Journal of the European Teacher Education Network
2019, Vol. 14, 1-15

Young children representing data

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Abstract
The study aimed to understand young children's ideas (4-6-year-olds, N=24) when collecting, organizing and representing data. It focuses on: 1) How do children understand data collection and organization? 2) What sort of difficulties do children have when representing data? A six sessions intervention was implemented, in which children worked with real objects and graphs (real and pictures). A qualitative analysis was used to explore children's ideas related to data representation. Results show that children are able to collect and organize data; and they can interpret information from a simple real graph and translate it into a pictogram, with understanding.

Key words: Analysing data; real graphs; pictograms; data in kindergarten

Introduction

Children demonstrate an interest in mathematics well before they enter school. Thus, in agreement with the National Council of Teacher of Mathematics (NCTM) (2000), kindergarten practices should be designed to capitalize on children's natural interest in mathematics to make their experience more engaging and beneficial. Frye at al. (2013) believe that the mathematics achievement of young children can be improved by placing more emphasis on mathematics instruction throughout the school day. Clements and Sarama (2018) high-quality learning results from formal and informal experiences during the preschool years, and 'informal' does not mean unplanned. Young learners possess innate mathematical abilities and it is surprisingly what they can achieve when given rich and cognitively demanding activities. Thus, in agreement with English (2016), it becomes relevant to expose what young learners can, rather than cannot do, and assist teachers in designing a richer array of early mathematical experiences.

According to the official document of curriculum guidelines for Portuguese Kindergarten "Orientações Curriculares para a Educação Pré-Escolar" (OCEPE), in this school level, mathematics should be explored on a daily basis through playing and games, involving situations that are meaningful for children (Silva, Marques, Mata, & Rosa, 2016). Children's exposure to mathematics should extend beyond number and operations to include a range of mathematics content areas, including geometry, patterns, measurement, and data analysis (Frye et al., 2013; NCTM, 2000). Giving young children experience in early mathematics content areas other than number and operations helps prepare them for the different mathematical subjects they will eventually encounter in school, such as algebra and statistics, and helps them view and

understand their world mathematically (Frye et al., 2013). Beyond this, the authors argue that it also creates a foundation for future mathematics learning, and children with strong backgrounds in these areas are more likely to succeed in the following grades. For example, early instruction in simple graphing exercises are the foundation for more advanced concepts such as statistics (Frye et al., 2013; NCTM, 2000). This article describes an intervention with young children to understand their ideas when collecting and representing data.

Collecting, organising and representing data in kindergarten

Children should be seen as active subjects in their learning process. They build actively their mathematical ideas interacting with physical and social environment around them and reflecting on these experiences (Baroody, 2002; NCTM, 2000). Starting from children's concepts and previous ideas is essential for the learning process, making learning meaningful to them. NCTM (2000) argues that children learn mathematical concepts through their daily activities. The informal comparing, classifying, sorting and organising objects, and counting activities can provide the mathematical beginnings for developing children's understanding of data (Castro & Rodrigues, 2008; NCTM, 2000). This is a rich domain fruitful to help children to understand the world around them.

The OCEPE highlights that in kindergarten, there are many opportunities to collect, organise and interpret data from everyday situations (Silva et al., 2016). Also NCTM (2000) refers that kindergarten mathematics should provide opportunities for children to formulate questions that can be addressed with data and collect, organise, and display relevant data to answer the questions. English (2012a) argues that the process begins with young children's inquiries and investigations of meaningful phenomena, progressing to deciding what is worthy of attention (i.e., identifying attributes of the phenomena), and then moving towards organising, structuring, visualizing, and representing data.

