On Interior Point Multidimensional Filter Line Search Methods for Constrained Optimization

M. FERNANDA P. COSTA
University of Minho
Dep. of Math. for Science and Technology
Campus de Azurem, 4800-058 Guimarães
PORTUGAL
mfc@mct.uminho.pt

EDITE M. G. P. FERNANDES
University of Minho
Dep. of Production and Systems
Campus de Gualtar, 4710-057 Braga
PORTUGAL
emgpf@dps.uminho.pt

Abstract: This paper aims to compare primal-dual interior point multidimensional filter line search methods for nonlinear programming. The multidimensional filter is based on the constraint violations and aims to enforce their convergence to zero. To prevent convergence to feasible nonoptimal points a standard monotone sufficient reduction condition is also imposed on the barrier function for a trial iterate to be acceptable. Nonmonotone reduction conditions that allow an increase in the filter entries and in the barrier function at each iteration are also implemented. Numerical experiments with both variants as well as a comparison with a merit function approach are reported.

Key–Words: Nonlinear optimization, Interior point method, filter line search method

1 Introduction

A primal-dual interior point method for nonlinear optimization that relies on a filter line search strategy to allow convergence from poor starting points is proposed. Some line search frameworks use penalty merit functions to enforce progress toward the solution.

Recently Fletcher and Leyffer [5] proposed a filter method, as an alternative to merit functions, as a tool to guarantee global convergence in algorithms for nonlinear constrained optimization [4, 6]. This technique incorporates the concept of nondominance to build a filter that is able to accept trial points if they improve the objective function or the constraints violation instead of a combination of those two measures defined by a merit function.

Thus the filter replaces the use of a merit function. The update of a penalty parameter that is associated with the penalization of the constraints in a merit function is avoided.

The filter technique has been already adapted to unconstrained optimization [7], as well as to interior point methods in different ways. A filter with two entries in a trust region context is presented in [11], while authors in [13, 14, 15] propose a filter line search approach. In these papers, the filter components are the barrier function and the norm of the constraints violation. Another extension of the filter approach to interior point methods defines three components for the filter measuring feasibility, comple-

mentarity and optimality [2, 3]. These measures come naturally from the optimality conditions of the barrier problem.

In this paper we propose a primal-dual interior point method with a multidimensional filter technique in a line search implementation. The goal of this multidimensional filter is to encourage convergence to primal feasibility points by enforcing every constraint violation to zero. For nearly feasible points, the method also guarantees a monotonic decrease of the barrier function.

However, enforcing monotonicity of the involved measures can slow the rate of convergence. So, a nonmonotone strategy, similar to that proposed in [8], is incorporated in the trial point acceptability conditions of the algorithm. The nonmonotone line search technique that has been successfully applied in an unconstrained optimization context [8, 9] and in systems of nonlinear equations [1] can be viewed as a generalization of the standard sufficient reduction conditions in the sense that the conditions that are used to define a new acceptable point are relaxed without affecting the convergence properties.

The paper is organized as follows. Section 2 briefly describes the interior point framework, Section 3 presents the herein proposed multidimensional filter line search method and Section 4 summarizes the nonmonotone sufficient reduction conditions. Section 5 contains the numerical experiments and the conclusions and future developments make Section 6.

2 The Interior Point Framework

Here we consider the formulation of a nonlinear constrained optimization problem with simple bounds as follows:

$$\min_{x\in\mathbb{R}^n} \quad F(x)$$
$$\text{s.t.} \quad b \leq h(x) \leq b + r \quad (1)$$
$$l \leq x \leq u$$

where $h_j : \mathbb{R}^n \rightarrow \mathbb{R}$ for $j = 1, \ldots, m$ and $F : \mathbb{R}^n \rightarrow \mathbb{R}$ are nonlinear and twice continuously differentiable functions. r is the vector of ranges on the constraints h, u and l are the vectors of upper and lower bounds on the variables and b is assumed to be a finite real vector. Elements of the vector r, l and u are real numbers subject to the following limitations: $0 \leq r_j \leq \infty$, $-\infty \leq l_i, u_i \leq \infty$ for $j = 1, \ldots, m$, $i = 1, \ldots, n$. Constraints of the form $b \leq h(x) \leq b + r$ are denoted by range constraints. When upper and lower bounds on the x variables do not exist, the vector x is considered free. Equality constraints are treated as range constraints with $r = 0$.

