Investment Decisions with Finite-Lived Collars*

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Abstract

The duration of most collar arrangements provided by governments to encourage early investment in infrastructure, renewable energy facilities, or other projects with social objectives are finite, not perpetual. We extend the previous literature on collar-style arrangements by providing an analytical solution for the idle and active values, as well as the investment triggers, for projects where collars are either finite-lived or retractable. What is the difference between these types of arrangements with their perpetual counterpart? Lots, including different vega signs, and substantially different values for different current price levels. Often, finite and retractable collars justify earlier investment timing than perpetual collars. In general, we demonstrate that the finite-lived and retractable versions have a significant impact on optimal behaviour, relative to the perpetual case. An important consideration when negotiating the floors, ceilings, and duration (or signalling the expected duration) of a finite or a retractable

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collar is the current price level of the output and its expected volatility over the life of the contract.

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1 Introduction

Collars represent a viable government policy for inducing investment by offering the investor protection against the downside risk of adverse cash-flows, which are reimbursed in part by foregoing abnormally favourable cash-flows (Adkins and Paxson, 2017). As an alternative to the traditional model of subsidized investment grants and taxation, collars provide governments with the benefit of only modifying the cash-flow structure without the disadvantage of incurring any upfront investment funding. In a world of uncertainty, collars can be evaluated within a real-option framework because of their option-like properties. This enables the value of the collar to be determined analytically along with tractable extensions where the collars are finite-lived and retractable.

There are several examples of finite collars evaluated using numerical methods. Couture and Gagnon (2010) describe a Spanish 2007 "variable premium" that involves a floor and cap, where the highest premium (over the market electricity price) is paid when the electricity price is low, and zero when the price exceeds a ceiling (so the facility owner receives all of the higher price). González (2008) provides some detail on these premium collars. de Miera et al. (2008) illustrate how a system where a government offering guarantees for infrastructure projects should involve a European collar. Brandão and Saraiva (2008) model a finite collar for a private-public partnership (PPP) toll road and compare the project values as a function of the floors and ceilings. Shan et al. (2010) show forecast traffic and tolls (both increasing over time) and collar option values with exercise prices increasing over time. Shaoul et al. (2012) describe several rail operating contracts in the UK that have been structured as layered downside and upside revenue sharing arrangements. Boomsma et al. (2012) provide analytical infinite option values and 10-year finite option values, using numerical methods. Abadie and Chamorro (2014) calculate numerically the option value for a floor. Fernandes et al. (2016) suggest a collar-type insurance for wind power in Brazil, where a wind generator has promised to supply power even during times where there is little wind.

Attarzadeh et al. (2017) provide an extensive review of some applications of collars in PPPs, with a case illustration based on a triangular fuzzy method. Buyukyoran and Gundes (2018) use a Monte Carlo simulation with constant traffic volatility and two periods of traffic growth. Options are based on annual average daily traffic times constant tolls for each of four separate periods, and "European" options are exercised each end year. Zapata Quimbayo et al. (2019) assume that traffic (tolls) are mean reverting.

Analytical studies on perpetual floors and ceilings presume the underlying factor follows a geometric Brownian process. Takashima et al. (2010) design a PPP deal involving government debt participation that incorporates a floor on the future maximum loss level where the investor has the right to sell back the project whenever adverse conditions emerge. Armada et al. (2012) make an analytical comparison of various subsidy policies including minimum revenue guarantees. Adkins and Paxson (2019) provide analytical solutions for perpetual collars, floors and ceilings, plus partial floors and ceilings, and show the sensitivity of these collars to changes in most of the parameter values. Barbosa et al. (2018) develop a model for a feed-in tariffs contract with perpetual and finite-lived minimum price guarantees (price-floor regime) with regulatory uncertainty.

Our major contribution is to extend the previous analysis of determining the idle (pre-investment) and active (post-investment) values for projects having a perpetual collar-style inducement to cases where the arrangement is either finite-lived or retractable. These extensions make our representation more realistic thereby enhancing the credibility of our findings. In general, we demonstrate that the finite-lived and retractable versions have a significant impact on optimal behaviour relative to the plain perpetual collar.

Specifically, we show that when compared to the perpetual collars, (i) finite and retractable collars provide substantially lower downside risk protection for the beneficiary of the collar when market prices are low; (ii) the finite and retractable collars sensitivity to volatility is smaller and, as for the perpetual case, can be non-monotonic; (iii) finite and retractable collars are less sensitive to changes in the price cap; (iv) a finite-lived and a retractable collar may induce earlier investments than a perpetual collar when the price floor and price cap are set sufficiently low, or the investment cost and the duration of the collar or the likelihood of not being retracted, for low price floors, are sufficiently high; (v) an optimal level of the price cap and the duration of the finite collar exists, limiting the capacity of those instruments to justify earlier investments; (vi) the value of the finite collar investment opportunity is always higher than the perpetual collar investment opportunity value, regardless of the triggers; (vii) retractable collars may have lower triggers than either finite or perpetual collars; (viii) naturally, the real option value of retractable collars is highly sensitive to the probability of withdrawal; and (ix) retractable collars can induce earlier and more valuable investments than finite collars, the same effect termed "flighty bird in hand" in Adkins and Paxson (2016).

Finally, we illustrate how our models can be used to determine an optimal investment subsidy along with the collar design, that induces investment for a given price level.

This paper is organized in the following way. In section 2, we outline the basic plain real option investment model, for later comparisons. In section 3, we show the analytical solution for perpetual collars, finite collars, and retractable collars. In section 4 further insights are gained by performing a numerical sensitivity analysis and discussing the implications and interpretations. Section 5 studies optimal incentives design, and section 6 concludes and suggests several extensions.

2 Plain investment opportunity

Let us start by presenting the well known solution for a plain perpetual investment opportunity (for details see, for instance, Dixit and Pindyck (1994)).

Consider a monopolistic firm with the option to invest in a project whose value depends on a single source of uncertainty that, in our case, corresponds to the unitary output price P, exogenously defined, which is assumed to follow a geometric Brownian motion process:[1]

$$dP = \alpha P dt + \sigma P dz, \tag{1}$$

where α and σ denote the risk-neutral drift and the volatility, respectively, and dz is an increment of the standard Wiener process. Additionally, $\alpha = r - \delta$, where r stands for the risk-free rate and δ is a return shortfall, or convenience yield. Assume the project requires an investment cost K, allowing the firm to produce a fixed output quantity Q. For the sake of simplicity, operating costs and taxes are not considered. After investing the value of the active project is:

$$V(P) = \frac{P}{\delta} Q. \tag{2}$$

where Q is a scaling factor that can be set to 1, without loss of generality.

Following the standard arguments, the value of a monopolistic opportunity to invest in this project, $F(P)$, is given by:

$$F(P) = \begin{cases} (V(P^*) - K)\left(\dfrac{P}{P^*}\right)^{\beta_1} & \text{for } P < P^* \\ V(P) - K & \text{for } P \geqslant P^* \end{cases} \tag{3}$$

where P^* corresponds to the investment trigger:

$$P^* = \frac{\beta_1}{\beta_1 - 1} \delta K, \tag{4}$$

and β_1 is the positive root of the characteristic quadratic equation $\frac{1}{2}\sigma^2\beta(\beta-1) + \alpha\beta - r = 0$, i.e.,

$$\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}. \tag{5}$$

3 The investment opportunity with a collar

We consider a government offering a concessionaire firm a collar arrangement that specifies certain limitations on the unit output price P. Specifically, the price floats freely subject

[1] This a common assumption in the literature. However other assumptions regarding the stochastic process occasionally appear as in Zapata Quimbayo et al. (2019), where traffic volume in a PPP is modelled as a mean-reverting process.

to a price floor P_L (corresponding to a low price) and a price cap P_H (corresponding to a high price), where $P_H \geqslant P_L$[2], so whenever $P_L \leqslant P < P_H$ the firm receives a unit price P; but, if $P < P_L$ or $P \geqslant P_H$ then the firm receives P_L or P_H, respectively. The firm receives the instantaneous revenue $R(P, P_L, P_H, Q) = \min\{\max\{P_L, P\}, P_H\}Q$ where Q denotes a constant output volume, that acts as a scaling factor that can be set to 1 without loss of generality. The collar corresponds to a portfolio of a floor and a cap, each composed of individual floorlets and caplets.[3]

In this arrangement, the government compensates the firm for low cash-flows while the firm foregoes abnormally high cash-flows. The government provides the firm with downside protection for $P < P_L$ by paying a subsidy $(P_L - P)$, but the firm has to transfer the excess profit $(P - P_H)$ to the government if $P > P_H$. By judiciously selecting P_L, P_H, the parties can moderate both the levels of downside protection and upside sacrifice, which consequently alters the pre-investment values for the collar arrangement as well as affecting its timing.

Initially, we present the solution for an investment opportunity with a perpetual collar, and then proceed to evaluate (i) finite-lived and (ii) retractable collars. In each case, we derive the pre- and post-investment values (idle and active values) as well as the impact of changing parameter values on the level of the trigger that justifies immediate investment.

3.1 Investments with perpetual collars

The solutions for an investment opportunity with a perpetual collar can be found in Adkins and Paxson (2017). Ignoring any operating costs, $V_p(P)$ denotes the value of an active project whose output price P is bounded by a price floor P_L and a price cap P_H. The solution for $V_p(P)$ satisfies the following non-homogeneous differential equation:

$$\frac{1}{2}\sigma^2 P^2 \frac{\partial^2 V_p(P)}{\partial P^2} + \alpha P \frac{\partial V_p(P)}{\partial P} - rV_p(P) + R(P) = 0, \tag{6}$$

where $R(P) = R(P, P_L, P_H) = \min\{\max\{P_L, P\}, P_H\}$ for convenience, and $R(P) = P_L$ for $P < P_L$, $R(P) = P$ for $P_L \leqslant P < P_H$, $R(P) = P_H$ for $P \geqslant P_H$, and

$$\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0. \tag{7}$$

.

[2]For the particular case where $P_L = P_H$, the collar reduces to a fixed price payment.

[3]Our collars have a ruthless exercise characteristic so that whenever $P < P_L$, the concessionaire receives P_L, and whenever $P > P_H$, the government receives $P - P_H$, while the concessionaire receives P_H. "Exercise" is ruthless, automatic, that is, occurs without either party taking an explicit action apart from making or receiving payments. Conceivably these payments could be made every minute, or half-hour for electricity in the UK according to the actual spot electricity rate, but perhaps conveniently payments might be settled monthly, quarterly or annually according to the legal accounting specification, based on averages. Once payments are made the contingent arrangement continues, in contrast to financial options.