High quality learning opportunities can emerge from formal and informal experiences provided in the kindergarten. Nevertheless, these informal experiences must be planed and idealised by the educator, as they have an intentionality. Children's daily life experiences while playing and exploring are very powerful. The educator should enhance enriching experiences for children in mathematics. Frye et al. (2013) recommend that kindergarten teachers should help children to collect and organise information, and then teach them to represent that information graphically. Kindergarten contexts are very rich in situations in which it makes sense to analyse data with young children. This requires the use of informal contexts to explore, with understanding, concepts and ideas that can be used later on formal settings.

Kindergarten children can collect data, organise and interpret information. The data collection can start from everyday situations that have aroused from the children's interest and curiosity. The educator should indicate the building process to the children, asking them how the data will be organised (in pictograms, bar graphs) and how data can be presented. It is essential that the children feel part of this process, in order to make this a meaningful learning experience for them. Starting from children's everyday

settings is a way to focus on their interests and use familiar contexts that are going to give sense to questions, such as “How many children prefer blue colour?”, “What is the children’s favourite fruit?”, or even “How many more objects are there in one group than in another?”.

Data collection should be carried out by the children, as this help them to develop flexibility concerning ways of representation and, later on, organisation of data into different groups (Castro & Rodrigues, 2008). In this process, the educator should provide guidance in order to accomplish a data collection focused on a precise issue. Children should also experience comparisons among data collections that they are carrying out in order to promote their reflexive reasoning (Frye et al., 2013).

After data collection, data organisation takes place. Children do this by recognising common qualities among the collected objects (such as colour, shape, number, for example). Then, these will be presented in the format they choose, tables or graphs. Portuguese kindergarten children are already familiar with the tables because it is something they fulfil on their daily routine when, for instance, they discuss the attendance map or when they register their working area for the day. Nevertheless, they may be less comfortable with graphing.

Data organisation and data representation has been revealed a versatile subject that allows to approach diverse contents from diverse areas, starting with the mathematics. Taking advantage of the children’s innate curiosity and providing them with tools to understand the world around them, it makes sense to explore data organisation and representation in early years (Baratta-Lorton, 1995; Frye at al., 2013; English, 2016; NCTM, 2000). "Since children feel a natural curiosity about the world around them, they often ask questions such as how many are they? How much is it? What kind is it? Or which of these are they? Such questions often provide opportunities for the initiation of the study of data and probability analysis." (NCTM, 2000, p.53). Collecting and organising information, such as creating graphs to display favourite animals or foods, allows children to find out more about one another (Frye et al., 2013).

According to Frye et al. (2013), educators should provide children with opportunities to count and sort familiar items to introduce them to the concept of organising and displaying information. This information can take the form of tangible objects, such as toys or blocks, or abstract concepts, such as characteristics (e.g., which children are 4 years old and which children are 5 years old) or preferences (e.g., favourite snacks, colours, or animals). For the authors, the goal of such activities is to demonstrate both the characteristics that distinguish the items and the total number in each set relative to other sets.

Once children are familiar with sorting and organising the information they have collected, they should learn to represent their information visually. Here graphs can have an important role. Graphing is a tool that helps young children to identify relationships. This skill develops naturally from children’s sorting and classifying activities, and also from comparing groups of objects (Baratta-Lorton, 1995). This means that young children can benefit from graph explorations, experiencing abstraction on data representation. Graphs allow children to summarize what they have

learned, and graphing provides an opportunity for children to share and discuss their findings (Frye et al., 2013).

Baratta-Lorton (1995) argues that the picture graphs use pictures and models to stand for real things. These graphs are more abstract than real graphs as a picture represents reality. Thus, the picture graphs are important because they establish a link between the real and the abstract, helping young children to understand symbolic graphs. Symbolic graphs use symbols to stand for real things, and this is the most abstract level of graphing as the symbols must be translated back into reality to have meaning.

Given the diversity of graphs, it becomes important to be able to create them, but also interpret and use them adequately. For Frye and colleagues (2013), educators can begin by introducing simple tallies and picture graphs to children, then teaching children to interpret the meaning of these graphs.

