Pedro Albuquerque Jerónimo do Rosário Dias **Impressions on Immigration and Economic Growth**

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Universidade do Minho
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Pedro Albuquerque Jerónimo do Rosário Dias

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Impressions on Immigration and Economic Growth

Tese de Doutoramento em Economia

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DECLARAÇÃO

Nome: Pedro Albuquerque Jerónimo do Rosário Dias

Endereço eletrónico: pajrdias@gmail.com

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Orientadora: Prof. Doutora Maria João Cabral de Almeida Ribeiro Thompson

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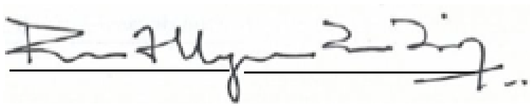
Assinatura: 

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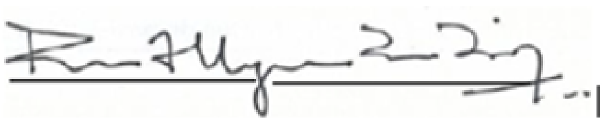
Signature: 

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Abstract

In this thesis we develop three models to examine the effects of demographic change on economic growth. Immigration is introduced as a source of demographic change. Our approach consists on extensions of the models of Solow (1957), Lucas (1988) and Romer (1990) for particular types of demographic change. Chapter 1 analyses the relationship between age structure and economic growth by augmenting the Gruescu's (2007) extension of the model of Solow (1957) for a technological specification that is connected to the age structure of the population. We conclude that countries were the ageing process is very advanced can reach a collapse trajectory due to technological regression and immigration can reverse that problem. Chapter 2 analyses the relationship between educational heterogeneity and economic growth. To allow for that, we extend the model of Lucas (1988) for unskilled labour, using the Mankiw et al. (1992) production function as in Robertson (2002). Bearing in mind that skilled and unskilled labour are imperfect substitutes and that the persistent increase of the share of formally educated people is a fact of development, we conclude that unskilled labour is critical for economic growth, therefore immigration policies centred on skill selection are strategically inadequate. Chapter 3 examines the impact of ethnic diversity on economic growth. We extend the Jones (1995) variant of the model of Romer (1990) for ethnic diversity, by assuming that ethnic diversity affects innovation and the productivity of labour. Ethnic diversity influences innovation through the channels of knowledge spillovers and redundancy of research projects, as well as augments or diminishes the productivity of labour through social capital. We conclude that ethnic diversity can be a decisive factor for economic growth and that multiethnic countries have a higher grow potential than the conservative. According to the institutional configuration of countries concerning to the inclusion of minorities, multiethnic immigration can favour or disfavour growth. The institutions, democratically elected, play a decisive role in respect to the socioeconomic outcomes of multiethnic immigration. Though the models on this thesis are suitable for any region that adheres to their postulates, we focus on the European Union and conclude that the pursuit of anti-immigration policies has a major harmful impact on economic growth.

Resumo

Nesta tese desenvolvemos três modelos para analisar os efeitos da mudança demográfica no crescimento económico. A imigração é introduzida como fonte de mudança demográfica. A nossa abordagem consiste em extensões dos modelos de Solow (1957), Lucas (1988) e Romer (1990) para tipos específicos de mudanças demográficas. O Capítulo 1 analisa a relação entre estrutura etária e crescimento económico, ampliando a extensão do modelo de Solow (1957) proposta por Gruescu (2007) para uma especificação tecnológica associada à estrutura etária da população. Concluímos que os países em adiantado processo de envelhecimento podem alcançar uma trajetória de colapso devido à regressão tecnológica e que a imigração pode reverter esse problema. O Capítulo 2 analisa a relação entre heterogeneidade educacional e crescimento económico. Para atingir esse objetivo, estendemos o modelo de Lucas (1988) para o trabalho não qualificado usando a função produção de Mankiw et al. (1992), como em Robertson (2002). Tendo presente que o trabalho qualificado e não qualificado são substitutos imperfeitos e que o aumento persistente da proporção de pessoas formalmente educadas é um facto do desenvolvimento, concluímos que o trabalho não qualificado é essencial para o crescimento económico e que, portanto, as políticas de imigração centradas na seleção de habilidades são estrategicamente inadequadas. O Capítulo 3 examina o impacto da diversidade étnica no crescimento económico. Estendemos a variante do modelo de Romer (1990) proposta por Jones (1995) à diversidade étnica, assumindo que, esta, afeta a inovação e a produtividade do trabalho. A diversidade étnica influencia a inovação através dos canais da difusão do conhecimento e da redundância dos projetos de pesquisa e aumenta ou diminui a produtividade do trabalho através do capital social. Concluímos que a diversidade étnica pode ser um fator decisivo para o crescimento económico e que os países multiétnicos têm maior potencial de crescimento do que os conservadores. De acordo com a configuração institucional dos países em relação à inclusão de minorias, a imigração multiétnica pode favorecer ou desfavorecer o crescimento. As instituições, democraticamente eleitas, desempenham um papel decisivo em relação aos resultados socioeconómicos da imigração multiétnica. Embora os modelos desta tese sejam adequados para qualquer região que adira aos seus postulados, concentramo-nos na União Europeia e concluímos que o prosseguimento de políticas anti-imigração tem um considerável impacto desfavorável ao crescimento económico.

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Dedication

To my mother Ana Isabel and to my father Basílio Dias

To my babies: Diogo, Bruno, Pedro, Alexandre and Gabriel

Introduction

The New Colossus

Not like the brazen giant of Greek fame,
With conquering limbs astride from land to land;
Here at our sea-washed, sunset gates shall stand
A mighty woman with a torch, whose flame
Is the imprisoned lightning, and her name
Mother of Exiles. From her beacon-hand
Glows world-wide welcome; her mild eyes command
The air-bridged harbour that twin cities frame.
“Keep ancient lands, your storied pomp!” cries she
With silent lips. “Give me your tired, your poor,
Your huddled masses yearning to breathe free,
The wretched refuse of your teeming shore.
Send these, the homeless, tempest-tost to me,
I lift my lamp beside the golden door!”

Emma Lazarus

Introduction

Migration is a timeless and everlasting human activity. Mobility is natural to human beings. The first humans were nomads moving across regions in search for food and fertile soils (Olmedo, 2002). Once sedentary, humans have continued their search for better food and more fertile soils, a metaphor for the search for better life conditions.

We are all committed to the diaspora to fulfil our aspirations. We still move across regions motivated by the search for better living conditions. Human mobility can be motivated by unemployment, low wages, extreme poverty, the escape from wars, epidemics, natural disasters, tyrannies, love relationships, or it can simply be an expression of free will.

International migrations have assumed many forms and their economic role has changed throughout History, from one country's reaction to exogenous shocks on to endogenous adjustments in its social conditions (Baycan and Nijkamp 2011). The increasing regional integration has intensified international migrations to unparalleled levels. International migrations constitute indeed the most prominent structural feature of globalization. The International Migration Report 2015's (United Nations, 2016) fresh figures are a clear expression of this fact[1].

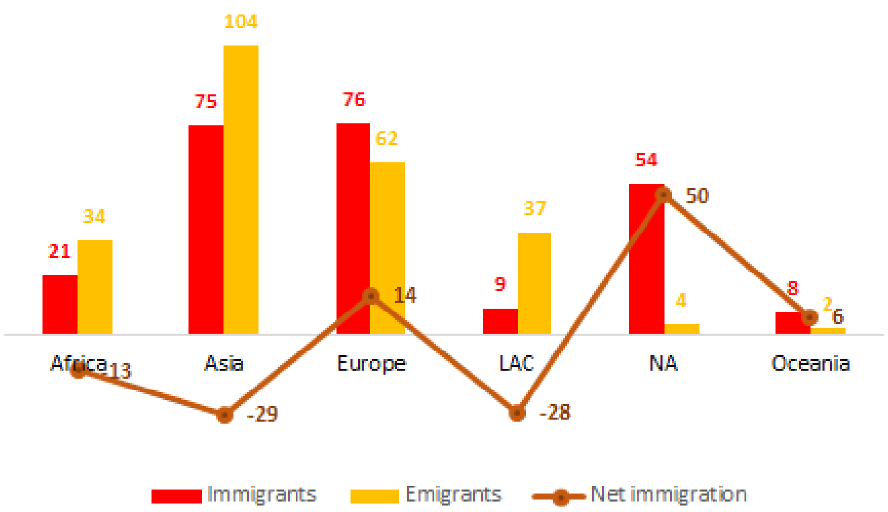
Between 1970 and 2015, the number of international migrants grew from 82 million into 244 million (United Nations, 2006, 2016). Filtering such data by region, we find that the migration trajectories have been changing. While in 1980 most migrants lived in the developing world, in 2015 most of them were living in the developed world. From 1980 to 2015, the migrant population increased from 48 million to 141 million in the most

[1] Some values are not directly taken from the Report but, instead, they are simple own computations over data presented in the Report.

advanced economies, while in the less advanced, the numbers grew from 52 million to 103 million (United Nations, 2016).

In 2015, Europe hosted 76 million immigrants, Asia, 75 million, Northern America, 54 million, Africa, 21 million, Latin America and the Caribbean, 9 million and the Oceania 8 million. Europe and Asia hosted almost the same number of immigrants, however, representing different net migration realities. Europe was a net immigration continent (for 14 million) and Asia was net emigration (for 29 million)[2]. The Northern America (for 50 million) and the Oceania (for 6 million) were also net immigration regions, while the Latin America and the Caribbean (for 28 million) and Africa (for 13 million) were net emigration (United Nations, 2016)[3] (Figure I.1).

Figure I.1 Continental distribution of immigrants, emigrants and net immigration in 2015 (Unit: Million Individuals).



Notes: NA = Northern America; LAC = Latin America and the Caribbean. Both belong to the same continent, though, they are treated separately by the United Nations. **Source:** Own presentation from United Nations (2016) data.

We can verify that, in our days, most international migrants move into countries of the same continent of their country of origin. In 2015, this was the experience of 52% of

[2] We refer to the past, although the numbers are so recent that they describe the present. Nevertheless, 2015 is already the past.

[3] The figures between parentheses are net migration values.

African international migrants, 60% of Asian, 66% of European, 59% of Oceanian, and 86% of Latin American and the Caribbean. Also, 58% of Northern American international migrants stayed within American continental borders.

Another fact that stands out from the figures is that Europe is the main destination of non-European migrants moving into a different continent from their own. Except for the Latin American and the Caribbean, the proportion of non-European international migrants that head to Europe rounds 20%. Regarding the Latin American and the Caribbean, 12% of the 13% that migrate intercontinentally move into Europe (Table I.1).

