize the relationship between problem posing and problem solving processes, realizing that it is relevant that these two elements appear interconnected, allowing the student to build a statement, try to solve it, reformulate its construction, and manage to reach a solution, in a dynamic and holistic process.

In another example of resolution provided by 1st graders, the student attended only to one element of the given picture to formulate the problem. Figure 3 shows Ricardo's resolutions of the task and Transcription 1 gives his explanation of the resolution.

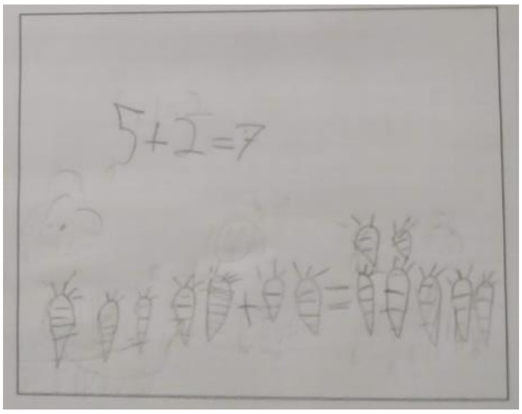


Figure 3: Problem statement created by Ricardo.

Ricardo – Ricardo had five carrots. And planted two more carrots. How many carrots does he have now?
Teacher – Very well, Ricardo. And how many do you have now?
Ricardo – I have seven!

Transcription 1 – Ricardo’s explanation of his resolution.

It is noteworthy that the Ricardo decided to focus on only one element of the picture, namely carrots, for the construction of his statement. Although this student opted for the use of addition, the use of higher numbers stands out, since they only had learned to operate with numbers up to ten, in their school context. It is also relevant to highlight a detail related to the awareness of the student in already realizing the need to incorporate a question in the production of problem statements. This characteristic is identified in literature as an indicator of progress (see Pinheiro & Vale, 2013).

A third formulation example is shown in Figure 4, in which Marco formulated the problem and then felt the need to revise it. The first attempt was “*Eu tinha três vasos mais cinco coelhos, fiquei com oito”* “I had three pots plus five rabbits, I had eight ”, but since it was not clear, it was asked to explain what he meant and then he said “*Eu quero saber com quantas coisas fico, três vasos mais cinco coelhos, fiquei com oito coisas*” “I want to know how many things I have, three pots plus five rabbits, I have eight things”.

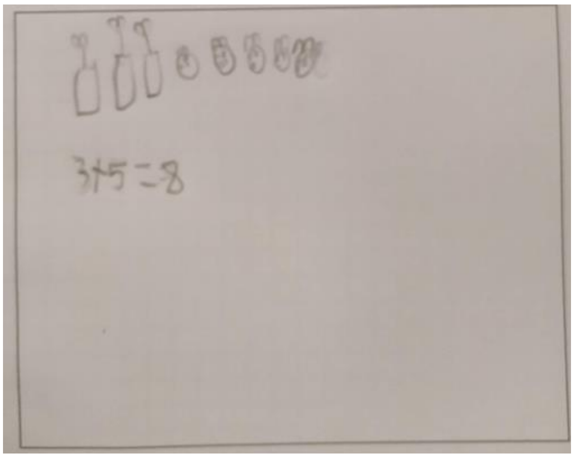


Figure 4: Problem statement created by Marco.

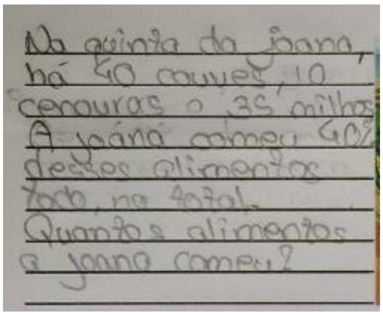
It should be noted that, in this case, Marco decided to formulate the problem based on two different elements, which led to confusion in the initial posing process. This made the student feel the need to reformulate his statement so that it became cohesive and coherent. Once again, it is also emphasized that the formulation was based on the learning that was being developed by the students, in this case, the development of the addition algorithm.

Many other resolutions were presented by the first-graders, but with some affinity to the ones presented here as examples. From the analysis of the different formulations presented, it was notorious that students build problems through the application of content they knew, creating scenarios that were meaningful to them.

The 6th - graders

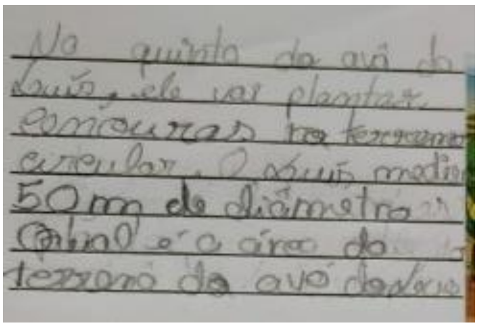
The same task was given to the 6th-graders to understand the impact of the picture with older students. The analysis carried out here focused on students' formulations will be presented based on students' written statement and their resolution of the problems created. In this context, and similarly, the fluency and novelty criteria in the formulations developed also stood out.

