A dynamic model of quality competition with endogenous prices[*]

Roberto Cellini[†] Luigi Siciliani[‡] Odd Rune Straume[§]

March 14, 2018

Abstract

We develop a dynamic model of price and quality competition in order to analyse the effects of competition on quality provision and to which extent an unregulated market is able to provide a socially optimal quality level. Our model combines a differential-game approach with a Hotelling spatial competition framework, and our analysis applies in particular to industries such as long-term care, health care, child care and education. If providers (nursing homes, hospitals, schools, nurseries) use closed-loop decision rules, which imply strategic interaction over time, we show that, although increased competition leads to higher quality in the steady state, quality provision is nevertheless lower than under open-loop rules, and also suboptimally low from a welfare perspective. Thus, our analysis identifies dynamic strategic interaction between competing providers as a potential source of inefficiency in quality provision.

Keywords: Differential-games; Competition; Quality.

JEL: H42; I11; L13.

[*]We thank two anonymous referees, Alessandra Buratto, Luca Grilli and Fabio Lamantia for valuable comments. The usual disclaimer applies. Odd Rune Straume acknowledges funding from COMPETE (ref. no. POCI-01-0145-FEDER-006683), with the FCT/MEC's (Fundação para a Ciência e a Tecnologia, I.P.) financial support through national funding and by the ERDF through the Operational Programme on Competitiveness and Internationalization – COMPETE 2020 under the PT2020 Partnership Agreement.

[†]Corresponding author. Department of Economics, University of Catania, Corso Italia 55 -I95129 Catania, Italy. E-mail: cellini@unict.it.

[‡]Department of Economics and Related Studies, University of York, Heslington, York YO10 5DD, UK. E-mail: luigi.siciliani@york.ac.uk.

[§]Department of Economics/NIPE, University of Minho, Campus de Gualtar, 4710-057 Braga, Portugal; and Department of Economics, University of Bergen, Norway. E-mail: o.r.straume@eeg.uminho.pt.

1 Introduction

Quality is a key aspect of services in long-term care, health care, child care and education, which in turn affects the way providers compete. In these industries, providers, i.e., nursing homes, hospitals, nurseries and schools (or universities), can attract more consumers not only by lowering the price, but also by increasing quality of the services they offer. For example, nursing homes can recruit more nurses, to increase time spent with residents, or recruit more qualified nurses to provide additional medical services; hospitals can provide better quality of care, both clinical quality and amenities; nurseries can introduce or improve didactic activities; schools can improve their curriculum or contact hours with pupils (students). These sectors count for a significant share of the economy and are the subject of intense political debates and policy reform.

In many OECD countries, providers in these industries compete on both quality and *price*. This is typically the case for the markets for long-term care, where nursing homes compete on quality and price in the US, the UK and several other European countries (e.g., France). This also applies to nurseries (kindergartens) and child care provision, although some countries impose relatively strict price regulation on nurseries (e.g., Norway). In the US, universities compete on both quality and price, and this is also the case in the UK where students currently pay significant fees, mostly funded through student loans. In the US, hospitals compete on both price and quality for patients who are not part of the Medicare and Medicaid programmes. This is also the case for private hospitals in many European countries offering specialist services and surgical treatments, and other health services which are not (or poorly) covered by public insurance, e.g., dental care. In England, before 1997 hospitals competed on quality and price to obtain a contract with the Health Authority. Since 1997 a fixed price system has been introduced, but there are current discussions to eliminate the fixed price regime and allow flexibility in pricing across hospitals to accommodate local needs.

If consumers (residents, patients, parents, students) make their purchasing decisions partly based on quality, a provider's incentive for attracting more demand by providing higher quality is positively related to the price of the product offered, so that price and quality decisions tend to interact in a way that makes the effect of competition on quality generally ambiguous. It is therefore of theoretical interest to analyse which factors can potentially determine whether competition has a positive or negative impact on quality provision.

Whether *competition* stimulates or stifles quality provision is also a question of great interest for policy makers. Many countries have recently introduced or enhanced public reporting of quality measures as a means to stimulate quality competition in these industries. There is a proliferation of quality indicators to compare providers, including publishing of rankings based on these indicators on a regular basis (e.g., 'league tables' of nursing homes, hospitals, schools and universities).[1] The aim is to make demand more responsive to quality and therefore stimulate providers to compete harder and offer better quality. However, the effect of such measures to increase competition might be very different depending on whether the providers also compete on prices, which tends to vary across countries. Our analysis therefore provides a relevant framework for studying the effects of such competition policies on quality in contexts where prices are not regulated. Furthermore, our analysis also provides insights on the effects of price deregulation on quality provision. In particular, we analyse whether free pricing can lead to a socially optimal quality provision or not, making the normative part of our analysis therefore highly relevant.

To answer our two key research questions, i.e., (i) if competition increases or reduces quality, and (ii) if quality is socially optimal, we make two main assumptions. First, we model demand within a *Hotelling* spatial framework. Second, we model quality as a *stock* variable. Both assumptions capture important features of industries such as long-term care, health care, child care and education.

In relation to the first, these are industries where the unit demand assumption of the Hotelling model is an appropriate description of consumer choice. In long-term care, child care and education markets, each consumer (resident, child, pupil/student) demands one admission to a nursing home, a nursery and a school/university, respectively. In health care markets, each consumer needs (demands) one surgical/medical treatment from a specific health care provider. The spatial dimension of the model also reflects the fact that geographical distance is a key

[1] Detailed examples of quality indicators in these industries are provided in Section 8.

determinant of demand in these industries, where consumers usually have a strong preference for the closest provider, unless providers that are located further away can offer a sufficiently higher quality or lower price.[2] Finally, the standard Hotelling assumption of fixed total demand, which we also adopt in our analysis, is a reasonable approximation for the above-mentioned industries, where total demand tends to be relatively inelastic.[3]

We model quality as a stock variable in a dynamic context, where quality provision requires investments and where quality is treated as a stock that can be increased over time only if the investment in quality is higher than its depreciation. This is a highly relevant feature of many dimensions of quality, since increased quality might require investments in new machinery (IT and communication systems, MRI machines) and additional training of the provider's workforce (nurses, doctors, teachers), for example. Given our dynamic set-up, we use a *differential-game* approach to derive the equilibrium price and quality provision[4] within a Hotelling framework, where two horizontally differentiated providers choose prices and quality investments over time.

Competition is a multifaceted concept, which within the proposed analytical framework we capture in three different ways. Each of these capture distinct features of the above-mentioned industries. Our three measures are: (i) the comparison of open-loop and closed-loop solutions (described in more detail below); (ii) a reduction in the transportation cost parameter as an inverse measure of competition intensity; this is a measure that captures the degree of product substitutability, which is one of the most commonly used measures of competition in markets with restricted entry (Vives, 2008)[5] and (iii) the degree of cross-shareholding (as used by, e.g., Symeonidis, 2000), which allows us to consider a (continuous) switch from monopoly to duopoly.

(i) Within the differential-game model, two different solution concepts are considered, corresponding to two different assumptions regarding the information set available to the players. We

[2] For example, empirical studies of the US for nursing homes suggest that distance to the provider is a key predictor of choice of nursing homes (Zwanziger et al., 2002; Shugarman and Brown, 2006; Grabowski et al., 2013; Rahman and Foster, 2015); this is also the case for hospitals where travelling distance and quality are the main predictors of hospital choice (Kessler and McClellan, 2000; Tay, 2003; Gutacker et al., 2016); and for schools (Hastings et al., 2005; Gibbons et al, 2008; Chumacero et al., 2011).

[3] See Werner et al. (2012) for nursing homes; Brekke et al. (2014) for hospitals.

[4] Price competition in oligopoly models, taking a differential-game approach, is studied in Vives (1985), Qiu (1997), Driskill and McCafferty (1989), Colombo and Labrecciosa (2015) among many others.

[5] A more standard measure of competition is the number of firms. However, in our dynamic setting, an n-provider model is not analytically solvable under feedback rules.

first derive the *open-loop* solution, where players are assumed to know the initial state (i.e., the initial quality stocks of the providers) but do not (or cannot) observe the evolution of states over time. This implies that each player has to decide its optimal dynamic plan at the beginning of the game and then stick to it forever. We compare this benchmark with a *closed-loop* solution, where each player can observe the dynamic evolution of states and therefore react to changes in the quality stock of the competitor. More specifically, we derive the Markovian closed-loop *feedback* solution, where the players' decisions at each point in time depend on the current state (which summarises the entire history of the game).

The closed-loop solution is in general more realistic but the nature of the industries considered (long term care, child care, health care and education) makes the open-loop solution potentially relevant beyond being a theoretical benchmark. For example, for hospitals and nursing homes, some states in the US have certificate of needs (CON) regulation which implies that providers might have to commit to investment plans that make the dynamic competitive framework perhaps more in line with the open-loop solution. Schools and universities may have to apply for planning permissions and lay out investment plans before they can be approved by the local municipalities. More generally, the heavily regulated nature of these industries suggest that competition based on open-loop versus closed-loop decision rules can to some extent be thought of as a policy choice. In this respect, our analysis also provides insights on the effects of regulatory measures that affect the strategic context faced by providers.

(ii) Competition intensity can be easily related to policies which introduce or enhance public reporting of quality measures, discussed above, to help consumers (residents, parents, patients, students) choose providers (nursing homes, hospitals, schools and universities). (iii) The degree of cross-shareholding can also be related to policy initiatives; although providers are supposed to compete, they are often also supposed to 'collaborate'. For example, hospitals and schools are encouraged or have to be part of local networks which encourage the spreading of good practice, share data, and design a common curriculum (Chone, 2016). These activities encourage, to some extent, providers to act as one entity, which is measured in the degree of cross-shareholding.

Our analysis produces three sets of results. First, steady-state quality in the open-loop solution does not depend on competition intensity, regardless of whether it is measured by product substitutability or cross-shareholding. This is in contrast to the closed-loop solution, where product substitutability (cross-shareholding) has a positive (ambiguous) effect on steady-state quality. Second, we find that steady-state quality is *lower* in the closed-loop than in the open-loop solution, for any degree of product substitutability and for any degree of cross-shareholding, as long as the marginal cost of quality and/or the transportation cost parameter is sufficiently high (to rule out multiplicity of equilibria). The reason is that, in the former case, each provider has an incentive to reduce current quality investments in order to dampen future price competition. This incentive is absent in the open-loop solution, where the players do not interact strategically over time. Third, and finally, we find that quality provision is socially optimal in the open-loop solution, which implies that the closed-loop solution is characterised by underprovision of quality in steady state.

We discuss how the first finding relates to the empirical evidence in Section 8. We extensively elaborate on the policy implications of our findings for the industries to which our analysis applies in the concluding Section 9. Finally, it should be emphasised that our analysis obviously also applies to many other industries where providers compete on both price and quality and where competition has a spatial dimension. One example is the airline industry, which is often modelled in a spatial competition framework since the time-scheduling of flights can be interpreted as locations on a time line.[6]

The rest of the paper is organised as follows. In the next section we present an overview and discussion of related theoretical literature. The model is then formally presented in Section 3. In Section 4 we solve the model under the assumption that players use open-loop decision rules. The open-loop equilibrium is then used as a benchmark for comparison with the closed-loop solution – where the players engage in dynamic strategic interaction – which is analysed in Section 5. The welfare properties of the two solutions are analysed and discussed in Section 6. In Section 7 we introduce two different extensions to our base analysis: (i) the degree of cross-shareholding as an alternative measure of competition intensity, and (ii) an alternative cost function with quality-dependent and convex production costs. In Section 8 we present and

[6] See, e.g., Borenstein and Netz (1999) and Salvanes et al. (2005).

discuss empirical evidence of the relationship between competition and quality in some of the industries for which our analysis applies. Policy implications and concluding remarks are offered in Section 9.

2 Review of the theoretical literature

Our paper contributes to the theoretical literature on the relationship between competition intensity and quality provision. This relationship is generally determined by two counteracting effects: (i) more competition increases the incentives to provide quality for given prices, but (ii) more competition also reduces the price-cost margin, which in turn reduces the incentives for quality provision. The relative strength of these two general effects depends on the specifics of the theoretical framework used.

Standard spatial competition models produce a well-known 'neutrality' result, where the two aforementioned effects exactly cancel each other out, and competition intensity (measured by transportation costs; i.e., product substitutability) has no effect on equilibrium quality provision.[7] Brekke, Siciliani and Straume (2010) have shown that the neutrality result is broken in the presence of income effects (where price changes affect the marginal utility of consumers), which creates a positive relationship between competition intensity and quality provision.[8] In the present paper we identify another factor which breaks this neutrality result, namely dynamic strategic interaction (as in the closed-loop solution).[9]

There is of course also a large literature studying price and quality competition within a vertical differentiation framework, where consumers differ in their willingness-to-pay for product quality, and where each firm produces either a single product of a given quality or a range of products of different qualities that are offered at different prices.[10] The present analysis is cast in

[7] See, e.g., Ma and Burgess (1993) for the case of competition on a Hotelling line, and Gravelle (1999) for the case of competition on a Salop circle.

[8] Using the number of firms/brands as a competition measure, Economides (1993) finds that more competition leads to lower quality provision. This result, which is driven by a negative relationship between the number of firms and firm-level demand, can also be reversed under more general utility and cost assumptions, as shown by Brekke, Siciliani and Straume (2010).

[9] There is also a small literature studing the relationship between competition and quality using a representative consumer framework (e.g., Sutton, 1996, Banker et al., 1998, Symeonidis, 2000). In this strand of the literature, more competition is typically found to have an ambiguous effect on equilibrium quality provision.

[10] Seminal contributions include Gabszewicz and Thisse (1979, 1980) and Shaked and Sutton (1982, 1983) for

a horizontal (rather than vertical) differentiation framework for two reasons. Firstly, and most crucially, a symmetric model is necessary for analytical tractability, which is a critical issue in differential games with feedback decision rules. Secondly, a symmetric model (i.e., based on horizontal differentiation) allows us to focus on symmetric equilibria, with equal quality across all firms/goods, which arguably makes it better suited for analysing the relationship between competition intensity and quality provision, since the latter concept is very precisely defined in a symmetric equilibrium. Models based on vertical differentiation, on the other hand, are arguably better suited for analysing questions related to asymmetric outcomes, such as incentives for quality differentiation.[11]

The relationship between competition and quality is closely related to the question of whether an unregulated market will produce a socially optimal quality provision. Our analysis also contributes towards answering this question. In a seminal paper, Spence (1975) showed that a monopolist will provide a quality level that is lower (higher) than the socially optimal level if the marginal valuation of quality is lower (higher) for the marginal than for the average consumer. In standard spatial competition models, the marginal valuation of quality is equal (in equilibrium) for the marginal and average consumer, implying socially optimal quality provision, if price and quality decisions are taken simultaneously (see, e.g., Ma and Burgess, 1993). This optimality result also carries over to the case of dynamic competition under open-loop rules (which implies an absence of truly dynamic strategic interaction). However, our finding of a suboptimal quality level in the closed-loop solution reveals that dynamic strategic interaction between competing firms creates an additional inefficiency that contributes to underprovision of quality.

The efficiency properties of the open-loop and closed-loop solutions, with respect to quality provision, have an interesting parallel in the efficiency properties of simultaneous-move and sequential-move versions of an equivalent one-shot game. Whereas simultaneous price and quality decisions yield socially efficient quality provision, as described above, sequential decision-making – where the firms can commit to quality choices before they set prices – yields lower,

the case of single-product firms and Mussa and Rosen (1978), Gal-Or (1983) and Champsaur and Rochet (1989) for the case of multi-product firms.