Table I.1 Distribution of international migrants by continent of origin (Unit: %).

	Africa	Asia	Europe	LAC	NA	Oceania
Africa	52	12	27		7	2
Asia	1	60	20		16	3
Europe	2	13	66	2	12	5
LAC		1	12	16	70	
NA	2	13	23	31	27	5
Oceania		6	19		15	59

Notes: NA = Northern America; LAC = Latin America and the Caribbean. Both belong to the same continent. **Source:** Own presentation from United Nations (2016) data.

Europe is clearly a net immigration continent, although with distinct inner realities. The Eastern, the Central and the Western Europe exhibit different migration balances. The transition economies of the Central and Eastern Europe are mostly net emigration countries, except for the Russian Federation, that is the destiny of many migrants of former Soviet Union countries. In fact, the Russian Federation is at the top of the European destiny countries, followed by Germany. In 2015, both the Russian Federation and Germany had 12 million immigrants. However, the Russian Federation had 11 million emigrants, 7 million more than Germany.

Amongst the 20 countries that receive 67% of worldwide migrants, there are 5 European Union members, hosting 55.3% of Europe's immigrants. These are: Germany (12 million), United Kingdom (9 million), France (8 million), Italy (6 million) and Spain (6 million) (United Nations, 2016). The European Union is, nowadays, a destiny region.

In this human mobility context, recent events motivate apprehension[4]. These are current news, with no citations required. In Germany, there were over 500 assaults on refuges in the first nine months of 2016. The anti-immigration political parties have substantially enlarged their electoral basis in France, Germany and Austria. In the United States, Donald Trump became president with electoral promises such as building a wall in the border with Mexico and obstructing Islamic immigration[5]. The governments of Hungary and Poland are openly pursuing anti-minorities constitutional reforms. Even non-extremist organizations and political parties have been issuing potentially xenophobic messages. In 2008, Italy’s statistics office blamed immigrants for the rise in the unemployment rate[6]. In 2010, Angela Merkel proclaimed the failure of multiculturalism. In 2013, David Cameron suggested that some immigrants had gone to the United Kingdom “for the wrong reasons”. The recent Brexit was profoundly motivated by anti-immigration feelings[7]. Hungary sharp wired its borders with Macedonia and a wall was built in Calais. Many other walls are increasingly being built, some physical, others regulatory, others (the worst) in the minds and in the hearts of European citizens.

The economic downturn led by the 2008 financial crisis, together with the refugee crisis and the recent Islamite terrorist attacks in European Union countries, facilitated the spread of anti-immigration spirits. These specific circumstances added the sensitive topic of Security to other common populist accusations to immigrants such as that immigrants do not respect theirs host countries’ social norms and that immigration

[4] From this point onwards, we will use Europe and European Union indifferently, if no ambiguity results from it.

[5] Donald Trump is not a president of a European country but, as allies and partners, the influence of the United States in Europe is very high.

[6] Thornhill et al. (2008).

[7] At the end of the first decade of the century, polls have demonstrated that a great majority of the British people believe that immigrants take away their jobs and reduce their wages (German Marshall Fund, 2007, 2008).

causes decreasing employment opportunities and wages, while increasing taxes, social transfers, social services, public expenditure, and so on. These claims have echoed throughout a significant proportion of the European public opinion, despite the total absence of scientific confirmation. On the contrary.

Despite non-consensual empirical evidence, some common denominators concerning the effects of immigration have been identified. Firstly, the effects of immigration depend on the time-horizon considered. While in the short-run, they can be positive, neutral or negative, in the long-run they promote the host country's economic growth and competitiveness. Secondly, when immigration has short-run negative effects, their extension is generally modest and controllable.

Furthermore, most of the credited institutions that study and deal with immigration policy (e.g. OCDE, IMO, UN) find a positive net effect of immigration on economic outcomes. It is frequently highlighted and emphasized[8]. Such findings do not mean that immigration is good for typical host countries in any circumstances, under all criteria. They mean that, recognizing the overall benefits of immigration, national immigration policies should articulate with the specific realities of each host country with a win-win outcome for both host and newcomers.

Table I.2 lists some reference empirical articles grouped according to their findings regarding the economic outcomes of immigration.

[8] The OECD Newsletter released in May 2014, summarizes the common conclusions achieved by the quoted institutions around the issue: https://www.oecd.org/migration/OECD%20Migration%20Policy%20Debates%20Numero%202.pdf.

Table I.2 Results of empirical studies concerning economic outcomes of immigration.

Natives' Are:	Wages and income per capita	Unemployment and employment opportunities	Social security and public finances
Influenced Positively	Dustmann et al. (2007) Ottaviano and Peri (2006) Manacorda et al. (2006) Hartog and Zorlu (2002) Winter-Ebmer and Zweimüller (1999) Bauer (1998) Winter-Ebmer and Zweimüller (1996) Jaeger (1996) De New and Zimmermann (1994)	Blanchflower et al. (2007) Konya (2000) Card (1990)	Hansen and Lofstrom (2003) Gott and Johnston (2002) Lee and Miller (2000) Lalonde and Topel (1997) Baker and Benjamin (1995) Borjas (1995) Passel and Clark (1994) Tienda and Jensen (1986) Blau (1984)
Unaffected	Ortega and Peri (2009) Carrasco et al. (2007) Ottaviano and Peri (2006) Dustmann et al. (2005) Hartog and Zorlu (2002) Cohen and Hsieh (2001) Vilhelmsson (2000) Ekberg (1999) Grant (1999) Bauer (1998) Bell (1997) Schoeni et al. (1996) LaLonde and Topel (1992) Butcher and Card (1991) LaLonde and Topel (1991) Card (1990) Carliner (1980) Chiswick (1978)	Carrasco et al. (2007) Gilpin et al. (2006) Dustmann et al. (2005) Angrist and Kugler (2003) Friedberg (2001) Vilhelmsson (2000) Ekberg (1999) Winter-Ebmer and Zweimüller (1999) Venturini (1999) Akbari and DeVoretz (1992) Altonji and Card (1991) Winegarden and Khor (1991) Altonji and Card (1991) Withers and Pope (1985)	Büchel and Frick (2005) Collado and Valera (2004) Moscarola (2003) Roodenburg et al. (2003) Storesletten (2000) Lee and Miller (2000) Ablett et al. (1999) Smith and Edmonton (1997)
Influenced Negatively	Dustmann et al. (2007) Ottaviano and Peri (2006) Borjas (2006) Manacorda et al. (2006) Borjas (2003) Hartog and Zorlu (2002) Card (2001) Cohen and Hsieh (2001) Bauer (1998) Jaeger (1996) De New and Zimmermann. (1994) LaLonde and Topel (1991) Borjas (1987) Grossman (1982)	Card (2001) Pischke and Velling (1997) Grossman (1982)	Rowthorn (2008) Büchel and Frick (2005) Hansen and Lofstrom (2003) Sinn and Werding (2001) Gustman and Steinmeier (2000) Borjas and Hilton (1996) Borjas (1995) Borjas (1994) Huddle (1993) Borjas and Trejo (1991)

Note: The underlined references correspond to articles that conclude for results of different signs when the dependent variable is controlled for specific attributes, such as skills, gender, origin, longevity in the host country, time-horizon, immigration cohort, etc.

The considerable number of references included in each column cell (that could evidently be much higher) is revealing of the multiple empirical results. Additionally, many of them have different results according to whether immigration is controlled for socioeconomic and demographic groups or sectors of employment (underlined papers).

Immigration is thus not a uniform category. It is indeed a heterogeneous phenomenon involving the interaction of natives' attributes with those of the immigrants, as well as the type of structure and dynamics of institutional, social and economic settings of the host country. Accordingly, immigration is too complex to be boxed in a "immigration is harmful for Europe" statement. Still, we will pose here a general statement that we wish to disentangle throughout the thesis, namely, that the spread of anti-immigration ideas and policy pressures in European Union can throw the European Union region into an economic growth trap.

With the present thesis we wish to contribute to the ongoing effort of accessing the impact of immigration on economic growth. Our analysis will be made through the lenses of the modern economic growth theory, whose three foundational models will be extended for our specific sets of assumptions.

As Europeans, we (the authors) share civilizational principles and values that are opposed to anti-immigration ideas. We also believe that, in matters involving human beings, ethical and moral deliberations superimpose economic outcomes as criteria. We will here abstain from such supra-economic considerations, but we still wish to remark that international migrations cannot be treated in the same way as international trade of goods and services or international capital flows. People are neither merchandise, money nor machinery. Indeed, humans have necessities, expectations, dreams, aspirations, human rights. All of which ultimately give rise to economic systems and hence should always be at the very core of all economic theory and policy constructions.

This thesis is composed of three developed models dedicated to the analysis of the impact of demographic change on economic growth. Immigration is, naturally, assumed to be a demographic force.

Our methodological approach consisted in extending three reference growth models, introducing types of demographic shifts. The chosen models, Solow (1956), Lucas (1988) and Romer (1990), are the three pillars of the new growth theory. They are analytically tractable with clear and distinct engines of growth.

In Solow (1956), the engine of growth is physical capital accumulation, in Lucas (1988), it is human capital accumulation and in Romer (1990) it is technological progress obtained through R&D activities[9]. The sequence adopted for our Chapters, from Solow (1956) to Lucas (1988) and onto Romer (1990), can be interpreted, and often is, as characterizing different stages of development of one economy. Assuming different relative weights throughout time, economic growth does, in reality, benefit from a combination of these three sources of growth, altogether, at any moment in time[10].

The remaining of the thesis is organized as follows.

In Chapter 1, we analyse the relationship between age structure and economic growth. We augment Gruescu's (2007) extension of Solow's (1956) model by linking the young-age and the old-age dependency ratios with technological progress. Then, implicitly, we assume that technology can regress in the post-industrial era, particularly because of a policy-wise unaddressed process of severe ageing.

With Chapter 2, we analyse the relationship between educational heterogeneity and economic growth. We follow Robertson's (2002) approach in extending Lucas' (1988) model to include unskilled labour in Mankiw et al.'s (1992) aggregate production function. Educational heterogeneity is introduced by considering skilled and unskilled labour that, as imperfect substitutes, provide services that are distinct. We also consider formal education as increasing in share with civilizational development throughout time. Such ensemble of assumptions allows us to understand natural patterns of migration flows.