Nowadays, there is a great variety of graphs and their success is so great that it is enough to observe the diversity of ways in which the information is described pictorially (bar graphs, lines, circles, Venn diagrams, flow diagrams, tree diagrams, pictograms) (Cazorla, 2002; Pinker, 1990). In this study pictograms were discussed, as they are highlighted in the curriculum guidelines for kindergarten in Portugal.

The relevance of this study

The Portuguese official curricular guidelines for Pre-school Education (Silva, Marques, Mata & Rosa, 2016) refers that in kindergarten there are several situations in which children can be involved in data collection, data organization and interpretation of data, from everyday situations and planned experiences. This document also points out that it is teachers' role to support the children in the posing questions, in data collection and organization using sets, tables, graphs, Venn diagrams, etc. According to this official document, kindergarten children are expected to use pictograms, bar graphs and simple tables to organize information previously collected by them, and also to interpret this information to answer the posing questions. Also NCTM (2000) refers that, in prekindergarten through grade 2, all children should "pose questions and gather data about themselves and their surroundings; and represent data using concrete objects, pictures, and graphs." (p.108).

English (2012a, 2012b) using the idea of data modelling defined as a developmental process, beginning with young children's inquiries of meaningful phenomena, progressing to identifying various attributes of the phenomena, and then moving towards organising, structuring, visualising, and representing data, underlies its relevance in early years mathematics curricula. English (2012b) points out that data modelling should be a fundamental component of early childhood curricula, yet there exists limited research on such modelling and how it can be fostered in the early school years. The author argues that data modelling provides rich opportunities to advance young children's statistical development and reveal their capabilities in dealing with challenging tasks.

Because data collection, data organization and interpretation of quantitative data are a novelty in the Portuguese curriculum for early years mathematics, it becomes important to share relevant experiences in this domain. As limited research has been developed regarding these issues in Portuguese reality, an exploratory study with kindergarten children was conducted to have an insight on children's ideas when representing data.

This study aimed to understand young children's ideas when collecting, organizing and representing data. It tries to address two questions: 1) How do children understand data collection and data organization? 2) What sort of difficulties do children have when representing data?

Methods

Qualitative methods were used to explore children's ideas related to collecting, organizing and representing data in kindergarten as one was looking for a description and interpretation of a learning phenomenon on their natural environment (see Bogdan & Bicklen, 2010; Merriam, 1998). A collective case study was conducted (see Yin, 2010) with a kindergarten group of children trying to understand "how" and "why" a phenomenon occurred.

The participants were 24 state-supported kindergarten children (4- to 6-year-olds), from Braga, Portugal. An intervention program was implemented, comprising 5 sessions: 1 session devoted to assess children's informal knowledge of data organization and representation; and 4 sessions focused on the exploration of children's ideas about data collection, organization and representation.

The tasks involved in the first and last sessions comprised the elaboration of graphs using real objects, pictures and symbols. During the exploration sessions children built simple tables and from it created the graphs (real, pictures and symbols), and they built the graphs and then, from it, created the tables. All the tasks were in agreement with the official curricula for mathematics kindergarten (see Silva et al., 2016), were adjusted to young children's contexts, and were an adaptation of Baratta-Lorton (1995) and Frye et al. (2013) tasks.

The tasks were implemented in kindergarten using children's daily routine, exploring their interests and curiosity. The session 1 was devoted to assess children's informal knowledge of data organization and representation, thus from a story about bears, children were challenged to find a way to represent their favourite bear colour building a real graph. This was a whole group task. The remaining sessions to explore children's ideas about data collection, organization and representation, smaller groups of 4 and 5 children each were organized to solve the tasks. All the tasks were given to children by the researcher, one of authors of this paper.

Familiar settings were used to organize and represent data on tables and graphs to give meaning to questions such as "how many girls are there in the room? And how many boys?", or "What is the fruit that children like the most?", or "how many children prefer the red colour?".

