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A problem-solving experience: The teacher's perspective

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Abstract. Problem solving is a skill that can be developed by students. For this to happen, the teacher must be prepared to teach classes in which they proposes that their students solve one or more problems. Teacher do not always feel ready and confident to have a class where the focus is on solving a problem. In this communication, the focus is on the planning and implementation of a class in which one intends to solve a mathematical problem. Thus, the experience of the first author is reported when planning a class, passing through the five practices that facilitate the discussion of mathematical tasks: anticipation, monitoring, selection, sequencing and connection.

Keywords: Problem Solving, Exploratory Teaching, Planning a Class, Discussion of Mathematical Tasks, Secondary School

Résumé. La résolution de problèmes est une compétence qui doit être développée par les étudiants. Pour ce faire, l'enseignant doit être prêt à donner des cours dans lesquels il/elle propose à ses élèves de résoudre un ou plusieurs problèmes. Les enseignants ne se sentient pas toujours prêts et confiants d'avoir une classe où l'accent est mis sur la résolution d'un problème. Dans cette communication, l'accent est mis sur la planification et la concrétisation d'une classe dans laquelle on entend résoudre un problème mathématique. Ainsi, l'expérience du premier auteur est rapportée lors de la planification d'un cours, en passant par les cinq pratiques qui facilitent la discussion des tâches mathématiques: anticipation, surveillance, sélection, séquence et connexion.

Mots clefs: Résolution de Problèmes, Enseignement Exploratoire, Planification de Cours, Discussion de Tâches Mathématiques, Enseignement Secondaire

1. Introduction

Solving problems is one of the ten areas of competence that must be developed over the twelve years of compulsory schooling in Portugal. Planning classes to engage students in tasks that promote understanding, solving problems and reasoning, and create opportunities to make sense and discussing processes and solutions in a relevant and enriching way for everyone is essential for the development of this area of competence (NCTM, 2017). In this way, students have the opportunity to develop their ability to solve problems, the critical spirit and the ability to expose and defend their ideas. Besides that, students can realize the difficulties they feel in solving a problem and share these difficulties with the class, allowing everyone to reflect on them and how to deal with them.

Accordng to Zimmermann (2016), if we want students to develop their problem-solving skills, teachers should be oriented on how they teach their classes. This author believes that teachers, in their initial education in order to become a teacher, should have "the opportunity to simulate little parts of lessons, teaching their fellow students solving a problem" (Zimmermann, 2016, p. 105), and their colleagues should react like they were pupils in the classroom. This way, future teachers

develop their ability to teach classes that involve problem solving as well as anticipating difficulties and questions that students can ask in the classroom.

Considering the relevance of problem solving in the classroom and the importance of planning a lesson carefully, this communication aims to share an experience of planning a class with focus on solving a problem. This sharing focuses on the teacher's concern with respect to lesson planning and achievement. In this paper, the focus will be on one of the problems that have been solved in small groups by the students involved in this research.

2. Context and method

This study was developed in a 11th year class of the Science and Technology course of a Portuguese public school. This group had 22 students and wasn't accustomed to solving problems before the intervention carried out by the first author in the context of this experience, held during the teaching internship. The students of this class, twelve girls and ten boys, were between 16 and 18 years old and, at the end of the school year, only two students had a negative grade in mathematics, and the average grade of the class was 14 points, in a maximum of 20.

In this one-month intervention, students solved problems individually and in small groups. Students were challenged to organize groups on the condition that they have to be separated into six groups of three students and one group of four. The groups were established by the students and the groups remained the same in all classes. Throughout the classes, there was no difference between the groups, with all groups working at a similar rhythm, with identical doubts and with all students actively contributing to solve the proposed problems.

Observing students' posture in class was a very important process because it allowed them to understand students' position on problem solving – initially, some students were not involved in problem solving, maintaining a defeatist attitude and not trying to understand the problem. Without this observation of the classes, it wouldn't be possible to perceive this attitude from some students, since the written productions do not help us to clearly identify these situations. Moreover, the observation also showed that this attitude was changing and that students who initially felt more insecure about their ability to solve problems, were feeling more secure and more willing to solve the proposed problems. Regarding student productions, all the resolutions that were made in the classroom were delivered to the teacher – in the case of group work, each group delivered a single resolution. In addition, when the problems were solved in groups, audio recorders were placed in all groups, to access the difficulties that were felt within the group and to better perceive the entire process of resolution developed.

The study with these students was based on a qualitative research. This type of research has some characteristics that are found in this study, according to Merriam (2009), for example: the researcher is the main instrument in the collection and analysis of data, the researcher builds ideas from observation and intuitive thoughts and the products of research is strongly descriptive.