[11] For example, Motta (1993) shows that incentives for vertical differentiation (in duopoly) are larger under Bertrand than under Cournot competition.

and therefore sub-optimal, quality provision in equilibrium.[12] The mechanism is similar to the one giving rise to different steady-state quality levels in the open-loop and closed-loop solutions of the dynamic model analysed in the present paper. Our analysis can therefore be seen as giving additional support to the sequential-move assumption in one-shot games. Even if price and quality choices are made simultaneously in each period of the game, dynamic strategic interaction (as in the closed-loop solution) will create the same type of incentives for underprovision of quality as in a one-shot game with sequential moves.

Our work also relates to studies which employ a differential-game approach. Piga (1998, 2000) analyses oligopolistic markets in which firms set price and advertising levels. Advertising has some characteristics that are similar to quality, and can be interpreted as a tool to increase the perceived product quality. However, the way advertising is modelled in these two studies is distinctly different from the way quality competition is modelled in the present paper. Importantly, advertising is modelled as a public good that increases market size. In contrast, quality investments have a business-stealing effect in our model. In Piga's models, the ranking of desirability of the outcomes depend on the information rule adopted (open-loop vs feedback).[13] Cellini et al. (2008) focus on persuasive advertising and compare the outcomes of price and quantity competition, and reach the conclusion that price competition entails more advertising.

Brekke et al. (2010) provide a model where oligopolistic firms (hospitals) set qualities in the presence of regulated prices (with a fixed price reimbursed by the government for every additional patient). Quality is also modelled as a stock variable and a Hotelling framework is used. They show that quality is lower under the closed-loop solution than under the open-loop solution when the marginal cost of production is increasing (to capture hospitals' smooth capacity constraints, given that hospital entry is heavily regulated). In contrast, the two solution concepts yield identical quality provision when the marginal cost of production is constant. In the current study we also find that quality is lower under the closed-loop solution. Critically,

[12] Ma and Burgess (1993) derive this result in the context of a Hotelling model, while Economides (1993) derive the equivalent result in the context of a Salop model.

[13] Like in the current study, Piga (1998) applies a Hotelling framework but, differently, market size (and not quality) is the state variable, which evolves over time according the amount of advertising undertaken by the two firms. In contrast, Piga (2000) presents a model with price as the state variable, in line with the assumption that prices are sticky.

this result is obtained under a *constant* marginal cost assumption and is due to the endogeneity of prices.[14]

Other studies of dynamic quality competition, combining a spatial competition framework with a differential-game approach, include Brekke et al. (2012), who consider quality competition with regulated prices when demand reacts sluggishly to quality changes, and Siciliani et al. (2013), who use a similar framework but with the additional assumption of motivated providers. There are several difference between these studies and the current one, the most important being the endogeneity of prices. Another paper using a spatial competition framework with quality as the choice variable, but applying different assumption about dynamics, is Cellini and Lamantia (2015), who study how the imposition of a minimum quality standard affects the equilibrium allocation and the dynamic properties of the system. Following Bischi et al. (2007) and Bischi et al. (2015), they also consider different behavioural rules adopted by the providers, apart from the open- or closed-loop rules.

Finally, investment in R&D which affects the production cost or product characteristics – with some parallels to investment in product quality – are studied by Hinloopen (2000, 2003) and Cellini and Lambertini (2005, 2009), among others. Intensity in R&D, and the incentive towards cooperative behaviour, depend on the form of market competition (price vs quantity competition) and the information structure, with a variety of possible outcomes. In general, more intense competition arises when the firms' choice variable is price (rather than quantity), leading to higher consumer surplus in steady-state equilibrium (as is well known, even from static games), and with closed- (rather than open-) loop information structures.

3 Model

Consider a market with two providers located at either end of the unit line $S = [0, 1]$. On this line segment there is a uniform distribution of consumers, with total mass normalised to 1. Assuming unit demand, the utility of a consumer who is located at $x \in S$ and buys from

[14] An analogous result is obtained by Brekke et al. (2012) when demand is modelled as sluggish and quality can be changed instantaneously under a fixed price regime.

Provider i, located at $z_i \in \{0, 1\}$, is given by

$$U(x, z_i) = kq_i - \tau |x - z_i| - p_i, \tag{1}$$

where q_i and p_i are the quality and price, respectively, of the good offered by Provider i, k is a parameter measuring the marginal willingness to pay for quality, and τ is the marginal transportation cost, which measures the degree of horizontal product differentiation. We assume full market coverage: each consumers buys one unit of the good, from the most preferred provider.

Since the distance between providers is equal to one, the consumer who is indifferent between Provider i and Provider j is located at x_i^D, implicitly given by

$$kq_i - \tau x_i^D - p_i = kq_j - \tau \left(1 - x_i^D\right) - p_j, \tag{2}$$

and explicitly given by $x_i^D = \frac{1}{2} + \frac{k(q_i - q_j)}{2\tau} - \frac{(p_i - p_j)}{2\tau}$, or in a more accurate way which takes into account the possible case of corner solutions:[15]

$$x_i^D = \max \left[0, \min \left[\frac{1}{2} + \frac{k(q_i - q_j)}{2\tau} - \frac{(p_i - p_j)}{2\tau}, 1\right]\right] \tag{3}$$

This is also the demand for Provider i, given the assumptions of (i) uniform consumer distribution with mass 1, (ii) exogenous locations of providers, and (iii) full market coverage.

A key parameter in our model is τ, which is given a relatively broad interpretation. On the one hand, τ reflects consumers' cost of travelling, which is an important aspect of the industries we have in mind, where distance is often a key predictor of consumers' choice of provider, as discussed in the previous section (and footnote 2). On the other hand, from (3) we see that demand responsiveness to quality or price changes depends unambiguously on τ, which explains why this parameter is a commonly used (inverse) measure of the intensity of competition in spatial competition models. Thus, we interpret τ as a broad measure of competition intensity that reflects factors beyond literal transportation costs.

[15] All equilibrium solutions in the models of this article are internal, and no specific parametric conditions are necessary to avoid corner solutions.

In order to ensure existence, uniqueness and stability of the steady-state solution in all models considered, we need to impose a parameter restriction that, in qualitative terms, implies that the cost of travelling (τ) is sufficiently large relative to the marginal willingness to pay for quality (k).[16] This parameter restriction implies that our analysis applies to industries in which travelling costs are sufficiently important. However, this is indeed a key characteristic of the specific industries we have in mind, since distance to the provider is a key determinant of consumer choice: residents, patients and pupils tend to choose the closest or local nursing home, hospital and school, respectively (see Introduction for references). For example, Gutacker et al. (2016) find that one standard deviation increase in health gains (which entails an large increase in quality) would increase patient willingness to travel by only about one kilometer. Similarly, for school choice, Hastings et al. (2005) find that an average student would choose the nearest school over a school three miles further away in which quality (measured by average test scores) were 1-2 standard deviations higher. This suggests that, in these industries, travelling distance is relatively more important than quality for consumers, which, in our model, translates into τ being large relative to k.

We assume that product quality changes over time, due to investment by providers and depreciation. Define $I(t)$ as the investment in quality at time t, and $\delta > 0$ as the depreciation rate of the quality stock. Analytically, the law of motion of quality is given by

$$\frac{dq_i(t)}{dt} := \dot{q}_i(t) = I_i(t) - \delta q_i(t). \tag{4}$$

Quality is therefore modelled as a stock variable which increases when investment in quality is higher that the depreciation rate. This is a plausible assumption. Quality is unlikely to change instantaneously. It requires investment in machines, skilled workforce, and careful planning. Each provider has a cost function $C(\cdot)$, which, at each point in time, depends on the quality investment, the quality stock, and output. For analytical tractability, the cost function is

[16] For example, in the open-loop model considered in Section 4, the critical point is a minimum and not a maximum if τ is sufficiently low. The explicit parameter restrictions needed for existence, uniqueness and stability are given along with the derived open-loop and closed-loop solutions in Sections 4 and 5, respectively.

parameterised as follows:[17]

$$C\left(x_i^D, I_i, q_i\right) = cx_i^D + \frac{1}{2}\left(\gamma I_i^2 + \beta q_i^2\right), \tag{5}$$

where $c > 0$, $\gamma > 0$ and $\beta > 0$. Thus, we assume constant marginal cost of production, and increasing and strictly convex costs of quality investments I_i. We also assume that each provider's costs are increasing and convex in the quality stock q_i.[18]

Assuming profit-maximising behaviour, the instantaneous objective function of Provider i is given by

$$\pi_i(t) = (p_i(t) - c)\, x_i^D\left(q_i(t), q_j(t), p_i(t), p_j(t)\right) - \frac{\gamma}{2} I_i(t)^2 - \frac{\beta}{2} q_i(t)^2, \tag{6}$$

and, defining ρ as the (constant, positive) preference discount rate, the objective function of Provider i over the infinite time horizon is

$$\int_0^{+\infty} \pi_i(t)\, e^{-\rho t} dt. \tag{7}$$

In the following we model the behaviour of providers, and find the corresponding equilibrium, under two alternative assumptions concerning the information set used by providers at each point in time. First, we model the *open-loop* strategy, where each provider sets its optimal investment plan at the start of the game and then sticks to it forever. Under this solution concept, the optimal value of the choice variables simply depends on time (and the value of state variables at the beginning of time). The open-loop solution concept requires minimal amount of information; in some instances, it has been criticised for being 'too static' in nature (Dockner et al., 2000, p. 30). However, as explained in the Introduction, in the industries that constitute the main applications of our analysis, providers are sometimes subjected to regulations enforcing long-term investment plans, which implies that the providers might operate in a strategic environment that resembles the open-loop setting.

The open-loop Nash equilibrium is, in general, only weakly time consistent (as defined, e.g.,

[17] Alternative parameterisations of the cost function are considered in Section 7.2.

[18] An intuitive justification for this assumption is that a higher quality level makes the required maintenance operations more and more demanding.

by Başar and Olsder, 1995). This is not the case under the *feedback closed-loop* strategy, where the choice variables set by players at any instant of time depend on the current value of the states. The feedback strategies are sometimes also labelled as 'Markovian', since only the current values of the states matter, irrespective of the past history – which is reflected in the current value of the state vector. The optimal strategies are commonly derived from the solution to Bellman's equation, and the Nash equilibrium under the feedback closed-loop strategy is strongly time consistent.

There are classes of differential games in which the closed-loop solution degenerates and the time path of the control variables coincides with the path of the open-loop solution. In these cases, even the open-loop solution is strongly time consistent. Examples of classes where this coincidence holds are linear state differential games, state-redundant games, state-separable games, and exponential games (Mehlmann, 1988, ch. 4, and Dockner et al., 2000, ch. 7, provide an overview). However, this is not the case in our present model, which has a linear-quadratic structure, and where the control variables follow different paths under open-loop and closed-loop solutions, and hence lead to different steady-state solutions.

A large body of theoretical and applied analyses compare the strategy and the equilibrium properties under the two solution concepts, which coincide only in some specific circumstances (see, e.g., Mehlman, 1988, Ch. 4; Dockner et al., 2000, Ch. 7). A variety of outcomes can emerge: while it is impossible, in general, to state which solution concept leads to the highest payoffs for the players, and which leads to the highest level of social welfare, it is arguably the case that the feedback closed-loop solution generally entails a stronger degree of competition, since players are able to respond at each point in time to the choice of their competitors.

In these models, it is usual to focus on the steady-state allocation, which can be interpreted as the counterpart of the equilibrium outcome of a static game. As shown below, both the open-loop and the feedback closed-loop equilibrium in our model lead the system to a steady state. Given our 'standard' assumptions concerning technology and demand, it is not surprising that the (symmetric) steady state we focus on is stable (in the saddle sense) under the open-loop rule, and it is globally stable under the feedback closed-loop rule.

4 Open-loop solution

Provider i's maximisation problem is given by

$$\underset{I_i(t),\, p_i(t)}{\text{Maximise}} \int_0^{+\infty} \pi_i\,(t)\, e^{-\rho t} dt, \tag{8}$$

$$\begin{aligned}
\text{subject to } \dot{q}_i(t) &= I_i(t) - \delta q_i(t), && (9)\\
\dot{q}_j(t) &= I_j(t) - \delta q_j(t), && (10)\\
q_i(0) &= q_{i0} > 0, && (11)\\
q_j(0) &= q_{j0} > 0. && (12)
\end{aligned}$$

Let $\mu_i(t)$ and $\mu_j(t)$ be the current value co-state variables associated with the two state equations. The current-value Hamiltonian is as follows, where time (t) is omitted to ease notation:

$$H_i = (p_i - c)\left(\frac{1}{2} + \frac{k\,(q_i - q_j)}{2\tau} - \frac{p_i - p_j}{2\tau}\right) - \frac{\gamma}{2} I_i^2 - \frac{\beta}{2} {q_i}^2 + \mu_i\,(I_i - \delta q_i) + \mu_j\,(I_j - \delta q_j)\,. \tag{13}$$

The solution satisfies the following conditions: (a) $\partial H_i/\partial I_i = 0$, (b) $\partial H_i/\partial p_i = 0$, (c) $\dot{\mu}_i = \rho\mu_i - \partial H_i/\partial q_i$, (d) $\dot{q}_i = \partial H_i/\partial \mu_i$, (e) $\dot{\mu}_j = \rho\mu_j - \partial H_i/\partial q_j$. More extensively, we have:

$$\begin{aligned}
\mu_i &= \gamma I_i, && (14)\\
0 &= \frac{1}{2} + \frac{k\,(q_i - q_j)}{2\tau} - \frac{(p_i - p_j)}{2\tau} - \frac{(p_i - c)}{2\tau}, && (15)\\
\dot{\mu}_i &= \mu_i\,(\delta + \rho) + \beta q_i - \frac{(p_i - c)\,k}{2\tau}, && (16)\\
\dot{\mu}_j &= (\delta + \rho)\,\mu_j + \frac{(p_i - c)\,k}{2\tau}, && (17)\\
\dot{q}_i &= I_i - \delta q_i, && (18)
\end{aligned}$$

to be considered along with the transversality condition $\lim_{t\to+\infty} e^{-\rho t}\mu_i(t) q_i(t) = 0$. The second-order conditions are satisfied if the Hamiltonian is concave in the control and state variables (Léonard and Van Long, 1992). We have $H_{I_i I_i} = -\,\gamma < 0$, $H_{p_i p_i} = -\frac{1}{\tau} < 0$ and $H_{q_i q_i} =$

$-\beta < 0$. Since $H_{I_i q_i} = H_{I_i p_i} = 0$, the only remaining condition needed to ensure concavity is $H_{p_i p_i} H_{q_i q_i} - (H_{q_i p_i})^2 = \frac{1}{4\tau^2}\left(4\tau\beta - k^2\right) > 0$.