Data collection was conducted using audio and video records, photographs, children's written productions and researcher field notes.

Results

Children's reactions to several tasks presented in the five sessions were analysed. However, due to length constrains, only three of these sessions were analysed here. To preserve children's anonymity, fictitious names were used in this analysis.

Session 1 was focused on children informal knowledge about data organization and representation. Children worked as a whole group all the session, and 22 children attended the session that day. After listening a story about bears, a box with small coloured rubber bears was given to the children. Each child was invited to choose one bear of his/her favourite colour (Figure 1), and the children were asked about how to organize all the bears, and they suggest to organize them by colours.

Figure 1. Choosing a bear to build a graph.

Then children were asked "how can we organise the bears to know the colours that were chosen by all the children?". One child said immediately "We can put all of them in the floor… like this" showing it with his bear and using other child's bear, putting them in a column. Accepting this suggestion, each child put his/hers bear on the floor and, with teachers' help, the real graph was created (Figure 2). The real graph was built on the carpet floor using rubber coloured bears that children used to play with.

Figure 2. Children creating a real graph with coloured bears.

From that graph made with bears, the educator asked children how that graph could be made in the classroom to be seen for everybody. Silence was made, and then one child suggested to draw it in the whiteboard. Accepting the suggestion, a bear of each colour was drew in the same line, and each child was asked to draw his/her bear in the whiteboard in respective colour column (Figure 3). From a real graph using coloured bears, the correspondent pictogram was made.


Figure 3. Children drawing his/her coloured bear in the appropriate column.

After the graph production, some interpretation questions were asked to the children. When asked which column had more bears, the children, first had problems with counting and fail to answer. Then they were asked to count again, slowly and loud, and they were able to give a correct response (Figure 4). Sara (5-year-olds) even mentioned "We saw that the red ones are more because they are 7!". When asked which columns had fewer bears, immediately a correct response was provided in chorus, referring the purple ones, explaining that there were only 3.


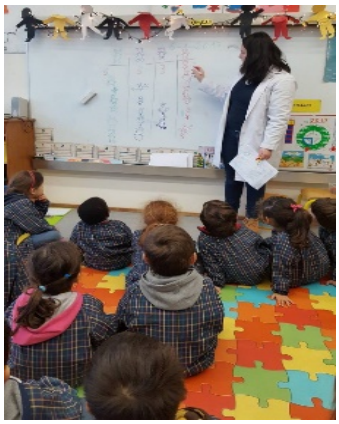

Figure 4 – Children counting the bears and answering some interpretation

Altogether, children had the opportunity to count the bears of all colours, and concluded that there were 6 blue bears, 6 greens, 3 purples and 7 red ones. Then, children were also asked if there were any columns with the same number of bears. Children at first answered "No!", but after a second thought corrected their answer. Three 5-year-olds children explained that "these are 6 and these are 6", pointing to the column of blue and

green bears, recognizing columns with the same number of bears. Transcription 1 gives evidence of that.

> Educator - Are there any columns with the same number?
> Several children - No!!
> Educator - No??!!! [Pause]
> Several children - Yes!!!
> Educator - Which ones?
> Sara (5yrs) - Those two [pointing to the whiteboard].
> Educator - Of which colours?
> Sara (5yrs) - There is a 6 on this side and a 6 on that side [pointing to the graph].
> Rui, Maria and Sofia (5yrs) - Green and Blue.
> Educator - So, green and blue columns have the same…
> Rui, Maria and Sofia (5yrs) - …number.

Transcription 1. Children recognizing columns with the same number of bears.

When building the pictogram, children were focused on the task. Some of the 4-year-olds seemed to be able to follow the discussion carried on, distinguishing where there were more and less bears. However, when asked "Why do you think so?", they could not explain, just pointed to the bears. Contrasting with this, several 5-year-olds children, when asked to justify their answer, were able to use counting to validate their answer. During this session, the children of 4-year-olds were able to represent their bear in the graph and to answer the questions about which column had more and which had less, but were not able to give any justification.