[9] Considering the exogenous growth models that describe economies driven by physical capital accumulation, Cass (1965) and Koopmans (1965) on Ramsey (1928) could be an alternative to Solow (1956). As for the endogenous human capital driven models, Rebelo (1991), Caballe and Santos (1993) and Barro and Sala-i-Martin (1995), for instance, could be alternatives to the modern formulation of Lucas (1988) on Usawa (1965). As for the endogenous R&D models, the models of vertical product innovation with creative destruction of Grossman and Helpman (1991) or Aghion and Howitt (1992) and of vertical product innovation with market structures of Smulders and van Klundert (1997), amongst many others, could be alternatives to Romer's (1990) model of horizontal product innovation. In fact, the richness of the wide variety of growth models that have been built upon specific assumptions and elaborate mathematical techniques over the last three decades is such that is impossible to mention (Thompson, 2008).

[10] We have in mind the net immigration economies such as those of the European Union, therefore, our focus is on developed economies.

In Chapter 3, we analyse the relationship between ethnic diversity and economic growth. We extend Jones' (1995) variant of Romer (1990) by introducing ethnic diversity with the purpose of conveying the idea that conservative institutional settings can hinder economic growth. We introduce ethnic diversity through the channels of creativity, of uniqueness of research projects and of social capital that enhances workers' productivity. Then, as ethnic diversity represents the cohabitation of individuals with different racial, religious and cultural backgrounds, the institutional characteristics of one society play a decisive role in socioeconomic outcomes.

Our Conclusion summarizes our findings regarding the socioeconomic consequences of current anti-immigration ideas and policy pressures in European Union.

Chapter 1

Age structure and Economic Growth

1.1 Introduction

The first methodical approach on the relationship between population and economic growth is often credited to Malthus (1798). According to Malthus, the natural size of populations is their own subsistence level. This conclusion was built upon the reality of his time, when resources grew slower than populations[11]. Then, an increase in the size of populations would lead to wars, famine, plagues and fertility behaviours to downsize populations. The negligible technological progress throughout centuries kept the populations' size tied to strong resource constraints. Malthus (1798) would still be correct if technology had not won the growth race over the population (Simon, 1977), with the Industrial Revolution[12].

Pre-industrial times were characterized by high fertility and high mortality rates. Life expectancies were low and infant mortality was high. The Industrial Revolution allowed for populations to grow simultaneously with their average income and living conditions improvements. Better public health, water and sanitation, personal hygiene, nutrition, led to decreases in infant mortality and life expectancy expansion. Such combination resulted in continuous increases in the population size through higher shares of the youngest. Populations grew in size and in youth. However, with time, fertility rates began declining as families could sustain their size with fewer new-borns[13]. It followed that populations' growth decelerated and their average age increased.

[11] Resources grew geometrically and populations grew exponentially.

[12] Whenever we refer to "The Industrial Revolution" not concretizing if it is the First, the Second or the Third, we are always referring to the First Industrial Revolution (1760 – 1840).

[13] In fact, the decline in fertility growth rates was also due to a complex combination of social, economic and demographic factors, such as the progressive emancipation of women, secularization, and so on. We have hence oversimplified.

Nowadays, in most industrialized countries, populations are stagnant (sometimes decreasing), and ageing. These countries are on the last stage of the post-industrial process known as the demographic transition[14].

Demographic transition consists in the change of a populations' age distribution. Such changes have implications on the share of the active population and on dependency ratios[15], with dramatic consequences on economies. Thus, while in pre-industrial times emphasis was given to the economic consequences of populations' size, arrival at the latest stages of demographic transition has shifted concerns to the economic consequences of the age-distribution dynamics[16].

This Chapter aims at analysing the consequences of age distribution on economic growth. In related literature, Yaari (1965), Blanchard, (1985), Nerlove et al. (1985), use overlapping generation frameworks with age-specific heterogeneity dynamics. Further theoretical and empirical contributions to this line of research introduce more accurate age-dependent structures and more realistic mortality designs (Heijdra and Romp, 2007; d'Albis, 2007).

In contrast to the above referred theoretical constructions, we have chosen not to endogenize the age-distribution dynamics. Instead, we have opted for the typical-agent approach. We build on Gruescu's (2007) extension of Solow's (1957) model. Our here developed model introduces to Gruescu's (2007) model: (i) a dependency ratio disaggregation into age-class components; and (ii) a specification for technological progress.

[14] See Bloom and Williamson (1998), Williamson (2001) and Lee (2003) for a comprehensive assessment of demographic transition.
[15] The dependency ratio is the average number of dependent individuals per worker.
[16] The literature concerning the relationship between demographic transition and economic growth is very vast, Birdsall (1988), Razin and Sadka (1995), Ehrlich and Lui (1997) and Galor and Weil (2000) providing excellent surveys on the matter.

In Gruescu's (2007) growth model, the dependency ratio is a simple ratio between the total number of dependent individuals and active population. Such measure implies no distinction between children and old people. We believe that the policy measures required for an economy whose dependents are mainly young are necessarily different from those required for an economy whose dependents are mainly old people. Hence, we propose an analysis of the impact of age structures on economic growth, disaggregating total dependence into young dependence and old dependence ratios.

With our introduced age-class disaggregation, the dependency ratio becomes the sum of its age-class dependency ratios, which renders the differential calculus for closed form solutions rather intricate. So, to deliver a mathematically friendly model, we create a stylized population whose size is proportional to the simple geometrical mean of the age-class dependency ratios.

The second main difference between Gruescu's model and ours is to do with the technological specification. In Gruescu (2007), like in Solow (1957), technological progress, which explains long-term growth, is captured by an exogenous parameter. In our proposed model, technology depends on the age structure, hence long-term growth depends on the dynamics of the population's age structure. Moreover, the model allows for, either, technological progress or technological regress.

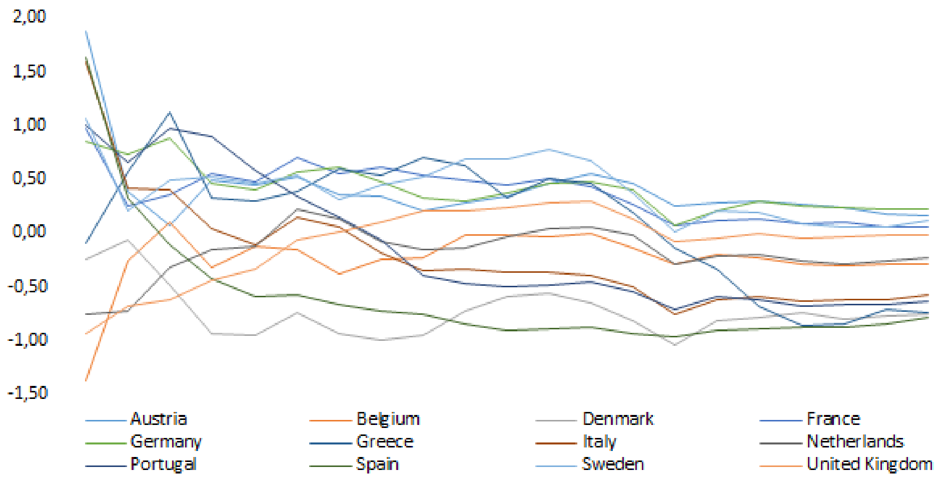
In pre-industrial times, technological regress has occurred mainly due to negative demographic shocks or land productivity downturns, which caused the neglect of unprofitable practices, consequently the loss of their knowledge (Aiyar et. al, 2008). In current times, knowledge cannot be lost, as our storing capacity is infinite. Still, our ability to accumulate and operate knowledge does depend on the economy's demographic structure. Severe ageing of populations together with stagnation or decline in their size can lead to generalized loss of ability to use knowledge.

Mankind has gone through many episodes of technological regress. The fall of the Roman Empire of the West is a classic example (King, 2000). The deterioration of the European aggregate output after the fifth century confirms it (Grantham, 1999).

Presently, the European Union is experiencing economic stagnation. According to recent research (e.g., IMF, 2015), our proposed conjecture of technological regress may be a possible explanation for the European Union weak growth. Assuming that Total Factor Productivity (TFP) is captured empirically by the rate of technological progress, we can hypothesise that there is, in fact, an ongoing process of technological regress in the European Union.

Figure 1.1 portrays the evolution of TFP in 12 selected European Union countries. It depicts quite well the suggestion of many authors (e.g., Kyoji et al., 2015, Van Ark, 2015) that advanced economies are facing sluggish TFP growth, which, in many cases, is negative. The observed TFP decline has indeed started years before the financial crisis of 2008, being regarded as one of the reasons for its depth and difficult recovery (e.g., Van Ark et al., 2008).

Figure 1.1 Total Factor Productivity for selected European Union members: 1995-2015.



Note: Own presentation over the Conference Board database[17].

[17] https://www.conference-board.org/data/economydatabase/

Modern growth theory does not predict technological regress. Technological adoption (e.g. Nelson and Phelps, 1966) and innovation (e.g., Romer, 1990; Aghion and Howitt, 1992) prevent technological regress from occurring. With the introduced framework, we can offer a possible theoretical explanation for the observed recent growth trends. Our model suggests that the above referred slow growth in the European Union, associated with technological regression, may be caused by factors related to these countries being in their latest stages of demographic transition.

While empirical findings are compatible with the supposition of underlying technological regress, the resulting policy recommendations to solve the problem have been neglecting the demographic dimension. With this Chapter we wish to convey the idea that demographic policy can play a major role in strategic growth policy, not only because we believe that it improves TFP, but also because it can contribute to the alleviation of supply-side and demand-side negative effects of ageing on economic growth.

In short, to analyse the opposite effects of rejuvenation and ageing on economic growth, our proposed model extends Gruescu's (2007) model by disaggregating the dependency ratio into young-age and old-age dependency ratios. We wish to convey the hypothesis that countries that undergo severe ageing processes can end up facing long-term negative growth rates due to technological regress. We propose possible solutions for this economic problem.

The remaining of this Chapter is organized as follows. Section 1.2 exposes the model's main assumptions. Section 1.3 solves for the equilibrium and discusses general results. Section 1.4 presents the transitional dynamics analysis and the adjustment paths after policy shocks. Section 1.5 dissertates on policy guidelines to the case of European Union. Section 1.6 concludes.

1.2 The Model

To examine the impact of age structures on economic growth, we augment Gruescu's (2007) extended version of Solow's (1957) model. Specifically, we disaggregate the dependency ratio by age classes to allow for demographic arrangements to reproduce the evolution of the population and to influence the technological parameter size. Technology thus becomes the channel through which age structures influence economic growth.

In our proposed model, technology can progress, stagnate or regress, and so can economies. We wish to convey the idea that political solutions to address adverse growth trajectories ought to focus on the reversion of adverse demographic arrangements on the origin.