In the following two examples present the most common type of problems that were obtained with these students, in which the formulations created were related to the mathematical contents that were being acquired by the students. Thus, in general, the problems created were related to algebra, namely to the rule of three (Figure 5), and to geometry, in area computation situations (Figure 6).



"At Joana's farm there are 40 cabbages, 10 carrots and 35 corn. Joana ate 40% of these foods in total. How much food did Joana eat?"

Figure 5: Problem statement created by Maria.



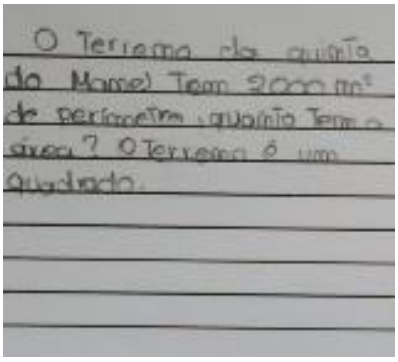
"At Luís's grandfather farm, he will plant carrots on the circular plot. Luís measured 50 meters in diameter. What is the area of the land of Luís's grandfather?"

Figure 6: Problem statement created by Elsa.

In both productions, it is possible to verify that the students already demonstrated awareness of the necessary elements for the formulation of problems. In some way, this fact would be expected since these students have already contacted and solved many mathematical problems, throughout their school career. Nevertheless, it is noteworthy that many students felt less confident in this task, as these sorts of challenges has not been very popular in their mathematical classes. However, it is emphasized that posing-problem tasks should be more frequent in school classes, giving more confidence to students to take risks and to create more out of the box situations.

It is possible to identify some common factors in the different productions created by students. One factor concerns to the relationship between problem posing and problem solving. The students intuitively relate the two processes to build a coherent mathematical reality. This leads to the emergence of a process of construction, reflection, reformulation, and resolution of problems, which will be fundamental to consolidate constructed learning and promote the discovery of new processes.

An example of the development of this process is shown in Figure 7. The process of revising the construction of the statement and awareness of the data, that were missing for the problem to be resolved, is clear:

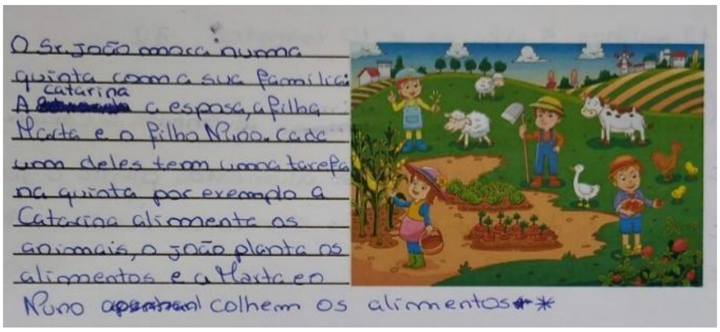


"The plot of Manuel's farm has 2000m of perimeter, how much area does it have? The plot is square."

Figure 7: Problem statement created by Sandra.

In this statement formulated by a student, the addition of the information that 'it is a square' stands out after the question of the problem. This data is essential to be able to solve the problem, but this information appears only added at the end, suggesting a process of final review and metacognition about the necessary data.

Another example of the posing process is related to the identification of the intermediate resolution of the problem, by the student, as the formulation was developed (Figure 8). Transcription 2 provides the translation of Raquel's problem statement.



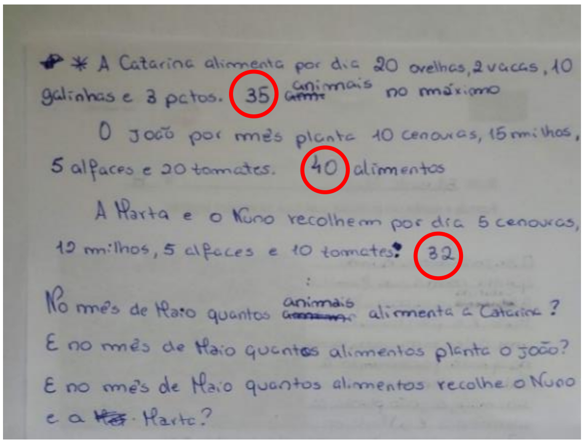


Figure 8: Problem statement created by Raquel.

"Mr. João lives on a farm with his family, Catarina, his wife, daughter Marta and son Nuno. Each of them has a task on the farm, for example, Catarina feeds the animals, João plants the food and Marta and Nuno collect the food.

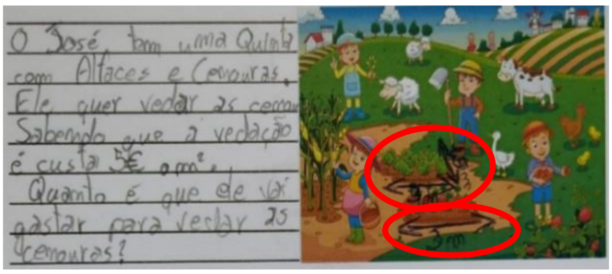
Catarina feeds 20 sheep, 2 cows, 10 chickens and 3 ducks a day. 35 animals maximum. João, per month, plants 10 carrots, 15 ears of corn, 5 le ttuces and 20 tomatoes. 40 foods. Marta and Nuno collect 5 carrots, 12 corn, 5 lettuces and 10 tomatoes a day. 32.