The general solution of (6) is:

$$V_p(P) = A_a P^{\beta_1} + A_b P^{\beta_2} \tag{8}$$

The solutions for the non-homogeneous part (the particular solutions) depend on where P stands in relation to P_L and P_H. Accordingly, the particular solution for $P < P_L$ is P_L/r, for $P \in [P_L, P_H)$ is P/δ, and for $P > P_H$ becomes P_H/r. Considering that $V_p(0) = 0$, then $A_b = 0$ for $P < P_L$. Additionally, given that $V_p(P)$ has an upside limit of P_H/r whenever $P \geqslant P_H$, then A_a must be set equal to 0 in this region. Putting together the solutions for all the regions we get:

$$V_p(P) = \begin{cases} A_{11}P^{\beta_1} + \dfrac{P_L}{r} & \text{for } P < P_L \\ A_{21}P^{\beta_1} + A_{22}P^{\beta_2} + \dfrac{P}{\delta} & \text{for } P_L \leqslant P < P_H \\ A_{32}P^{\beta_2} + \dfrac{P_H}{r} & \text{for } P \geqslant P_H \end{cases} \tag{9}$$

The constants $A_{11}, A_{21}, A_{22}, A_{32}$ are found by ensuring that $V_p(P)$ is continuous and continuously differentiable along P. The solutions for the constants are as follows:[4]

$$A_{11} = \frac{\left(P_H^{1-\beta_1} - P_L^{1-\beta_1}\right)}{\beta_1 - \beta_2}\left(\frac{\beta_2 - 1}{\delta} - \frac{\beta_2}{r}\right) \tag{10}$$

$$A_{21} = \frac{P_H^{1-\beta_1}}{\beta_1 - \beta_2}\left(\frac{\beta_2 - 1}{\delta} - \frac{\beta_2}{r}\right) \tag{11}$$

$$A_{22} = -\frac{P_L^{1-\beta_2}}{\beta_1 - \beta_2}\left(\frac{\beta_1 - 1}{\delta} - \frac{\beta_1}{r}\right) \tag{12}$$

$$A_{32} = \frac{\left(P_H^{1-\beta_2} - P_L^{1-\beta_2}\right)}{\beta_1 - \beta_2}\left(\frac{\beta_1 - 1}{\delta} - \frac{\beta_1}{r}\right) \tag{13}$$

Moving back to the idle stage, the value of the perpetual American real option to invest in a project with a perpetual collar, $F_p(P)$ must satisfy the following ordinary differential equation:

$$\frac{1}{2}\sigma^2 P^2 \frac{\partial^2 F_p(P)}{\partial P^2} + \alpha P \frac{\partial F_p(P)}{\partial P} - rF_p(P) = 0. \tag{14}$$

The general solution has the form $F_p(P) = B_a P^{\beta_1} + B_b P^{\beta_2}$. Considering that $F_p(0) = 0$ then we set $B_b = 0$. The arbitrary constant B_a is found using the value matching condition (VM):

$$F_p(P_p^*) = B_a P_p^{*\beta_1} = V_p(P_p^*) - K, \tag{15}$$

[4]These solutions are obtained equalizing the value and the derivatives of the first and second branches of Equation (9) at P_L, and of the second and third branches at P_H.

i.e.,

$$B_a = \left(V_p(P_p^*) - K\right)\left(\frac{1}{P_p^*}\right)^{\beta_1}, \tag{16}$$

and the investment trigger, P_p^*, is obtained by solving the smooth pasting condition (SP):

$$\beta_1 B_a {P_p^*}^{\beta_1 - 1} = V_p'(P_p^*).$$

Multiplying both sides by P_p^* and using the VM condition, we get:

$$\beta_1(V_p(P_p^*) - K) = V_p'(P_p^*)P_p^* \tag{17}$$

The price floor must be lower than Kr, otherwise the government guarantees that the investment will never return a value less than K.[5] Notice that the trigger P_p^* can be either below or above P_H (but above P_L), which means the VM and the SP can be placed in all of the domain $P \in [P_L, \infty)$. Considering this domain from Equation (9), the trigger is:

$$P_p^* = \left(\frac{\beta_1}{(\beta_1 - \beta_2)A_{32}}\left(K - \frac{P_H}{r}\right)\right)^{\frac{1}{\beta_2}} > P_H, \qquad \text{for } K \geqslant K_p^H, \tag{18}$$

where[6]

$$K_p^H = \frac{P_H^{\beta_2}}{\beta_1}\left(P_H^{1-\beta_2} - P_L^{1-\beta_2}\right)\left(\frac{\beta_1 - 1}{\delta} - \frac{\beta_1}{r}\right) + \frac{P_H}{r}, \tag{19}$$

and it, P_p^*, is found by solving numerically the following equation for the remaining cases:

$$(\beta_1 - \beta_2)A_{22}{P_p^*}^{\beta_2} + (\beta_1 - 1)\frac{P_p^*}{\delta} - \beta_1 K = 0, \qquad \text{for } K_p^0 = P_L/r < K < K_p^H. \tag{20}$$

From Equation (18) and as illustrated in our numerical example, the investment trigger can exceed the price cap P_H for appropriate market conditions such as high volatility or project-specific conditions such as a high investment cost. For example, we know from the standard model that the investment trigger increases with underlying volatility to mitigate the chance of implementing a project having a future adverse value. Similarly, the collar investment trigger exceeds the price cap for certain high volatilities to lessen the chance of P falling beneath P_H and indeed P_L.

[5]When $P_L \to Kr$ in the domain $[P_L, P_H)$, Equation (17) becomes:

$$-\left(\frac{P_p^*}{P_L}\right)^{\beta_2} P_L\left(\frac{\beta_1 - 1}{\delta} - \frac{\beta_1}{r}\right) + (\beta_1 - 1)\frac{P_p^*}{\delta} - \beta_1\frac{P_L}{r} = 0$$

whose only valid solution is $P_p^* = P_L$ (other possible real solutions are negative). Therefore, for any set of parameters, $\lim_{P_L \to Kr} P_p^* = P_L$. Notice that if $P_L = Kr$, the net present value of the project $(V_p(P) - K)$ is positive, and, in the limit, zero when $P \to 0$. A higher price floor produces a risk-free investment opportunity (a positive NPV for every P), inducing investment for any P (the threshold becomes zero). Additionally, there is no economic reason to offer a floor above Kr since the zero threshold is achieved at $P_L = Kr$.

[6]K_p^H is found solving, using Equation (18), $P_p^*(K_p^H) = P_H$.

Accordingly, the solution for $F_p(P)$ is:

$$F_p(P) = \begin{cases} \left(V_p(P_p^*) - K\right)\left(\dfrac{P}{P_p^*}\right)^{\beta_1} & \text{for } P < P_p^* \\ V_p(P) - K & \text{for } P \geqslant P_p^* \end{cases} \tag{21}$$

3.2 Investments with finite-lived collars

We now evaluate a collar investment that is finite-lived. As before, a government offers a collar contract that guarantees the firm a minimum price P_L while limiting the gains to a maximum P_H, but confined to a finite duration $T < \infty$ years. The collar contract remains in force during the period $(t^*,\ t^* + T)$, where t^* is the investment timing, but lapses at period T. Thereafter, the firm's profits depend vary according to the stochastic behaviour of P.

Let us start with the solution for the active project. Immediately after being undertaken, the value of a project protected by a collar that lasts for T years corresponds to the finite integral of European caplets and floorlets. The individual caplets and floorlets can be continuously exercised during the T years, just by being in the money. The finite integral is equivalent to a portfolio that includes: (i) a long position in a perpetual collar, (ii) a short position in a forward-start perpetual collar (that start after T years). As suggested by Shackleton and Wojakowski (2007), this is possible because the individual caplets and floorlets contained within the integral are independent, and, therefore, can be computed as finite collar integral as the difference between the perpetual collar and the risk neutral expectation of the forward start perpetual collar.

Additionally, if the project lasts beyond the collar period, we need to add (iii) a long position in the expected profits that will start after T years. Combining (i) and (ii) replicates the finite-collar, whereas (iii) captures the value in operating the project without restrictions in P perpetually after the end of the collar.[7]

Accordingly, the value of an active project with a finite-lived collar is given by:

$$V_f(P,T) = V_p(P) - S(P,T) + \frac{P}{\delta}e^{-\delta T}. \tag{22}$$

The first term, $V_p(P)$, is as presented in equation (9). The second term, $S(P,T)$, represents the present value of a forward-start perpetual collar (a collar that starts in the

[7] Our generic model assumes a perpetual concession with a finite collar. Naturally, the model also applies for finite concessions with a duration the same or less than the finite collar. In that case component (iii) should be ignored.

future moment T), which is given by:[8]

$$\begin{aligned}
S(P,T) &= A_{11}P^{\beta_1}N(-d_{\beta_1}(P,P_L)) + \frac{P_L}{r}e^{-rT}N(-d_0(P,P_L)) \\
&+ A_{21}P^{\beta_1}\left(N(d_{\beta_1}(P,P_L)) - N(d_{\beta_1}(P,P_H))\right) \\
&+ A_{22}P^{\beta_2}\left(N(d_{\beta_2}(P,P_L)) - N(d_{\beta_2}(P,P_H))\right) \\
&+ \frac{P}{\delta}e^{-\delta T}\left(N(d_1(P,P_L)) - N(d_1(P,P_H))\right) \\
&+ A_{32}P^{\beta_2}N(d_{\beta_2}(P,P_H)) + \frac{P_H}{r}e^{-rT}N(d_0(P,P_H)),
\end{aligned} \tag{23}$$

where $N(.)$ is the standard normal cumulative distribution, and

$$d_\beta(P,x) = \frac{\ln P - \ln x + \left(r - \delta + (\beta - 0.5)\sigma^2\right)T}{\sigma\sqrt{T}}\ , \qquad \beta \in \{0,1,\beta_1,\beta_2\},\ x \in \{P_L, P_H\}. \tag{24}$$

Naturally, in Equation (23) the negative sign represents the short position in the forward-start perpetual collar. Finally, the last term represents the present value of the expected profits that will start after T.

The value of the option to invest in a project with a finite collar, $F_f(P)$ must satisfy the following ordinary differential equation:

$$\frac{1}{2}\sigma^2 P^2 \frac{\partial^2 F_f(P)}{\partial P^2} + \alpha P \frac{\partial F_f(P)}{\partial P} - rF_f(P) = 0, \tag{25}$$

whose general solution is $F_f(P) = C_a P^{\beta_1} + C_b P^{\beta_2}$. Considering the boundary at $P = 0$ ($F_f(0) = 0$) we set $C_b = 0$. The arbitrary constant C_a and the investment trigger P_f^* are found using the value VM and SP conditions:

$$C_a P_f^{*\beta_1} = V_f(P_f^*, T) - K \tag{26}$$

$$\beta_1 C_a P_f^{*\beta_1 - 1} = V_f'(P_f^*). \tag{27}$$

For the finite collar, no restrictions are required for the VM and SP, meaning that the transition between the idle and the active stages can occur for any P $\left(P_L \lesseqqgtr P_f^* \lesseqqgtr P_H\right)$. However, the price floor must be lower than $Kr/(1 - e^{-rT})$, otherwise it produces a risk-free profit.[9] The SP condition (27), allows us to find the trigger P_f^* and can be reduced to:[10]

$$\beta_1(V_f(P_f^*, T) - K) - V_f'(P_f^*)P_f^* = 0. \tag{28}$$

[8] See Appendix A.

[9] The maximum value of P_L is found by solving $V_f(0) - K = 0$. Notice that $V_p(0) = \frac{P_L}{r}$ and $S(0) = \frac{P_L}{r}e^{-rT}$.

[10] Multiplying both sides by P_f^* and using the VM condition.