1.2.1 Demographics

The size of the population P_t at time t is:

$$P_t = P_{1t} + L_t + P_{2t} \qquad (1.01)$$

where P_{1t} refers to the size of the young population (children, students), L_t represents the number of workers; and P_{2t} stands for the size of the older population (retirees).

Dividing both sides of (1.01) by P_t and rearranging, we compute dependency ratio as:

$$d_t - 1 = d_{1t} + d_{2t} \qquad (1.02)$$

where d_{1t} is the young-age and d_{2t}, is the old-age dependency ratios.

Variable d_{1t} is the computation of the average number of young dependent individuals per worker and the d_{2t}, the average number of old dependent individuals per worker, i.e.:

$$d_{jt} = \frac{P_{jt}}{L_t} \tag{1.03}$$

with $j = 1,2$.

The population-labour ratio d_t is given by:

$$d_t = \frac{P_t}{L_t} \tag{1.04}$$

or, alternatively, by:

$$d_t = 1 + d_{1t} + d_{2t} \tag{1.05}$$

As Gruescu (2007), we also introduce de population-labour ratio in the model of Solow (1957); however, as an alternative of using the aggregate d_t, we use its disaggregated form $1 + d_{1t} + d_{2t}$.

Since this option can lead to mathematical complexity, mainly concerning the attainment of explicit/closed-form solutions, we assume that the population is reproduced by the following law:

$$P_t = \delta d_{1t}^{0.5} d_{2t}^{0.5} L_t$$

then, using its standardized P_t/δ version, it becomes:

$$P_t = d_{1t}^{0.5} d_{2t}^{0.5} L_t \tag{1.06}$$

Log-time-differentiating (1.06), we get the population's growth rate:

$$p_t = 0.5\left(g_{d_{1t}} + g_{d_{2t}}\right) + n_t$$

where n_t is the growth rate of the labour force and the $g_{d_{jt}}$ represent the growth rates of dependency ratios $d_{jt}, j = 1,2$.

Log-time-differentiating (1.03), the growth rates of the dependency ratios d_{jt} become:

$$g_{d_{jt}} = p_{jt} - n_t \tag{1.07}$$

where p_{jt} is the growth rate of subpopulation P_{jt}, $j = 1,2$.

Assuming that p_{jt} are constant and equal to p_j, the growth rate of the population p is:

$$p = 0.5(p_1 + p_2) \tag{1.08}$$

As life expectancies have been growing over time, we assume restriction $p_2 > 0.$.

According to (1.08), n_t does not affect p. It is inherently included in p_1 and p_2; then, we establish for the growth rate of the labour force:

$$n = \eta(p_1 - p_2) \tag{1.09}$$

where $0 < \eta < 1$. It means that the growth rate of the labour force increases if $p_1 > p_2$.

Inserting (1.09) in (1.07) we obtain the growth rates of the age-class dependency ratios:

$$g_{d_1} = (1 - \eta)p_1 + \eta p_2 \tag{1.10}$$

$$g_{d_2} = (1 + \eta)p_2 - \eta p_1 \tag{1.11}$$

and of the total dependency ratio:

$$g_d = (0.5 - \eta)p_1 + (0.5 + \eta)p_2 \tag{1.12}$$

Equations (1.10) to (1.12) describe our model's assumptions for the evolution of the dependency ratios with the age structure.

When the growth rate of the young population increases 1 percentage point, the young-age dependency ratio increases by $1 - \eta$ points and the old-age dependency ratio falls by η points. Then the total dependency ratio increases or diminishes by $0.5 - \eta$, when η is lower or higher than 0.5, respectively.

As $0 < \eta < 1$, then $-0.5 < g_d < 0.5$. On the other hand, an increase of 1 percentage point in the older population cause an increase in η on g_{d_1} and of $1 + \eta$ on g_{d_2} , leading to a total increase of $0.5 + \eta$ in g_d.

Summing up, variations in the size of the non-economically active population originate diverse outcomes concerning the evolution of the dependency ratios, consequently on social security, on taxes and on public budgets.

While an increase in the younger population can lead to decreases in the total dependency ratio g_d (when $\eta > 0.5$), an increase in the older population always leads to an increase in g_d. Even when $\eta < 0.5$, the impact of increases in p_1 is substantially lower than that of increases in p_2[18].

[18] As we will see later, this result adheres to the European Union's experience.

1.2.2 Technology

While P_{1t} represents the future composition and amount of skills of the labour force, P_{2t} represents the abilities withdrew from the labour force. Then we assume that the relationship between the technological level and the demographic setting is:

$$A_t = \left(\frac{P_{1t}}{P_{2t}}\right)^{\tau} \tag{1.13}$$

where we assume $\tau > 0$.

Equation (1.13) is equivalent to:

$$A_t = \left(\frac{d_{1t}}{d_{2t}}\right)^{\tau} \tag{1.14}$$

Log-time-differentiating (1.13), we obtain the growth rate of technology as:

$$g_A = \tau(p_1 - p_2) \tag{1.15}$$

1.2.3 Production

The economy produces output Y_t according to the labour-augmenting Cobb-Douglas technology:

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha} \tag{1.16}$$

with $0 < \alpha < 1$.

As usual, K_t refers to physical capital, A_t is the technological level given by (1.13) or (1.14) and L_t represents the size of the labour force.

1.3 Equilibrium

We will now derive and characterize analytically the model's results for its steady state.

Depreciation of physical capital is not contemplated in this setting. Additionally, to emphasize the growth effects of age structure, we do not consider Solow's residual.

Inserting (1.14) in (1.16), the production function becomes:

$$Y_t = K_t^{\alpha}\left[\left(\frac{d_{1t}}{d_{2t}}\right)^{\tau} L_t\right]^{1-\alpha} \tag{1.17}$$

Using (1.04) and dividing both members by P_t, we get the output per-capita production function:

$$y_t = \left[\frac{1}{d_t}\left(\frac{d_{1t}}{d_{2t}}\right)^{\tau}\right]^{1-\alpha} k_t^{\alpha} \tag{1.18}$$

Without physical capital depreciation, aggregate investment is:

$$\dot{K}_t = sY_t$$

where $0 \leq s \leq 1$ is the exogenous saving rate, then:

$$\frac{\dot{K}_t}{P_t} = sy_t$$

and knowing that:

$$\frac{\dot{K}_t}{P_t} = \dot{k}_t + pk_t$$

we get the fundamental dynamic equation for physical capital per-capita as:

$$\dot{k}_t = sy_t - pk_t$$

which in this context is equivalent to:

$$\dot{k}_t = s\left[\frac{1}{d_t}\left(\frac{d_{1t}}{d_{2t}}\right)^{\tau}\right]^{1-\alpha} k_t^{\alpha} - 0.5(p_1 + p_2)k_t \qquad (1.19)$$

Dividing both members of (1.19) by k_t, we compute the growth rate of physical capital:

$$g_{k_t} = s\left[\frac{1}{d_t}\left(\frac{d_{1t}}{d_{2t}}\right)^{\tau}\right]^{1-\alpha} k_t^{\alpha-1} - 0.5(p_1 + p_2) \qquad (1.20)$$

In steady state, g_{k_t} is constant, that is, $\dot{g}_{k_t} = 0$.

Deriving $\dot{g}_{k_t}$:

$$\dot{g}_{k_t} = (\tau - 0.5)(1-\alpha)s\left[\frac{1}{d_t}\left(\frac{d_{1t}}{d_{2t}}\right)^{\tau}\right]^{1-\alpha} k_t^{\alpha-1}\frac{\dot{d}_{1t}}{d_{1t}} -$$

$$-(\tau + 0.5)(1-\alpha)s\left[\frac{1}{d_t}\left(\frac{d_{1t}}{d_{2t}}\right)^{\tau}\right]^{1-\alpha} k_t^{\alpha-1}\frac{\dot{d}_{2t}}{d_{2t}} -$$

$$-(1-\alpha)s\left[\frac{1}{d_t}\left(\frac{d_{1t}}{d_{2t}}\right)^{\tau}\right]^{1-\alpha} k_t^{\alpha-1}\frac{\dot{k}_t}{k_t}$$

then:

$$\dot{g}_{k_t} = (1-\alpha)s\left[\frac{1}{d_t}\left(\frac{d_{1t}}{d_{2t}}\right)^{\tau}\right]^{1-\alpha} k_t^{\alpha-1}\left[(\tau - 0.5)g_{d_{1t}} - (\tau + 0.5)g_{d_{2t}} - g_{k_t}\right]$$

As:

$$(1-\alpha)s\left[\frac{1}{d_t}\left(\frac{d_{1t}}{d_{2t}}\right)^{\tau}\right]^{1-\alpha}k_t^{\alpha-1}\neq 0$$

for $\dot{g}_{k_t}=0$ we must have:

$$(\tau-0.5)g_{d_{1t}}-(\tau+0.5)g_{d_{2t}}-g_{k_t}=0$$

i.e.:

$$(\tau-0.5)[(1-\eta)p_1+\eta p_2]-(\tau+0.5)[(1+\eta)p_2-\eta p_1]-g_{k_t}=0\Leftrightarrow$$

$$\Leftrightarrow[(\tau-0.5)(1-\eta)+(\tau+0.5)\eta]p_1+[(\tau-0.5)\eta-(\tau+0.5)(1+\eta)]p_1-g_{k_t}=0$$

The steady state growth rate of physical capital per-capita is then:

$$g_{k_t^*}=(\tau-0.5+\eta)p_1-(\tau+0.5+\eta)p_2 \qquad (1.21)$$

In steady state $g_{k_t}=g_{k_t^*}$.