From (14) and (16) we derive

$$\dot{I}_i = (\delta + \rho) I_i + \frac{\beta}{\gamma} q_i - \frac{(p_i - c) k}{2\tau\gamma}, \tag{19}$$

and from (15) we derive

$$p_i = \frac{c + \tau + k(q_i - q_j) + p_j}{2}. \tag{20}$$

Symmetrically, the problem of Provider j yields $p_j = [c + \tau + k(q_j - q_i) + p_i]/2$. Hence, in Nash equilibrium, where both (20) and the symmetric solution for Provider j hold simultaneously, it must be the case that

$$p_i = c + \tau + \frac{k(q_i - q_j)}{3} \text{ and } p_j = c + \tau + \frac{k(q_j - q_i)}{3}. \tag{21}$$

In the steady state we have $\dot{q}_i = 0$, $q_i = q_j = q^{OL}$ and $p_i = p_j = p^{OL}$, implying

$$p^{OL} = c + \tau, \tag{22}$$

$$q^{OL} = \frac{k}{2\left(\gamma\delta\left(\delta + \rho\right) + \beta\right)}, \tag{23}$$

and

$$I^{OL} = \delta q^{OL}. \tag{24}$$

Regarding the adjustment around the steady state, $\dot{q}_h$, $\dot{I}_h$ and $\dot{p}_h$ are linear functions of q_h, I_h and p_h $(h = i, j)$, as evidenced by (18), (19) and the differentiation of (21). The complete dynamics of the system can be characterised by six differential equations. Consider, however, that $\dot{p}_h$ $(h = i, j)$ are proportional to $\dot{q}_h$. Thus, some information regarding the dynamics can be obtained by focussing the attention on the two-variable system made by $\dot{q}_i$ and $\dot{I}_i$, whose Jacobian matrix is

$$J = \frac{\partial(\dot{I}_i, \dot{q}_i)}{\partial(I_i, q_i)} = \begin{bmatrix} (\delta+\rho) & \frac{6\tau\beta - k^2}{6\tau\gamma} \\ 1 & -\delta \end{bmatrix}. \tag{25}$$

This matrix has a positive trace, ρ, and a negative determinant if $4\tau\beta > k^2$, which makes the steady state locally stable in the saddle sense.[19] Under this condition, the locus $\dot{I} = 0$ is negatively sloped in the (I, q)-space and the locus $\dot{q} = 0$ is positively sloped. The saddle path is negatively sloped. Thus, if $q(0) > q^{OL}$, quality decreases over time toward the steady state, while investment increases; in this case, $p(0) > p^{OL}$ and the price decreases over time.

The comparative statics results regarding the steady-state levels of price and quality are intuitive. The steady-state price equates the sum of marginal production and transportation cost. This result is analogous to the Nash equilibrium of an equivalent static model. Steady-state investment and quality are also decreasing in the marginal cost of quality (β) and investment (γ), and decreasing in the time preference discount rate (ρ). Notice also that a higher depreciation rate of quality (δ) is associated with lower steady-state quality, while the effect on investment can be non-monotonic and depends on the exact parameter configuration.

The most interesting characteristic of the open-loop solution, though, is the fact that the steady-state quality is not a function the degree of product substitutability. Applying the standard interpretation of τ as being an inverse measure of competition intensity, we obtain the following result:

Proposition 1 *When the providers use open-loop decision rules, steady-state quality does not depend on the intensity of competition in the market.*

All else equal, stronger competition increases the elasticity of (provider-specific) demand with respect to both price and quality, which leads to lower prices but has two counteracting effects on quality provision: a positive direct effect and an indirect negative effect, since a lower price reduces the incentive to increase quality. In standard spatial competition models, in a static setting, these two effects exactly cancel each other, implying that competition intensity does not

[19] The condition $4\tau\beta > k^2$ is sufficient but not necessary. This condition also ensures positive steady-state profits since $\pi^{OL} = \frac{\tau}{2} - \frac{k^2(\gamma\delta^2+\beta)}{8(\gamma\delta(\delta+\rho)+\beta)^2} > 0$ iff $4\tau\beta > k^2 \frac{(\gamma\delta^2+\beta)\beta}{(\gamma\delta^2+\beta+\rho\gamma\delta)^2}$ where $\frac{(\gamma\delta^2+\beta)\beta}{(\gamma\delta^2+\beta+\rho\gamma\delta)^2} < 1$.

affect equilibrium quality provision.[20] Proposition 1 confirms that this 'neutrality' result carries over to a dynamic setting, as long as the providers use open-loop decision rules. This is perhaps not all that surprising, given the somewhat 'static' nature of the open-loop solution, where the optimal investment plan is decided once and for all at the outset of the game.

5 Closed-loop solution

In this section we present the closed-loop solution, where each provider knows not only the initial state of the system, but can also observe (and therefore react to) the quality stock of the competing provider in all periods. More specifically, we present the closed-loop *feedback* solution, where the players – at each point in time – make decisions by taking into account the current value of states (which summarises the entire past history of the game). While the closed-loop feedback solution is strongly time-consistent, and therefore arguably a more appealing solution concept in a context of dynamic competition, this solution is also considerably more complicated to calculate. In this section we therefore present directly the optimal dynamic decision rules in the closed-loop feedback solution and relegate the derivation of these rules to Appendix A.1.

If the parameters β and/or τ are sufficiently large relative to k, which we will henceforth assume is the case, there is a unique globally asymptotically stable closed-loop solution. The optimal pricing rule for Provider i in this solution is given by

$$p_i(t) := \Phi_i^{CL}(q_i(t), q_j(t)) = c + \tau + \frac{k(q_i(t) - q_j(t))}{3}. \tag{26}$$

At each point in time, there is a positive relationship between the quality stock and the price charged by each provider. All else equal, higher quality implies higher demand, which makes demand less price elastic and therefore increases the profit-maximising price. Obviously, an increase in the competitor's quality level has the opposite effect. Since the two providers optimal pricing rules are symmetric, it follows that

[20] See, e.g., Ma and Burgess (1993) for the case of Hotelling competition and Gravelle (1999) for the case of Salop competition.

$$p_i(t) - p_j(t) = \frac{2k(q_i(t) - q_j(t))}{3}. \tag{27}$$

Thus, at each point in time, the provider with higher quality charges a higher price.

The optimal quality investment rule for Provider i in the closed-loop solution is

$$I_i(t) := \phi^{CL}(q_i(t), q_j(t)) = \frac{1}{\gamma}(\alpha_1 + \alpha_3 q_i + \alpha_5 q_j), \tag{28}$$

where

$$\alpha_1 = \frac{k\gamma}{3(\gamma(\delta + \rho) - \alpha_3)} > 0, \tag{29}$$

$$\alpha_3 = s\gamma - \sqrt{\frac{\gamma}{54}\left(4\sqrt{(y-2g)\,y} + (5y - 2g)\right)} < 0, \tag{30}$$

and

$$\alpha_5 = -\frac{1}{2}\sqrt{\frac{4\gamma}{27}\left(y - g - \sqrt{y(y-2g)}\right)} < 0, \tag{31}$$

and where $y := 6\left(s^2\gamma + \beta\right)$, $s := \delta + \frac{1}{2}\rho$ and $g := \frac{k^2}{\tau}$. The negative sign of α_3 is assumed to ensure global asymptotic stability. For the solution to be real, we must also assume that $y \geq 2g$. Finally, given that $\alpha_3 < 0$, the slightly stricter condition $y \geq \frac{8}{3}g$ is sufficient (but not necessary) to ensure that the solution is unique (multiplicity of equilibria is further discussed below). As for the equilibrium dynamics off the steady state, these are similar to the open-loop solution (details omitted but available from authors).

In qualitative terms, the conditions $\alpha_3 < 0$ and $y \geq \frac{8}{3}g$, or, more explicitly,

$$9\tau\left(\left(\delta + \frac{\rho}{2}\right)^2\gamma + \beta\right) \geq 4k^2$$

are both satisfied if the transportation cost parameter τ is sufficiently large relative to the marginal valuation of quality, k. Notice that, in qualitative terms, this condition is analogous to the condition $4\tau\beta - k^2 > 0$ that ensures concavity of (13) and therefore existence of the open-loop solution. In fact, it is easily shown that the condition $4\tau\beta - k^2 > 0$ is sufficient to ensure stability and uniqueness also of the closed-loop solution if the marginal cost of quality investments γ is sufficiently large relative to the marginal cost of the (maintenance of the) quality stock β, which is plausible.

The key property of the quality investment rule given by (28), is the negative sign of α_5, which implies that quality investments are *intertemporal strategic substitutes*;[21] the higher the quality stock of a given provider, the lower the optimal investment level of the competing provider. The intuition for this property is related to the interaction between price and quality investment choices. All else equal, an increase in the quality stock of Provider j leads to reduced demand for Provider i, and this provider will therefore optimally reduce its price, as shown by (26). However, this price reduction implies a lower price-cost margin for Provider i, which in turn implies a reduction in the marginal profit gain of attracting more demand by increasing quality. Provider i will therefore respond by reducing its quality investments.[22]

5.1 Steady state

In the steady state, where $q_i = q_j$, equilibrium *prices* in the closed-loop solution are given by

$$p^{CL} = c + \tau \tag{32}$$

and are therefore equal to the steady-state prices in the open-loop solution (and to the equilibrium prices in an equivalent static game). Steady-state *quality* in the closed-loop solution is implicitly given by the steady-state condition $I_i = \delta q_i$, and explicitly given by[23]

$$q^{CL} = \frac{k\gamma}{3\left(\gamma\left(\delta+\rho\right)-\alpha_3\right)\left(\gamma\delta-\left(\alpha_3+\alpha_5\right)\right)}. \tag{33}$$

[21] As defined by Jun and Vives (2004), intertemporal strategic substitutability implies that the control of each player responds negatively to the state of the other player.

[22] In a static model of price and quality competition, Brekke et al. (2017) show that the strategic substitutability of quality choices holds for more general demand functions, and also holds for the case of variable (output-dependent) quality costs, as long as the effect of higher quality on marginal production costs is not too strong.

[23] It is worth noticing that we are restricting our attention to the case in which, at the steady state, qualities are the same, and hence quality differences (if any) disappear over time. However, as is usual in these kinds of differential games, we cannot exclude *a priori* that other, asymmetric, equilibria do exist (see, e.g. Dockner et al., 2000; Jorgenssen and Zaccour, 2004).

In addition to $\alpha_3 < 0$, global asymptotic stability also requires $\alpha_3 + \alpha_5 < 0$ and $\alpha_3 - \alpha_5 < 0$, which also ensures that quality is positive. Notice that, since $\alpha_5 < 0$, the condition $\alpha_3 < 0$ ensures that

$$\alpha_3 + \alpha_5 = s\gamma - \sqrt{\frac{\gamma y}{6}} < 0. \tag{34}$$

How does steady-state quality under feedback rules depend on the degree of competition (inversely measured by τ)? Since $\alpha_3 + \alpha_5$ does not depend on g, and therefore does not depend on τ, it is relatively straightforward to see that

$$\frac{\partial q^{CL}}{\partial \tau} = \underset{+}{\frac{\partial q^{CL}}{\partial \alpha_3}} \underset{+}{\frac{\partial \alpha_3}{\partial g}} \underset{-}{\frac{\partial g}{\partial \tau}} < 0. \tag{35}$$

Thus:

Proposition 2 *When providers adopt feedback closed-loop decision rules, steady-state quality is increasing in the degree of competition.*

As previously explained, increased competition has two counteracting effects on the providers' incentives to invest in quality. For given prices, demand becomes more quality elastic, which increases the profit-gain of quality investments. On the other hand, demand also becomes more price elastic, leading to lower prices, which in turn dampens incentives for quality investments. In contrast to the open-loop case, where these two effects exactly cancel each other in steady state, the first (direct) effect dominates the second (indirect) effect under dynamic competition with feedback rules, yielding a positive relationship between competition intensity and quality provision in steady state.

5.2 Comparison of closed loop and open loop

We have already seen that steady-state prices are equal under both solution concepts. However, steady-state quality provisions differ between the two solution concepts. A comparison (proof in Appendix A.2) yields the following result:

Proposition 3 *Steady-state quality is lower in the closed-loop solution than in the open-loop* *solution.*

This result is perhaps somewhat surprising. Although higher competition intensity leads to higher steady-state quality levels in the closed-loop solution, as shown in Proposition 2, quality provision is nevertheless always lower in the arguably more 'competitive' strategic environment – when the players use feedback closed-loop rules – than in the open-loop setting.

The intuition behind this result is related to how current quality investments affect future price competition. Suppose that, at time t, Provider i has a higher quality level than Provider j (i.e., $q_i(t) > q_j(t)$). The optimal pricing rule, given by (26), then dictates that Provider j should 'compensate' for the lower quality stock by setting a lower price than Provider i. In other words, higher quality investments by one provider today will trigger stronger price competition from the other provider in the future, which – all else equal – dampens the incentives for quality investments. Thus, when the providers use feedback decision rules and can, at each point in time, adjust their investment and price decisions according to the evolution of states, each provider has a strategic incentive to reduce its quality investments in order to dampen future price competition from the rival provider.[24] This is in contrast to the open-loop solution, where there is no strategic interaction over time, and where the above-mentioned strategic effect is not present. This explains why steady-state quality is lower in the closed-loop solution than in the open-loop solution. Finally, note that since profits are decreasing in quality, the condition required for profits being positive under the open-loop solution (see footnote 17) implies that profits are also positive under the closed-loop solution.

The result in Proposition 3 and the intuition behind it has a striking analogy in the difference between simultaneous and sequential decisions in a one-shot version of the game. As shown by Ma and Burgess (1993), equilibrium quality is lower when quality and price decisions are made sequentially rather than simultaneously, and the reason is precisely the strategic incentive to lower quality in order to dampen price competition when quality decisions are made before

[24] Colombo and Labrecciosa (2015) present a differential game of oligopoly, in which a similar mechanism is at work. They consider the case in which firms have to use a renewable productive asset, and show that the decision on current price taken by a player affects the future incentive of opponents to move their own price: this dynamic interdependence can lead Bertrand competition to be less efficient than Cournot competition.

prices are set.[25] This suggests that, in the case at hand, simultaneous-move and sequential-move games in a static setting provide results which are reasonable parallels of the open-loop and the closed-loop solutions, respectively, in a dynamic setting.

5.3 Multiplicity of equilibria

For completeness we discuss the possibility of multiplicity of equilibria, which we have ruled out above by assuming that the marginal cost of quality or the transportation costs are sufficiently large relative to consumers' marginal valuation of quality. If $2g < y < 8g/3$ (or, more explicitly, $\frac{k^2}{\tau} < 3\left(\left(\delta + \frac{1}{2}\rho\right)^2 \gamma + \beta\right) < \frac{4}{3}\frac{k^2}{\tau}$) another solution candidate *might* co-exist (solution S5 in Appendix A.1) with the one described above (solution S3 in Appendix A.1) for a subset of this parameter space. More precisely, the global stability condition requires $\alpha_3 - \alpha_5 < 0$, which is satisfied only if $\left(3y - 4g - 6\sqrt{3B}\right)\frac{\sqrt{E}}{4g} > s\gamma$, where $B = \frac{1}{6}\left(\frac{y}{2} - g\right)y > 0$, and $E = \frac{4\gamma}{27}\left(y - g + 2\sqrt{3B}\right) > 0$. This global stability condition is always satisfied under the unrealistic assumption that the marginal cost of investments tends to zero (i.e. $\gamma \to 0$). The condition is highly non-linear in γ. We can show through numerical simulations that when the marginal cost of investment γ is sufficiently high then multiplicity does not arise. If multiplicity does arise over a small set of parameter values, the alternative solution suggests that i) quality decreases with competition, rather than increases with competition as suggested by Proposition 2 (see Appendix A.3 for proof); ii) quality can be higher under closed-loop solution than the open-loop solution (while it is always lower under the closed-loop solution in Proposition 3; proof omitted and obtained through a numerical example). Therefore, the results are reversed under the alternative solution. To keep our analysis focused we impose the sufficient condition that the marginal cost of quality or the transportation costs are sufficiently large relative to consumers' marginal valuation of quality, which eliminates the possibility of multiple equilibria.