In May, how many animals does Catarina feed? And in May, how many foods does João plant? And in May, how much food does Nuno and Marta collect?"

Transcription 2: Translation of the problem statement.

In this statement, the presence of numbers highlighted with red circles stands out. These numbers were written throughout the formulation process, not when solving the problem (which was developed in pencil), indicating that the student was solving the problem mentally, as he was formulating it, in order to be able to control the data and processes used. This process of self-regulation and metacognition is central to mathematical learning process and to autonomously develop their mathematical knowledge.

Another interesting case arises from the formulation of a student who decided to integrate part of the chosen data to pose the problem in the image provided, replicating the typical model found in school textbooks (Figure 9). This problem also represents another theme that emerged in the students' formulations, creating situations related to measurement problems.



"José has a farm with lettuce and carrots. He wants to protect the carrots. Knowing that the fence costs € 5 per m². How much will he spend to protect the carrots?"

Figure 9: Problem statement created by Matias.

The example provided in the Figure 9 contains some of the essential data of the problem statement, needed for its resolution, which have been integrated into the picture and omitted in the text. This student, in his formulation process, decided to adopt a more sophisticated approach to data presentation, which reveals a process of conscious reflection on the most efficient way of transmitting the premises of the problem to the solver.

Final remarks

Developing creativity in mathematics is not a simple task. Like many other branches of the educational field, it is a process that takes time to develop, practice to engage and adequate instruments, such as problem-posing tasks, to be effective (Ayllón, Gómez, & Ballesta-Claver, 2016). This study suggests that both 1st and 6th -graders understood the problematic of posing a mathematical problem, and all the premises involved. Converging with previous research referred by Silver (1997) and Aktaş (2016), the students of our study, by creating an original situation, consciously manipulated the data provided by the image and their mathematical knowledge to create a meaningful problem statement and find a way to resolve it.

Thus, analysing the categories of mathematical criteria of Silver (1997), fluency was identified as a transversal property, understanding that every student created problem statements that fit the requirements of the task. Flexibility was also recognized in the student's productions, especially among the 6th-graders, identifying several problem themes and mathematical issues. As for novelty, it was also acknowledged in numerous students' productions, in both grades, either in the creation of original mathematical situations or in the innovative use of problem statement elements, such as the conscious attention of posing an explicit question in the production of problem statements (see Pinheiro & Vale, 2013), in the 1st grade, or the integration of part of the problem data posed in the image provided, as a sophisticated method of data presenting, in the 6th grade.

Throughout the analysis of the problems posed by the students, it was possible to identify two types of intentions that the students transposed to their problem formulation process: the systematization and the construction of new learnings.

Regarding the systematization of learning, it is noticed that students build and pose their problems through the application of content they know, integrating and modifying their knowledge for systematizing learnings (Harpen & Presmeg, 2013). Thus, creativity in the application of already known contents and processes stands out, building a situation coherent with their use. This conscious manipulation of the contents contributes to the construction of a deeper knowledge, allowing the learning process to become more meaningful. This was identified in both 1st- and 6th-grades, as a transversal way of thinking: using problem-posing tasks as a holistic task of combining creative thinking and known procedures.

Concerning the construction of new learnings, it appears that the problem posing can also serve as a starting point through the conceptualization of innovative situations and their conscious exploration (Harpen & Presmeg, 2013). Here, creativity plays a major role in conceptualizing new situations, which enable and promote the construction of new learnings (Agić & Rešić, 2015). This action allows students to understand the scientific nature of research and discovery of mathematical learning, challenging them to build innovative and original situations, in search of new knowledge and more sophisticated learning processes. This process was more evident in the 6th grade, as a result of students being more experienced and felt more comfortable in taking risks. However, creativity development is as continuous process, and starting early with tasks which promote its evolution, especially in primary school, will contribute to create a new approach of the mathematics learning process (Haddad, 2012).

Addressing the identified difficulties, it is important to highlight that the complexity of the formulated problems, in general, tended to a level slightly below their capacities, both in the primary and middle schools. Similarly, the typology of conceptualized problems was not very varied, lacking the presence of more examples of problems involving different procedures and investigations. These difficulties were previously identified in the literature (see Pinheiro & Vale, 2013), and were already expected. Somehow, one of the reasons that may explain this phenomenon links to the low frequency of this type of activity in the students' daily mathematics class routines, which means that they still do not feel free to risk more daring proposals.

Thus, it is imperative that problem-posing tasks take a more protagonist role in a modern perspective of mathematical education (Vale & Pimentel, 2004, Boavida *et al.*, 2008; Palhares, 1997; Harpen & Presmeg, 2013), and teachers must be aware of its strengths in order to propose them more frequently, promoting a conscious development and arousing the interest in mathematics. By solving and posing problems, students may see mathematics with fresh eyes, in a more meaningful and autonomous learning process.

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