Then, from (1.20) and (1.21) together, we get:

$$s\left[\frac{1}{d_t}\left(\frac{d_{1t}}{d_{2t}}\right)^{\tau}\right]^{1-\alpha}k_t^{\alpha-1}-p=(\tau+\eta)(p_1-p_2)-p$$

The steady state levels of physical capital per-capita are then:

$$k_t^*=\frac{1}{d_t}\left(\frac{d_{1t}}{d_{2t}}\right)^{\tau}\left[\frac{s}{(\tau+\eta)(p_1-p_2)}\right]^{\frac{1}{1-\alpha}} \qquad (1.22)$$

Log-time-differentiating equation (1.18):

$$g_{y_t} = \alpha g_{k_t} + (1-\alpha)[(\tau+\eta)(p_1 - p_2) - 0.5(p_1 + p_2)]$$

and recalling (1.21):

$$g_{y_t} = \alpha g_{k_t} + (1-\alpha) g_{k_t^*} \tag{1.23}$$

meaning that the steady state growth rate of output per-capita is equal to the growth rate of physical capital per-capita:

$$g_{y_t^*} = g_{k_t^*} \tag{1.24}$$

Introducing (1.22) in (1.18) we obtain the steady state levels of output per-capita:

$$y_t^* = \left[\frac{1}{d_t}\left(\frac{d_{1t}}{d_{2t}}\right)^{\tau}\right]^{1-\alpha} \left\{\frac{1}{d_t}\left(\frac{d_{1t}}{d_{2t}}\right)^{\tau} \left[\frac{\eta s}{n(\tau+\eta)}\right]^{\frac{1}{1-\alpha}}\right\}^{\alpha}$$

then:

$$y_t^* = \frac{1}{d_t}\left(\frac{d_{1t}}{d_{2t}}\right)^{\tau} \left[\frac{s}{(\tau+\eta)(p_1 - p_2)}\right]^{\frac{\alpha}{1-\alpha}} \tag{1.25}$$

As consumption is a proportion of output; i.e.:

$$c_t = (1-s) y_t \tag{1.26}$$

it follows that the steady state growth rate of consumption per-capita is equal to the growth rate of output per-capita; i.e.:

$$g_{c_t} = g_{y_t} \tag{1.27}$$

and:

$$g_{c_t^*} = g_{y_t^*} \tag{1.28}$$

From (1.25) and (1.26) it follows that the steady state levels of the consumption per-capita are:

$$c_t^* = \frac{1-s}{d_t}\left(\frac{d_{1t}}{d_{2t}}\right)^{\tau}\left[\frac{s}{(\tau+\eta)(p_1-p_2)}\right]^{\frac{\alpha}{1-\alpha}} \tag{1.29}$$

The model's steady state is characterized by equations (1.21), (1.22), (1.24), (1.25), (1.28) and (1.29).

1.4 Transitional Dynamics

1.4.1 Growth Trajectories

Firstly, we must note that the existence of a steady state requires some restrictions on the parameters. We choose not to restrict our parameter values, as it is our wish to show how, under certain demographic conditions, one economy can collapse.

Let us consider the variables in per-capita units.

According to (1.21), for $g_{k_t^*} = 0$:

$$p_1 = \frac{\tau+\eta+0.5}{\tau+\eta-0.5}p_2$$

We will therefore use:

$$p_1 = \beta p_2 \tag{1.30}$$

with:

$$\beta = \frac{\tau + \eta + 0.5}{\tau + \eta - 0.5} \tag{1.31}$$

as a central condition of the model.

Indeed, because it represents stagnation, (1.30) describes the frontier between sustainability and unsustainability of economic growth, that is, between positive growth (growth) and negative growth (collapse).

Given the production function (1.18):

$$y_t = \left[\frac{1}{d_t}\left(\frac{d_{1t}}{d_{2t}}\right)^{\tau}\right]^{1-\alpha} k_t^{\alpha}$$

the evolution of output per-capita with time can be described, in general terms, as:

$$\frac{\partial y_t}{\partial t} = \frac{\partial y_t}{\partial k_t}\frac{\partial k_t}{\partial t} + \frac{\partial y_t}{\partial d_{1t}}\frac{\partial d_{1t}}{\partial t} + \frac{\partial y_t}{\partial d_{2t}}\frac{\partial d_{2t}}{\partial t}$$

which is equivalent to equation (1.23):

$$g_{y_t} = \alpha g_{k_t} + (1-\alpha) g_{k_t^*}$$

Equation (1.23) explains the different growth trajectories, according to the relative position of p_1 and βp_2.

From (1.18), we get the marginal productivity of physical capital as:

$$\frac{\partial y_t}{\partial k_t} = \alpha \left[\frac{1}{d_t}\left(\frac{d_{1t}}{d_{2t}}\right)^{\tau}\right]^{1-\alpha} k_t^{\alpha-1}$$

and

$$\frac{\partial^2 y_t}{\partial k_t^2} = \alpha(\alpha-1) \left[\frac{1}{d_t}\left(\frac{d_{1t}}{d_{2t}}\right)^{\tau}\right]^{1-\alpha} k_t^{\alpha-2}$$

At least for the initial levels of k_t, we know that:

$$\frac{\partial y_t}{\partial k_t} > 0$$

and:

$$\frac{\partial^2 y_t}{\partial k_t^2} < 0$$

meaning that, for that range of k_t, the production function exhibits diminishing returns to physical capital.

As the growth rate of physical capital is given by (1.20), recall:

$$g_{k_t} = s \left[\frac{1}{d_t}\left(\frac{d_{1t}}{d_{2t}}\right)^{\tau}\right]^{1-\alpha} k_t^{\alpha-1} - 0.5(p_1 + p_2)$$

it becomes equivalent to:

$$g_{k_t} = \frac{s}{\alpha}\frac{\partial y_t}{\partial k_t} - 0.5(p_1 + p_2)$$

and consequently, for the initial values of k_t, when we are sure that the diminishing returns hold as k_t and $g_{k_t} > 0$, as $p = 0.5(p_1 + p_2)$ is constant, the component αg_{k_t} of g_{y_t} in equation (1.23) exhibits diminishing returns on k_t.

Then, as $(1-\alpha)g_{k_t^*}$ is a constant, the g_{y_t} starts at high values and decreases, convexly, at least for the initial values of positive values of k_t, because:

$$\frac{\partial g_{y_t}}{\partial k_t} = \frac{s}{\alpha}\frac{\partial^2 y_t}{\partial k_t^2} < 0$$

and:

$$\frac{\partial^2 g_{y_t}}{\partial k_t^2} = \frac{s}{\alpha}\frac{\partial^3 y_t}{\partial k_t^3} = s(\alpha-1)(\alpha-2)\left[\frac{1}{d_t}\left(\frac{d_{1t}}{d_{2t}}\right)^{\tau}\right]^{1-\alpha} k_t^{\alpha-3} > 0$$

Given the behaviour of component αg_{k_t}, we are ready to make a full description of the economy, according to the demographic setting in case.

Next, we analyse the three scenarios, viz., when $p_1 = \beta p_2$ (Case 1), when $p_1 > \beta p_2$ (Case 2) and when $p_1 < \beta p_2$ (Case 3).

Case 1: $\boldsymbol{p_1 = \beta p_2}$

When $p_1 = \beta p_2$, $g_{k_t^*} = 0$, then $g_{y_t} = \alpha g_{k_t}$.

In this case, the theoretical $k_t^* > 0$, because $p_1 > p_2$, as $\beta > 1$. Then the problem of the range of k_t does not arise (See Figure 1.2).

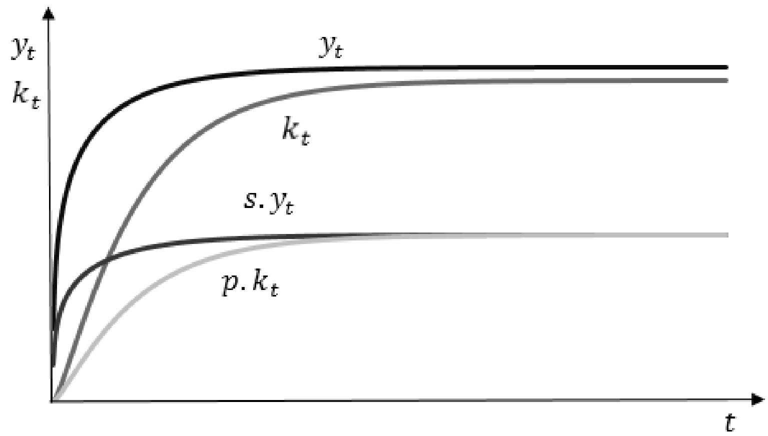
Starting from $k_0 < k_t^*$, the first g_{y_t} are the highest. The first marginal amounts of y_t are relatively high.

As k_t grows, new additions of k_t lead to lower $g_{y_t} > 0$ because of diminishing marginal returns to physical capital. So y_t increases, increasingly less, and so do savings sy_t. Same for the stock of physical.

As the effective depreciation pk_t is linear in k_t, it grows faster than the savings sy_t, implying that, somewhere in time, the sy_t and pk_t will equalize.

Then $g_{k_t} = 0$, as well as $g_{y_t} = \alpha g_{k_t} = 0$. The steady state of the economy is characterized by stagnation of output, physical capital and consumption per-capita.

Figure 1.2 Time-evolution of output, physical capital, savings and effective depreciation for the case of a long-term zero-growth economy. Variables per-capita.



Case 2: $p_1 > \beta p_2$

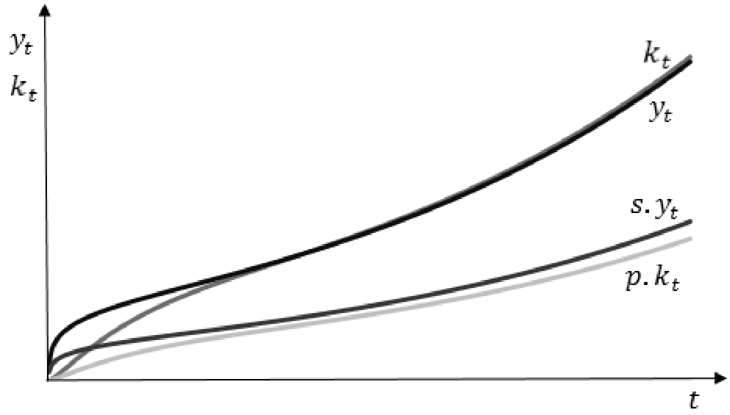
In this case the problem of the range of k_t, also, does not arise. As $g_{k_t^*} > 0$, then k_t^* is positive and rising on the steady state.

When $p_1 > \beta p_2$, the relationship $g_{y_t} = \alpha g_{k_t} + (1-\alpha) g_{k_t^*}$ involves a diminishing returns to capital component αg_{k_t} and a positive constant demographically determined

$(1-\alpha)g_{k_t^*} > 0$. Consequently, the growth rate of output per-capita g_{y_t} decreases convexly up to the point when $g_{k_t} = g_{k_t^*}$, leading to the result: $g_{y_t^*} = g_{k_t^*} > 0$.

This process is depicted in Figure 1.3.

Figure 1.3 Time-evolution of output, physical capital, savings and effective depreciation for the case of a long-term positive-growth economy. Variables per-capita.