Appendix B provides the equations that need to be solved to obtain P_f^*.

The value of the option to invest in the project granted with a finite-lived collar (F_f) is:

$$F_f(P) = \begin{cases} (V_f(P_f^*, T) - K)\left(\dfrac{P}{P_f^*}\right)^{\beta_1} & \text{for } P < P_f^* \\ V_f(P, T) - K & \text{for } P \geqslant P_f^* \end{cases} \tag{29}$$

3.3 Investments with retractable collars

In the setting of the previous section the government offers a finite collar that will be honoured after the investment takes place. Alternatively, a temporary collar may be offered, without specifying a date when it will be retracted (Adkins and Paxson, 2016). This corresponds to a perpetual collar that will be withdrawn at an unknown date, both for idle and active projects, which become an American option to invest in a project with random-lived conditions. If the collar is withdrawn before the firm invests, the investment opportunity becomes only dependent on the output market price, whereas if the firm invests before the collar is withdrawn, it benefits from the collar protection only until the retraction (random) date. This kind of policy uncertainty occurs in renewable energy investments, as governments often change their support schemes.[11] In line with Dixit and Pindyck (1994, ch. 9), Hassett and Metcalf (1999) and, among others, Adkins and Paxson (2019), we model policy uncertainty as a Poisson jump, with a probability of occurrence of λdt over the short interval dt.[12]

The value of the active project, $V_r(P)$, is the solution to the following differential equation:

$$\frac{1}{2}\sigma^2 P^2 \frac{\partial^2 V_r(P)}{\partial P^2} + \alpha P \frac{\partial V_r(P)}{\partial P} - rV_r(P) + R(P) + \lambda\left[V(P) - V_r(P)\right] = 0, \tag{30}$$

where $R(P) \equiv R(P, P_L, P_H) = \min\{\max\{P_L, P\}, P_H\}$. The last term of the left-hand side of the equation represents the loss in value that is expected due to the likelihood of the collar withdrawal occurring in the next instant.

The solution for the homogeneous part of equation (30) has the form $D_a P^{\eta_1} + D_b P^{\eta_2}$,

[11] Ritzenhofen and Spinler (2016) argue that governments revisit their policies in response to technology innovations and budget constraints.

[12] Under the Poisson process, the expected time to the collar withdrawal is $E[T] = 1/\lambda$. Poisson processes have also been used, e.g. by Merton (1976), to model discontinuous stock returns as a mixed Brownian motion/Poisson process.

where

$$\eta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2(r+\lambda)}{\sigma^2}} \tag{31}$$

$$\eta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2(r+\lambda)}{\sigma^2}}. \tag{32}$$

.

As before, the solutions for the non-homogeneous part (the particular solutions) depend where P stands in relation to P_L and P_H and also of the possible transition from $V_r(P)$ to $V(P)$. The particular solutions are the same as for the perpetual case, with an additional term that accounts for the possible collar retraction: $\lambda/(\delta+\lambda) \times P/\delta$.

The solutions for the three regions are:

$$V_r(P) = \begin{cases} D_{11}P^{\eta_1} + \dfrac{\lambda}{\delta+\lambda}\dfrac{P}{\delta} + \dfrac{P_L}{r+\lambda} & \text{for } P < P_L \\ D_{21}P^{\eta_1} + D_{22}P^{\eta_2} + \dfrac{P}{\delta} & \text{for } P_L \leqslant P < P_H \\ D_{32}P^{\eta_2} + \dfrac{\lambda}{\delta+\lambda}\dfrac{P}{\delta} + \dfrac{P_H}{r+\lambda} & \text{for } P \geqslant P_H \end{cases} \tag{33}$$

The solution for the constants $D_{11}, D_{21}, D_{22}, D_{32}$ are found by ensuring that $V_r(P)$ is continuous and continuously differentiable along P:

$$D_{11} = \frac{\left(P_H^{1-\eta_1} - P_L^{1-\eta_1}\right)}{\eta_1 - \eta_2}\left(\frac{\eta_2 - 1}{\delta+\lambda} - \frac{\eta_2}{r+\lambda}\right) \tag{34}$$

$$D_{21} = \frac{P_H^{1-\eta_1}}{\eta_1 - \eta_2}\left(\frac{\eta_2 - 1}{\delta+\lambda} - \frac{\eta_2}{r+\lambda}\right) \tag{35}$$

$$D_{22} = -\frac{P_L^{1-\eta_2}}{\eta_1 - \eta_2}\left(\frac{\eta_1 - 1}{\delta+\lambda} - \frac{\eta_1}{r+\lambda}\right) \tag{36}$$

$$D_{32} = \frac{\left(P_H^{1-\eta_2} - P_L^{1-\eta_2}\right)}{\eta_1 - \eta_2}\left(\frac{\eta_1 - 1}{\delta+\lambda} - \frac{\eta_1}{r+\lambda}\right) \tag{37}$$

Naturally, for the two extreme values of λ, the value of $V_r(P)$ converges to the expected values, i.e. when $\lambda \to 0$, the collar will never be retracted and, therefore, the value converges to the value of the perpetual collar ($V_r(P) \to V_p(P)$), and when $\lambda \to \infty$, the collar will be retracted immediately, and the value converges to the plain active project value ($V_r(P) \to V(P)$).[13]

The value of the idle project must be the solution to the following ordinary differential

[13]Notice that $\lim_{\lambda\to 0}\eta_1 = \beta_1$, $\lim_{\lambda\to 0}\eta_2 = \beta_2$, $\lim_{\lambda\to\infty} D_{11} = \lim_{\lambda\to\infty} D_{21} = \lim_{\lambda\to\infty} D_{22} = \lim_{\lambda\to\infty} D_{32} = 0$.

equation:

$$\frac{1}{2}\sigma^2 P^2 \frac{\partial^2 F_r(P)}{\partial P^2} + \alpha P \frac{\partial F_r(P)}{\partial P} - rF_r(P) + \lambda\left[F(P) - F_r(P)\right] = 0. \tag{38}$$

The solution, considering the boundary condition at $P = 0$, is:

$$F_r(P) = E_a P^{\eta_1} + \frac{\lambda}{r+\lambda} F(P), \tag{39}$$

where

$$F(P) = HP^{\beta_1} = (V(P^*) - K)\left(\frac{1}{P^*}\right)^{\beta_1} P^{\beta_1}. \tag{40}$$

The arbitrary constant E_a is found using the VM condition:

$$F_r(P_r^*) = E_a P_r^{*\eta_1} + \frac{\lambda}{r+\lambda} HP_r^{*\beta_1} = V_r(P_r^*) - K, \tag{41}$$

i.e.,

$$E_a = \left(V_r(P_r^*) - K - \frac{\lambda}{r+\lambda} HP_r^{*\beta_1}\right)\left(\frac{1}{P_r^*}\right)^{\eta_1}, \tag{42}$$

and the investment trigger, P_r^*, is obtained by solving the smooth pasting condition (SP):

$$\eta_1(V_r(P_r^*) - K) + (\beta_1 - \eta_1)\frac{\lambda}{r+\lambda} HP_r^{*\beta_1} = V_r'(P_r^*)P_r^*. \tag{43}$$

Appendix C provides the equations that need to be solved to obtain P_r^*.

4 Comparative statics

Some important features of the model are analysed with a numerical example. Consider an investment option for which the base-case parameters in Table 1 apply.

Parameter	Description	Value
P	Current price of the output	\$2
P_L	Price floor	\$2
P_H	Price cap	\$6
σ	Volatility	0.2
r	Risk-free rate	0.04
δ	Return shortfall	0.03
Q	Output quantity	1
K	Investment cost	\$70
T	Duration of the collar (years)	10
λ	Arrival rate of the collar withdrawal	0.1

Table 1: The base case parameters.

As stated before, the value of active project with a collar can be decomposed into the value of the plain active project and a portfolio of short positions in the caplets and long positions in floorlets. Table 2 shows the value of the active project for different values of the state variable and different collar maturities for the finite case (Panel A) and different withdrawal arrival rates for the retractable case (Panel B). The protection provided by the floor decreases as P increases, while the opposite occurs for the penalty coming from the price cap. Additionally, we can see that adding a collar to the plain active project can create or destroy value for the firm. Specifically, when P is closer to the price floor (P_L), the collar adds value, and the opposite occurs when P is closer to the price cap (P_H). Finally the effect of the duration of the finite collar or the likelihood of the (retractable) collar withdrawal also has an ambiguous effect.

Panel A - Finite collars

	$T = 10$			$T = 25$			$T = \infty$		
	$P = \$2$	$P = \$3$	$P = \$6$	$P = \$2$	$P = \$3$	$P = \$6$	$P = \$2$	$P = \$3$	$P = \$6$
Plain	\$66.67	\$100.00	\$200.00	\$66.67	\$100.00	\$200.00	\$66.67	\$100.00	\$200.00
Short Caplets	-\$0.09	-\$0.64	-\$9.39	-\$1.56	-\$4.96	-\$29.51	-\$14.16	-\$28.05	-\$90.28
Long Floorlets	\$2.34	\$0.67	\$0.04	\$6.27	\$2.88	\$0.63	\$13.43	\$8.30	\$3.65
Collar	\$68.91	\$100.03	\$190.65	\$71.37	\$97.92	\$171.13	\$65.93	\$80.25	\$113.37

Panel B - Retractable collars

	$\lambda = 0.1; E[T] = 10$			$\lambda = 0.04; E[T] = 25$			$\lambda = 0; E[T] = \infty$		
	$P = \$2$	$P = \$3$	$P = \$6$	$P = \$2$	$P = \$3$	$P = \$6$	$P = \$2$	$P = \$3$	$P = \$6$
Plain	\$66.67	\$100.00	\$200.00	\$66.67	\$100.00	\$200.00	\$66.67	\$100.00	\$200.00
Short Caplets	-\$0.42	-\$1.36	-\$10.18	-\$2.15	-\$5.40	-\$25.96	-\$14.16	-\$28.05	-\$90.28
Long Floorlets	\$2.29	\$0.86	\$0.16	\$5.08	\$2.48	\$0.73	\$13.43	\$8.30	\$3.65
Collar	\$68.54	\$99.51	\$189.99	\$69.59	\$97.09	\$174.77	\$65.93	\$80.25	\$113.37

Table 2: Decomposition of the value of an active project with a collar, for finite durations ($T = 10$ and $T = 25$, or $\lambda = 0.1$ and $\lambda = 0.04$) and for the perpetual case ($T = \infty$ or $\lambda = 0$). The results are for $P = \$2$, \$3 and \$6, $P_L = \$2$, $P_H = \$6$, $\sigma = 0.2$, $r = 0.04$, $\delta = 0.03$, $Q = 1$, $K = \$70$.

Figure 1 illustrates these effects and compares the value of the active project with the intrinsic value of the collar (assuming that P stays forever at the current level) and the plain project (without collar) for different P values (moneyness of the collar).