[25] Notice that the strategy of reducing quality in order to dampen price competition does not 'succeed' in equilibrium, in the sense that steady-state prices in the closed-loop solution are identical to the ones on the open-loop solution. This is also true for the equivalent simultaneous-move and sequential-move versions of the one-shot game. The reason is of course the symmetric nature of the game, where the effects of unilateral quality reductions on prices are cancelled out in equilibrium, since both firms face exactly the same incentives.

6 Welfare

We define social welfare as the discounted present value of the sum of aggregate consumer surplus and profits accruing over the infinite time horizon. Since total demand is fixed, this is equivalent to aggregate gross consumer utility minus the total costs of production, transportation and quality provision.[26] We derive the first-best optimal solution by letting the social planner choose the quality investment and market share for each provider, in order to maximise social welfare. Formally, this problem is given by

$$\underset{I_i(t),\, I_j(t),\, x_i^D(t)}{\text{Maximise}} \quad W = \int_0^{+\infty} \left[\begin{array}{c} \int_0^{x_i^D(t)} \left(v - \tau x + kq_i(t)\right) dx + \int_{x_i^D(t)}^{1} \left(v - \tau\left(1 - x\right) + kq_j(t)\right) dx \\ -c - \frac{\gamma}{2} I_i(t)^2 - \frac{\beta}{2} q_i(t)^2 - \frac{\gamma}{2} I_j(t)^2 - \frac{\beta}{2} q_j(t)^2 \end{array} \right] e^{-\rho t} dt, \tag{36}$$

$$\begin{aligned} \text{subject to} \quad \dot{q_i}(t) &= I_i(t) - \delta q_i(t), & (37)\\ \dot{q_j}(t) &= I_j(t) - \delta q_j(t), & (38)\\ q_i(0) &= q_{i0} > 0, & (39)\\ q_j(0) &= q_{j0} > 0. & (40) \end{aligned}$$

Let $\mu_i(t)$ and $\mu_j(t)$ be the current value co-state variables associated with the two state equations. The current-value Hamiltonian is:

$$H = v - c - \frac{\tau}{2} + kq_i x_i^D + kq_j \left(1 - x_i^D\right) - \frac{\gamma}{2}\left(I_i^2 + I_j{}^2\right) - \frac{\beta}{2}\left(q_i{}^2 + q_j{}^2\right) + \mu_i\left(I_i - \delta q_i\right) + \mu_j\left(I_j - \delta q_j\right). \tag{41}$$

[26] Notice that social welfare does not depend directly on prices, which are here just instruments of surplus distribution between firms and consumers, with no efficiency losses involved.

The solution is given by (a) $\partial H/\partial I_i = 0$, (b) $\partial H/\partial I_j = 0$, (c) $\partial H/\partial x_i^D = 0$, (d) $\dot{\mu_i} = \rho\mu_i - \partial H/\partial q_i$, (e) $\dot{\mu_j} = \rho\mu_j - \partial H/\partial q_j$, (f) $\dot{q_i} = \partial H_i/\partial \mu_i$, (h) $\dot{q_j} = \partial H/\partial \mu_j$, or more extensively,

$$\mu_i = \gamma I_i, \quad \mu_j = \gamma I_j, \tag{42}$$

$$0 = k(q_i - q_j), \tag{43}$$

$$\dot{\mu_i} = (\rho + \delta)\mu_i + \beta q_i - k x_i^D, \tag{44}$$

$$\dot{\mu_j} = (\rho + \delta)\mu_j + \beta q_j - k(1 - x_i^D), \tag{45}$$

$$\dot{q_i} = I_i - \delta q_i, \quad \dot{q_j} = I_j - \delta q_j. \tag{46}$$

In the symmetric steady state we have: $\mu^* = \gamma I^*$, $(\rho + \delta)\mu^* + \beta q^* - \frac{k}{2} = 0$ and $q^* = \frac{I^*}{\delta}$, which gives

$$q^* = \frac{k}{2(\delta\gamma(\rho + \delta) + \beta)} = q^{OL}. \tag{47}$$

Therefore, steady-state quality under open-loop decision rules coincides with the first-best steady-state quality level. Considering the result in Proposition 3, the following result follows immediately:

Proposition 4 *Compared with the first-best optimal level, quality is optimally provided in the open-loop solution and is underprovided in the closed-loop solution.*

The welfare-optimal quality provision in the open-loop solution is explained by a combination of linear provider-level demand and fixed total demand, which implies that consumers' marginal and average valuations of quality are identical. As demonstrated by Spence (1975) in a monopoly setting, whether quality is over- or under-provided depends on the difference between marginal and average willingness-to-pay for quality. However, the result of Proposition 4 shows that dynamic strategic interaction with feedback decision rules creates an inefficiency that leads to underprovision of quality in the closed-loop solution.[27]

[27] The fact that time, and more specifically competition over time, can be a source of inefficient equilibria is well known in different contexts; for instance, Cellini and Lambertini (1998) show that accumulation of capital over time could be a source of inefficient market allocation in a differential game framework. Furthermore, Araujo and Guimaraes (2015) show that time can be a source of inefficiency in an oligopoly market with delay options.

The welfare properties of the open-loop and closed-loop solutions mimic the welfare properties of the Hotelling model with price and quality competition in a one-shot game, where quality is optimally provided with simultaneous decision making, whereas sequential quality and price decisions imply an underprovision of quality in equilibrium (as shown by Ma and Burgess, 1993). This should come as no great surprise, since we have already established the equivalence between open-loop and closed-loop in a dynamic setting and, respectively, the simultaneous-move and sequential-move versions of the one-shot game.[28]

7 Extensions

In this section we extend our main analysis along two different dimensions. First, we investigate whether our main results are robust to an alternative measure of competition intensity based on the degree of cross-shareholding. Second, we consider an alternative cost function where production costs are convex and also depend on the quality level.

7.1 Cross-shareholding

Suppose that the instantaneous objective function of Provider i is given by $\pi_i(t) + \theta\pi_j(t)$, where $\theta \in (0,1)$ measures the degree of cross-shareholding. A reduction of θ captures, in a continuous way, a transition from monopoly to duopoly and is therefore an alternative measure of competition intensity, as used by, e.g., Symeonidis (2000) and Vives (2008).[29]

[28] However, as established by Proposition 2, a relevant difference between the steady-state quality in the closed-loop solution and the equilibrium quality in the sequential-move one-shot game is that the former depends on the degree of product substitutability while the latter does not. Put differently, the 'neutrality' result obtained by static games no longer holds if we consider dynamic competition.

[29] The Hotelling assumptions of full market coverage and non-binding reservation prices are not compatible with monopoly, which would yield infinitely high prices. A finite upper bound on equilibrium prices therefore requires a strictly positive value of θ.

7.1.1 Open-loop solution

The current-value Hamiltonian is now given by

$$\begin{aligned} H_i &= (p_i - c)\left(\frac{1}{2} + \frac{k(q_i - q_j)}{2\tau} - \frac{p_i - p_j}{2\tau}\right) \\ &\quad +\theta(p_j - c)\left(\frac{1}{2} + \frac{k(q_j - q_i)}{2\tau} - \frac{p_j - p_i}{2\tau}\right) \\ &\quad -\frac{\gamma}{2}I_i^2 - \frac{\beta}{2}{q_i}^2 + \mu_i(I_i - \delta q_i) + \mu_j(I_j - \delta q_j). \end{aligned} \tag{48}$$

By deriving optimality conditions equivalent to those given in Section 4, the steady state values of price and quality are given by[30]

$$p^{OL} = c + \frac{\tau}{1-\theta}, \tag{49}$$

$$q^{OL} = \frac{k}{2(\beta + \gamma\delta(\delta + \rho))}. \tag{50}$$

We see that an increase in competition (measured by a reduction of cross-shareholding) leads to a lower price but has no effect on steady-state quality provision, which is identical to the one derived in Section 4. Thus:

Proposition 5 *When providers use open-loop decision rules, steady-state quality does not depend on the intensity of competition, regardless of whether this is measured by product substitutability or by the degree of cross-shareholding between the providers.*

7.1.2 Closed-loop solution

As in Section 5, if the parameters β and/or τ are sufficiently large relative to k, there is a unique globally asymptotically stable closed-loop solution (see Appendix A.5 for details of the derivation). The optimal pricing rule for Provider i in this solution is given by

$$p_i(t) = c + \frac{\tau}{1-\theta} + \frac{k(q_i(t) - q_j(t))}{3+\theta}. \tag{51}$$

[30] The second-order conditions, which also guarantee positive steady-state profits, are identical to the ones of the corresponding problem in Section 4.

The optimal investment rule is still given by (28), but with the following two re-parameterisations: (i) α_1, as defined by (29), is replaced by

$$\widehat{\alpha}_1 := \frac{\alpha_1}{3+\theta}, \tag{52}$$

and (ii) the parameter g in (30) and (31) is replaced by

$$\widehat{g} := \frac{9\left(1+\theta\right)^2}{\left(3+\theta\right)^2} g. \tag{53}$$

The steady-state value of quality is therefore still given by (33), but with the two above-mentioned re-parameterisations. This solution has the following properties (see Appendix A.5 for a proof):

Proposition 6 *When providers use closed-loop decision rules:*

(i) A reduction in cross-shareholding between the providers has an a priori ambiguous effect on steady-state quality provision.

(ii) Steady-state quality provision is lower than in the open-loop solution for all degrees of cross-shareholding.

Thus, whereas quality provision is still lower in the closed-loop than in the open-loop solution, increased competition – measured by a reduction in cross-shareholding – now has an *a priori* indeterminate effect on steady-state quality provision in the closed-loop solution. This is caused by the presence of two counteracting effects: a lower value of θ reduces α_1 but increases α_3, leading to an ambiguous overall effect.

7.2 Convex and quality-dependent production costs

Let us now investigate if and how our main results in Sections 3-5 depend on the assumed cost structure. Suppose that the cost function of Provider i is given by

$$C\left(x_i^D, I_i, q_i\right) = c_1 q_i x_i^D + \frac{1}{2}\left(c_2\left(x_i^D\right)^2 + \gamma I_i^2 + \beta q_i^2\right), \tag{54}$$

where $c_1, c_2 > 0$. This specification implies that marginal production costs are increasing in both output and quality.

7.2.1 Open-loop solution

The current-value Hamiltonian is now given by

$$\begin{aligned} H_i &= (p_i - c_1 q_i)\left(\frac{1}{2} + \frac{k(q_i - q_j)}{2\tau} - \frac{p_i - p_j}{2\tau}\right) \\ &\quad -\frac{c_2}{2}\left(\frac{1}{2} + \frac{k(q_i - q_j)}{2\tau} - \frac{p_i - p_j}{2\tau}\right)^2 \\ &\quad -\frac{\gamma}{2} I_i^2 - \frac{\beta}{2} {q_i}^2 + \mu_i (I_i - \delta q_i) + \mu_j (I_j - \delta q_j). \end{aligned} \tag{55}$$

By deriving optimality conditions equivalent to those given in Section 4, the steady-state values of price and quality are given by[31]

$$p^{OL} = \tau + \frac{c_2}{2} + c_1 q^{OL}, \tag{56}$$

$$q^{OL} = \frac{k - c_1}{2(\beta + \gamma\delta(\delta + \rho))}. \tag{57}$$

In contrast to the open-loop solution derived in Section 4, the steady-state price is now increasing in the quality level. This is caused by the assumption of quality-dependent production costs ($c_1 > 0$). Higher quality increases marginal costs, which in turns leads to a price increase. However, as in Section 4, steady-state quality does not depend on the intensity of competition, as measured by the degree of product substitutability. Thus:

Proposition 7 *Proposition 1 holds with both quality-dependent ($c_1 > 0$) and convex ($c_2 > 0$)*

[31] The second-order conditions are satisfied if $\beta > (k - c_1)/(4\tau + c_2)$. The additional condition

$$\tau > \frac{(\beta + \gamma\delta^2)(k - c_1)^2 - \beta c_2 (\beta + 2\gamma\delta(\delta + \rho)) + \gamma^2\delta^2 c_2 (\delta + \rho)^2}{4\beta(\beta + 2\gamma\delta(\delta + \rho)) + 4\gamma^2\delta^2(\delta + \rho)^2}$$

ensure positive steady-state profits, given by

$$\pi^{OL} = \frac{\beta(4\tau + c_2)(\beta + 2\gamma\delta(\delta + \rho)) + \gamma^2\delta^2(\delta + \rho)^2(4\tau + c_2) - (k - c_1)^2(\beta + \gamma\delta^2)}{8(\beta + \gamma\delta(\delta + \rho))^2}.$$

production costs.

7.2.2 Closed-loop solution

It can be shown (see Appendix A.6) that the closed-loop solution is identical to the one presented in Section 4, but with the following three re-parameterisations: (i) the parameter k is replaced by $\widetilde{k} := k - c_1$ in all expressions where it appears, (ii) α_1, as defined by (29), is replaced by

$$\widetilde{\alpha}_1 := \frac{3\left(4\tau + c_2\right)}{4\left(3\tau + c_2\right)}\alpha_1 \tag{58}$$

and (ii) the parameter g in (30) and (31) is replaced by

$$\widetilde{g} := \frac{9\tau\left(4\tau + c_2\right)}{4\left(3\tau + c_2\right)^2} g. \tag{59}$$

This allows us to state the following results (see Appendix A.7 for a proof):

Proposition 8 *(i) Proposition 2 holds with quality-dependent production costs ($c_1 > 0$).*

(ii) Proposition 2 holds with convex production costs if c_2 is sufficiently small. Otherwise, the relationship between competition intensity and steady-state quality in the closed-loop solution is ambiguous.

(iii) Proposition 3 and 4 hold with both quality-dependent ($c_1 > 0$) and convex ($c_2 > 0$) production costs.

These results confirm that, overall, the results derived from our base model do not depend crucially on the assumed cost structure of the providers. The only exception is the relationship between competition intensity and quality provision in the closed-loop solution, where the sign of this relationship depends on the degree of production cost convexity. When seen in conjunction, the results in this section suggest that our most robust result is the underprovision of quality in the closed-loop solution.

8 Discussion on empirical evidence

In this section we relate our results to the empirical evidence and to possible empirical strategies which could be used in future studies to test the predictions of our model. Our analysis provides the clear-cut prediction that competition increases quality under the closed-loop solution, but not under the open-loop solution. Therefore, empirical evidence which finds that more competition (e.g., public reporting) increases quality gives support to the hypothesis that providers compete under the closed-loop rule. Instead, a non-significant effect supports the hypothesis that providers compete under the open-loop rule.

In the introduction, we have also argued that certificate of needs (CON) regulation in the US or similar regulatory requirements in Europe imply that providers might have to commit to investment plans, an environment which is reminiscent of the open-loop solution. Therefore, the implication for empirical work is that the effect of competition on quality should be weaker in the presence of such regulations or for industries which are more heavily regulated. However, our model also predicts that quality is higher under the open loop solution: empirically, under this interpretation, this would be consistent with studies that test whether the introduction of CON regulation increased quality. Finally, our model implies that policies which introduce or encourage partnerships and collaborations across providers, a measure of the degree of cross-shareholding, might increase or reduce quality.