This section briefly describes an infeasible primal-dual interior point method for solving (1). We refer to [12] for details. Adding nonnegative slack variables w, p, g and t, problem (1) becomes

$$\min \quad F(x)$$
$$\text{s.t.} \quad h(x) - w = b$$
$$h(x) + p = b + r$$
$$x - g = l \quad (2)$$
$$x + t = u$$
$$w, p, g, t \geq 0.$$

Incorporating the nonnegativity constraints in logarithmic barrier terms in the objective function, problem (2) is transformed into

$$\min b_\mu \equiv F(x) - \mu \sum_{j=1}^{m} \ln(w_j) - \mu \sum_{j=1}^{m} \ln(p_j) - $$
$$- \mu \sum_{i=1}^{n} \ln(g_i) - \mu \sum_{i=1}^{n} \ln(t_i)$$
$$(3)$$

subject to the same set of equality constraints, where b_μ is the barrier function and μ is a positive barrier parameter. This is the barrier problem associated with (1). Under acceptable assumptions, the sequence of solutions of the barrier problem converges to the solution of the problem (1) when $\mu \searrow 0$. Thus primal-dual interior point methods aim to solve a sequence of barrier problems for a positive decreasing sequence of μ values. Optimality conditions for the barrier pro-

blem produce the standard primal-dual system

$$\nabla F(x) - \nabla h(x)^{\mathrm{T}} y - z + s = 0$$
$$y + q - v = 0$$
$$\mathbf{W}\mathbf{V}e_1 = \mu e_1$$
$$\mathbf{P}\mathbf{Q}e_1 = \mu e_1$$
$$\mathbf{G}\mathbf{Z}e_2 = \mu e_2$$
$$\mathbf{T}\mathbf{S}e_2 = \mu e_2 \quad (4)$$
$$h(x) - w - b = 0$$
$$w + p - r = 0$$
$$x - g - l = 0$$
$$x + t - u = 0,$$

where v, q, z and s are the dual variables, $\nabla F(x)$ is the gradient vector of $F(x)$, $\nabla h(x)$ is the Jacobian matrix of the constraints h, $\mathbf{W} = \text{diag}(w_j)$, $\mathbf{P} = \text{diag}(p_j)$, $\mathbf{G} = \text{diag}(g_i)$, $\mathbf{T} = \text{diag}(t_i)$, $\mathbf{V} = \text{diag}(v_j)$, $\mathbf{Q} = \text{diag}(q_j)$, $\mathbf{Z} = \text{diag}(z_i)$ and $\mathbf{S} = \text{diag}(s_i)$ are diagonal matrices, $y = v - q$, e_1 and e_2 are m and n vectors of all ones. The first two equations define the conditions of dual feasibility, the next four equations are the μ-complementarity conditions and the last four define the primal feasibility. This is a nonlinear system of $5n + 5m$ equations in $5n + 5m$ unknowns.

Applying the Newton's method to solve (4), we obtain the system of equations

$$-\mathbf{H}(x, y)\Delta x + \nabla h(x)^{\mathrm{T}}\Delta y + \Delta z - \Delta s =$$
$$= \nabla F(x) - \nabla h(x)^{\mathrm{T}} y - z + s$$
$$\Delta y - \Delta q + \Delta v = y + q - v$$
$$\Delta w + \mathbf{V}^{-1}\mathbf{W}\Delta v = \mu \mathbf{V}^{-1}e_1 - w$$
$$\mathbf{P}^{-1}\mathbf{Q}\Delta p + \Delta q = \mu \mathbf{P}^{-1}e_1 - q$$
$$\mathbf{G}^{-1}\mathbf{Z}\Delta g + \Delta z = \mu \mathbf{G}^{-1}e_2 - z \quad (5)$$
$$\mathbf{T}^{-1}\mathbf{S}\Delta t + \Delta s = \mu \mathbf{T}^{-1}e_2 - s$$
$$\nabla h(x)\Delta x - \Delta w = w + b - h(x)$$
$$\Delta p + \Delta w = r - w - p$$
$$\Delta x - \Delta g = l - x + g$$
$$\Delta x + \Delta t = u - x - t$$

for the Newton directions $\Delta x, \Delta s, \Delta z, \Delta g, \Delta t$, $\Delta y, \Delta w, \Delta p, \Delta q$ and Δv, where $\mathbf{H}(x, y) = \nabla^2 F(x) - \sum_{j=1}^{m} y_j \nabla^2 h_j(x)$.