[Figure 1 about here]

In Figure 2 we can analyse the effects of the main parameters on the value of the active project in more detail. Figures 2(a)–2(b) show that a higher return shortfall reduces the value of the active projects in all cases. In general, the perpetual collar has a lower value because it imposes a perpetual cap, except when the return shortfall is high and the current level of P is far from the cap (Figure 2(a)). Figures 2(c)–2(d) show different sensitivities in respect to volatility. For $P = \$2$ the value of the perpetual collar is shown to be non-monotonic. For a low volatility, the moneyness of the long put options protection (floorlets) dominates. As the volatility increases, the probability for entering in the in-the-money region of the short call options position (caplets) increases, decreasing the value of the perpetual collar. The effect of the short call position dominates after a certain level of P, which depends on the volatility. Therefore, for low price and low uncertainty levels, the value of the perpetual collar increases before reaching an inflection point. In general, the value of the perpetual collar stays below that of finite-lived collar,[14] and the difference increases with σ and P. Economically, the high values of these two parameters increase

[14]However, it is possible to show that for a sufficiently high δ, the value of the active project with a perpetual collar is higher than that of the finite-lived case, at the base parameter values.

the likelihood of being above P_H, which penalizes the firm while being subject to the cap. For the case of the finite-lived, it is possible for the firm to escape from the cap penalty after the collar period ends, which enhances its value when compared to the case of the perpetual collar.

[Figure 2 about here]

Additionally, in 2(e)–2(f) and 2(g)–2(h) we see the value of the perpetual, finite and retractable collars increase as P_H and P_L increase. Interestingly, for low (high) price floors, the finite and retractable (perpetual) collars are more valuable. The level of P_L for the separation region increases as P increases, which means the current level of P is critical in negotiating the floor level and the finite collar duration. Additionally, the finite and retractable collars are not too sensitive to increases in P_H, so there would be little advantage after an initial arrangement of renegotiating the ceiling if the finite collar duration is short.

[Figure 3 about here]

Figure 3 shows the effect of the (expected) duration of the finite and retractable collars. In 3(a)–3(b) we see that the value of the active finite collar may increase or decrease as duration T increases (producing a non-monotonic effect), depending on the level of P. The finite collar value approaches the perpetual collar value as T approaches ∞. Referring to the option concept of Θ (the sensitivity of the option value with respect to remaining time to expiration) we see that, when P is closer to P_H, the short position in the call options dominates, producing an overall value increase in the active finite collar as T decreases, i.e., $\Theta < 0$ (Figure 3(b)). In practical terms, due to the short position, the firm benefits as the call options approach the maturity date. On the other hand, whenever the long position in the put option becomes dominant (if P is closer to P_L and the duration is not too long) it produces an overall value increase as T increases, revealing the traditional $\Theta > 0$ (Figure 3(a)). The value of the retractable collar is higher than the value of a finite collar for long maturities if the price level is distant from the price floor. Short-maturity finite collars are less valuable than retractable collars for low levels of P. Finally, in 3(c)–3(d) we see that the effects of the expected duration ($1/\lambda$) of a retractable collar are similar to those of the duration of the finite-lived collar.

In summary, when compared to the perpetual collars, (i) finite and retractable collars provide substantially less downside risk protection for the concessionaire when P is low; (ii) the finite and retractable collars sensitivity to volatility is smaller and both can be non-monotonic; (iii) finite and retractable collars are less sensitive to changes in the cap P_H.

We now analyse the effect of the parameter values on the investment timing and the value of the investment opportunity. Table 3 shows the investment triggers for different maturities of the finite collar (Panel A) and different arrival rate of the retractable collars withdrawal (Panel B), both for two price floors. As expected a higher price floor hastens investment. Somewhat surprisingly the effect of the finite collar maturity can be non-monotonic for the lower price floor ($P_L = 1$). As we will see in the figures below, the likelihood of the collar withdrawal can have the same non-monotonic effect.

Panel A - Finite collars

	$T = 10$		$T = 25$		$T = \infty$	
	$P_L = 1$	$P_L = 2$	$P_L = 1$	$P_L = 2$	$P_L = 1$	$P_L = 2$
Finite	\$4.788	\$4.764	\$4.580	\$4.421	\$5.037	\$4.519
Perpetual	\$5.037	\$4.519	\$5.037	\$4.519	\$5.037	\$4.519

Panel B - Retractable collars

	$\lambda = 0.1; E[T] = 10$		$\lambda = 0.04; E[T] = 25$		$\lambda = 0; E[T] = \infty$	
	$P_L = 1$	$P_L = 2$	$P_L = 1$	$P_L = 2$	$P_L = 1$	$P_L = 2$
Retractable	\$4.178	\$4.128	\$4.244	\$4.097	\$5.037	\$4.519
Perpetual	\$5.037	\$4.519	\$5.037	\$4.519	\$5.037	\$4.519

Table 3: Investment triggers for finite durations ($T = 10$ and $T = 25$, or $\lambda = 0.1$ and $\lambda = 0.04$) and for the perpetual case ($T = \infty$ or $\lambda = 0$). The results are for $\sigma = 0.2$, $P_L = \$1$ and $P_L = \$2$, $P_H = \$6$, $r = 0.04$, $\delta = 0.03$, $Q = 1$, $K = \$70$.

Figures 4, 5, and 6 allow us to go deeper into the comparative static analysis of the impact of the main parameter values on the investment triggers, showing the results for finite, retractable and perpetual collars as well as for the plain (without collar) investment opportunity. The impact of the price floor is analysed in Figure 4(a). As stated before, a higher P_L hastens investment, and for a high P_L the investment trigger for the finite collar stays below the price floor.[15] The figure also reveals which arrangement is more effective for hastening the investment for different levels of P_L, namely that the finite and retractable collars are preferable only for low levels of the price floor.

[Figure 4 about here]

We see in Table 3 the level of the price floor can produce different impacts of the maturity on the investment timing. By depicting in the following figures the effects of the parameter values for two price floors ($P_L = 1$ and $P_L = 2$) we are able to show that the level of P_L may have a significant impact on the investment timing order of the finite, retractable and perpetual collars, and also on their relation with the plain investment opportunity.

[15] This occurs for the P_L that solves Equation (57) or (74), for the finite and retractable collars, respectively.

Figures 4(b)–4(c) depict the well known effect of uncertainty on investment timing (a higher volatility implies a higher trigger). In particular, it shows that the trigger for a finite-lived and a retractable collar can be smaller that of the perpetual collar for a low price floor ($P_L = 1$) or they can lie between those of a perpetual collar and a plain investment for a higher price floor ($P_L = 2$) and high volatilities. Additionally, the triggers, depending on the levels of uncertainty, can be placed below or above the price cap P_H. As we already said, there is economic reasoning for the latter situation. When the uncertainty is significant, the investor may find it optimal to wait and only invest for a sufficiently large P, knowing that the firm only receives P_H if the price trigger surpasses the cap, to mitigate the larger probability of significantly lower values of P in the future, due to the high volatility. For a low price floor, a perpetual collar may not be sufficient to hasten investment (the trigger is above the plain project trigger) for a high volatility (Figure 4(b)).

Figures 4(d)–4(e) report the obvious positive relation between the investment cost and the triggers. However, depending on the level of investment one type of collar contract (either finite/retractable or perpetual) can be preferred in hastening investment. For low investment costs when $P_L = 2$ a perpetual collar should be set, otherwise a finite collar and particularly a retractable collar is more effective. This figure also shows that for relatively expensive projects, the investment takes place at output prices above the cap.

[Figure 5 about here]

Figure 5 highlights the effect of the return shortfall on the investment trigger. Figure 5(a) shows that a higher investment shortfall deters investment. This is produced by two opposing effects: on the one hand, a higher δ reduces the active project value (see Figures 2(a) and 2(b)) and, on the other hand, a higher δ increases the cost of waiting. The former effect dominates for the base case parameters.

Figures 5(b)–5(c) show the impact of the price cap P_H on the trigger and its effectiveness in promoting investment. For the base case δ, the retractable collar is the most effective in hastening investment. However, for a higher return shortfall ($\delta = 0.04$), three regions appear. For a low P_H, the plain project (without collar) is preferable for hastening the investment, as its trigger remains below the other triggers. For intermediate levels of the price cap, the retractable collar is more effective than a perpetual collar, whereas for high P_H the perpetual collar is the one that contributes more in promoting early investment.

In any case, it is interesting to note that the effect of the price cap on the finite collar is non-monotonic. Initially a higher P_H is able to induce earlier investments but after a certain level of P_H the opposite effect occurs. There seems to exist a limit to the capacity of inducing earlier investment by reducing the price cap, i.e. there is an optimal level of P_H given the other parameter values. A government should not impose a lower price cap if its objective is to promote investment, otherwise the reverse effect will occur.

[Figure 6 about here]

As suggested in Table 3, the (expected) duration of the collar may have a non-monotonic effect on the investment timing, which is shown in more detail in Figure 6. As for the price cap, there is an optimal duration of the finite collar that induces the earliest investment. If the collar lasts long enough, a finite-lived or a retractable collar are able to induce earlier investments than a perpetual collar. A retractable collar seems to be more effective in promoting investment than a finite collar for shorter durations. A long-lived collar is required in order for the finite collar to be preferable. Retractable collars are only more effective than finite collars in prompting investment if the likelihood of the collar withdrawal is not too high. In fact, for a duration of 10 years, the equivalent arrival rate ($\lambda = 0.1$) makes the retractable collar more attractive to promote investment.

In summary, a finite-lived or a retractable collar may induce earlier investments than a perpetual collar when the price floor and price cap are set sufficiently low, or when the investment cost and the duration of the collar or the likelihood of not being retracted, for low price floors, are sufficiently high. Furthermore an optimal level of the price cap and the duration of the finite collar exist, limiting the capacity of those instruments to promote investment when a finite collar is used. Retractable collars are usually more effective in promoting investment, possibly indicating that political uncertainty pays.

Figure 7 shows that the value of a finite or retractable collar increases as the uncertainty increases. However, the effect of uncertainty is ambiguous for the perpetual collar. The justification is similar to the one presented for the active project value (see Figure 4(c)).[16] Figure 7 also reveals that the value of the finite collar dominates that of the perpetual collar, regardless of the level of the triggers (Figures 4 and 5). However, the value of the investment opportunity when the collar is retractable may be smaller than that of a perpetual collar when the volatility is low.

[Figure 7 about here]

Retractable collars are usually more effective in promoting investment (Figures 4–6). Figure 8 shows that they can not only be more effective (lower triggers), but also more valuable for the firm, which is equivalent to being less costly to the government.[17] A retractable collar has a lower trigger if $\lambda < 2.96$ (Figure 8(a)), and simultaneously a higher value for the firm if $\lambda > 0.77$ (Figure 8(b)). If the perceived risk of retraction is in this region, the firm invests sooner and is less penalised by the retractable collar than when offered a finite collar.

[Figure 8 about here]

[16]The remaining figures of the sensitivity analysis are available upon request.

[17]The cost for the government is the difference between the value of the plain investment opportunity and the value of the investment opportunity with the collar.

5 Designing investment incentives

Our models can easily be extended to accommodate a mix of investment incentives that can be offered as a package. For an exogenous investment cost K, the thresholds are obtained as a function of the price floors and caps. Notice, however, that the threshold P^* can be an input, and $K^*(P^*, P_L, P_H)$ can be obtained as an output, corresponding to the investment cost that prompts investment at P^*. This is useful, for instance, to determine the amount of investment subsidy, S^*, that reduces the investment cost to $K - S^* = K^*(P_0, P_L, P_H)$, making immediate investment optimal (where P_0 is the current level of P). The triplet $\{S^*, P_L, P_H\}$ can be manipulated to obtain the desired outcome, ensuring that the restrictions on K are not violated ($K \geqslant K_i^0, i \in \{p, f, r\}$).