Our model therefore provides a theoretical framework for testing the effect of competition on quality in industries such as long-term care, health care, child care and education. There is a proliferation of quality data in these sectors which are increasingly used for empirical work. For nursing homes, common quality measures include number of residents with pressure sores, decubitis ulcers, dehydration and urinary tract infection (outcome measures), catheters, feeding tubes and physical restraints (process measures); and total number of nurses, proportion of registered nurses on total nursing staff (structural measures) (see, e.g., Grabowski and Hirth, 2003; Grabowski and Angelelli, 2004). For hospitals, common quality measures include risk-adjusted mortality rates, either overall or for specific conditions (heart attack, stroke, hip fracture), and 30-day re-admission rates, either overall or for specific treatments (e.g., hip and knee replacement, coronary bypass) (Gaynor and Town, 2011). For schools and university, quality can be measured with test scores (Hoxby, 2000; Gibbons, Silva and Machin, 2008; OECD, 2014), student satisfaction, and employment outcomes (e.g., salaries or graduate employment rates for a given subject).

Though limited, the body of empirical evidence on nursing homes seems consistent with our findings. Zhao (2016) evaluates the effect of public reporting which improved access to information of nursing homes along key dimensions. The study finds that while the effect of competition on nursing home quality is positive and small, this effect becomes significantly stronger with public reporting. These results are in line with the older study by Grabowski and Town (2011), which introduced quality measures on the Nursing Home Compare website, during 1999-2005. They find that nursing homes facing greater competition improved their quality more compared to facilities in less competitive markets once public reporting was introduced, though public reporting itself had a small effect on quality.

The literature from the US on hospital competition outside of Medicare and Medicaid where prices are not fixed, also tends to find that competition increases quality, but not always. Gowrisankaran and Town (2003) find that risk-adjusted mortality is higher in more competitive areas in Los Angeles county. Sohn and Rathouz (2003) also find that mortality for patients receiving coronary angioplasty (PTCA) are lower for hospitals facing more competition. Escarce et al. (2006) find that hospitals in more competitive areas had lower AMI, hip fracture and stroke mortality in California, New York, but not in Wisconsin, suggesting heterogeneity across states. However, Mukamel et al. (2002) find that competition, as measured by the introduction of selective contracting, increased risk-adjusted mortality. Two studies from England, also suggest that competition reduces quality when prices were not fixed in the early nineties (Propper et al., 2004; Burgess et al., 2008), though the hospital sector remained heavily regulated and quality indicators were absent making it difficult for hospitals to compete on quality (see Gaynor and Town, 2011, for a fuller review of the literature).

Within the education sector in the US, the extensive review by Belfleld and Levin (2003) suggests that school competition is generally associated with an increase in educational outcomes, e.g., test scores, across a range of institutional settings but the effect is quantitatively modest.

9 Policy implications and concluding remarks

Our analysis has several policy implications for industries such as long term care, health care, child care and education. Our key finding suggests that competition policies in these industries, such as those that facilitate public reporting and comparison across providers, can be useful in increasing quality, and this seems consistent with empirical evidence reviewed in the previous section. The result is new and in contrast with previous static models which show that within a spatial framework, a key feature of the industries at stake, competition does not affect quality and this is due to competition also reducing price and therefore weakening the incentives to compete on quality (e.g., Ma and Burgess, 1993). It is the dynamic strategic interaction which makes the direct effect to compete on quality stronger than the indirect effect through a reduction in price, so that competition ultimately increases quality. The static models would instead predict that public reporting would be an ineffective policy to raise quality.

This positive finding comes however with a disappointing result. We show that quality is always under-provided under the more realistic closed-loop solution. Although public reporting makes the gap between first-best and actual quality smaller, quality is never as high as the one desired by the regulator (or social planner). One possibility would be for the regulator to induce the providers to mimic the open-loop solution, where quality is instead provided at the socially optimal level. This could be achieved by regulation which induces providers to make and commit to long-term investment plans which cannot be frequently revised. Such regulation raises however the issue of enforcement of such plans which could probably be achieved by a strong regulator with credible monitoring tools.

A more extreme form of regulation would be to introduce fixed prices since in this case, more competition always increases quality (Brekke et al., 2010). Moreover, the regulator can always set the price at a level which is high enough to induce the optimal quality provision and combine it with a lump-sum transfer to ensure providers do not make profits. In practice, however, fixed price regulation often follows automated rules which set prices equal to average costs (as for hospitals within Medicare and Medicaid, and other European countries) and these are unlikely to lead to optimal quality provision (Kristensen et al., 2016). In addition, fixed price regulation has also other known limitations, e.g., in relation to the ability to accommodate for heterogeneity in costs and other parameters (Laffont and Tirole, 1993), and the risk of regulatory and political capture.

Our model has been tailor-made to fit a particular set of industries, in which quality is a key concern and policies to stimulate competition are widespread. However, given that competition on both price and quality is prevalent in a wide range of industries, a natural question to ask is whether the main results from our analysis also apply under alternative analytical frameworks that are not based on spatial competition with unit demand. The answer is yes, to some extent, but only under particular conditions. For example, using a Shubik-Levitan demand system (Shubik and Levitan, 1980), which is based on a representative consumer approach, steady-state quality provision tend to be lower in the closed-loop than in the open-loop, as we find in the present analysis, if the degree of product substitutability is sufficiently high.[32] However, we should stress that the relatively clear-cut results derived from our analysis rely to some extent on the particular assumptions of the modelling framework. This means that, whereas our results are arguably highly relevant for the particular industries that we highlight, they do not necessarily apply to all other industries more generally.

References

[1] Araujo, L., B. Guimaraes, 2015. Intertemporal coordination with delay options. *Journal of Economic Theory*, **157**, 793–810.

[2] Banker, R.D., I. Khosla, K.K. Sinha, 1998. Quality and competition. *Management Science*, **44**, 1179–1192.

[3] Başar, T., Olsder G.J., 1995 (2nd ed), *Dynamic Noncooperative Game Theory*, London: Academic Press.

[32] This and other results based on a representative consumer framework are available upon request.

[4] Belfleld, C., H. Levin, 2003. The Effects of Competition between Schools on Educational Outcomes: A review for the United States. *Review of Educational Research*, **72**, 279-341.

[5] Bischi, G.I., F. Lamantia, D. Radi, 2015. An evolutionary Cournot model with limited market knowledge. *Journal of Economic Behavior and Organization*, **116**, 219–238.

[6] Bischi, G.I., F. A.K. Naimzada, L. Sbragia, 2007. Oligopoly games with local monopolistic approximation. *Journal of Economic Behavior and Organization*, **62**, 371–388.

[7] Borenstein, S., J. Netz, 1999. Why do all the flights leave at 8 am?: Competition and departure-time differentiation in airline markets, *International Journal of Industrial Organization*, **17**, 611-640.

[8] Brekke, K., R. Cellini, L. Siciliani, O.R. Straume, 2010. Competition and quality in health care markets: a differential game approach. *Journal of Health Economics*, **29**, 508–523.

[9] Brekke, K., R. Cellini, L. Siciliani, O.R. Straume, 2012. Competition in regulated markets with sluggish beliefs about quality. *Journal of Economics and Management Strategy*, **21**, 131–78.

[10] Brekke, K.R., L. Siciliani, O.R. Straume, 2010. Price and quality in spatial competition. *Regional Science and Urban Economics*, **40**, 471–480.

[11] Brekke, K.R., H. Gravelle, L. Siciliani, O.R. Straume, 2014. Patient Choice, Mobility and Competition Among Health Care Providers, Chapter 1, pages 1-26, in R. Levaggi and M. Montefiori (eds.), *Health Care Provision and Patient Mobility, Developments in Health Economics and Public Policy*, 12, Berlin: Springer-Verlag.

[12] Brekke, K.R., L. Siciliani, O.R. Straume, 2017. Horizontal mergers and product quality. *Canadian Journal of Economics*, **50**, 1063–1103.

[13] Burgess, S., C. Propper, D. Gossage, 2008. Competition and quality: evidence from the NHS internal market 1991-9. *Economic Journal*, **118**, 138–170.

[14] Cellini R., F. Lamantia, 2015. Quality competition in markets with regulated prices and minimum quality standards. *Journal of Evolutionary Economics*, **25**, 345–70.

[15] Cellini R., L. Lambertini, 1998. A dynamic model of differentiated oligopoly with capital accumulation. *Journal of Economic Theory*, **83**, 145–55.

[16] Cellini, R., L. Lambertini, 2005. R&D incentives and market structure: a dynamic analysis. *Journal of Optimization Theory and Applications*, **126**, 85–96.

[17] Cellini, R., L. Lambertini, 2009. Dynamic R&D with spillover: Competition vs. cooperation. *Journal of Economic Dynamics and Control*, **33**, 568–82.

[18] Cellini, R., L. Lambertini, A. Mantovani, 2008. Persuasive advertising under Bertrand competition: a differential game. *Operations Research letters*, **36**, 381–84.

[19] Champsaur, P., J.-C. Rochet, 1989. Multiproduct duopolists. *Econometrica*, **57**, 533–557.

[20] Choné, P., 2017. Competition policy for health care provision in France. *Health Policy*, **121**, 111-118.

[21] Chumacero, R.A., D. Gomez, R. Paredes, 2011. I would walk 500 miles (if it paid): vouchers and school choice in Chile. *Economics Educaction Review*, **30**, 1103–1114.

[22] Colombo, L., P. Labrecciosa, 2015. On the Markovian efficiency of Bertrand and Cournot equilibira. *Journal of Economic Theory*, **155**, 332–58.

[23] Dockner, E.J, S. Jørgensen, N. Van Long, G. Sorger, 2000. *Differential Games in Economics and Management Science*. Cambridge, Cambridge University Press.

[24] Driskill R.A., S. McCafferty, 1989. Dynamic duopoly with adjustment costs: a differential game approach. *Journal of Economic Theory*, **49,** 324–38.

[25] Economides, N., 1993. Quality variations in the circular model of variety-differentiated products. *Regional Science and Urban Economics*, **23**, 235–257.

[26] Escarce, J. J., A.K. Jain, J. Rogowski. 2006. Hospital competition, managed care, and mortality after hospitalization for medical conditions: Evidence from three states. *Medical Care Research and Review*, **63**, 112-140S.

[27] Gabszewicz, J.J., J.-F. Thisse, 1979. Price competition, quality and income disparities. *Journal of Economic Theory*, **20**, 340–359.

[28] Gabszewicz, J.J., J.-F. Thisse, 1980. Entry (and exit) in a differentiated industry. *Journal of Economic Theory*, **22**, 327–338.

[29] Gal-Or, E., 1983. Quality and quantity competition. *Bell Journal of Economics*, **14**, 590–600.

[30] Gibbons, S., S. Machin, O. Silva, 2008. Choice, competition, and pupil achievement, *Journal of the European Economic Association*, **6**, 912–947.

[31] Gaynor, M., R.J. Town, 2011. Competition in Health Care Markets, Handbook of Health Economics, Volume 2, chapter 9, 499-637.

[32] Gowrisankaran, G., R. Town. 2003. Competition, payers, and hospital quality. *Health Services Research*, **38**, 1403-1422.

[33] Grabowski, D.C., V. Angelelli 2004. The Relationship of Medicaid Payment Rates, Bed Constraint Policies, and Risk- Adjusted Pressure Ulcers. *Health Service Research*, **39**, 793-812.

[34] Grabowski, D.C., R.A., Hirth, 2003. Competitive spillovers across non-profit and for-profit nursing homes. *Journal of Health Economics*, **22**, 1-22.

[35] Grabowski, D.C., R.J. Town, 2011. Does information matter? Competition, quality, and the impact of nursing home report cards, *Health Services Research*, **46**, 1698-1719.

[36] Grabowski, D.C., Z. Feng, R. Hirth, M. Rahman, V. Mor, 2013. Effect of nursing home ownership on the quality of post-acute care: an instrumental variablesapproach. *Journal of Health Economics*, **32**, 12–21.

[37] Gravelle, H., 1999. Capitation contracts: access and quality. *Journal of Health Economics*, **18**, 315–340.

[38] Gutacker, N., L. Siciliani, G. Moscelli, H. Gravelle, 2016. Choice of hospital: which type of quality matters? *Journal of Health Economics*, **50**, 230-246.

[39] Hastings, J., T. Kane, D.O. Staiger, 2005. Parental Preferences and School Competition: Evidence from a Public School Choice Program, National Bureau of Economic Research working paper #11805, November.

[40] Hinloopen, J., 2000. Strategic R&D co-operatives. *Research in Economics*, **54**, 153–85.

[41] Hinloopen, J., 2003. R&D efficiency gains due to cooperation. *Journal of Economics*, **80**, 107–25.

[42] Hoxby, C., 2000. Does Competition Among Public Schools Benefit Students and Taxpayers? *American Economic Review*, **90**, 1209-1238.

[43] Jorgensen, S., G. Zaccour, 2004. *Differential games in marketing*, Dordrecht: Kluwer Academic Publication.

[44] Jun, B., X. Vives, 2004. Strategic incentives in dynamic oligopoly. *Journal of Economic Theory*, **116**, 249–81.

[45] Kessler, D., M. McClellan, 2000. Is hospital competition socially wasteful? *Quarterly Journal of Economics*, **115**, 577–615.

[46] Kristensen, S., L. Siciliani, M. Sutton, 2016. Optimal Price-Setting in Pay for Performance Schemes in Health Care. *Journal of Economic Behavior & Organization*, **123**, 57-77.

[47] Laffont, J.J., J. Tirole. 1993. *A Theory of Incentives in Procurement and Regulation*, Cambridge: MIT Press.

[48] Léonard, D., N. van Long, 1992. Optimal Control Theory and Static Optimization in Economics, Cambridge University Press, Cambridge.

[49] Ma, C.A., J.F. Burgess, 1993. Quality competition, welfare and regulation. *Journal of Economics*, **58**, 153–73.

[50] Mehlmann, A., 1988. *Applied differential games*, New York: Plenum Press.

[51] Motta, M., 1993. Endogenous quality choice: price vs. quantity competition. *Journal of Industrial Economics*, **41**, 113–131.

[52] Mukamel, D., J.A. Zwanziger, A. Bamezai, 2002. Hospital competition, resource allocation and quality of care. *BMC Health Services Research*, **2**, 10-18.

[53] Mussa, M., S. Rosen, 1978. Monopoly and product quality. *Journal of Economic Theory*, **18**, 301–317.

[54] OECD, 2014. PISA 2012 Results in Focus.What 15-year-olds know and what they can do with what they know. OECD Publishing, Paris, France.

[55] Piga, C.A.G., 1998. A dynamic model of advertising and product differentiation. *Review of Industrial Organization*, **13**, 509-22.

[56] Piga, C.A.G., 2000. Competition in a duopoly with sticky price and advertising. *International Journal of Industrial Organization*, **18**, 595–614.

[57] Propper, C., S. Burgess, K. Green, 2004. Does competition between hospitals improve the quality of care? Hospital death rates and the NHS internal market. *Journal of Public Economics*, **88**, 1247-1272.

[58] Qiu, L., 1997. On the dynamic efficinecy of Bertrand and Cournot equilibria. *Journal of Economic Theory*, **75,** 213-29.