For easy of presentation we introduce the following notations:

$$\rho \equiv w + b - h(x)$$
$$\alpha \equiv r - w - p$$
$$\nu \equiv l - x + g \quad (6)$$
$$\tau \equiv u - x - t.$$

In a line search context, and after the search directions have been computed, the algorithm proceeds iteratively, choosing a step size $\overline{\alpha}_k \in (0, \overline{\alpha}_k^{\max}]$ at

each iteration and determining the new iterates by $\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \overline{\alpha}_k \boldsymbol{\Delta} x_k$, $\boldsymbol{s}_{k+1} = \boldsymbol{s}_k + \overline{\alpha}_k \boldsymbol{\Delta} s_k, \ldots,$ $\boldsymbol{v}_{k+1} = \boldsymbol{v}_k + \overline{\alpha}_k \boldsymbol{\Delta} v_k$. $\overline{\alpha}_k^{\max}$ is the longest step size that can be taken, with an upper bound of 1, along these directions to assure the nonnegativity of the slack and dual variables.

Implementation details to provide initial values for all the variables in this interior point paradigm as well as to solve system (5) and to compute μ and $\overline{\alpha}^{\max}$ are described in [12].

The procedure that decides which trial step size is accepted in this interior point method is a multidimensional filter line search method.

3 Multidimensional Filter Line Search Method

In this section we briefly present the main ideas of a multidimensional entry filter method based on a line search approach. To abbreviate the notation we set

$$
\begin{aligned}
\boldsymbol{u} &= (\boldsymbol{x}, \boldsymbol{w}, \boldsymbol{p}, \boldsymbol{g}, \boldsymbol{t}, \boldsymbol{y}, \boldsymbol{v}, \boldsymbol{q}, \boldsymbol{s}, \boldsymbol{z}), \\
\boldsymbol{u}^1 &= (\boldsymbol{x}, \boldsymbol{w}, \boldsymbol{p}, \boldsymbol{g}, \boldsymbol{t}), \\
\boldsymbol{\Delta} &= (\boldsymbol{\Delta} x, \boldsymbol{\Delta} w, \boldsymbol{\Delta} p, \boldsymbol{\Delta} g, \boldsymbol{\Delta} t, \boldsymbol{\Delta} y, \boldsymbol{\Delta} v, \boldsymbol{\Delta} q, \boldsymbol{\Delta} s, \boldsymbol{\Delta} z), \\
\boldsymbol{\Delta}^1 &= (\boldsymbol{\Delta} x, \boldsymbol{\Delta} w, \boldsymbol{\Delta} p, \boldsymbol{\Delta} g, \boldsymbol{\Delta} t), \\
\overline{\boldsymbol{h}} &= (\boldsymbol{\rho}, \boldsymbol{\alpha}, \boldsymbol{\upsilon}, \boldsymbol{\tau}).
\end{aligned}
$$

To adapt the methodology of a multidimensional filter as given in [7] to this interior point method, and to encourage convergence to feasible points, we propose the use of the $2m + 2n$ elements of the vector $\overline{\boldsymbol{h}}$ as the filter entries. These components measure primal infeasibility as stated in the optimality conditions (4).

The notion of filter is based on that of dominance. In our case, a point $\boldsymbol{u}_+$ is said to dominate a point $\boldsymbol{u}_-$ whenever

$$
|\overline{h}_i(\boldsymbol{u}_+^1)| \le |\overline{h}_i(\boldsymbol{u}_-^1)| \quad \text{for all } i = 1, \ldots, 2m + 2n.
$$

Filter methods aim to accept a new trial point if it is not dominated by any other point in the filter.