To illustrate, let us consider the cases of perpetual and retractable collars. For the perpetual case, the optimal subsidy depends whether $P_0 \geqslant P_H$ or $P_L \leqslant P_0 < P_H$. For the first case, the optimal subsidy is obtained by setting $P_p^* = P_0$ and $K = K - S_p^*$ in Equation 18, and solving for S_p^*, leading to:

$$S_p^* = K - K_p^*(P_0, P_L, P_H) = K - \left(\frac{P_H}{r} + \frac{(\beta_1 - \beta_2) A_{32} P_0^{\beta_2}}{\beta_1} \right) \tag{44}$$

provided that $S_p^* \leqslant K - K_p^H$.

The optimal subsidy for the other case is obtained using the same procedure in Equation (20):

$$S_p^* = K - K_p^*(P_0, P_L, P_H) = K - \frac{1}{\beta_1} \left((\beta_1 - \beta_2) A_{22} {P_0}^{\beta_2} + (\beta_1 - 1) \frac{P_0}{\delta} \right) \tag{45}$$

subject to $K - K_p^0 > S_p^* > K - K_p^H$.

Let us now analyse the case where a subsidy is offered along with a retractable collar. The first step for determining S_r^* is to find K_r^L and K_r^H (see equations (74) and (75) in Appendix C). The proper equation for finding S_r^* lies in one of the three regions presented in (73): in the upper region if $K - K_r^0 > S_r^* > K - K_r^L$, in the middle if $K - K_r^L \geqslant S_r^* > K - K_r^H$, or in the bottom if $S_r^* \leqslant K - K_r^H$. In (73), replace P_r^* by P_0, H by $\frac{K-S^*}{\beta_1 - 1} \left(\frac{\beta_1}{\beta_1 - 1} \delta (K - S_r^*) \right)^{-\beta_1}$, and K by $(K - S_r^*)$ in the proper equation, and solve numerically for S_r^*. Notice that the solution requires an interactive procedure in order to ensure that S_r^* respects the corresponding condition for the region.

6 Conclusion

In this paper we extend the previous literature on collar-style arrangements by determining the idle and active values, as well as the investment triggers, for projects where these arrangements are either finite-lived or retractable. These extensions reveal to be more realistic (when compared to the perpetual collar arrangements) and useful for decision makers when setting the specific conditions of the contract. In general, we demonstrate that the finite-lived and retractable versions have a significant impact on optimal behaviour, relative to the plain perpetual collar.

Our numerical results show that there are lots of differences between perpetual and finite/retractable collars. (i) both finite and retractable collars provide less downside risk protection for the beneficiary of the collar when market prices are low, but allow for substantial upside benefits when prices are very high; (ii) the finite and retractable collars are not very sensitive to volatility changes, but, like perpetual collars, can be non-monotonic; (iii) finite and retractable collars are hardly sensitive to changes in the price cap, especially at low prices; (iv) finite and retractable collars may motivate earlier investments either when the specified price cap is low, and/or the price floor is low, especially with short duration of the collar or low likelihood of not being withdrawn; (v) there is an optimal level of the price cap and the duration of the finite collar, which results in a low trigger, so these (and some other policy instruments) should not necessarily be considered separately; (vi) the value of the finite collar investment opportunity is always higher than the perpetual collar investment opportunity value, regardless of the triggers; (vii) retractable collars may have lower triggers than either finite or perpetual collars; (viii) the real option value of retractable collars is highly sensitive to the probability of withdrawal; and (ix) retractable collars can induce earlier and more valuable investments than finite collars.

A critical consideration for negotiating the floors, ceilings, and duration (or signalling the expected duration) of finite or retractable collars is the current price level and expected volatility over the life of the contract. Additionally, we show that the contract can be designed and complemented with investment subsides for a desired outcome, such as immediate investment.

For future research several additional topics could be addressed. Our model can easily accommodate taxation, tax relief, or revenue subsidies. Two stochastic factors can be considered (for instance, the quantity of output Q may also have a random behaviour). However, allowing for stochastic traffic or price volatility, traffic volatility decreasing over time in steps, or uncertain concession duration (apart from the retractable approach) would be challenging for an analytical model. Different stepped downside and upside sharing arrangements, as in the Adkins and Paxson (2019) Section 6 on partial collars, could be implemented for finite collars. Additionally, the finite and the uncertain (or retractable) duration can also be applied to several other investment incentives and in different contexts (domestic investment or FDI). Competition can also be included, particularly in the context of mixed markets. Finally, it would be of most interest to apply the models to real world situations, perhaps as supplements or substitutes for numerical methods.

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A The value of a forward start collar

Shackleton and Wojakowski (2007) value separately caps and floors. Following similar arguments the value of a forward start collar is given by:

$$S(P,T) = e^{-rT}\mathbf{E}_0^Q\left[V_p(P_T)\right], \tag{46}$$

where P_T if the price P at time T.

The value of perpetual collar starting at level P_T is given by Equation (9):

$$V_p(P_T) = \begin{cases} A_{11}P^{\beta_1} + \dfrac{P_L}{r} & \text{for } P_T < P_L \\ A_{21}P^{\beta_1} + A_{22}P^{\beta_2} + \dfrac{P}{\delta} & \text{for } P_L \leqslant P_T < P_H \\ A_{32}P^{\beta_2} + \dfrac{P_H}{r} & \text{for } P_T \geqslant P_H \end{cases} \tag{47}$$

or in a compact notation:

$$\begin{aligned} V_p(P_T) = &\left(A_{11}{P_T}^{\beta_1} + \frac{P_L}{r}\right)\mathbf{1}_{P_T<P_L} \\ &+ \left(A_{21}{P_T}^{\beta_1} + A_{22}{P_T}^{\beta_2} + \frac{P_T}{\delta}\right)\mathbf{1}_{P_L\leqslant P_T<P_H} \\ &+ \left(A_{32}{P_T}^{\beta_2} + \frac{P_H}{r}\right)\mathbf{1}_{P_T\geqslant P_H}, \end{aligned} \tag{48}$$

where the indicator $\mathbf{1}_{condition}$ equals 1 if the *condition* is met or 0 otherwise.

From the Appendix A of Shackleton and Wojakowski (2007):

$$e^{-rT}\mathbf{E}_0^Q\left[P_T^\beta \mathbf{1}_{P_T<P_L}\right] = e^{q(\beta)T}P^\beta N(-d_\beta(P,P_L)) \tag{49}$$

$$\begin{aligned} e^{-rT}\mathbf{E}_0^Q\left[P_T^\beta \mathbf{1}_{P_L\leqslant P_T<P_H}\right] &= e^{-rT}\mathbf{E}_0^Q\left[P_T^\beta \mathbf{1}_{P_T\geqslant P_L}\right] - e^{-rT}\mathbf{E}_0^Q\left[P_T^\beta \mathbf{1}_{P_T\geqslant P_H}\right] \\ &= e^{q(\beta)T}P^\beta\left(N(d_\beta(P,P_L)) - N(d_\beta(P,P_H))\right) \end{aligned} \tag{50}$$

$$e^{-rT}\mathbf{E}_0^Q\left[P_T^\beta \mathbf{1}_{P_T\geqslant P_H}\right] = e^{q(\beta)T}P^\beta N(d_\beta(P,P_H)), \tag{51}$$

where

$$d_\beta(P,x) = \frac{\ln P - \ln x + \left(r - \delta + (\beta - 0.5)\sigma^2\right)T}{\sigma\sqrt{T}}, \quad \beta \in \{0,1,\beta_1,\beta_2\},\ x \in \{P_L, P_H\}; \tag{52}$$

$$q(0) = -r, \quad q(1) = -\delta, \quad q(\beta_1) = 0, \quad q(\beta_2) = 0.$$

Rearranging, we obtain:

$$
\begin{aligned}
S(P,T) = {} & A_{11}P^{\beta_1}N(-d_{\beta_1}(P,P_L)) + \frac{P_L}{r}e^{-rT}N(-d_1(P,P_L)) \\
& + A_{21}P^{\beta_1}\left(N(d_{\beta_1}(P,P_L)) - N(d_{\beta_1}(P,P_H))\right) \\
& + A_{22}P^{\beta_2}\left(N(d_{\beta_2}(P,P_L)) - N(d_{\beta_2}(P,P_H))\right) \\
& + \frac{P}{\delta}e^{-\delta T}\left(N(d_0(P,P_L)) - N(d_0(P,P_H))\right) \\
& + A_{32}P^{\beta_2}N(d_{\beta_2}(P,P_H)) + \frac{P_H}{r}e^{-rT}N(d_1(P,P_H)). \qquad (53)
\end{aligned}
$$

B The investment trigger equations for finite-lived collars

Considering the three regions of V_p (Equation (9)) and depending on the value of K, Equation (28) becomes one of the following three equations, which must be solved to find the investment trigger P_f^*:

$$
Z(P_f^*) + \begin{cases}
(\beta_1 - 1)\dfrac{P_f^*}{\delta}e^{-\delta T} + \beta_1\left(\dfrac{P_L}{r} - K\right) = 0 & \text{for } K_f^0 < K < K_f^L \\
(\beta_1 - \beta_2)\,A_{22}{P_f^*}^{\beta_2} + (\beta_1 - 1)\left(\dfrac{P_f^*}{\delta} + \dfrac{P_f^*}{\delta}e^{-\delta T}\right) & \\
\qquad -\beta_1 K = 0 & \text{for } K_f^L \leqslant K < K_f^H \\
(\beta_1 - \beta_2)\,A_{32}{P_f^*}^{\beta_2} + (\beta_1 - 1)\dfrac{P_f^*}{\delta}e^{-\delta T} & \\
\qquad +\beta_1\left(\dfrac{P_H}{r} - K\right) = 0 & \text{for } K \geqslant K_f^H
\end{cases} \qquad (54)
$$

where

$$
\begin{aligned}
Z(P) =& -\beta_1 S(P,T) - S'(P)P \\
=& -(\beta_1-\beta_2)\Big[A_{22}P^{\beta_2}\left(N(d_{\beta_2}(P,P_L)) - N(d_{\beta_2}(P,P_H))\right) \\
& + A_{32}P^{\beta_2}N(d_{\beta_2}(P,P_H))\Big] \\
& -(\beta_1-1)\frac{P}{\delta}e^{-\delta T}\left(N(d_1(P,P_L)) - N(d_1(P,P_H))\right) \\
& -\beta_1\left[\frac{P_L}{r}e^{-rT}N(-d_0(P,P_L)) + \frac{P_H}{r}e^{-rT}N(d_0(P,P_H))\right]
\end{aligned} \tag{55}
$$

$$
K_f^0 = \frac{P_L}{r}\left(1-e^{-rT}\right) \tag{56}
$$

$$
K_f^L = \frac{1}{\beta_1}\left(Z(P_L) + (\beta_1-1)\frac{P_L}{\delta}e^{-\delta T} + \beta_1\frac{P_L}{r}\right) \tag{57}
$$

$$
K_f^H = \frac{1}{\beta_1}\left(Z(P_H) + (\beta_1-\beta_2)A_{32}{P_H}^{\beta_2} + (\beta_1-1)\frac{P_H}{\delta}e^{-\delta T} + \beta_1\frac{P_H}{r}\right). \tag{58}
$$

Appendix B.1 provides the derivative of the forward start collar, $S'(P)$.