[59] Rahman, M., A.D. Foster, 2015. Racial segregation and quality of care disparity in US nursing homes. *Journal of Health Economics*, **39**, 1–16.

[60] Salvanes, K.G., F. Steen, L. Sørgard, 2005. Hotelling in the air? Flight departures in Norway, *Regional Science and Urban Economics*, **35**, 193-213.

[61] Shaked, A., J. Sutton, 1982. Relaxing price competition through product differentiation. *Review of Economic Studies*, **49**, 3–13.

[62] Shaked, A., J. Sutton, 1983. Natural oligopolies. *Econometrica*, **51**, 1469–1483.

[63] Shubik, M., R. Levitan, 1980. *Market Structure and Behavior*. Cambridge, MA, Harvard University Press.

[64] Shugarman, L.R., J.A. Brown, 2006. Nursing Home Selection: How Do Consumers Choose? Volume I: Findings from Focus Groups of Consumers and Information Intermediaries. Washington, DC. Prepared for Office of Disability, Aging and Long-Term Care Policy, Office of the Assistant Secretary for Planning and Evaluation, U.S. Department of Health and Human Services, Contract #HHS-100-03-0023.

[65] Siciliani, L., O.R. Straume, R. Cellini, 2013. Quality competition with motivated providers and sluggish demand. *Journal of Economic Dynamics and Control*, **37**, 2041–71.

[66] Sohn, M.-W., P.J., Rathouz, 2003. Competition among hospitals and quality of care: Hospital-level analysis. University of Chicago.

[67] Spence, M., 1975. Monopoly, quality and regulation. *Bell Journal of Economics*, **6**, 417–429.

[68] Sutton, J., 1996. Technology and market structure. *European Economic Review*, **40**, 511–530.

[69] Symeonidis, G., 2000. Price and nonprice competition with endogenous market structure. *Journal of Economics & Management Strategy*, **9**, 53–83.

[70] Tay, A., 2003. Assessing competition in hospital care markets: the importance of accounting for quality differentiation. *RAND Journal of Economics*, **34**, 786–814.

[71] Vives, X., 1985. Efficiency of Bertrand and ournot equilibria with product differentiation. *Journal of Economic Theory*, **36**, 166–75.

[72] Vives, X., 2008. Innovation and competitive pressure. *Journal of Industrial Economics*, **56**, 419–469.

[73] Werner, R.M., E.C. Norton, R.T. Konetzka, D. Polsky, 2012. Do consumers respond to publicly reported quality information? Evidence from nursing homes. *Journal of Health Economics*, **31**, 50–61.

[74] Zhao, X., 2016. Competition, information, and quality: Evidence from nursing homes. *Journal of Health Economics*, **49**, 136-152.

[75] Zwanziger, J., D.B. Mukamel, et al., 2002. Use of resident-origin data to define nursing home market boundaries. *Inquiry*, **39**, 56–66.

(Online) Appendix

A.1. Derivation of the closed-loop solution in Section 4

The provider's instantaneous objective function is

$$(p_i - c)\left(\frac{1}{2} + \frac{k(q_i - q_j)}{2\tau} - \frac{p_i - p_j}{2\tau}\right) - \frac{\gamma}{2}I_i^2 - \frac{\beta}{2}{q_i}^2 \tag{A1}$$

which – faced with the linear dynamic constraint – gives rise to a linear-quadratic problem. Hence, we define the value function as

$$V^i(q_i, q_j) = \alpha_0 + \alpha_1 q_i + \alpha_2 q_j + (\alpha_3/2)q_i^2 + (\alpha_4/2)q_j^2 + \alpha_5 q_i q_j. \tag{A2}$$

Define $I_i = \phi_i(q_i, q_j)$ and $I_j = \phi_j(q_i, q_j)$. The value function has to satisfy the Hamilton-Jacobi-Bellman (HJB) equation:

$$\rho V^i(q_i, q_j) = \max \left\{ \begin{array}{c} (p_i - c)\left(\frac{1}{2} + \frac{k(q_i - q_j)}{2\tau} - \frac{p_i - p_j}{2\tau}\right) - \frac{\gamma}{2}I_i^2 - \frac{\beta}{2}{q_i}^2 \\ +V^i_{q_i}(q_i, q_j)(I_i - \delta q_i) + V^i_{q_j}(q_i, q_j)(I_j - \delta q_j) \end{array} \right\}. \tag{A3}$$

Maximisation of the right-hand-side with respect to I_i yields $V^i_{q_i} = \gamma I_i$, which after substitution gives

$$I_i = \phi_i(q_i, q_j) = \frac{\alpha_1 + \alpha_3 q_i + \alpha_5 q_j}{\gamma}. \tag{A4}$$

Similarly, we obtain

$$I_j = \phi_j(q_i, q_j) = \frac{\alpha_1 + \alpha_3 q_j + \alpha_5 q_i}{\gamma}. \tag{A5}$$

Maximisation of the right-hand-side with respect to p_i and p_j yields

$$\frac{1}{2} + \frac{k(q_i - q_j)}{2\tau} - \frac{p_i - p_j}{2\tau} - (p_i - c)\frac{1}{2\tau} = 0, \tag{A6}$$

$$\frac{1}{2} + \frac{k(q_j - q_i)}{2\tau} - \frac{p_j - p_i}{2\tau} - (p_j - c)\frac{1}{2\tau} = 0, \tag{A7}$$

from which we obtain the simple expression:

$$
\begin{aligned}
p_i &= \Phi_i(q_i, q_j) = c + \tau + \frac{k\left(q_i - q_j\right)}{3}, && \text{(A8)}\\
p_j &= \Phi_j(q_i, q_j) = c + \tau - \frac{k\left(q_i - q_j\right)}{3}. && \text{(A9)}
\end{aligned}
$$

By substituting $I_i = \phi_i(q_i, q_j)$, $I_j = \phi_j(q_i, q_j)$, $V^i_{q_i}(q_i, q_j) = \alpha_1 + \alpha_3 q_i + \alpha_5 q_j$ and $V^i_{q_j} = \alpha_2 + \alpha_4 q_j + \alpha_5 q_i$ into the HJB equation, we obtain

$$
\rho V^i(q_i, q_j) = \left\{ \begin{array}{c} \left(\tau + \frac{k(q_i - q_j)}{3}\right)\left(\frac{1}{2} + \frac{k(q_i - q_j)}{6\tau}\right) \\ -\frac{1}{2\gamma}\left(\alpha_1 + \alpha_3 q_i + \alpha_5 q_j\right)^2 - \frac{\beta}{2} {q_i}^2 \\ + \left(\alpha_1 + \alpha_3 q_i + \alpha_5 q_j\right)\left(\frac{\alpha_1 + \alpha_3 q_i + \alpha_5 q_j}{\gamma} - \delta q_i\right) \\ + \left(\alpha_2 + \alpha_4 q_j + \alpha_5 q_i\right)\left(\frac{\alpha_1 + \alpha_3 q_j + \alpha_5 q_i}{\gamma} - \delta q_j\right) \end{array} \right\}, \tag{A10}
$$

and, after substitution of V^i, we obtain

$$
\begin{aligned}
& \left(\rho\alpha_0 - \frac{1}{2}\tau - \frac{1}{2\gamma}\alpha_1^2 - \frac{1}{\gamma}\alpha_1\alpha_2\right) \\
& + q_i\left(\alpha_1\left(\delta + \rho\right) - \frac{1}{3}k - \frac{1}{\gamma}\alpha_1\alpha_3 - \frac{1}{\gamma}\alpha_2\alpha_5 - \frac{1}{\gamma}\alpha_1\alpha_5\right) \\
& + q_j\left(\alpha_2\left(\delta + \rho\right) + \frac{1}{3}k - \frac{1}{\gamma}\alpha_2\alpha_3 - \frac{1}{\gamma}\alpha_1\alpha_4 - \frac{1}{\gamma}\alpha_1\alpha_5\right) \\
& + q_i^2\left(\alpha_3\left(\delta + \frac{1}{2}\rho\right) - \frac{1}{2\gamma}\alpha_3^2 - \frac{1}{\gamma}\alpha_5^2 + \frac{1}{2}\beta - \frac{1}{18}\frac{k^2}{\tau}\right) \\
& + q_j^2\left(\alpha_4\left(\delta + \frac{1}{2}\rho\right) - \frac{1}{\gamma}\alpha_3\alpha_4 - \frac{1}{2\gamma}\alpha_5^2 - \frac{1}{18}\frac{k^2}{\tau}\right) \\
& + q_i q_j\left(\left(2\delta + \rho\right)\alpha_5 + \frac{1}{9}\frac{k^2}{\tau} - \frac{2}{\gamma}\alpha_3\alpha_5 - \frac{1}{\gamma}\alpha_4\alpha_5\right) \\
= \;& 0
\end{aligned} \tag{A11}
$$

For the equality to hold, the terms in brackets in the above equation have to be equal to zero. Notice that the last three terms do not depend on α_0, α_1 and α_2, but only on α_3, α_4 and α_5. We therefore focus on the following system of three equations in three unknowns (α_3, α_4 and

α_5):

$$\alpha_3\left(\delta+\frac{1}{2}\rho\right)-\frac{1}{2\gamma}\alpha_3^2-\frac{1}{\gamma}\alpha_5^2+\frac{1}{2}\beta-\frac{1}{18}\frac{k^2}{\tau} = 0, \tag{A12}$$

$$\alpha_4\left(\delta+\frac{1}{2}\rho\right)-\frac{1}{\gamma}\alpha_3\alpha_4-\frac{1}{2\gamma}\alpha_5^2-\frac{1}{18}\frac{k^2}{\tau} = 0, \tag{A13}$$

$$\alpha_5\left(2\delta+\rho\right)+\frac{1}{9}\frac{k^2}{\tau}-\frac{2}{\gamma}\alpha_3\alpha_5-\frac{1}{\gamma}\alpha_4\alpha_5 = 0. \tag{A14}$$

Define $g := \frac{k^2}{\tau}$ and $s := \left(\delta+\frac{1}{2}\rho\right)$. We can re-write the system more succinctly as

$$s\alpha_3-\frac{1}{2\gamma}\alpha_3^2-\frac{1}{\gamma}\alpha_5^2+\frac{1}{2}\beta-\frac{1}{18}g = 0, \tag{A15}$$

$$s\alpha_4-\frac{1}{\gamma}\alpha_3\alpha_4-\frac{1}{2\gamma}\alpha_5^2-\frac{1}{18}g = 0, \tag{A16}$$

$$2s\alpha_5+\frac{1}{9}g-\frac{2}{\gamma}\alpha_3\alpha_5-\frac{1}{\gamma}\alpha_4\alpha_5 = 0. \tag{A17}$$

Define

$$A := \sqrt{\gamma\left(\frac{3}{2}y-g\right)} > 0, \tag{A18}$$

$$B := \frac{1}{6}\left(\frac{y}{2}-g\right)y > 0, \tag{A19}$$

$$C := \frac{4\gamma}{27}\left(y-g-2\sqrt{3B}\right) > 0, \tag{A20}$$

$$E := \frac{4\gamma}{27}\left(y-g+2\sqrt{3B}\right) > 0, \tag{A21}$$

where $y := 6\left(s^2\gamma+\beta\right)$, and where the condition $y > 2g$ ensures that these parameters (and therefore the possible solutions) are real. The positive sign of C is confirmed by noticing that

$$y-g > 2\sqrt{3B} \Leftrightarrow (y-g)^2 > \left(2\sqrt{3\left(\frac{1}{6}\left(\frac{y}{2}-g\right)y\right)}\right)^2, \tag{A22}$$

which always holds since

$$(y-g)^2-\left(2\sqrt{3\left(\frac{1}{6}\left(\frac{y}{2}-g\right)y\right)}\right)^2 = g^2 > 0. \tag{A23}$$

There are six possible solutions to (A15)-(A17), given by:

$$\alpha_3 = s\gamma - \frac{1}{9}A,\ \alpha_4 = \frac{2\,(6y+5g)}{9\,(6y-4g)}A,\ \alpha_5 = \frac{2}{9}A \tag{S1}$$

$$\alpha_3 = s\gamma + \frac{1}{9}A,\ \alpha_4 = -\frac{2\,(6y+5g)}{9\,(6y-4g)}A,\ \alpha_5 = -\frac{2}{9}A \tag{S2}$$

$$\alpha_3 = s\gamma - \left(\frac{6y-5g}{4g} - \frac{81}{16g\gamma}C\right)\sqrt{C}, \alpha_4 = \frac{1}{2}\sqrt{C}, \alpha_5 = -\frac{1}{2}\sqrt{C} \tag{S3}$$

$$\alpha_3 = s\gamma + \left(\frac{6y-5g}{4g} - \frac{81}{16g\gamma}C\right)\sqrt{C}, \alpha_4 = -\frac{1}{2}\sqrt{C}, \alpha_5 = \frac{1}{2}\sqrt{C} \tag{S4}$$

$$\alpha_3 = s\gamma - \left(\frac{6y-5g}{4g} - \frac{81}{16g\gamma}E\right)\sqrt{E}, \alpha_4 = \frac{1}{2}\sqrt{E}, \alpha_5 = -\frac{1}{2}\sqrt{E} \tag{S5}$$

$$\alpha_3 = s\gamma + \left(\frac{6y-5g}{4g} - \frac{81}{16g\gamma}E\right)\sqrt{E}, \alpha_4 = -\frac{1}{2}\sqrt{E}, \alpha_5 = \frac{1}{2}\sqrt{E} \tag{S6}$$

Global asymptotic stability requires $\alpha_3 < 0$, $\alpha_3 + \alpha_5 < 0$ and $\alpha_3 - \alpha_5 < 0$. We can immediately eliminate (S2) because $\alpha_3 > 0$. The same is true for (S1), since $\alpha_3 + \alpha_5 = s\gamma + \frac{1}{9}A > 0$. Regarding (S4), notice that a sufficient condition for $\alpha_3 > 0$ is

$$\frac{6y-5g}{4g} - \frac{81}{16g\gamma}C = \frac{1}{4g}\left(3y - 2g + 6\sqrt{3B}\right) > 0, \tag{A24}$$

which always holds for $y > 2g$. Similarly, regarding (S6), a sufficient condition for $\alpha_3 > 0$ is

$$\frac{6y-5g}{4g} - \frac{81}{16g\gamma}E = \frac{1}{4g}\left(3y - 2g - 6\sqrt{3B}\right) > 0, \tag{A25}$$

which always holds since

$$(3y-2g)^2 - \left(6\sqrt{3B}\right)^2 = 2g\,(2g+3y) > 0. \tag{A26}$$

Thus, (S4) and (S6) can also be ruled out because $\alpha_3 > 0$. In the two remaining solutions – (S3) and (S5) – we have $\alpha_5 < 0$, implying that $\alpha_3 + \alpha_5 < \alpha_3 < \alpha_3 - \alpha_5$. For these two solutions, the conditions for global asymptotic stability therefore reduce to $\alpha_3 - \alpha_5 < 0$. For (S5) we have

$$\alpha_3 - \alpha_5 = s\gamma - \left(6y - 7g - \frac{81}{4\gamma}E\right)\frac{\sqrt{E}}{4g}, \tag{A27}$$

where

$$6y - 7g - \frac{81}{4\gamma}E = 3y - 4g - 6\sqrt{3B}. \tag{A28}$$

A necessary (but not sufficient) condition for $\alpha_3 - \alpha_5 < 0$ is

$$(3y - 4g)^2 - \left(6\sqrt{3B}\right)^2 = -2g\,(3y - 8g) > 0, \tag{A29}$$

which is violated for $y > \frac{8}{3}g$. Thus, the condition $y > \frac{8}{3}g$ is sufficient (but not necessary) to rule out (S5).