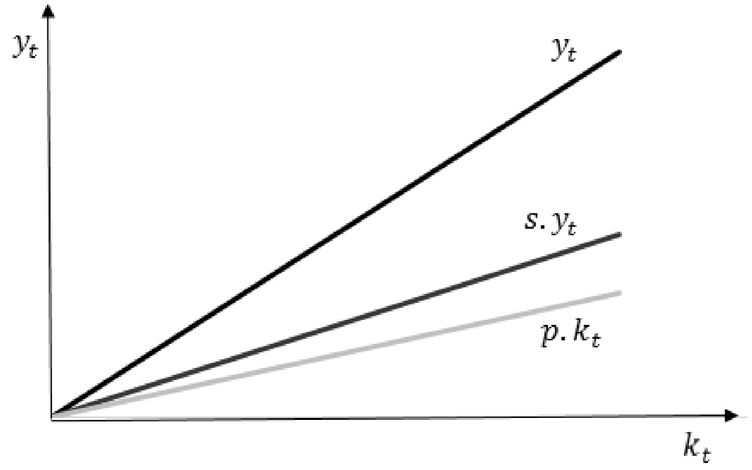


The transition process is basically similar to the case when $p_1 = \beta p_2$, although the demographic setting ensures that, once the diminishing returns on physical capital are through, savings continue to be higher than effective depreciation.

This means that, when $p_1 > \beta p_2$, the contribution of rejuvenation to the technological progress surpasses the technological erosion due to ageing and the erosion due to the increase in the dependent population.

In other words, when $p_1 > \beta p_2$, the diminishing returns of physical capital end up being surpassed by the positive contribution of the demographic trends. Figure 1.4 illustrates this mechanism: The evolution of y_t with k_t is like that exhibited by the AK model.

Figure 1.4 Evolution of output, savings and effective depreciation with physical capital for the case of a long-term positive-growth economy. Variables per-capita.



Case 3: $\boldsymbol{p_1 < \beta p_2}$

When $p_1 < \beta p_2$, equation $g_{y_t} = \alpha g_{k_t} + (1-\alpha) g_{k_t^*}$ has an initially positive decreasing component αg_{k_t} plus a negative constant $(1-\alpha) g_{k_t^*}$, since on the steady state (that does not exist) $g_{k_t^*} < 0$.

Then, while:

$$g_{k_t} > -\frac{(1-\alpha)}{\alpha} g_{k_t^*}$$

the αg_{k_t} is higher than $(1-\alpha) g_{k_t^*}$ in absolute value. Hence $g_{y_t} > 0$, and the economy grows.

When:

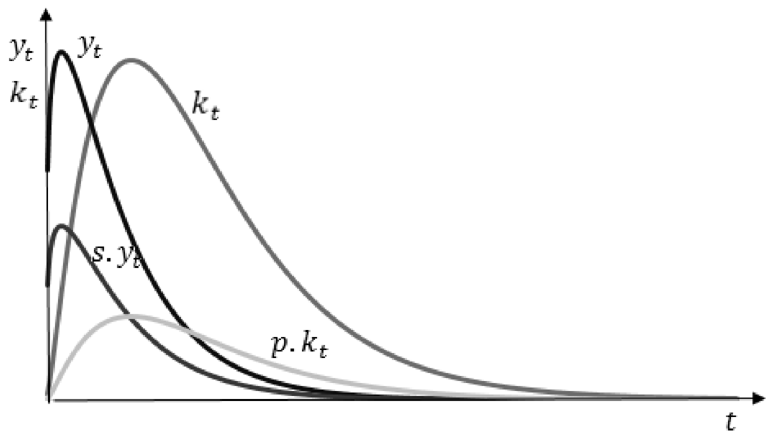
$$g_{k_t} < -\frac{(1-\alpha)}{\alpha} g_{k_t^*}$$

the negative effects of the demographic setting overcome eventual positive (though decreasing) augments of output and output starts to fall.

The decrease in output y_t leads to a decrease in savings sy_t faster than that of effective depreciation pk_t. Consequently, the economy starts decapitalizing continuously and, if no demographic measures are taken to reverse the situation, it tends to collapse.

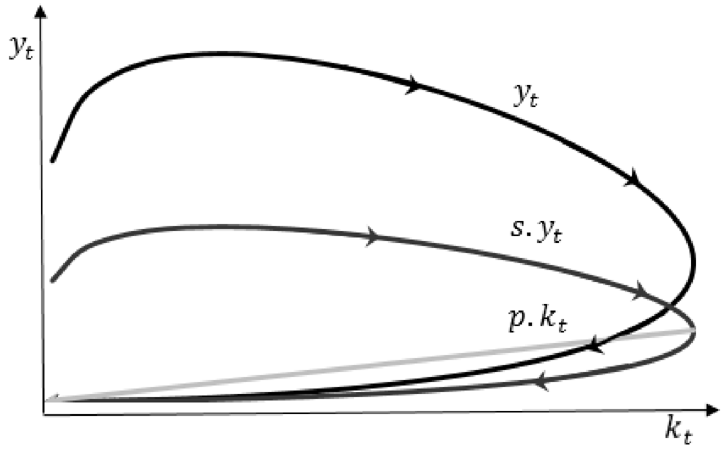
This process is depicted in Figure 1.5.

Figure 1.5 Time-evolution of output, physical capital, savings and effective depreciation in the case of a long-term negative-growth economy. Variables per-capita.



This result in particular was our proposed hypothesis that when a country undergoes a severe ageing process, it will end up experiencing long-term negative growth rates due to technological regress. Nothing else done, such economy tends to collapse, following the pattern exhibited on Figure 1.6.

Figure 1.6 Evolution of output, savings and effective depreciation with physical capital in the case of a long-term negative-growth economy. Variables per-capita.



1.4.2 Policy Experiments

According to its demographic setting, one economy will exhibit positive growth, stagnation or a collapsing trajectory. The dynamics underlying such evolutions can be summarized by the depiction of the growth rate of physical capital with physical capital, as the economy is driven by physical capital accumulation.

In cases 1 and 2, that is when $p_1 = \beta p_2$ and when $p_1 > \beta p_2$, we know that:

$$\frac{\partial g_{k_t}}{\partial k_t} < 0$$

and:

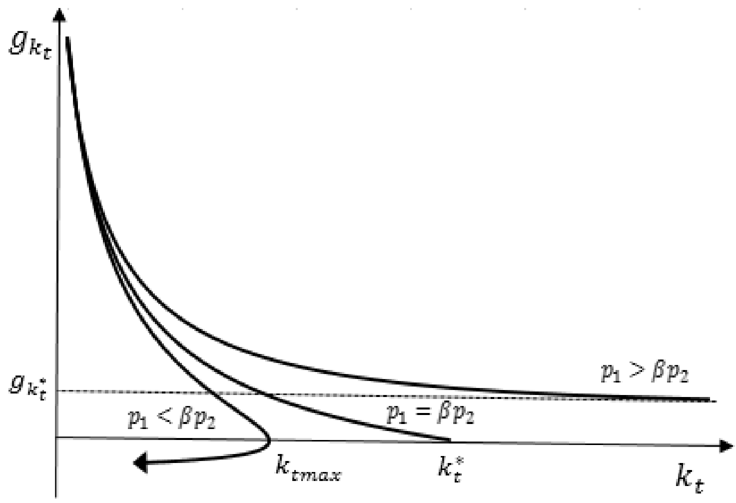
$$\frac{\partial^2 g_{k_t}}{\partial k_t^2} > 0$$

Then the shape of g_{k_t} will be convex and decreasing, so that for $p_1 = \beta p_2$, g_{k_t} will converge to $g_{k_t^*} = 0$ at $k_t = k_t^*$; and for $p_1 > \beta p_2$, we have $g_{k_t^*} > 0$.

In case 3, that is, when $p_1 < \beta p_2$, the behavior of g_{k_t} will be the same until it reaches the point when $g_{k_t} = 0$, which corresponds to the highest level of physical capital achieved. Then g_{k_t} decreases with k_t, tending to $g_{k_t^*} < 0$ and $k_t^* < 0$, respectively.

These growth rates are exhibited in Figure 1.7.

Figure 1.7 Evolution of the growth rates of physical capital with physical capital for the different demographic settings.



Savings Policy

The derivative of g_{k_t} equation (1.20) in order to s, is:

$$\frac{\partial g_{k_t}}{\partial s} = \left[\frac{1}{d_t}\left(\frac{d_{1t}}{d_{2t}}\right)^{\tau}\right]^{1-\alpha} k_t^{\alpha-1} > 0$$

meaning that increasing(decreasing) the savings rate s will shift g_{k_t} to a curve upward(downward), followed by a decrease(increase) up to the original $g_{k_t^*}$.

The conclusion that changes in the savings rate do not affect the steady state growth rates is a natural consequence of the steady state growth rate equation (1.21) recalling:

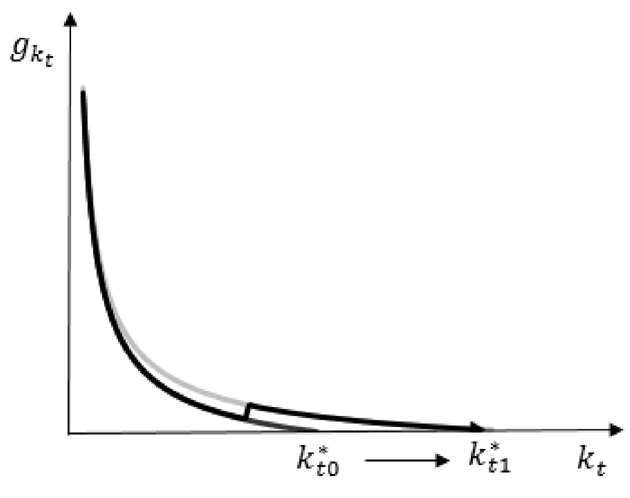
$$g_{k_t^*} = (\tau - 0.5 + \eta)p_1 - (\tau + 0.5 + \eta)p_2$$

The equation of $g_{k_t^*}$ does not include s, thus $g_{k_t^*}$ is insensitive to savings policy. Although $g_{k_t^*}$ does not change, level k_t^* changes.

Let us look at the case of an increase in s, knowing that a decrease will exhibit exactly the opposite evolution.

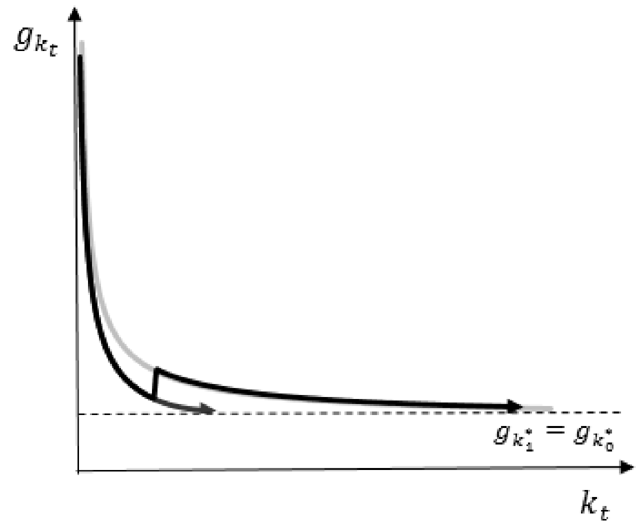
Case 1: When $p_1 = \beta p_2$, an increase in s leads to a higher value of g_{k_t}, leading to a higher growth of k_t up to its new steady state level k_t^*. Then $g_{k_1^*} = g_{k_0^*} = 0$ and $k_1^* > k_0^*$ (See Figure 1.8).