B.1 Partial derivative of the forward start collar

Starting with the value of the forward start collar (Equation (23)) and noting that:

$$
N(d) = \int_{-\infty}^{d}\frac{1}{\sqrt{2\pi}}e^{-\frac{u^2}{2}}du \tag{59}
$$

$$
\frac{\partial d_\beta(P,x)}{\partial P} = \frac{1}{P\sigma\sqrt{T}} \tag{60}
$$

$$
\frac{\partial N\left(\pm d_\beta(P,x)\right)}{\partial P} = \frac{\partial\left(\pm N\left(d_\beta(P,x)\right)\right)}{\partial d_\beta(P,x)}\frac{\partial d_\beta(P,x)}{\partial P} = \pm\frac{1}{P\sigma\sqrt{2\pi T}}e^{-\frac{d_\beta(P,x)^2}{2}} \tag{61}
$$

$$
\begin{aligned}
\frac{\partial P^\beta N\left(\pm d_\beta(P,x)\right)}{\partial P} &= \frac{N\left(\pm d_\beta(P,x)\right)\partial P^\beta}{\partial P} + \frac{P^\beta\partial N\left(\pm d_\beta(P,x)\right)}{\partial P} \\
&= \beta P^{\beta-1}N\left(\pm d_\beta(P,x)\right) \pm \frac{P^{\beta-1}}{\sigma\sqrt{2\pi T}}e^{-\frac{d_\beta(P,x)^2}{2}}
\end{aligned} \tag{62}
$$

$$\frac{\partial\left[\frac{P_L}{r}e^{-rT}N\left(-d_0\left(P,P_L\right)\right)+\frac{P_H}{r}e^{-rT}N\left(d_0\left(P,P_H\right)\right)\right]}{\partial P}$$
$$=\frac{e^{-rT}}{rP\sigma\sqrt{2\pi T}}\left[P_H e^{-\frac{1}{2}d_0(P,P_H)^2}-P_L e^{-\frac{1}{2}d_0(P,P_L)^2}\right] \tag{63}$$

$$\frac{\partial\left[\frac{P}{\delta}e^{-\delta T}\left(N\left(d_1\left(P,P_L\right)\right)-N\left(d_1\left(P,P_H\right)\right)\right)\right]}{\partial P}$$
$$=\frac{1}{\delta}e^{-\delta T}\left[N\left(d_1\left(P,P_L\right)\right)-N\left(d_1\left(P,P_H\right)\right)\right]$$
$$+\frac{e^{-\delta T}}{\delta\sigma\sqrt{2\pi T}}\left[e^{-\frac{1}{2}d_1(P,P_L)^2}-e^{-\frac{1}{2}d_1(P,P_H)^2}\right] \tag{64}$$

$$\frac{\partial\left[A_{21}P^{\beta_1}\left(N\left(d_\beta\left(P,P_L\right)\right)-N\left(d_{\beta_1}\left(P,P_H\right)\right)\right)\right]}{\partial P}$$
$$=\beta_1 A_{21}P^{\beta_1-1}\left(N\left(d_{\beta_1}\left(P,P_L\right)\right)-N\left(d_{\beta_1}\left(P,P_H\right)\right)\right)$$
$$+\frac{A_{21}P^{\beta_1-1}}{\sigma\sqrt{2\pi T}}\left[e^{-\frac{1}{2}d_\beta(P,P_L)^2}-e^{-\frac{1}{2}d_\beta(P,P_H)^2}\right] \tag{65}$$

$$\frac{\partial\left[A_{22}P^{\beta_2}\left(N\left(d_{\beta_2}\left(P,P_L\right)\right)-N\left(d_{\beta_2}\left(P,P_H\right)\right)\right)\right]}{\partial P}$$
$$=\beta_2 A_{22}P^{\beta_2-1}\left(N\left(d_{\beta_2}\left(P,P_L\right)\right)-N\left(d_{\beta_2}\left(P,P_H\right)\right)\right)$$
$$+\frac{A_{22}P^{\beta_2-1}}{\sigma\sqrt{2\pi T}}\left[e^{-\frac{1}{2}d_{\beta_2}(P,P_L)^2}-e^{-\frac{1}{2}d_{\beta_2}(P,P_H)^2}\right] \tag{66}$$

$$\frac{\partial\left[A_{11}P^{\beta_1}N\left(-d_{\beta_1}\left(P,P_L\right)\right)+A_{32}P^{\beta_2}N\left(d_{\beta_2}\left(P,P_H\right)\right)\right]}{\partial P}$$
$$\beta_{11}A_{11}P^{\beta_1-1}N\left(-d_\beta\left(P,P_L\right)\right)-\frac{A_{11}P^{\beta_1-1}}{\sigma\sqrt{2\pi T}}e^{-\frac{1}{2}d_{\beta_1}(P,P_2)^2}$$
$$+\beta_2 A_{32}P^{\beta_2-1}N\left(d_{\beta_2}\left(P,P_H\right)\right)+\frac{A_{32}P^{\beta_2-1}}{\sigma\sqrt{2\pi T}}e^{-\frac{1}{2}d_{\beta_2}(P,P_H)^2} \tag{67}$$

$$d_\beta\left(P,x\right)=\frac{\ln P-\ln x+\left(r-\delta+(\beta-0.5)\sigma^2\right)T}{\sigma\sqrt{T}}=d_0\left(P,x\right)+\beta\sigma\sqrt{T} \tag{68}$$

$$
\begin{aligned}
d_\beta\left(P,x\right)^2 &= \left(d_0\left(P,x\right)+\beta\sigma\sqrt{T}\right)^2\\
&= d_0\left(P,x\right)^2 + 2d_0\left(P,x\right)\beta\sigma\sqrt{T}+\beta^2\sigma^2 T\\
&= d_0\left(P,x\right)^2 + 2\beta\left(\ln P-\ln x\right)+2\left(0.5\sigma^2\beta(\beta-1)+(r-\delta)\beta\right)T\\
&= d_0\left(P,x\right)^2 + 2\beta\left(\ln P-\ln x\right)+2rT
\end{aligned}
\tag{69}
$$

Using the above substitutions, gather the terms involving $e^{-\frac{1}{2}d_0(P,P_L)^2}$

$$
\begin{aligned}
&-\frac{e^{-rT}}{rP\sigma\sqrt{2\pi T}}e^{-\frac{1}{2}d_0(P,P_L)^2}+\frac{e^{-\delta T}}{\delta\sigma\sqrt{2\pi T}}e^{-\frac{1}{2}d_1(P,P_L)^2}+\frac{A_{21}P^{\beta_1-1}}{\sigma\sqrt{2\pi T}}e^{-\frac{1}{2}d_{\beta_1}(P,P_L)^2}\\
&+\frac{A_{22}P^{\beta_2-1}}{\sigma\sqrt{2\pi T}}e^{-\frac{1}{2}d_{\beta_2}(P,P_L)^2}-\frac{A_{11}P^{\beta_1-1}}{\sigma\sqrt{2\pi T}}e^{-\frac{1}{2}d_{\beta_1}(P,P_L)^2}\\
=&-\frac{P_L e^{-rT}}{rP\sigma\sqrt{2\pi T}}e^{-\frac{1}{2}d_0(P,P_L)^2}+\frac{P_L e^{-\frac{1}{2}d_0(P,P_L)^2}}{P\sigma\sqrt{2\pi T}}e^{-\frac{1}{2}d_0(P,P_L)^2}+\frac{A_{21}P_L^{\beta_1}e^{-rT}}{P\sigma\sqrt{2\pi T}}e^{-\lambda d_0(P,P_L)^2}\\
&+\frac{A_{22}P_L^{\beta_2}e^{-rT}}{P\sigma\sqrt{2\pi T}}e^{-\frac{1}{2}d_0(P,P_L)^2}-\frac{A_{11}P_L^{\beta_1}e^{-rT}}{P\sigma\sqrt{2\pi T}}e^{-\frac{1}{2}d_a(P,P_L)^2}\\
=&\frac{e^{-rT}e^{-\frac{1}{2}d_0(P,P_L)^2}}{P\sigma\sqrt{2\pi T}}\left[-\frac{P_L}{r}+\frac{P_L}{\delta}+A_{21}P_L^{\beta_1}+A_{22}P_L^{\beta_2}-A_{11}P_L^{\beta_1}\right]\\
=&0,
\end{aligned}
\tag{70}
$$

after substituting for A_{11}, A_{21}, A_{22}.

Similarly, gathering the terms involving $e^{-\frac{1}{2}d_0(P,P_H)^2}$

$$
\begin{aligned}
&\frac{P_H e^{-rT}}{rP\sigma\sqrt{2\pi T}}e^{-\frac{1}{2}d_0(P,P_H)^2}-\frac{e^{-\delta T}}{\delta\sigma\sqrt{2\pi T}}e^{-\frac{1}{2}d_1(P,P_H)^2}-\frac{A_{21}P^{\beta_1-1}}{\sigma\sqrt{2\pi T}}e^{-\frac{1}{2}d_{\beta_1}(P,P_H)^2}\\
&-\frac{A_{22}P^{\beta_2-1}}{\sigma\sqrt{2\pi T}}e^{-\frac{1}{2}d_{\beta_2}(P,P_H)^2}+\frac{A_{32}P^{\beta_2-1}}{\sigma\sqrt{2\pi T}}e^{-\frac{1}{2}d_{\beta_2}(P,P_H)^2}\\
=&\frac{P_H e^{-rT}}{rP\sigma\sqrt{2\pi T}}e^{-\frac{1}{2}d_0(P,P_H)^2}-\frac{P_H e^{-rT}}{\delta P\sigma\sqrt{2\pi T}}e^{-\frac{1}{2}d_0(P,P_H)^2}-\frac{A_{21}P_H^{\beta_1}e^{-rT}}{P\sigma\sqrt{2\pi T}}e^{-\frac{1}{2}d_0(P,P_H)^2}\\
&-\frac{A_{22}P_H^{\beta_2}e^{-rT}}{P_\sigma\sqrt{2\pi T}}e^{-\frac{1}{2}d_0(P,P_H)^2}+\frac{A_{32}P_H^{\beta_2}e^{-rT}}{P\sigma\sqrt{2\pi T}}e^{-\frac{1}{2}d_0(P,P_H)^2}\\
=&\frac{e^{-rT}e^{-\frac{1}{2}d_0(P,P_k)^2}}{P\sigma\sqrt{2\pi T}}\left[\frac{P_H}{r}-\frac{P_H}{\delta}-A_{21}P_H^{\beta_1}-A_{22}P_H^{\beta_2}+A_{32}P_H^{\beta_2}\right]\\
=&0.
\end{aligned}
\tag{71}
$$