3. Theoretical background

Although they point out that there isn't a unique way to work problem solving in the classroom, Onuchic and Allevato (2011) present a possible script for a problem solving class. In this script, the lesson plan goes through nine points: problem preparation individual reading, group reading, solving the problem, observation and encouragement, registration of resolutions in the blackboard, plenary, consensus, formalization of content. This plan consists of thinking in advance about the problem that is to be worked out, so that it serves the purpose (like working on a certain content taught or discovering new content). Then, in the classroom, the teacher distributes the problem to all students, who read individually and then form groups and re-read the problem together. The problem starts to be solved and, as the students solve it, the teacher must observe the work of each group, clarifying possible doubts, launching new questions and trying to bridge the difficulties experienced by the students. Once completed, one representative from each group records the resolution obtained on the blackboard, so that the whole group knows what the process followed by

that group was. This register gives the opportunity to listen to the ideas of colleagues and create a discussion that allows the development of the critical thinking of each student. From this discussion, a consensus should be established regarding the solution of the proposed problem and, finally, the concepts that were constructed by solving that problem formalized. This script established by Onuchic and Allevato (2011) seems to follow the five facilitating practices of discussion about a mathematical task established by Stein, Engle, Smith and Hughes (2008). These five practices include anticipating student responses, monitoring students' work while they solve the problem, selecting the resolution that are relevant to show the class, establishing a sequence for that presentation, and ending it with the connection between different responses and ideas.

3. Experience of planning a lesson

The lesson used to solve this problem took 90 minutes. This problem was solved after the topic "Principle of Mathematical Induction" (PMI), inserted in the chapter on Sequences, has been taught. The problem chosen had the title "Cuts in the pizza", and its statement can be seen in figure 1.

> *At a pizzeria, a group of friends made the following request:*
> *"We want a circular pizza with only 8 cuts and as many slices as possible."*
> *1. Assuming that the slices can have different sizes and that the cuts have to be done in a straight line, how many slices can be obtained to satisfy the group's request?*
> *2. What if the group had ordered the pizza with* n *cuts? What would be the maximum number of slices?*

Figure 1. Statement of the problem proposed to students. This statement has been adapted from Oliveira, H., Canavarro, A. P., & Menezes, L. (2013). Cortes na pizza (ensino secundário) – caso multimedia. In Site do Projeto P3M, Práticas Profissionais de Professores de Matemática. Available at: http://p3m.ie.ul.pt/caso-4-cortes-na-piza-ensino-secundario

When this lesson was planned, an anticipation was made regarding what might happen in the classroom and a record of what was expected to be obtained in response to the problem written down. In task 1, the students were expected to arrive at the "37 slices" answer, and in task 2 the class should define a recursive sequence, with the first term 2 and each subsequent term is the sum of the previous one and n ($u_{n+1} = u_n + n$). The objective was for students to realize that the maximum number of slices would be obtained by: (1) intersecting cuts and (2) each new cut should intersect with all existing cuts, but not at intersection points. To achieve this goal, students were expected to use a strategy of drawing a picture, followed by attempt and error, until they realized the existence of a pattern. Regarding the difficulties, it was expected that some students would tend to think that the cuts should pass through the center of the pizza which would prevent them from reaching the correct result. In this process of anticipating the class, it was further defined that the presentation and discussion of group resolutions would be done in an increasing order of solutions, if there were different answers. Besides that, if there was still class time, the problem could be extended to specific cuts, such as parallel cuts and perpendicular cuts.

The students entered the room and sat down according to the previously established groups. The class started with the teacher distributing the problem statement to all the students and recommending that they read it carefully. She also stated that they would have 30 minutes to solve the problem so that the discussion about it would be made later. During that time, the teacher went through all the groups, giving place to the monitoring phase and realizing the situation in which they were, what difficulties and what results were being obtained. The initial 30 minutes had to be

increased to 50 minutes because the students weren't able to reach an answer to the whole problem, using the time only for to answer the task 1. Only one group was able to present a response to the two tasks, and this was the only group that presented a correct answer to task 1. Throughout this monitoring, there were few questions put to the teacher, due to the spirit of mutual aid between the students while working in group. When the doubts that appears were questions of the whole group, the teacher was called upon to intervene and tried to help the group without giving answers, encouraging the students to work together and discuss ideas with each other. At this stage, questions have emerged from the students, such as "should the slices have the same geometric shape?" and "should the slices be triangular?". The teacher asked the students if this was written in the statement, and they said no. With this clarification, it was intended that the students understood that they should be restricted to what is provided by the statement, and work from that base. Sometimes, students get too "stuck" to certain concepts and ideas, which prevents them from successfully solving a problem. One of the groups was so focused on the fact that the slices had to pass through the center of the pizza that, although the teacher encouraged them to think better about their resolution, the group could not get out of that idea.