Figure 1.8 The effect of a permanent increase in the saving rate in a long-term zero-growth economy.



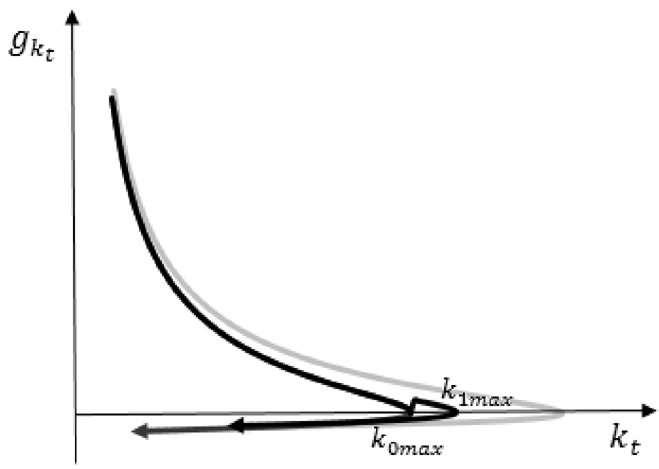
Case 2: If $p_1 > \beta p_2$, an increase in s will lead to a new trajectory of growth of k_t^*, placing the new balanced growth path always beyond the original steady state levels (Figure 1.9).

Figure 1.9 The effect of a permanent increase in the saving rate in a long-term positive-growth economy.



Case 3: Finally, when $p_1 < \beta p_2$, an increase in s leads to a delay of the collapse; i.e., it is like if the economy was running to the abyss, though, some steps behind (See Figure 1.10).

Figure 1.10 The effect of a permanent increase in the saving rate in a long-term negative-growth economy.



Summing up, in what concerns steady state growth trajectories, investment (savings) policies are neutral.

Demographic Policy

In what follows, we will use notations x_{t0} and x_{t1} to name the initial and final states of any variable x_t. We will be ignoring the short-term consequences of any type of the above mentioned demographic shocks.

When the growth rates of subpopulations P_{1t} and P_{2t} vary, the economy experiences a demographic shock and transits from one steady state to another (if it exists) or, at least for another growth path.

Concerning the steady state growth rates, we deduct that $g_{k^*_{t1}} = g_{k^*_{t0}}$ is equivalent to:

$$(\tau - 0.5 + \eta)p_{11} - (\tau + 0.5 + \eta)p_{21} = (\tau - 0.5 + \eta)p_{10} - (\tau + 0.5 + \eta)p_{20} \Leftrightarrow$$

$$\Leftrightarrow (\tau - 0.5 + \eta)(p_{11} - p_{10}) = (\tau + 0.5 + \eta)(p_{21} - p_{20})$$

Defining:

$$\Delta p_j = p_{j1} - p_{j0}$$

with $j = 1{,}2$ and Δp_j representing the percentage points more in the growth rate of the age structure j, we conclude that:

(i) $g_{k^*_{t1}} = g_{k^*_{t0}}$ when:

$$\Delta p_1 = \beta \Delta p_2 \qquad (1.32)$$

(ii) $g_{k_{t1}^*} > g_{k_{t0}^*}$ when:

$$\Delta p_1 > \beta \Delta p_2 \tag{1.33}$$

(iii) $g_{k_{t1}^*} < g_{k_{t0}^*}$ when:

$$\Delta p_1 < \beta \Delta p_2 \tag{1.34}$$

The value of β is given by equation (1.31), recalling:

$$\beta = \frac{\tau + \eta + 0.5}{\tau + \eta - 0.5}$$

If a demographic shock leads to $g_{k_{t1}^*} > g_{k_{t0}^*}$, we know with certainty that $k_{t1}^* > k_{t0}^*$. Likewise, if $g_{k_{t1}^*} < g_{k_{t0}^*}$, we have $k_{t1}^* < k_{t0}^*$. Even when one economy does not have a steady state before and after the demographic change (i.e., when $p_{1t} < \beta p_{2t}$, with $t = 0{,}1$), conditions $g_{k_{t1}^*} > g_{k_{t0}^*}$ and $g_{k_{t1}^*} < g_{k_{t0}^*}$ mean a slower and a faster decapitalization, respectively. The long-term consequences of the demographic shocks of type (1.33) and (1.34) have thus been identified. However, when the demographic change is of type (1.32), all we know is that $g_{k_t^*}$ remains in its initial level; we do not know the position of the final k_t^* relative to the initial.

To understand what happens during the adjustment after a demographic shock, as well as the variation on the levels of k_t^* after a demographic shock of type (1.32), let us recall, once again, equation (1.20):

$$g_{k_t} = s\left[\frac{1}{d_t}\left(\frac{d_{1t}}{d_{2t}}\right)^{\tau}\right]^{1-\alpha} k_t^{\alpha-1} - 0.5(p_1 + p_2)$$

Defining:

$$\Sigma_t = \left[\frac{1}{d_0}\left(\frac{d_{10}}{d_{20}}\right)^{\tau}\right]^{1-\alpha} k_t^{\alpha-1}$$

equation (1.20) becomes:

$$g_{k_t} = \Sigma_t e^{(1-\alpha)[(\tau-0.5+\eta)p_1-(\tau+0.5+\eta)p_2]t} - 0.5(p_1+p_2)$$

The total differential of g_{k_t} for changes on p_1 and p_2 is:

$$\Delta g_{k_t} = \frac{\partial g_{k_t}}{\partial p_1}\Delta p_1 + \frac{\partial g_{k_t}}{\partial p_2}\Delta p_2$$

equivalent to:

$$\Delta g_{k_t} = \left[(1-\alpha)(\tau+\eta-0.5)t\Sigma_t e^{(1-\alpha)[(\tau-0.5+\eta)p_1-(\tau+0.5+\eta)p_2]t} - 0.5\right]\Delta p_1 - \left[(1-\alpha)(\tau+\eta+0.5)t\Sigma_t e^{(1-\alpha)[(\tau-0.5+\eta)p_1-(\tau+0.5+\eta)p_2]t} - 0.5\right]\Delta p_2$$

Then, at the moment of the demographic shock $t \approx 0$, we can consider that:

$$\Delta g_{k_t} \approx -0.5(\Delta p_1 + \Delta p_2) = -\Delta p \tag{1.35}$$

This result tells us that right after the shock, the growth rate jumps in the opposite direction of that of the population growth rate shock.

If $\Delta p_1 + \Delta p_2$ is negative (positive), g_{k_t} jumps to a value above (under) its initial curve. This happens due to a higher effective depreciation sensitivity relative to savings sensitivity, to the demographic shock, immediately after the shock. The instantaneous impact of demographic change in the growth rate of physical capital is mostly reflected on the variation of effective depreciation[19]. Then, savings start to grow faster than depreciation and the adjustment to the final steady state values proceeds.

Table 1.1 summarizes the short-term and long-term effects of shifts on the age structure of a country's population, given an initial demographic arrangement.

Table 1.1 Short-term and long-term results of demographic change for different initial demographic conditions

Initial conditions	Demographic change	Final conditions	Transition	Steady state	
				Growth rate	Levels
$\boldsymbol{p_{10} = \beta p_{20}}$	$\Delta p_1 = \beta \Delta p_2 < 0$	$p_{11} = \beta p_{21}$	$g_{k_t} \uparrow$	$g_{k_t^*} \rightarrow$	$k_t^* \uparrow$
	$\Delta p_1 = \beta \Delta p_2 > 0$		$g_{k_t} \downarrow$		$k_t^* \downarrow$
	$\Delta p_1 > \beta \Delta p_2 < 0$	$p_{11} > \beta p_{21}$	$g_{k_t} \uparrow$	$g_{k_t^*} \uparrow$	$k_t^* \uparrow$
	$\Delta p_1 > \beta \Delta p_2 > 0$		$g_{k_t} \downarrow$		
	$\Delta p_1 < \beta \Delta p_2 < 0$	$p_{11} < \beta p_{21}$	$g_{k_t} \uparrow$	$g_{k_t^*} \downarrow$	$k_t^* \downarrow$
	$\Delta p_1 < \beta \Delta p_2 > 0$		$g_{k_t} \downarrow$		
$\boldsymbol{p_{10} > \beta p_{20}}$	$\Delta p_1 = \beta \Delta p_2 < 0$	$p_{11} > \beta p_{21}$	$g_{k_t} \uparrow$	$g_{k_t^*} \rightarrow$	$k_t^* \uparrow$
	$\Delta p_1 = \beta \Delta p_2 > 0$	$p_{11} > \beta p_{21}$	$g_{k_t} \downarrow$		$k_t^* \downarrow$
	$\Delta p_1 > \beta \Delta p_2 < 0$	$p_{11} > \beta p_{21}$	$g_{k_t} \uparrow$	$g_{k_t^*} \uparrow$	$k_t^* \uparrow$
	$\Delta p_1 > \beta \Delta p_2 > 0$	$p_{11} > \beta p_{21}$	$g_{k_t} \downarrow$		
	$\Delta p_1 < \beta \Delta p_2 < 0$	$p_{11} ? \beta p_{21}$	$g_{k_t} \uparrow$	$g_{k_t^*} \downarrow$	$k_t^* \downarrow$
	$\Delta p_1 < \beta \Delta p_2 > 0$	$p_{11} ? \beta p_{21}$	$g_{k_t} \downarrow$	$g_{k_t^*} \downarrow$	$k_t^* \downarrow$
$\boldsymbol{p_{10} < \beta p_{20}}$	$\Delta p_1 = \beta \Delta p_2 < 0$	$p_{11} < \beta p_{21}$	$g_{k_t} \uparrow$	$g_{k_t^*} \rightarrow$	$k_t^* \uparrow$
	$\Delta p_1 = \beta \Delta p_2 > 0$	$p_{11} < \beta p_{21}$	$g_{k_t} \downarrow$		$k_t^* \downarrow$
	$\Delta p_1 > \beta \Delta p_2 < 0$	$p_{11} ? \beta p_{21}$	$g_{k_t} \uparrow$	$g_{k_t^*} \uparrow$	$k_t^* \uparrow$
	$\Delta p_1 > \beta \Delta p_2 > 0$	$p_{11} ? \beta p_{21}$	$g_{k_t} \downarrow$		
	$\Delta p_1 < \beta \Delta p_2 < 0$	$p_{11} < \beta p_{21}$	$g_{k_t} \uparrow$	$g_{k_t^*} \downarrow$	$k_t^* \downarrow$
	$\Delta p_1 < \beta \Delta p_2 > 0$	$p_{11} < \beta p_{21}$	$g_{k_t} \downarrow$	$g_{k_t^*} \downarrow$	$k_t^* \downarrow$

Notation: ↑ = increase; → = maintenance; ↓ = decrease; ? = undetermined.