The remaining parts reduce to:

$$\begin{aligned}S'(P,T) =&\beta_1 A_{11} P^{\beta_1 - 1} N(d_{\beta_1}(P,P_L)) + \frac{1}{\delta} e^{-\delta T} \left(N(d_1(P,P_L)) - N(d_1(P,P_H))\right) \\ &+ \beta_1 A_{21} P^{\beta_1 - 1} \left(N(d_{\beta_1}(P,P_L)) - N(d_{\beta_1}(P,P_H))\right) \\ &+ \beta_2 \left[A_{22} P^{\beta_2 - 1} \left(N(d_{\beta_2}(P,P_L)) - N(d_{\beta_2}(P,P_H))\right) + A_{32} P^{\beta_2 - 1} N(d_{\beta_2}(P,P_H))\right].\end{aligned} \tag{72}$$

C The investment trigger equations for retractable collars

Notice that V_r has three regions coming from V_p (Equation (9)), and, therefore, Equation (43) reduces to one of the following three equations:

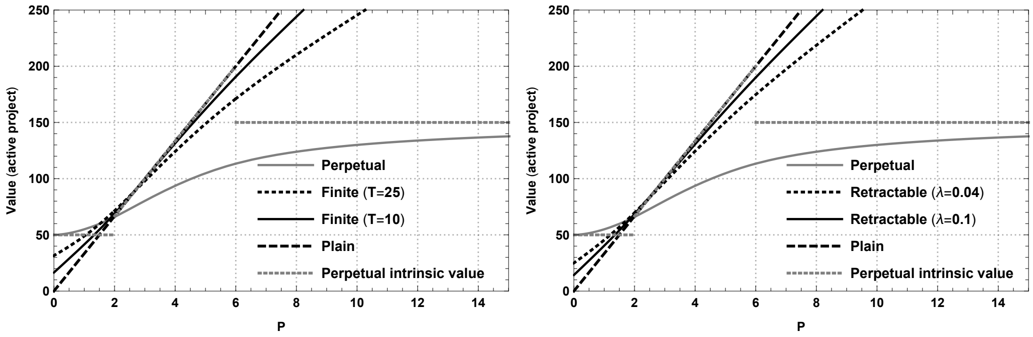
$$\begin{cases} (\beta_1 - \eta_1)\dfrac{\lambda}{r+\lambda} H P_r^{*\beta_1} + (\eta_1 - 1)\dfrac{\delta}{\delta+\lambda}\dfrac{P_r^*}{\delta} \\ \qquad + \eta_1 \left(\dfrac{P_L}{r+\lambda} - K\right) = 0 & \text{for } K_r^0 < K < K_r^L \\ (\eta_1 - \eta_2) D_{22} P_r^{*\eta_2} + (\beta_1 - \eta_1)\dfrac{\lambda}{r+\lambda} H P_r^{*\beta_1} \\ \qquad + (\eta_1 - 1)\dfrac{P_r^*}{\delta} - \eta_1 K = 0 & \text{for } K_r^L \leqslant K < K_r^H \\ (\eta_1 - \eta_2) D_{32} P_r^{*\eta_2} + (\beta_1 - \eta_1)\dfrac{\lambda}{r+\lambda} H P_r^{*\beta_1} \\ \qquad + (\eta_1 - 1)\dfrac{\delta}{\delta+\lambda}\dfrac{P_r^*}{\delta} + \eta_1 \left(\dfrac{P_H}{r+\lambda} - K\right) = 0 & \text{for } K \geqslant K_r^H \end{cases} \tag{73}$$

where $H P_r^{*\beta_1} = F(P_r^*)$ is defined by Equation (40), with $H \equiv (V(P^*) - K)(P^*)^{-\beta_1}$, $K_r^0 = P_L/(r+\lambda)$, and K_r^L and K_r^H are found solving numerically the following nonlinear equations:

$$(\beta_1 - \eta_1)\frac{\lambda}{r+\lambda}\frac{K_r^L}{\beta_1 - 1}\left(\frac{(\beta_1 - 1)P_L}{\beta_1 \delta K_r^L}\right)^{\beta_1} + (\eta_1 - 1)\frac{\delta}{\delta+\lambda}\frac{P_L}{\delta} + \eta_1\left(\frac{P_L}{r+\lambda} - K_r^L\right) = 0, \tag{74}$$

$$\begin{aligned}(\eta_1 - \eta_2) D_{32} {P_H}^{\eta_2} + (\beta_1 - \eta_1)\frac{\lambda}{r+\lambda}\frac{K_r^H}{\beta_1 - 1}\left(\frac{(\beta_1 - 1)P_H}{\beta_1 \delta K_r^H}\right)^{\beta_1} + (\eta_1 - 1)\frac{\delta}{\delta+\lambda}\frac{P_H}{\delta} \\ + \eta_1\left(\frac{P_H}{r+\lambda} - K_r^H\right) = 0.\end{aligned} \tag{75}$$

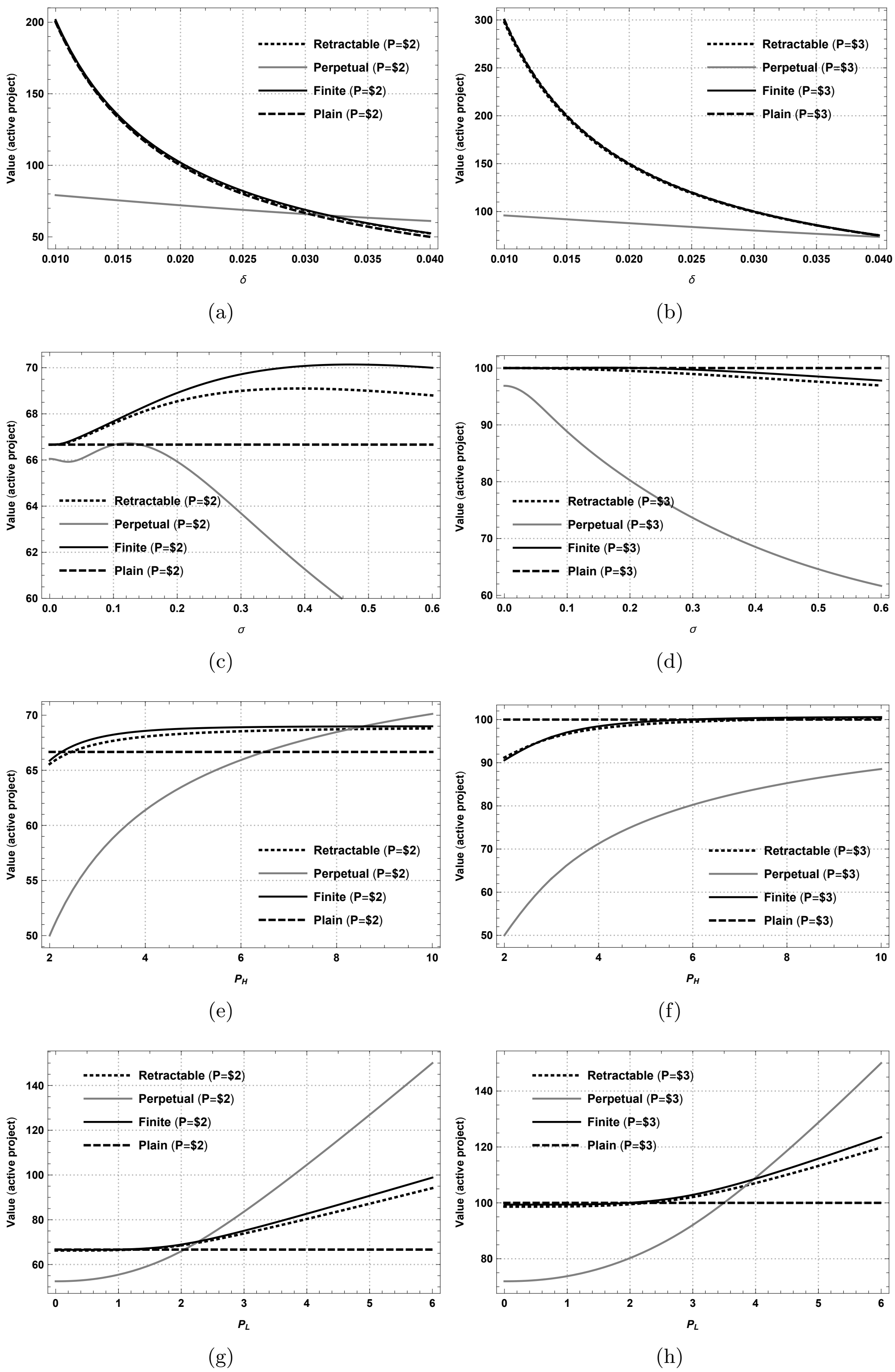
Figures



(a) Finite (b) Retractable

$P_L = \$2; P_H = \$6; \sigma = 0.2; r = 0.04; \delta = 0.03; Q = 1; K = \$70.$

Figure 1: The active project value: 1(a) for finite collars with different maturities ($T = 10, 25$) and 1(b) for retractable collars with different arrival rates of the collar withdrawal ($\lambda = 0.04, 0.1$); for both cases the perpetual collar, the intrinsic value of the collar (assuming that P stays forever at the current level) and the plain project (without collar) are included.



(a) (b)

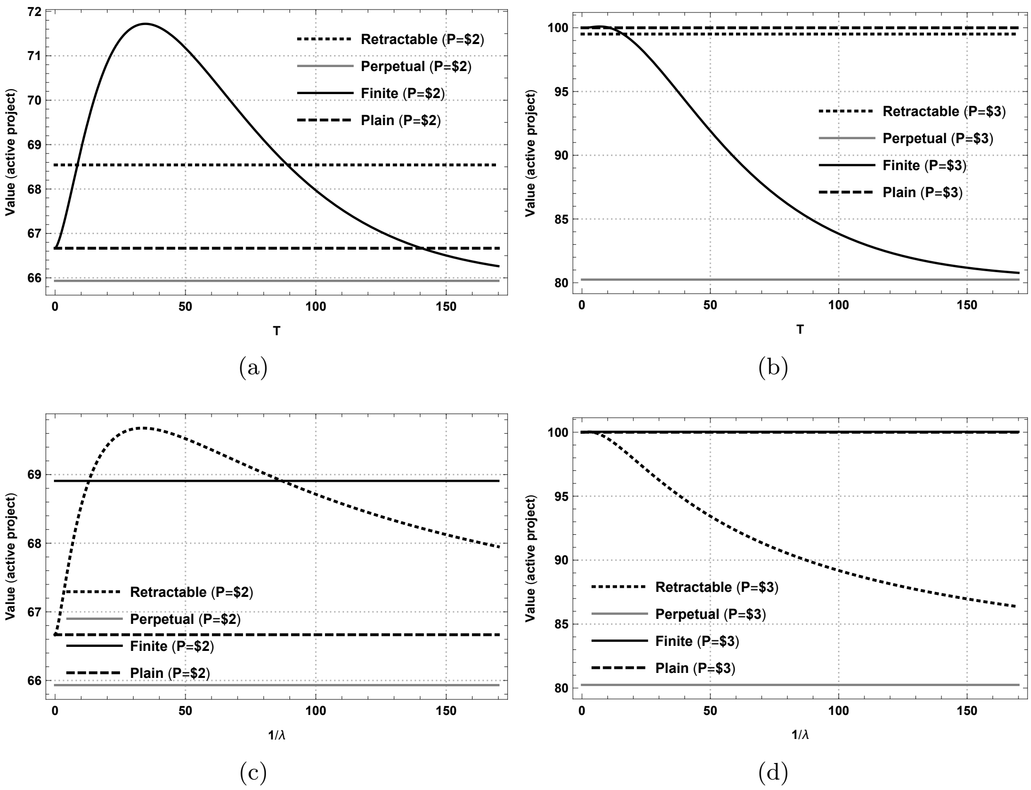
(c) (d)