In session 2, the children solved the task "My favourite smarties". The children were divided in groups of five, according to age. A structure to create a graph was previously given by the educator, and they had to fill it to organise their favourite smarties by colour (Figure 5). Then, they had to register the information and make a data display (Figure 6), translating the real graph into a picture graph. This task was given to all groups of children, and different graphs were produced.

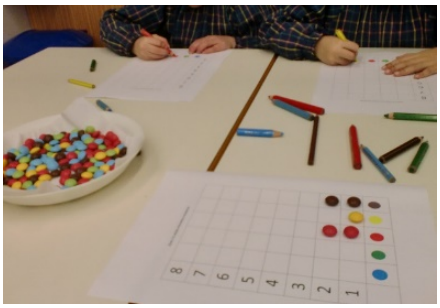

Figure 5. Graph created with children's favourite smarties (4-year-olds).

Almost all the children participated in the real graph production with no difficulties. The majority of 4-year-olds children succeeded in this task. However, not all of them succeeded at first in translating the real graph into the picture graph. A child found

difficult this translation and explained that was distracted when doing the representation, colouring all the row, producing an incorrect resolution (Figure 7).

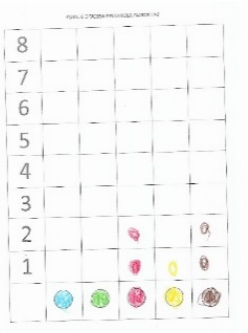

Figure 6. Register made by a 4-year-olds child.

Figure 7. Incorrect resolution made by a 4-year-olds child.

After that, some interpretation questions were asked. Transcription 2 presents an example of such moment.

Educator - Looking at this graph, what is your favourite smarty?
[Children speak all at the same time saying what they had chosen]
What can we see in this graph?
Carol (4yrs) - Ah… numbers.
Educator - Numbers... so which one has more numbers?
Maria (4yrs) - Brown!
Educator - And....?
Maria (4yrs) - Red!
Educator – So, your favourite colours are...?
Maria (4yrs) – Red and brown.

Transcription 2. Children interpreting the graph created by them.

All the children succeeded in this task. From the observed children's enthusiasm, one can say that they enjoyed this task, and they could answer to almost all the

interpretations questions. The other group of 4-year-olds also created a graph (Figure 8), but some children failed the interpretation questions (see Transcription 3).

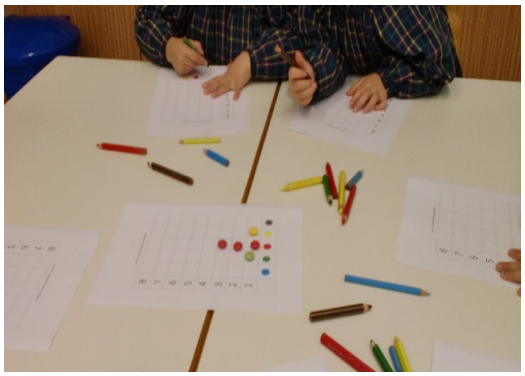

Figure 8. Graph of children's favourite smarties made by another group (4-year-olds).

Educator - So, which is your favourite smarty?
Lia (4yrs) - The red and the green.
Carol (4yrs) - No, no, it is green and yellow!
Educator - Which of them has fewer?
Carol (4yrs) - It has 1 [pointing to green].
Educator - How many are red?
Carol (4yrs) - Three!

Transcription 3. Children interpreting the graph created by them.

Lia had some difficulties with the notion of 'few' and 'more', and also with numbers to represent quantities. Carol usually had no problems with counting and with numbers, but fails her first answer. However, she was able to correct herself.