After a search direction $\boldsymbol{\Delta}_k$ has been computed, the line search method considers a backtracking procedure, where a decreasing sequence of step sizes $\overline{\alpha}_{k,l} \in (0, \overline{\alpha}_k^{\max}]$ ($l = 0, 1, \ldots$), with $\lim_l \overline{\alpha}_{k,l} = 0$, is tried until an acceptance criterion is satisfied. Here, we use l to denote the iteration counter for the inner loop.

3.1 Sufficient reduction

Line search methods that use a merit function ensure sufficient progress toward the solution by imposing

that the merit function value at each new iterate satisfies an Armijo condition with respect to the current iterate.

Adopting this idea, we consider the trial iterate $\boldsymbol{u}_k(\overline{\alpha}_{k,l}) = \boldsymbol{u}_k + \overline{\alpha}_{k,l} \boldsymbol{\Delta}_k$ during the backtracking line search technique to be acceptable if it leads to sufficient progress in one of the $2m + 2n$ following measures compared to the current iterate, i.e., if

$$
|\overline{h}_i(\boldsymbol{u}_k^1(\overline{\alpha}_{k,l}))| \le (1 - \gamma_f) |\overline{h}_i(\boldsymbol{u}_k^1)| \tag{7}
$$

holds for some $i \in I$ and fixed constant $\gamma_f \in (0, 1)$, where $I = \{1, \ldots, 2m + 2n\}$.

However, to prevent convergence to a feasible point that is nonoptimal, and whenever, for the current iterate, we have

$$
|\overline{h}_i(\boldsymbol{u}_k^1)| \le \overline{h}_i^{\min} \text{ and } \boldsymbol{\nabla} b_\mu(\boldsymbol{u}_k^1)^{\mathrm{T}} \boldsymbol{\Delta}_k^1 < 0 \tag{8}
$$

for some $\overline{h}_i^{\min} \in (0, \infty]$, for all $i \in I$, the following sufficient reduction criterion on the barrier function is imposed on the trial point $\boldsymbol{u}_k^1(\overline{\alpha}_{k,l})$:

$$
b_\mu(\boldsymbol{u}_k^1(\overline{\alpha}_{k,l})) \le b_\mu(\boldsymbol{u}_k^1) + \eta_b \overline{\alpha}_{k,l} \boldsymbol{\nabla} b_\mu(\boldsymbol{u}_k^1)^{\mathrm{T}} \boldsymbol{\Delta}_k^1 \tag{9}
$$

instead of (7). Here $\eta_b \in (0, 0.5)$ is a constant. According to a previous publication on filter methods [4], we call a trial step size $\overline{\alpha}_{k,l}$ for which (8) holds, a "b_μ-step". Similarly, if a "b_μ-step" is accepted as the final step size $\overline{\alpha}_k$ in iteration k, we refer to k as a "b_μ-type iteration".

3.2 The filter definition

At each iteration k, the algorithm also maintains a filter, here denoted by

$$
\overline{F}_k \subseteq \left\{ \overline{\boldsymbol{h}} \in \mathrm{I\!R}^{2m+2n} : |\overline{h}_i| \ge 0, \text{ for all } i \in I \right\}.
$$

Following the ideas in [13, 14, 15], the filter here is not defined by a list but as a set $\overline{F}_k$ that contains those combinations of values of $\overline{h}_1, \ldots, \overline{h}_{2m+2n}$ that are prohibited for a successful trial point in iteration k. Thus a trial point $\boldsymbol{u}_k(\overline{\alpha}_{k,l})$ is rejected, if

$$
\left(\overline{h}_1(\boldsymbol{u}_k^1(\overline{\alpha}_{k,l})), \ldots, \overline{h}_{2m+2n}(\boldsymbol{u}_k^1(\overline{\alpha}_{k,l})) \right) \in \overline{F}_k.
$$

At the beginning of the optimization, the filter is initialized to

$$
\overline{F}_k \subseteq \left\{ \overline{\boldsymbol{h}} \in \mathrm{I\!R}^{2m+2n} : |\overline{h}_i| \ge \overline{h}_i^{\max}, \text{ for all } i \in I \right\} \tag{10}
$$

for sufficiently large positive constants $\overline{h}_1^{\max}, \ldots, \overline{h}_{2m+2n}^{\max}$, so that the algorithm will never accept trial points that have values of $\overline{h}_1, \ldots, \overline{h}_{2m+2n}$ larger than $\overline{h}_1^{\max}, \ldots, \overline{h}_{2m+2n}^{\max}$, respectively.