(e) (f)

(g) (h)

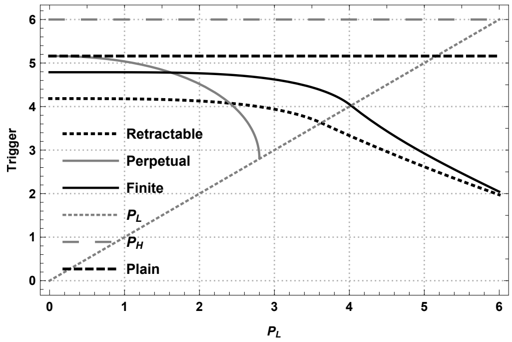
$P = \$2$ and $\$3; P_L = \$2; P_H = \$6; \sigma = 0.2; r = 0.04; \delta = 0.03; Q = 1; K = \$70; T = 10; \lambda = 0.1.$

Figure 2: The sensitivity analysis of the effect of the main parameters : 2(a)–2(b) for the rate of return shortfall, 2(c)–2(d) for the impact volatility, 2(e)–2(f) for the impact of the price cap, 2(g)–2(h) for price floor.

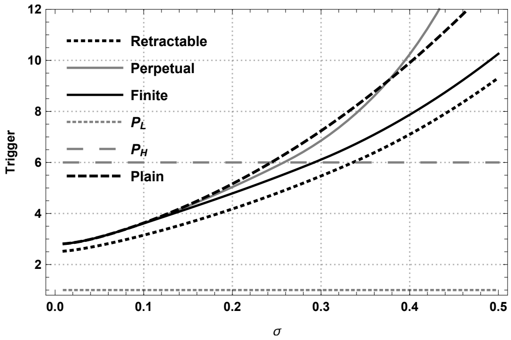


$P = \$2,$ and $\$3; P_L = \$2; P_H = \$6; \sigma = 0.2; r = 0.04; \delta = 0.03; Q = 1; K = \$70; T = 10; \lambda = 0.1.$

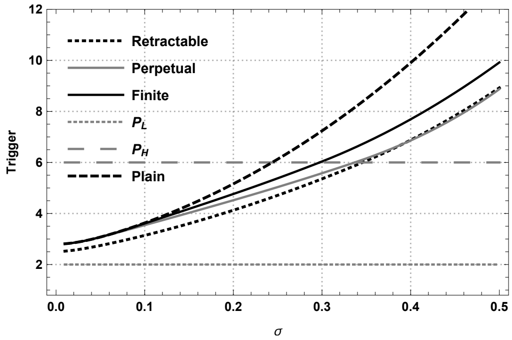
Figure 3: The sensitivity analysis of the effect of the (expected) duration of the collar on the project active value: 3(a)–3(b) for the duration of the finite collar, and 3(c)–3(d) for the expected duration of the retractable collar.



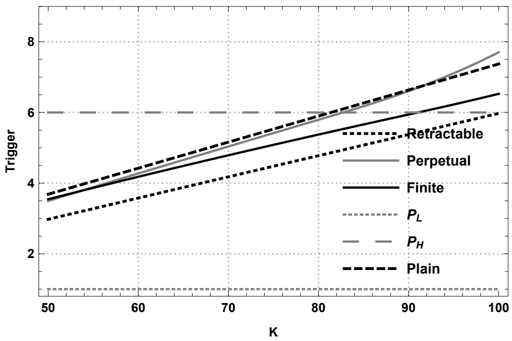
(a)



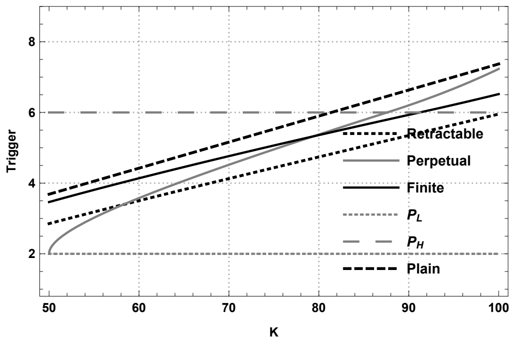
(b) $P_L = 1$



(c) $P_L = 2$



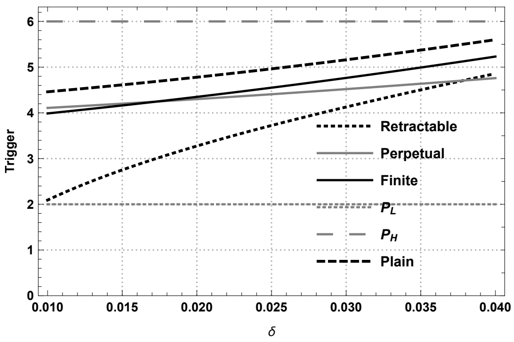
(d) $P_L = 1$



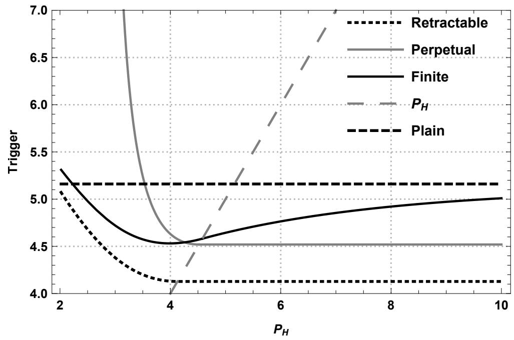
(e) $P_L = 2$

$P_H = \$6; \sigma = 0.2; r = 0.04; \delta = 0.03; Q = 1; K = \$70; T = 10; \lambda = 0.1.$

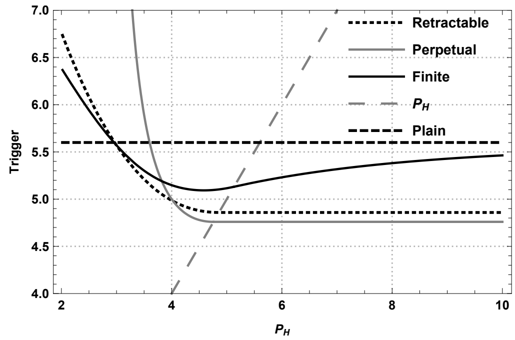
Figure 4: The sensitivity analysis of the effect on the investment trigger of the price floor (4(a)), volatility (4(b)–4(c)) and the investment cost (4(d)–4(e)).



(a)



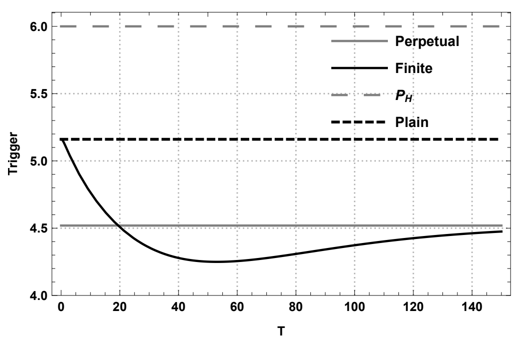
(b) $\delta = 0.03$



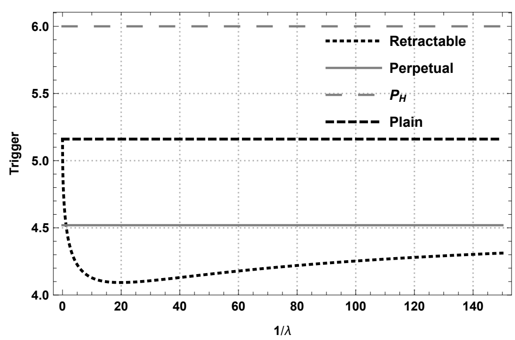
(c) $\delta = 0.04$

$P_L = \$2; \sigma = 0.2; r = 0.04; Q = 1; K = \$70; T = 10; \lambda = 0.1.$

Figure 5: The sensitivity analysis of the effect on the investment trigger of the return shortfall (5(a)), and the price cap (5(b)–5(c)).



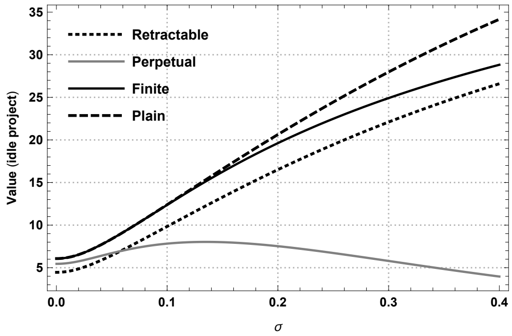
(a)



(b)

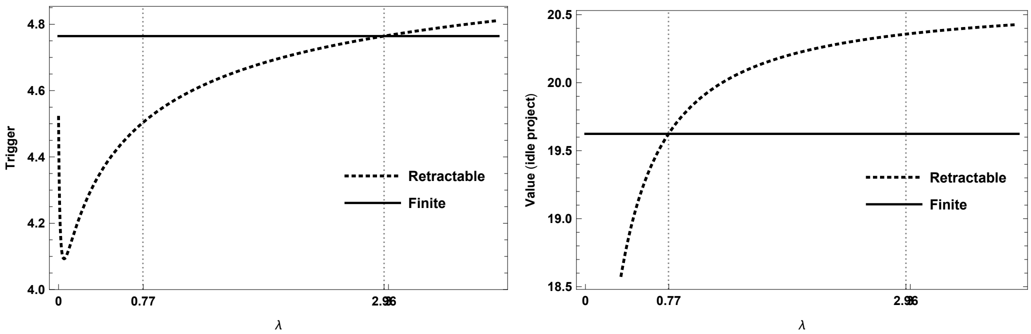
$P_L = \$2; P_H = \$6; \sigma = 0.2; r = 0.04; \delta = 0.03; Q = 1; K = \$70; T = 10; \lambda = 0.1.$

Figure 6: The sensitivity analysis of the effect on the investment trigger of the (expected) duration of the collar on the project active value: 6(a) for the duration of the finite collar, and 6(b) for the expected duration of the retractable collar.



$P = \$2; P_L = \$2; P_H = \$6; r = 0.04; \delta = 0.03; Q = 1; K = \$70; T = 10; \lambda = 0.1.$

Figure 7: The sensitivity analysis of the effect of volatility on the project value.



(a) $\sigma = 0.2$ (b) $\sigma = 0.2$

$P = \$2; P_L = \$2; P_H = \$6; \sigma = 0.2; r = 0.04; \delta = 0.03; Q = 1; K = \$70; T = 10.$

Figure 8: A comparison of the trigger and project value for the finite and retractable collars, for different λ.