Finally, for the only solution left, (S3), we have

$$\alpha_3 - \alpha_5 = s\gamma - \left(6y - 7g - \frac{81}{4\gamma}C\right)\frac{\sqrt{C}}{4g}. \tag{A30}$$

Global stability requires that the expression in (A30) is negative. Since

$$\frac{\partial\,(\alpha_3 - \alpha_5)}{\partial y} = -\frac{\gamma\left(3g + \frac{27}{4\gamma}C\right)\sqrt{3}}{36\sqrt{12B}\sqrt{\frac{27}{4}C}} < 0 \tag{A31}$$

and

$$\frac{\partial\,(\alpha_3 - \alpha_5)}{\partial g} = \frac{\gamma\left(\frac{27}{4\gamma}C + g\right)\sqrt{3}}{9\sqrt{12B}\sqrt{\frac{27}{4}C}} > 0, \tag{A32}$$

it follows that (A30) is negative if y is sufficiently large relative to g. Given the definitions of y and g, this condition requires that β and/or τ must be sufficiently large relative to k. This condition is similar to the sufficient condition required to have the saddle dynamics in the open-loop solution. It also ensures $y > \frac{8}{3}g$, which is a sufficient condition for (S3) to be the unique globally asymptotically stable closed-loop solution.

In the steady state closed-loop solution we have

$$\begin{aligned} I_i &= \frac{\alpha_1 + \alpha_3 q_i + \alpha_5 q_j}{\gamma}, && \text{(A33)} \\ I_i &= \delta q_i, && \text{(A34)} \\ q^{CL} &= \frac{\alpha_1}{\gamma\delta - \alpha_3 - \alpha_5}, && \text{(A35)} \end{aligned}$$

where α_3 and α_5 are given by (S3). From the second and third line in (A11) we can define the following system of two equations in α_1 and α_2:

$$\alpha_1 (\delta + \rho) - \frac{k}{3} - \frac{\alpha_1 \alpha_3}{\gamma} - \frac{\alpha_2 \alpha_5}{\gamma} - \frac{\alpha_1 \alpha_5}{\gamma} = 0, \quad \text{(A36)}$$

$$\alpha_2 (\delta + \rho) + \frac{k}{3} - \frac{\alpha_2 \alpha_3}{\gamma} - \frac{\alpha_1 \alpha_4}{\gamma} - \frac{\alpha_1 \alpha_5}{\gamma} = 0. \quad \text{(A37)}$$

Solving this system yields the following solution for α_1:

$$\alpha_1 = \frac{k\gamma \left(\alpha_3 + \alpha_5 - (\delta + \rho)\gamma\right)}{3\left((\delta + \rho)\left(2\gamma\alpha_3 + \gamma\alpha_5 - (\delta + \rho)\gamma^2\right) - \alpha_3^2 + \alpha_5^2 - \alpha_3\alpha_5 + \alpha_4\alpha_5\right)}. \quad \text{(A38)}$$

From (S3), note that $\alpha_4 = -\alpha_5$. We can therefore re-write α_1 as

$$\alpha_1 = \frac{k\gamma \left(\alpha_3 + \alpha_5 - (\delta + \rho)\gamma\right)}{3\left(\left(\gamma(\delta + \rho)\left((2\alpha_3 + \alpha_5) - (\delta + \rho)\gamma\right)\right) - \alpha_3(\alpha_5 + \alpha_3)\right)} = \frac{k\gamma}{3\left(\gamma(\delta + \rho) - \alpha_3\right)}, \quad \text{(A39)}$$

so that

$$q^{CL} = \frac{\alpha_1}{\gamma\delta - \alpha_3 - \alpha_5} = \frac{k\gamma}{3\left(\gamma(\delta + \rho) - \alpha_3\right)\left(\gamma\delta - \alpha_3 - \alpha_5\right)}. \quad \text{(A40)}$$

A.2. Proof of Proposition 2

The closed-loop solution requires $g \leq \frac{y}{2}$. Since g is monotonically decreasing in τ while y does not depend on τ, this implies that the closed-loop solution exists for sufficiently high values of τ. At the lower limit of τ, implicitly given by $y = 2g$, steady-state quality in the closed-loop solution is

$$q^{CL}\big|_{y=2g} = \frac{k}{\left(\frac{2k^2}{3\tau} - \frac{\gamma\rho^2}{2}\right) + \frac{\rho}{4}\left(\sqrt{2\gamma\left(\frac{2k^2}{3\tau} - \frac{\gamma\rho^2}{2}\right) + (\gamma\rho)^2} - \gamma\rho\right)}, \tag{A41}$$

whereas steady-state quality in the open-loop solution is

$$q^{OL}\big|_{y=2g} = \frac{k}{\frac{2k^2}{3\tau} - \frac{\gamma\rho^2}{2}}. \tag{A42}$$

A straightforward comparison of (A41) and (A42) shows that

$$q^{OL}\big|_{y=2g} > q^{CL}\big|_{y=2g} \quad if \quad \sqrt{2\gamma\left(\frac{2k^2}{3\tau} - \frac{\gamma\rho^2}{2}\right) + (\gamma\rho)^2} - \gamma\rho > 0, \tag{A43}$$

or, equivalently,

$$q^{OL}\big|_{y=2g} > q^{CL}\big|_{y=2g} \quad if \quad \sqrt{\frac{2\gamma k}{q^{OL}\big|_{y=2g}} + (\gamma\rho)^2} - \gamma\rho > 0, \tag{A44}$$

which always holds. Since q^{OL} is independent of τ while q^{CL} is monotonically decreasing in τ, it follows that $q^{OL} > q^{CL}$ for all $g \leq \frac{y}{2}$. *Q.E.D.*

A.3. Multiplicity of equilibria

If $2g < y < 8g/3$ then solution S5 (in Appendix A.1) co-exists with S3 for a subset of this parameter space. The global stability condition requires $\alpha_3 - \alpha_5 < 0$, which is satisfied if $\left(3y - 4g - 6\sqrt{3B}\right)\frac{\sqrt{E}}{4g} > s\gamma$. This is always the case for $\gamma \to 0$. Through numerical simulations we can show this is not the case for γ sufficiently high (omitted).

Solution S5 is given by

$$\alpha_3 = s\gamma - \left(\frac{6y - 5g}{4g} - \frac{81}{16g\gamma}E\right)\sqrt{E}, \alpha_4 = \frac{1}{2}\sqrt{E}, \alpha_5 = -\frac{1}{2}\sqrt{E}. \tag{A45}$$

Steady-state quality in this equilibrium is then given by $q^{CL} = \frac{k\gamma}{3(\gamma(\delta+\rho)-\alpha_3)(\gamma\delta-\alpha_3-\alpha_5)}$, which is structurally identical to S3. Notice also that $\alpha_3 + \alpha_5 = s\gamma - \sqrt{\frac{\gamma y}{6}}$ in both S3 and S5. Thus, $\alpha_3 + \alpha_5$ is independent of g (and therefore independent of τ) also in S5. We can then easily characterise the relationship between q^{CL} and τ in S5, which is given by

$$\frac{\partial q^{CL}}{\partial \tau} = \underset{+}{\frac{\partial q^{CL}}{\partial \alpha_3}} \underset{-}{\frac{\partial \alpha_3}{\partial g}} \underset{-}{\frac{\partial g}{\partial \tau}} > 0. \tag{A46}$$

This relationship is reversed compared to S3, since, in S5,

$$\frac{\partial \alpha_3}{\partial g} = -\frac{\sqrt{6}\gamma}{18} \frac{2\sqrt{y(y-2g)} - (y-2g)}{(y-2g)\sqrt{\gamma\left(5y - 2g - 4\sqrt{y(y-2g)}\right)}} < 0. \tag{A47}$$

Therefore, competition (lower transportation costs) reduce quality under S5.

A.4. The closed-loop solution with cross-shareholding

The problem is still linear-quadratic with a value function as in (A2). The derivation follows the logic of Section A.1 and some steps are therefore skipped. The HJB equation is now

$$\rho V^i(q_i, q_j) = \max \left\{ \begin{array}{c} (p_i - c)\left(\frac{1}{2} + \frac{k(q_i - q_j)}{2\tau} - \frac{p_i - p_j}{2\tau}\right) \\ +\theta\,(p_j - c)\left(\frac{1}{2} + \frac{k(q_j - q_i)}{2\tau} - \frac{p_j - p_i}{2\tau}\right) - \frac{\gamma}{2} I_i^2 - \frac{\beta}{2} {q_i}^2 \\ +V^i_{q_i}(q_i, q_j)\,(I_i - \delta q_i) + V^i_{q_j}(q_i, q_j)\,(I_j - \delta q_j) \end{array} \right\}. \tag{A48}$$

Maximisation with respect to prices yields

$$p_i = c + \frac{\tau}{1-\theta} + \frac{k\,(q_i - q_j)}{3+\theta}. \tag{A49}$$

The HJB equation can therefore be written as

$$\rho V^i(q_i, q_j) = \left\{ \begin{array}{c} \left(\frac{\tau}{1-\theta} + \frac{k(q_i - q_j)}{3+\theta}\right)\left(\frac{1}{2} + \frac{k(1+\theta)(q_i - q_j)}{2\tau(3+\theta)}\right) \\ +\theta\left(\frac{\tau}{1-\theta} - \frac{k(q_i - q_j)}{3+\theta}\right)\left(\frac{1}{2} - \frac{k(1+\theta)(q_i - q_j)}{2\tau(3+\theta)}\right) \\ -\frac{1}{2\gamma}\,(\alpha_1 + \alpha_3 q_i + \alpha_5 q_j)^2 - \frac{\beta}{2} {q_i}^2 \\ +\,(\alpha_1 + \alpha_3 q_i + \alpha_5 q_j)\left(\frac{\alpha_1 + \alpha_3 q_i + \alpha_5 q_j}{\gamma} - \delta q_i\right) \\ +\,(\alpha_2 + \alpha_4 q_j + \alpha_5 q_i)\left(\frac{\alpha_1 + \alpha_3 q_j + \alpha_5 q_i}{\gamma} - \delta q_j\right) \end{array} \right\}, \tag{A50}$$

which, after substitution of V^i, yields

$$
\begin{aligned}
& \tau(\theta+3)^2\left[(1-\theta)\left(2\gamma\rho\alpha_0-\alpha_1^2-2\alpha_1\alpha_2\right)-\tau\gamma(1+\theta)\right] \\
& +2\tau(3+\theta)(1-\theta)q_i\left[\gamma(\delta+\rho)(3+\theta)\alpha_1-(3+\theta)(\alpha_1\alpha_3+\alpha_1\alpha_5+\alpha_2\alpha_5)-k\gamma\right] \\
& +2\tau(3+\theta)(1-\theta)q_j\left[\gamma(\delta+\rho)(3+\theta)\alpha_2+k\gamma-(3+\theta)(\alpha_1\alpha_4+\alpha_2\alpha_3+\alpha_1\alpha_5)\right] \\
& +(1-\theta)q_i^2\left[\begin{array}{c}\gamma\left(\tau(3+\theta)^2\beta-(1+\theta)^2k^2\right) \\ +\tau\gamma(3+\theta)^2(2\delta+\rho)\alpha_3-\tau(3+\theta)^2\left(\alpha_3^2+2\alpha_5^2\right)\end{array}\right] \qquad \text{(A51)} \\
& +(1-\theta)q_j^2\left[\tau\gamma(3+\theta)^2(2\delta+\rho)\alpha_4-\gamma(\theta+1)^2k^2-\tau(3+\theta)^2\left(2\alpha_3\alpha_4+\alpha_5^2\right)\right] \\
& +2(1-\theta)q_iq_j\left[\gamma(1+\theta)^2k^2+\tau\gamma(3+\theta)^2(2\delta+\rho)\alpha_5-\tau(3+\theta)^2(2\alpha_3+\alpha_4)\alpha_5\right] \\
= \;& 0.
\end{aligned}
$$

The parameters α_3, α_4 and α_5 are found by solving the system of equations defined by the last three terms in (A51) being simultaneously equal to zero. This system can be written as

$$\frac{\beta}{2}-\frac{(1+\theta)^2}{2(3+\theta)^2}\frac{k^2}{\tau}+\left(\delta+\frac{\rho}{2}\right)\alpha_3-\frac{1}{2\gamma}\alpha_3^2-\frac{1}{\gamma}\alpha_5^2 = 0, \qquad \text{(A52)}$$

$$\left(\delta+\frac{\rho}{2}\right)\alpha_4-\frac{(1+\theta)^2}{2(3+\theta)^2}\frac{k^2}{\tau}-\frac{1}{\gamma}\alpha_3\alpha_4-\frac{1}{2\gamma}\alpha_5^2 = 0, \qquad \text{(A53)}$$

$$\frac{(1+\theta)^2}{(3+\theta)^2}\frac{k^2}{\tau}+(2\delta+\rho)\alpha_5-\frac{2}{\gamma}\alpha_3\alpha_5-\frac{1}{\gamma}\alpha_4\alpha_5 = 0. \qquad \text{(A54)}$$

This system is identical to (A15)-(A17) if we redefine g as $\widehat{g} := \frac{9(1+\theta)^2}{(3+\theta)^2}\frac{k^2}{\tau}$, which implies that the unique solution to the system is given by (S3) with $g=\widehat{g}$. Thus, steady-state quality in the closed-loop solution is given by (A35), where α_3 and α_5 are given by (S3) with $g=\widehat{g}$. The parameter α_1 is found by solving

$$\frac{(3+\theta)}{3}(\delta+\rho)\alpha_1-\frac{k}{3}-\frac{(3+\theta)}{3\gamma}(\alpha_1\alpha_3+\alpha_1\alpha_5+\alpha_2\alpha_5) = 0, \qquad \text{(A55)}$$

$$\frac{(3+\theta)}{3}(\delta+\rho)\alpha_2+\frac{k}{3}-\frac{(3+\theta)}{3\gamma}(\alpha_1\alpha_4+\alpha_2\alpha_3+\alpha_1\alpha_5) = 0, \qquad \text{(A56)}$$

which, when using the fact that $\alpha_4 = -\alpha_5$, yields

$$\alpha_1 = \frac{k\gamma}{(3+\theta)\left(\gamma\left(\delta+\rho\right)-\alpha_3\right)}, \tag{A57}$$

which is equal to the equilibrium value of α_1 in (A39) multiplied by $1/(3+\theta)$.

A.5. Proof of Proposition 6

(i): Note first that $\alpha_3 + \alpha_5$ does not depend on $\widehat{g}$, and therefore does not depend on θ. On the other hand, θ affects both α_1 (directly) and α_3 (through the effect on $\widehat{g}$). The overall effect is therefore given by

$$\frac{\partial q^{CL}}{\partial\theta} = \frac{\partial q^{CL}}{\partial\alpha_1}\frac{\partial\alpha_1}{\partial\theta} + \frac{\partial q^{CL}}{\partial\alpha_3}\frac{\partial\alpha_3}{\partial\widehat{g}}\frac{\partial\widehat{g}}{\partial\theta}. \tag{A58}$$

It is easy to verify that $\partial q^{CL}/\partial\alpha_1 > 0$, $\partial q^{CL}/\partial\alpha_3 > 0$, $\partial\alpha_1/\partial\theta < 0$, $\partial\alpha_3/\partial\widehat{g} > 0$ and $\partial\widehat{g}/\partial\theta > 0$. Thus, the first term in (A58) is negative whereas the second term is positive, making the sign of $\partial q^{CL}/\partial\theta$ *a priori* indeterminate.