After the 50 minutes, each group delivered its resolution and the teacher established an order, which was already thought through the anticipation phase and also in the monitoring phase of the groups. In the case of this class, the selection phase was omitted. This is because the number of resolutions was reduced (one per group, so there were seven resolutions), and six different answers were obtained. In the sequencing phase, the sequence previously thought following the increasing order of solutions was followed. In the case of the two groups with the same solution, the sequence was established according to the depth of the response of each group – first presented the group that only arrived at the solution through figures and then the other group explained their attempts to find some pattern that would allow them to respond to task 2. By starting the presentation and discussion of results from the lowest value solution, students had the opportunity to participate more actively, asking their colleagues questions about why they hadn't tried other strategies. For example, the first group got the answer "16 slices" because they just tried to make cuts that went through the center of the pizza (figure 2).

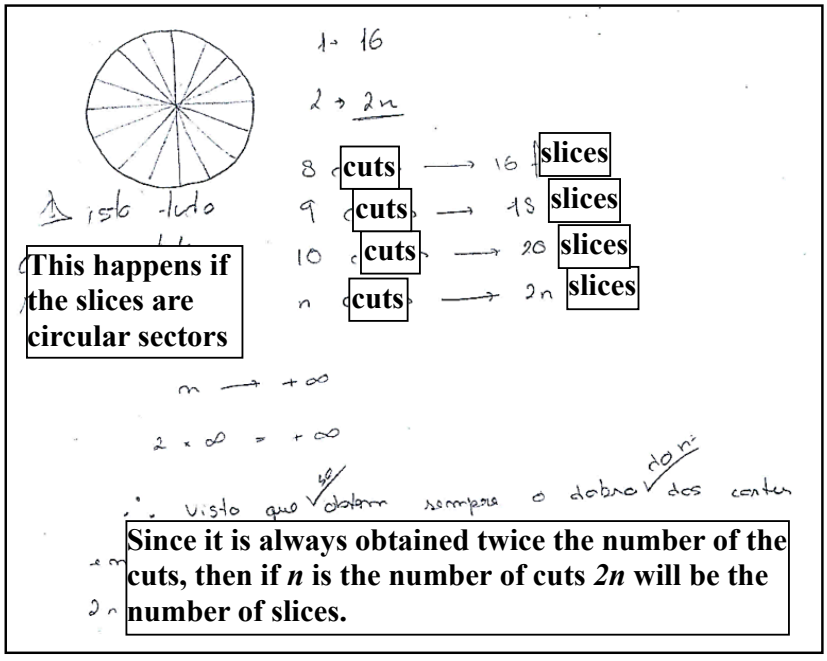

Figure 2. Resolution of the problem presented by the group with the lowest value solution.

They introduced their idea and one student from another group asked why they didn't try to

make cuts that didn't pass in the center of the pizza, making the discussion richer and increasing her critical ability. In this way, the students interacted with their colleagues, to give new ideas and to understand why the answer was not correct, helping to give meaning to the pre-established sequence. In the end, it was noticeable that this entire phase of presentation and discussion of results merged with the connection phase, as students were able to access the strategies of all groups, perceive all ideas and understand what failed in most cases. In addition to all the skills they developed around problem solving, students were also able to apply the knowledge learned about the PMI. This is because the last group presented the recursive sequence that responded to task 2, and yet another group raised a hypothesis for the general term of the same sequence. With these two ways of defining the same sequence, the students applied the PMI to show that such was the general term of the sequence.

4. Conclusion

Planning a lesson is a challenging process. Planning a lesson that consists of solving a problem is an even more challenging process. The plan of a class of this type is related to the practice of exploratory teaching, following the five phases proposed by Stein, Engle, Smith and Hughes (2008). It is necessary to make a good selection of the problem that we are going to propose to the students, and this selection depends on the objective that we intend. We may have a problem that relates to the content taught, and then we must be careful to see if that is indeed a problem, or whether it will become an exercise. If we want to have a richer discussion after solving a problem, we need to choose a problem that allows for different types of resolution. All this careful selection of problems is part of the anticipation phase of a lesson. At this stage, the teacher tries to select a good problem, to anticipate the doubts and questions of the students, which strategies can be used and how to select and sequence the answers that will be presented to the whole class. As we move on to the lesson, the monitoring phase allows the teacher to perceive the situation of each student, to understand what the doubts are, and to begin to identify the answers that will be relevant when presenting the results. Arriving the discussion stage, the role of the teacher is very important in selecting which groups should present their resolutions, and in what order. This sequence should start with incorrect or incomplete answers, and only in a final phase we can present a more correct resolution, allowing the whole group to follow the progression and realize that there are normal failures and that we can learn from mistakes. Finally, we go through the connections phase, because the discussion is important so that the students know the different strategies and ideas that could be used, and can apply them in future problems.

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