The filter is augmented, using the update formula

$$\overline{F}_{k+1} = \overline{F}_k \cup \left\{\overline{h} \in \mathbb{R}^{2m+2n} : \right.$$
$$\left. |\overline{h}_i| > (1 - \gamma_f)\,|\overline{h}_i(\boldsymbol{u}_k^1)|, \text{ for all } i \in I \right\} \tag{11}$$

after every iteration in which the accepted trial step size satisfies (7). On the other hand, if (8) and (9) hold for the accepted step size, the filter remains unchanged.

Finally, in some iterations it is not possible to find a trial step size $\overline{\alpha}_{k,l}$ that satisfies the above criteria. If the backtracking multidimensional line search technique finds a trial step size $\overline{\alpha}_{k,l} < \overline{\alpha}^{\min}$, the filter is reset to the initial set.

Our interior point multidimensional filter line search algorithm for solving constrained optimization problems is as follows:

Algorithm 1

Given: Starting point $\boldsymbol{u}_0$, constants $\overline{h}_i^{\min} > 0, \overline{h}_i^{\max} > 0$, for all $i \in I$, $\gamma_f \in (0, 1)$, $\eta_b \in (0, 0.5)$, $\overline{\alpha}^{\min} > 0$.

1. *Initialize.* Initialize the filter (using (10)) and set $k \leftarrow 0$.

2. *Check convergence.* Stop if the relative measures of primal and dual infeasibilities are less or equal to 10^{-4}.

3. *Compute search direction.* Compute the search direction $\boldsymbol{\Delta}_k$ from the linear system (5).

4. *Backtracking line search.*

    4.1 *Initialize line search.* Compute the longest step size $\overline{\alpha}_k^{\max}$ to ensure positivity of slack and dual variables. Set $l \leftarrow 0$, $\overline{\alpha}_{k,l} = \overline{\alpha}_k^{\max}$.

    4.2 *Compute new trial point.* If the trial step size becomes too small, i.e., $\overline{\alpha}_{k,l} < \overline{\alpha}^{\min}$, go to step 8. Otherwise, compute the trial point $\boldsymbol{u}_k(\overline{\alpha}_{k,l})$.

    4.3 *Check acceptability to the filter.* If $\left(\overline{h}_1(\boldsymbol{u}_k^1(\overline{\alpha}_{k,l})), \ldots, \overline{h}_{2m+2n}(\boldsymbol{u}_k^1(\overline{\alpha}_{k,l}))\right) \in \overline{F}_k$, reject the trial step size and go to step 4.6.

    4.4 *Check sufficient decrease with respect to current iterate.* If (7) holds, accept the trial step and go to step 5. Otherwise go to step 4.6.

    4.5 *Check sufficient decrease with respect to current iterate.* If $\overline{\alpha}_{k,l}$ is a b_μ-step size ((8) holds) and the Armijo condition (9) for the b_μ function holds, accept the trial step and go to step 5.

    4.6 *Choose new trial step size.* Set $\overline{\alpha}_{k,l+1} = \overline{\alpha}_{k,l}/2$, $l \leftarrow l + 1$, and go back to step 4.2.

5. *Accept trial point.* Set $\overline{\alpha}_k \leftarrow \overline{\alpha}_{k,l}$ and $\boldsymbol{u}_k \leftarrow \boldsymbol{u}_k(\overline{\alpha}_k)$.

6. *Augment the filter if necessary.* If k is not a b_μ-type iteration, augment the filter using (11). Otherwise, leave the filter unchanged.

7. *Continue with next iteration.* Increase the iteration counter $k \leftarrow k + 1$ and go back to step 2.

8. *Reset the filter.* Reset the filter using (10) and continue with the regular iteration in step 7.

4 Nonmonotone sufficient reduction conditions

Nonmonotone line search methods that use a merit function ensure sufficient progress toward the solution by imposing that the merit function value at each new iterate satisfies an Armijo condition with respect to the maximum merit function value of a prefixed number of previous iterates. We refer to [8] for details. Thus, the standard condition which implies a monotonic decrease of the merit function is relaxed without affecting the global convergence.