The 5- and 6-year-olds children solved these tasks with no difficulty, and could answer correctly to the interpretation questions. These tasks allowed children to produce different charts and to talk about their productions. The enthusiasm was so great that they wanted to work with bigger quantities. So, each group chose two favourite colours resulting in a graph with more data displayed. Figure 9 illustrates such an example. As colour was in analysis in these tasks, it was also an opportunity for children to revise the colours and also counting skills (Transcription 4).

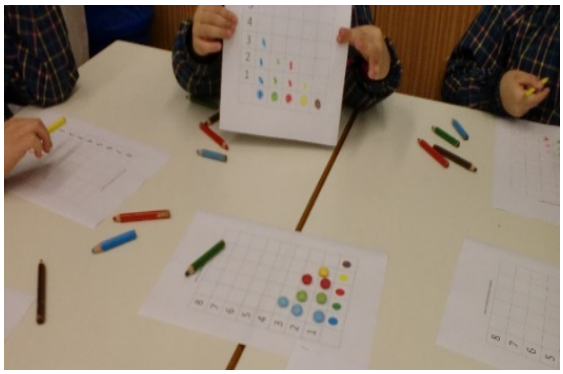

Figure 9. Graph of children's favourite smarties (5-6-year-olds).

Educator - Which is your favourite smarty?
Rui (5yrs) - The blue [All agree with him by physical expressions].
Educator - Which on has fewer?
Maria (5yrs) - Yellow [Answering immediately].
Educator - How do you know?
Maria (5yrs) – It has one!
Educator – What about the blue ones?
Maria (5yrs) – Three.
Educator – Are there more green or blue smarties?
Rui (5yrs) – Blue!
Educator – So, altogether how many smarties do you think we have here?
Maria (5yrs) – Eight.
Maria & Patti (5yrs) – Eight!

Transcription 4. Five-year-olds children interpreting the graph created by them.

In all the groups, it was possible to observe different resolution times used by children. Some finished the graphs quite fast, others took a little more time due to distraction issues or also due to perfection in drawing when representing data.

The tasks of sessions 1 and 2 allowed children to use numbers to quantify, to compare quantities and numbers, giving them an opportunity to establish relations between quantities.

Session 3 started with a conversation about their transportation to the kindergarten, as before they were analysing means of transportation. In this session, the children were organized in groups of eight with mixed ages of 4-5-year-olds and 5-6-year-olds.

In each group, children were asked about the mean of transportation used to reach kindergarten. Easily the group realised that they came by bus, by car, and walking. Then, they were challenged to create a picture graph about their transportation to kindergarten.

The educator prepared a cardboard to analyse "How did you come to school?" with the children, to help them to organize the data. For each group, each child marked her/his transportation by drawing (Figures 10 & 11), and then answered some interpretation questions presented by the educator.

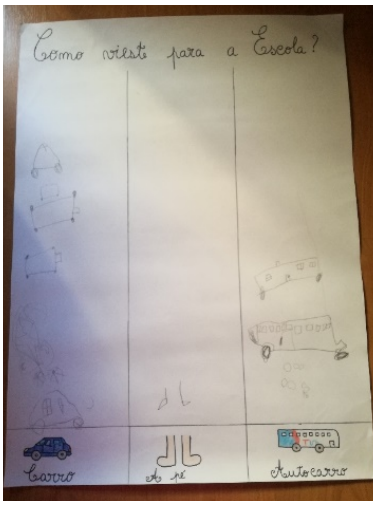

Figure 10. Children representing their means of transportation (4-5-year-olds).

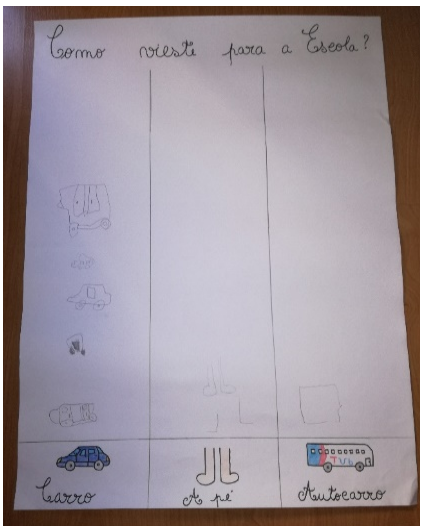

Figure 11. Children representing their means of transportation (5-6-year-olds).