(ii): The closed-loop solution requires $\widehat{g} \leq \frac{y}{2}$. Since $\widehat{g}$ is monotonically decreasing in τ while y does not depend on τ, this implies that the closed-loop solution exists for sufficiently high values of τ. At the lower limit of τ, implicitly given by $y = 2\widehat{g}$, steady-state quality in the closed-loop solution is

$$q^{CL}\big|_{y=2\widehat{g}} = \frac{4k\gamma}{(\theta+3)\left(\frac{3(\theta+1)}{(\theta+3)}\sqrt{\frac{4k^2\gamma}{3\tau}} - \gamma\rho\right)\left(\frac{2(\theta+1)}{(\theta+3)}\sqrt{\frac{4k^2\gamma}{3\tau}} + \gamma\rho\right)}, \tag{A59}$$

whereas steady-state quality in the open-loop solution is

$$q^{OL}\big|_{y=2\widehat{g}} = \frac{k}{\frac{6k^2}{\tau}\frac{(\theta+1)^2}{(\theta+3)^2} - \gamma\rho^2}. \tag{A60}$$

After some manipulations, (A59) can be expressed as

$$q^{CL}\big|_{y=2\widehat{g}} = \frac{k}{\frac{\theta+3}{3}\left(\frac{6k^2}{\tau}\frac{(\theta+1)^2}{(\theta+3)^2} - \gamma\rho^2 + \rho\left(\frac{\gamma\rho}{4} + \frac{3(\theta+1)}{4(\theta+3)}\sqrt{\frac{4k^2\gamma}{3\tau}}\right)\right)}. \tag{A61}$$

It is immediately evident that the denominator in (A61) is larger than the denominator in (A60), implying that $q^{CL}\big|_{y=2\widehat{g}} < q^{OL}\big|_{y=2\widehat{g}}$. Since q^{OL} is independent of τ while q^{CL} is monotonically decreasing in τ, it follows that $q^{OL} > q^{CL}$ for all $\widehat{g} \leq \frac{y}{2}$. *Q.E.D.*

A.6. The closed-loop solution with convex and quality-dependent production costs

The problem is still linear-quadratic with a value function as in (A2). The derivation follows the logic of Section A.1 and some steps are therefore skipped. The HJB equation is now

$$\rho V^i(q_i, q_j) = \max \left\{ \begin{array}{c} (p_i - c_1 q_i)\left(\frac{1}{2} + \frac{k(q_i - q_j)}{2\tau} - \frac{p_i - p_j}{2\tau}\right) \\ -\frac{c_2}{2}\left(\frac{1}{2} + \frac{k(q_i - q_j)}{2\tau} - \frac{p_i - p_j}{2\tau}\right)^2 - \frac{\gamma}{2} I_i^2 - \frac{\beta}{2} {q_i}^2 \\ +V^i_{q_i}(q_i, q_j)(I_i - \delta q_i) + V^i_{q_j}(q_i, q_j)(I_j - \delta q_j) \end{array} \right\}. \tag{A62}$$

Maximisation with respect to prices yields

$$p_i = \frac{(3\tau + c_2)(2\tau + c_2) + k(2\tau + c_2)(q_i - q_j) + c_1(2\tau(2q_i + q_j) + c_2(q_i + q_j))}{6\tau + 2c_2}. \tag{A63}$$

The HJB equation can therefore be written as

$$\rho V^i(q_i, q_j) = \left\{ \begin{array}{c} \frac{(4\tau + c_2)\left(3\tau + c_2 + \widetilde{k}(q_i - q_j)\right)^2}{8(3\tau + c_2)^2} \\ -\frac{1}{2\gamma}(\alpha_1 + \alpha_3 q_i + \alpha_5 q_j)^2 - \frac{\beta}{2} {q_i}^2 \\ +(\alpha_1 + \alpha_3 q_i + \alpha_5 q_j)\left(\frac{\alpha_1 + \alpha_3 q_i + \alpha_5 q_j}{\gamma} - \delta q_i\right) \\ +(\alpha_2 + \alpha_4 q_j + \alpha_5 q_i)\left(\frac{\alpha_1 + \alpha_3 q_j + \alpha_5 q_i}{\gamma} - \delta q_j\right) \end{array} \right\}, \tag{A64}$$

where $\widetilde{k} := k - c_1$, and which, after substitution of V^i, yields

$$
\begin{aligned}
& (3\tau + c_2)^2 \left[4\alpha_1^2 + 4\tau\gamma + 8\alpha_1\alpha_2 + \gamma c_2 - 8\gamma\rho\alpha_0\right] \\
& +2q_i (3\tau + c_2) \left[\begin{array}{c} \gamma (4\tau + c_2) \widetilde{k} - 4\gamma (\delta + \rho) (3\tau + c_2) \alpha_1 \\ +4 (3\tau + c_2) (\alpha_1\alpha_3 + \alpha_1\alpha_5 + \alpha_2\alpha_5) \end{array} \right] \\
& +2q_j (3\tau + c_2) \left[\begin{array}{c} 4 (3\tau + c_2) (\alpha_1\alpha_4 + \alpha_1\alpha_5 + \alpha_2\alpha_3) \\ -4\gamma (\delta + \rho) (3\tau + c_2) \alpha_2 - \gamma (4\tau + c_2) \widetilde{k} \end{array} \right] \\
& +q_i^2 \left[\begin{array}{c} \gamma \left(\widetilde{k}^2 (4\tau + c_2) - 4\beta (3\tau + c_2)^2\right) \\ -4\gamma (3\tau + c_2)^2 (2\delta + \rho) \alpha_3 + 4 (3\tau + c_2)^2 \left(\alpha_3^2 + 2\alpha_5^2\right) \end{array} \right] \\
& +q_j^2 \left[\begin{array}{c} \gamma\widetilde{k}^2 (4\tau + c_2) - 4\gamma (3\tau + c_2)^2 (2\delta + \rho) \alpha_4 \\ +4 (3\tau + c_2)^2 \left(\alpha_5^2 + 2\alpha_3\alpha_4\right) \end{array} \right] \\
& +2q_iq_j \left[\begin{array}{c} 4 (3\tau + c_2)^2 (\alpha_4\alpha_5 + 2\alpha_3\alpha_5) - \gamma\widetilde{k}^2 (4\tau + c_2) \\ -4\gamma (3\tau + c_2)^2 (2\delta + \rho) \alpha_5 \end{array} \right] \\
= \; & 0
\end{aligned}
\tag{A65}
$$

The parameters α_3, α_4 and α_5 are found by solving the system of equations defined by the last three terms in (A65) being simultaneously equal to zero. This system can be written as

$$\alpha_3 \left(\delta + \frac{1}{2}\rho\right) - \frac{1}{2\gamma} \left(\alpha_3^2 + 2\alpha_5^2\right) + \frac{1}{2}\beta - \frac{\widetilde{k}^2}{18\tau} \frac{9\tau (4\tau + c_2)}{4 (3\tau + c_2)^2} = 0, \tag{A66}$$

$$\alpha_4 \left(\delta + \frac{1}{2}\rho\right) - \frac{1}{\gamma} \left(\alpha_3\alpha_4 + \frac{1}{2}\alpha_5^2\right) - \frac{\widetilde{k}^2}{18\tau} \frac{9\tau (4\tau + c_2)}{4 (3\tau + c_2)^2} = 0, \tag{A67}$$

$$\alpha_5 (2\delta + \rho) - \frac{1}{\gamma} (\alpha_4\alpha_5 + 2\alpha_3\alpha_5) + \frac{\widetilde{k}^2}{9\tau} \frac{9\tau (4\tau + c_2)}{4 (3\tau + c_2)^2} = 0, \tag{A68}$$

which is identical to (A15)-(A17) if, in (A15)-(A17), k is replaced by $\widetilde{k}$ and g is replaced by $\widetilde{g} := \frac{9\tau(4\tau+c_2)}{4(3\tau+c_2)^2}$. Thus, with these re-parameterisations, for $y > \frac{8}{3}\widetilde{g}$ the unique solution to (A66)-

(A68) is given by (S3) in Section A.1. Finally, the parameter α_1 is derived from the system

$$\gamma\left(4\tau+c_2\right)\widetilde{k}-4\gamma\left(\delta+\rho\right)\left(3\tau+c_2\right)\alpha_1+4\left(3\tau+c_2\right)\left(\alpha_1\alpha_3+\alpha_1\alpha_5+\alpha_2\alpha_5\right) = 0, \quad \text{(A69)}$$

$$4\left(3\tau+c_2\right)\left(\alpha_1\alpha_4+\alpha_1\alpha_5+\alpha_2\alpha_3\right)-4\gamma\left(\delta+\rho\right)\left(3\tau+c_2\right)\alpha_2-\gamma\left(4\tau+c_2\right)\widetilde{k} = 0, \quad \text{(A70)}$$

which, using the fact that $\alpha_4=-\alpha_5$, yields

$$\alpha_1=\frac{\gamma\widetilde{k}\left(4\tau+c_2\right)}{4\left(3\tau+c_2\right)\left(\gamma\left(\delta+\rho\right)-\alpha_3\right)}, \quad \text{(A71)}$$

which, with the re-parameterisation $k=\widetilde{k}$, is equal to the equilibrium value of α_1 in (A39) multiplied by $\frac{3(4\tau+c_2)}{4(3\tau+c_2)}$.

A.7. Proof of Proposition 8

(i) If $c_2=0$, the only difference between the closed-loop solutions for $c_1=0$ and $c_1>0$ is the re-parameterisation $\widetilde{k}:=k-c_1$, which does not affect the result in Proposition 2.

(ii) If $c_2>0$, steady-state quality is given by

$$q^{CL}=\frac{\alpha_1}{\gamma\delta-\alpha_3-\alpha_5}, \quad \text{(A72)}$$

where α_1 is given by (A71) and where α_3 and α_5 are given by (S3), with g replaced by $\widetilde{g}:=\frac{9\tau(4\tau+c_2)}{4(3\tau+c_2)^2}g$ and k replaced by $\widetilde{k}:=k-c_1$. Since $\alpha_3+\alpha_5$ does not depend on $\widetilde{g}$, the effect of product substitutability on steady-state quality is given by

$$\frac{\partial q^{CL}}{\partial\tau}=\frac{\partial q^{CL}}{\partial\alpha_1}\frac{\partial\alpha_1}{\partial\tau}+\frac{\partial q^{CL}}{\partial\alpha_3}\frac{\partial\alpha_3}{\partial\widetilde{g}}\frac{\partial\widetilde{g}}{\partial\tau}, \quad \text{(A73)}$$

where

$$\frac{\partial\alpha_1}{\partial\tau}=\frac{\gamma c_2\widetilde{k}}{4\left(3\tau+c_2\right)^2\left(\gamma\left(\delta+\rho\right)-\alpha_3\right)}>0 \quad \text{(A74)}$$

and

$$\frac{\partial\widetilde{g}}{\partial\tau}=-\frac{9\left(6\tau+c_2\right)\widetilde{k}^2}{2\left(3\tau+c_2\right)^3}<0. \quad \text{(A75)}$$

Since $\frac{\partial q^{CL}}{\partial \alpha_1} > 0$, $\frac{\partial q^{CL}}{\partial \alpha_3} > 0$ and $\frac{\partial \alpha_3}{\partial \widetilde{g}} > 0$, the first term in (A73) is positive, whereas the second term is negative. The first term vanishes if c_2 approaches zero. Thus, by continuity, Proposition 2 still holds if c_2 is sufficiently small. For sufficiently high values of c_2, it is easily confirmed, by numerical examples, that $\partial q^{CL}/\partial \tau$ can take positive values.

(iii) Proposition 3 is confirmed by noting, from (57), that q^{OL} does not depend on c_2. We already know that $q^{CL} < q^{OL}$ for $c_2 = 0$, which implies constant marginal production costs. Thus, $q^{CL} < q^{OL}$ also for $c_2 > 0$ if $\partial q^{CL}/\partial c_2 < 0$. This is true, since

$$\frac{\partial \alpha_1}{\partial c_2} = -\frac{3k\gamma\sqrt{Y_1}\begin{pmatrix} 10\tau\left(3\tau + c_2\right)\left(4\beta + \gamma\left(2\delta + \rho\right)^2\right) \\ +3k^2\left(4\tau + c_2\right) + 6\tau\rho\sqrt{Y_1} \end{pmatrix}}{4Y_1\left(\sqrt{Y_1} + 3\gamma\rho\left(3\tau + c_2\right)\right)^2} < 0 \tag{A76}$$

and

$$\frac{\partial\left(\alpha_3 + \alpha_5\right)}{\partial c_2} = -\frac{k^2\gamma\left(\sqrt{Y_2} + \sqrt{Y_1}\right)\left(5\tau + c_2\right)\sqrt{Y_2}\sqrt{Y_1}}{4\left(3\tau + c_2\right)^2 Y_2 Y_1} < 0, \tag{A77}$$

where

$$Y_1 := \frac{2}{3}\left(5y - 2\widetilde{g}\right)\gamma\left(3\tau + c_2\right)^2 > 0 \tag{A78}$$

and

$$Y_2 := \frac{4}{3}\left(y - \widetilde{g}\right)\gamma\left(3\tau + c_2\right)^2 > 0. \tag{A79}$$

Finally, given that $q^{CL} < q^{OL}$, Proposition 4 is confirmed by showing that q^{OL} still coincides with the steady-state level associated with the welfare maximisation problem given by (36). The current-value Hamiltonian for this problem is

$$\begin{aligned} H \;=\;& v - \frac{\tau}{2} + kq_i x_i^D + kq_j\left(1 - x_i^D\right) \\ & -c_1\left(q_i x_i^D + q_j\left(1 - x_i^D\right)\right) - \frac{c_2}{2}\left(\left(x_i^D\right)^2 + \left(1 - x_i^D\right)^2\right) \\ & -\frac{\gamma}{2}\left(I_i^2 + {I_j}^2\right) - \frac{\beta}{2}\left({q_i}^2 + {q_j}^2\right) + \mu_i\left(I_i - \delta q_i\right) + \mu_j\left(I_j - \delta q_j\right). \end{aligned} \tag{A80}$$

The optimality conditions are equal to (42)-(46) in Section 6 if we everywhere replace k by $k - c_1$

and also replace (43) by

$$0 = (k - c_1)(q_i - q_j) - c_2 \left(2x_i^D - 1\right). \tag{A81}$$

In the symmetric steady state we have: $\mu^* = \gamma I^*$, $(\rho + \delta)\mu^* + \beta q^* - \frac{(k-c_1)}{2} = 0$ and $q^* = \frac{I^*}{\delta}$, which gives

$$q^* = \frac{k - c_1}{2(\delta\gamma(\rho + \delta) + \beta)} = q^{OL}. \tag{A82}$$

Therefore, steady-state quality under open-loop decision rules coincides with the first-best steady-state quality level also under the assumptions of convex and quality-dependent production costs. *Q.E.D.*