Adapting this idea to the multidimensional filter approach, a trial iterate $\boldsymbol{u}_k(\overline{\alpha}_{k,l}) = \boldsymbol{u}_k + \overline{\alpha}_{k,l}\boldsymbol{\Delta}_k$ in the backtracking nonmonotone line search technique is considered acceptable, if it leads to sufficient progress in one of the following measures, when compared to a previous iterate that yields the maximum value of the corresponding measure within the last $m(k)$ iterates, where $m(0) = 0$ and $0 \leq m(k) \leq \min\{m(k-1) + 1, M\}, k \geq 1$ (M is a nonnegative integer), i.e., if

$$|\overline{h}_i(\boldsymbol{u}_k^1(\overline{\alpha}_{k,l}))| \leq (1 - \gamma_f)\,\overline{h}_{i,\max} \tag{12}$$

holds for some $i \in I$ where

$$\overline{h}_{i,\max} = \max_{0 \leq \overline{j} \leq m(k)} |\overline{h}_i(\boldsymbol{u}_{k-\overline{j}}^1)|.$$

The acceptability Armijo condition imposed on the barrier function at the trial iterate $\boldsymbol{u}_k^1(\overline{\alpha}_{k,l})$ is also relaxed and given by

$$b_\mu(\boldsymbol{u}_k^1(\overline{\alpha}_{k,l})) \leq \max_{0 \leq \overline{j} \leq m(k)} b_\mu(\boldsymbol{u}_{k-\overline{j}}^1) + \eta_b \overline{\alpha}_{k,l} \boldsymbol{\nabla} b_\mu(\boldsymbol{u}_k^1)^{\mathrm{T}} \boldsymbol{\Delta}_k^1 \tag{13}$$

instead of (9).

Table 1: Numerical results

| | Filter method | | | l_2 merit function | |
Problem	N_{it}	N_{feas}	N_{b_μ}	N_{it}	N_{mf}
HS1	24	27	26	24	27
HS2	42	44	25	42	44
HS3	1	2	0	1	2
HS4	5	6	2	5	6
HS5	12	120	119	9	26
HS6	8	9	1	8	9
HS10	(13)	(14)	(0)	(13)	(14)
HS11	8	23	15	8	9
HS12	15	32	17	15	16
HS14	8	9	0	8	9
HS15	(9)	(10)	(2)	(18)	(20)
HS16	(11)	(12)	(1)	(11)	(12)
HS17	5	6	2	7	10
HS18	11	12	0	12	76
HS19	25	26	0	31	48
HS20	(22)	(23)	1	(22)	(25)
HS21	5	6	0	5	6
HS23	26	29	4	31	78
HS24	15	16	11	16	19
HS27	16	17	0	17	70
HS28	6	7	0	6	7
HS30	8	9	0	8	9
HS31	13	24	14	13	14
HS32	8	9	2	8	34
HS33	10	11	2	10	11

Table 2: Numerical results (cont.)

| | Filter method | | | l_2 merit function | |
Problem	N_{it}	N_{feas}	N_{b_μ}	N_{it}	N_{mf}
HS34	10	11	2	10	12
HS35	2	3	0	2	3
HS36	(9)	(10)	(2)	(9)	(10)
HS37	(14)	(71)	(70)	(13)	(14)
HS38	33	95	94	25	38
HS41	(26)	(27)	(21)	(9)	(11)
HS42	12	13	0	12	13
HS43	12	13	2	12	13
HS44	19	20	1	19	20
HS45	4	5	2	4	5
HS46	15	37	22	15	194
HS48	7	8	1	7	8
HS49	14	15	7	14	15
HS50	14	15	7	14	64
HS51	6	7	0	6	57
HS52	9	10	0	9	10
HS53	8	9	1	9	14
HS55	11	12	2	11	12
HS60	12	56	46	11	18
HS63	11	26	16	11	12
HS64	(54)	(271)	(232)	(55)	(235)
HS65	9	10	2	9	10
HS76	4	5	0	7	8
HS77	(18)	(35)	(19)	(18)	(48)
HS79	13	14	3	11	65