Easily, the children could recognise which means of transportation were more popular among them, referring that the majority was coming to school by car and only a few walked to school. To justify this argument, they used counting. Transcription 5 shows an extract of conversation that suggests that 4-5-year-olds children had no difficulties in interpreting this information.

Educator – What do we want to know here?
Carol (4yrs) – How we got here.
Anna (5yrs) – ... yes, to the school.
[Pause, while children were drawing]
Daniel (5yrs) – I think cars are going to win!
Educator – Why do you think so?
Daniel (5yrs) – Because there are a lot of cars [pointing to the graph].

Transcription 5. Children interpreting the graph created by them (4-5-year-olds).

After building the graph, the children were asked interpretation questions to assess their understanding of data representation. Transcription 6 suggests that even the 4-year-olds could understand the information provided in the graph.

Educator – Looking at the graph, which column do you think that has more?
Carol (4yrs) – The cars one.
Educator – How do you know?
Carol (4yrs) – Because it has six!
Educator – How do you know they are six?
Carol (4yrs) – Here it has six, here it has one and here has three [pointing to the graph].
Educator – Are there more children coming by car or walking?
Manuel (4yrs) – By car, it is higher, it has more.

Transcription 6. Children interpreting the graph created by them (4-5-year-olds).

These tasks allowed children to answer "how many are there" questions and reflect on which categories had more and which had less, comparing quantities.

Similarly, the group of 5-6-year-olds succeeded in the graph creation (see Figure 11) and revealed no difficulties in understanding the data representation. Even 4-year-olds children, who were previously struggling with explanations, were now justifying their answers relying on counting. Results show that children are able to collect, organize data and interpret information presented in distinct ways provided by graphs, using real objects or pictures.

Final remarks

This study aimed to understand children's ideas about data representation. For it, real graphs and picture graphs (see Baratta-Lorton, 1995) were explored. Always starting from children's daily situations, it was possible to involve children in data collection,

organization and data representation. Because the situations were meaningful for children, it was possible to keep them motivated and active in their learning process.

Graphing can be a problem-solving tool used to help young children see relationships, which is a skill that develops naturally from comparing groups of objects activities. From these activities children were able to answer "how many are there" questions, and explain which group has more and which has less. These ideas can be seen as foundations for more sophisticate reasoning of "how many more objects" there are in one group than in another.

This work allowed children to establish a connection between the real graphs and picture graphs. The graphing experiences used here helped children to move from the concrete, using real graphs to represent data, to the picture graphs to represent data. As argued by Silva et al (2016), children learn to mathematize their informal experiences by abstracting and using mathematical ideas to create representations of meaningful situations for them.

This group of children was very heterogeneous with children from 4- to 6-year-olds. It was interesting to observe that some younger children (4-year-olds) could make a correct interpretation of the graphs similarly to the older ones (5-6-year-olds), in spite of finding more difficult to justify their answers verbally than the older ones. Children were asked to justify all their answers as a way to incentive their verbal communication skills, and also because, in agreement with Silva et al. (2016), to communicate the developed mathematical processes help children organize her/his reasoning and improve their representation abilities. However, to present a justification seemed to be the most difficult challenge for younger children when solving these tasks.

Data organization and representation is a powerful theme in early years mathematics. Evidences show that graphs exploration can be a rich opportunity for young children to develop their reasoning, problem solving and communication skills were given here. However, more research and experiences need to be carried about these issues to have a better insight of children's development when using data analysis processes in kindergarten.

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