The inclusion of the nonmonotone strategy also affects the updating of the filter which in this case is:

$$\overline{F}_{k+1} = \overline{F}_k \cup \left\{ \overline{\boldsymbol{h}} \in \mathbb{R}^{2m+2n} : \right.$$
$$\left. |\overline{h}_i| > (1 - \gamma_f)\overline{h}_{i,\max}, \text{ for all } i \in I \right\}. \tag{14}$$

The corresponding algorithm for the interior point multidimensional filter method with the nonmonotone line search strategy is similar to Algorithm 1. The references to equations (7), (9) and (11) should be replaced by (12), (13) and (14) respectively. Furthermore, $m(0) = 0$ should be included in step 1 and the updating formula $m(k) = \min\{m(k - 1) + 1, M\}$ should precede the increase of the iteration counter k in step 7.

5 Numerical experiments

We tested this interior point framework with a multidimensional filter line search technique on 50 small constrained problems from the Hock and Schittkowski (HS) collection [10]. The tests were done in double precision arithmetic with a Pentium 4 and Fortran 90.

First, we compare the standard monotone line search strategy (Algorithm 1) with a similar interior point method based on a l_2 merit function approach. Tables 1 and 2 summarize the results.

Some of the chosen values for the constants are similar to the ones used in [15]: $\overline{h}_i^{\min} = 10^{-4} \max\left\{1, |\overline{h}_i(\boldsymbol{u}_0^1)|\right\}$, $\overline{h}_i^{\max} = 10^4 \max\left\{1, |\overline{h}_i(\boldsymbol{u}_0^1)|\right\}$, for all $i \in I$, $\gamma_f = 10^{-5}$, $\eta_b = 10^{-4}$, $\overline{\alpha}^{\min} = 10^{-12}$.

The algorithms use a symmetric positive definite quasi-Newton BFGS approximation to the matrix $\mathbf{H}$. In the first iteration, the iterative process uses a positive definite approximation to $\nabla^2 F(\boldsymbol{x}_0)$. However, when this matrix is nearly singular or ill-conditioned the identity matrix is used instead. Results inside parenthesis in the tables were obtained with the identity matrix in the first iteration.

The tables report on values of the number of iterations (N_{it}) needed to achieve a solution according

Table 3: Different filter nonmonotone results

| Problem | Filter method | | |
	N_{it}	N_{feas}	N_{b_μ}
HS1	32	35	34
HS5	14	104	103
HS11	8	9	1
HS12	15	16	1
HS15	(17)	(18)	(0)
HS23	27	28	4
HS31	13	14	4
HS37	(14)	(16)	(15)
HS38	32	40	39
HS46	15	16	1
HS60	12	13	3
HS64	(60)	(165)	(123)
HS77	(18)	(19)	(3)

Table 4: Cumulative results

	N_{it}	N_{feas}	N_{b_μ}
Filter method (monotone)	672	1301	796
Filter method (nonmonotone)	696	964	447
l_2 merit function	670	1510	1510

to the convergence tolerance referred to in Algorithm 1, the number of $\overline{h}$ evaluations (N_{feas}) and the number of b_μ evaluations (N_{b_μ}) in the filter method, and the number of merit function evaluations (N_{mf}) in the merit function approach.

We may observe that the two methods behave similarly on almost 50% of the problems. On the remaining problems, for example, HS5, HS11, HS12, HS31, HS37, HS38, HS41, HS60, HS63 and HS64 the number of function $\overline{h}$ evaluations in the filter method exceeds the number of merit function evaluations in the merit function approach. The opposite situation occurs on the problems HS15, HS17, HS18, HS19, HS20, HS23, HS24, HS27, HS32, HS34, HS46, HS50, HS51, HS53, HS76, HS77 and HS79. We remark that the l_2 merit function depends on b_μ and $\overline{h}$ meaning that r merit function evaluations correspond to r b_μ evaluations and r $\overline{h}$ evaluations.

We now compare the monotone and the nonmonotone variants of the multidimensional filter method. For the implementation of the nonmonotone line search strategy, we considered $M = 5$. The obtained results were similar for the majority of the problems. Table 3 contains the list of the problems with different results.

Based on the cumulative results illustrated in the Table 4 we may observe that although the two filter method variants require more iterations than the merit function approach they gain in the number of barrier function and constraints evaluations, in particular the nonmonotone variant where significant reductions on the N_{feas} and N_{b_μ} are registered.

A comparison based on CPU time could be more appropriate in this case. However, as these prelimi-nary numerical experiments involve small problems, CPU time registering is not an easy task. We intend to go on testing the herein proposed method using larger problems and in this situation CPU time measuring will be adopted.

6 Conclusions

We present an interior point multidimensional filter line search method for solving a nonlinear constrained optimization problem. The novelty here is that a certain emphasis is given to primal feasibility measures in order to consider a new iterate to be acceptable. Doing this we avoid the need for a feasibility restoration phase. However, whenever a trial iterate is considered feasible, according to a given tolerance, and the appropriate search direction is descent for the barrier function b_μ, the algorithm imposes on the trial iterate an Armijo condition in order to be acceptable.

Relaxed acceptability conditions were also imposed on the trial iterates breaking the standard monotonicity requirements.

Both algorithms were tested with a set of small problems and compared with a similar interior point method based on a merit function approach. The numerical experimentations favor the multidimensional filter line search method in particular its nonmonotone version.

Future developments will focus on the global convergence analysis of the algorithm. The extension of the multidimensional filter to an interior point method needs an adequate theoretical understanding.

References:

[1] L. Bonettini, A nonmonotone inexact Newton method, *Opt. Meth. and Soft.* 20, 2005, 475–491.

[2] M.F.P. Costa and E.M.G.P. Fernandes, An interior point filter approach for nonliear programming, in T.E. Simos, G. Psihoyios and Ch. Tsi-

touras (eds.) *ICNAAM 2006*, Wiley-VCH, 2006, pp. 369–372.

[3] M.F.P. Costa and E.M.G.P. Fernandes, Practical implementation of an interior point nonmonotone line search filter method, in R. Criado, D. Estep, M.A. Pérez García and J. Vigo-Aguiar (eds.) *CMMSE-2006*, 2006, pp. 245–255.

[4] R. Fletcher, N.I.M. Gould, S. Leyffer, Ph.L. Toint and A. Wächter, Global convergence of a trust-region SQP-filter algorithm for general nonlinear programming, *SIAM J. Optim.* 13, 2002, pp. 635–659.

[5] R. Fletcher and S. Leyffer, Nonlinear programming without a penalty function, *Math. Prog.* 91, 2002, pp. 239–269.

[6] R. Fletcher, S. Leyffer and Ph.L. Toint, On the global convergence of a filter-SQP algorithm, *SIAM J. Optim.* 13, 2002, pp. 44–59.

[7] N.I.M. Gould, C. Sainvitu and Ph.L. Toint, A filter-trust-region method for unconstrained optimization, *SIAM J. Optim.* 16, 2005, pp. 341–357.

[8] L. Grippo, F. Lampariello and S. Lucidi, A nonmonotone line search technique for Newton's method. *SIAM J. Num. Anal.* 23, 1986, 707–716.

[9] L. Grippo, F. Lampariello and S. Lucidi, A class of nonmonotone stabilization methods in unconstrained optimization. *Numer. Math.* 59, 1991, 779–805.

[10] W. Hock and K. Schittkowski,*Test Examples for Nonlinear Programming*, Springer–Verlag, 1981.

[11] M. Ulbrich, S. Ulbrich and L.N. Vicente, A globally convergent primal-dual interior-point filter method for nonlinear programming, *Math. Prog.* 100, 2004, pp. 379–410.

[12] R.J. Vanderbei, LOQO: An interior-code for quadratic programming, *Tech. Rep., Princeton University, School of Engineering and Applied Science,* US, 1998.

[13] A. Wächter and L.T. Biegler, Line search filter methods for nonlinear programming: motivation and global convergence, *SIAM J. Optim.* 16, 2005, pp. 1–31.

[14] A. Wächter and L.T. Biegler, Line search filter methods for nonlinear programming: local convergence, *SIAM J. Optim.* 16, 2005, pp. 32–48.

[15] A. Wächter and L.T. Biegler, On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming, *Math. Prog.* 106, 2006, pp. 25–57.