[19] We must remark that, theoretically, when the decrease in the population is accentuated, a perverse effect that consists in the growth of the physical capital due to a lot less effective depreciation, can occur. This is as perverse as a zero-growth steady state or balanced deficits with no population. These scenarios are theoretical and do not fit reality, hence they are implicitly excluded in our analysis.

1.5 Notes on the European Union

Our here developed model explores the effects of age structure on economic growth. According to the model, the evolution of the age structures of the population are determinant factors of the steady state growth trajectories of countries. We use the expression "steady state" also for the cases in which it does not exist, one of the possible outcomes of the model.

Condition (1.30), $p_1 = \beta p_2$, corresponds to the frontier between positive and negative long-term growth.

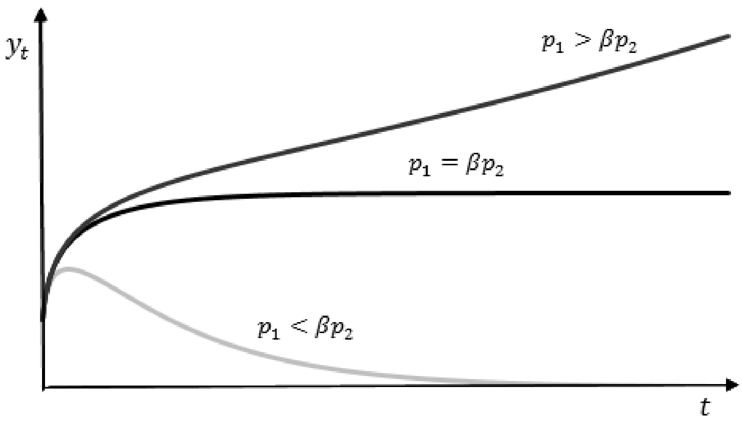
The different growth trajectories, exhibited in Figure 1.11, identify a major motive of concern for the case $p_1 < \beta p_2$. In fact, when $p_1 < \beta p_2$, the economy follows a collapse trajectory. After a relatively short period of increase, output per-capita y_t decays continuously.

The case of $p_1 = \beta p_2$ itself is not a comfortable one either. It is on the edge, hence, a very small decrease in p_1, or increase in p_2, will lead the economy to its collapse trajectory. Such possible scenarios, sustain our belief that demographic policy ought to be brought into the centre of political agenda of the European Union.

As an illustration, we will next examine some European Union demographic trends regarding age structures. We wish to characterize demographically the European Union and place it in a particular path of economic growth in our proposed model.

The here proposed model prescribes implicitly general policy guidelines in the last part of this Section.

Figure 1.11 Evolution of output per-capita for different demographic settings.



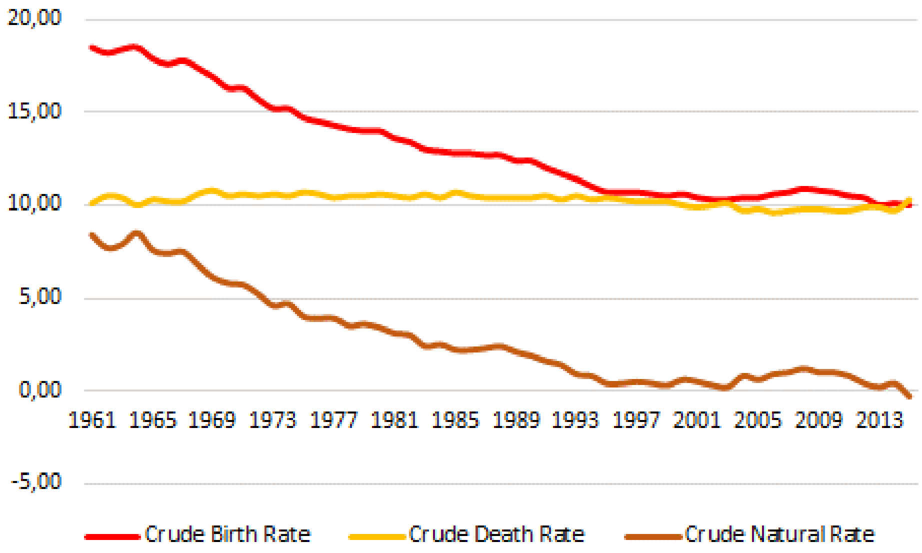
1.5.1 Demographic Trends in the European Union

Crude natural rates

Considering the period between 1961 and 2015, we find that the yearly crude birth rates have decreased from 18.5% in 1961 to 10% in 2015, while the crude death rates have remained stable at values between 9% and 11%. This combination has originated a continuous decrease in the crude natural rate, from 8.1% in 1961 to -0.3% in 2015 (See Figure 1.12).

The crude natural growth rate of the European native population has exhibited, since 1994, values below 1%, with no exception. In 2015, the native population of the European Union, which includes the descendants of immigrants and the immigrants naturalized, has started to decrease (See Figure 1.12).

Figure 1.12 Crude birth rates, crude death rates and crude natural rates for the 28 members of the European Union together: 1961-2015 (%).



Note: Own presentation from Eurostat data.

According to data collected for a sample of 11 European Union countries, the crude natural rates have been decreasing, almost continuously, throughout the decades under observation. In this dimension, some countries exhibit very worrisome natural rates indeed. Concerning the time trend, the worst case of the sample appears to be Germany, which has exhibited negative crude natural rates since the 1980s. Then we have the case of Italy that, after almost stagnation in 1990, exhibits consecutively negative crude natural rates (See Table 1.2).

Considering the rates registered in 2015 for the 11 countries sample, the worst cases are those of Italy and Greece with -2.7%, Germany with -2.3%, Portugal with -2.2% and Spain with -0.1%. All the other countries like Austria (0.3%), Finland (0.5%), Belgium (1%), Denmark (1%) and The Netherlands (1.4%) have exhibited stagnation/very low growth of population[20] (See Table 1.2). These findings are in line with the graphic illustration in Figure 1.12 of general downsizing of the population.

[20] Crude natural growth rates in parenthesis.

Table 1.2 Crude natural rates for 11 selected European Union countries: 1960, 1970, 1980, 1990, 2000, 2008 and 2015 (%).

	1960	1970	1980	1990	2000	2008	2015
Belgium	4,3	2,4	1,1	2,0	1,1	2,1	1,0
Denmark	7,1	4,6	0,3	0,5	1,7	1,9	1,0
Germany	5,3	0,9	-1,1	-0,2	-0,9	-2,0	-2,3
Greece	11,6	8,1	6,3	0,8	-0,2	0,9	-2,7
Spain	13,1	11,3	7,5	1,8	0,9	2,9	-0,1
France	6,5	6,0	4,7	4,2	4,1	4,2	2,8
Italy	8,6	7,1	1,5	0,5	-0,2	-0,1	-2,7
Netherlands	13,2	9,9	4,7	4,6	4,2	3,0	1,4
Austria	5,2	1,8	-0,2	1,0	0,2	0,3	0,2
Portugal	13,4	10,1	6,5	1,4	1,4	0,0	-2,2
Finland	9,6	4,4	3,9	3,1	1,4	2,0	0,5

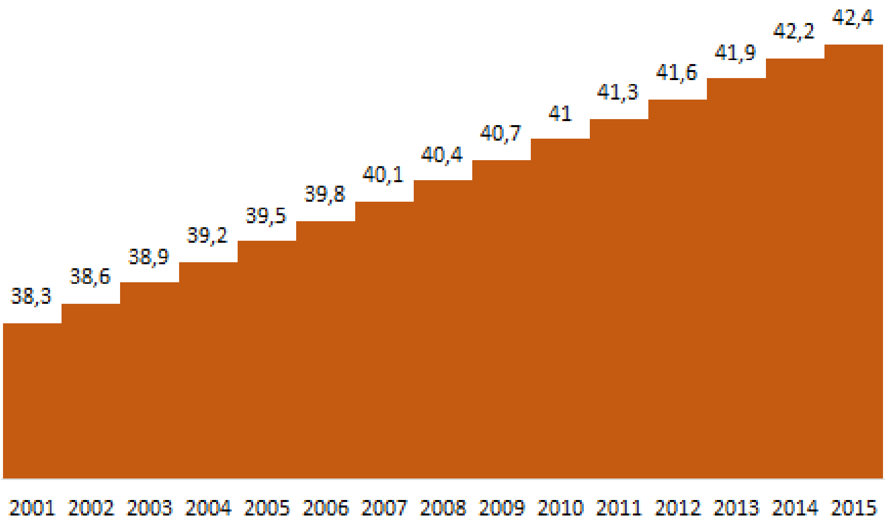
Note: Own presentation from Eurostat data.

Median age

The motives for concern are not only related with the decay in the size of the European population. The advanced ageing is very worrying. Ageing together with decay in size of a country's population have, inexorably, devastating economic outcomes. Labour supply shortages, social security and public debt are expected to worsen, causing socioeconomic unsustainability. Figure 1.13 depicts the evolution of the median age in the European Union, over the last 15 years.

The median age of the resident population in the European Union has increased for 4.1 years from 2001 to 2015, from 38.3 to 42.4 years old. The yearly growth is steady, and evolves like the steps of a staircase, according to an arithmetic progression with common difference of 0.3 years each year (with the unique exception of 0.2 years between 2014 and 2015) (See Figure 1.13).

Figure 1.13 Median age of the population in the European Union: 2001-2015 (Years Old).



Note: Own presentation from Eurostat data.

Looking at the national level for our sample of 11 countries, we find that the youngest country (France) exhibits a median age of 41.2 years. The oldest countries are Germany with 45.9, Italy with 45.1, Portugal with 43.5, Greece with 43.4 and Austria with 43.0. Finland exhibits the same median age of the European Union with 42.4, close to Spain, 0.1 years younger (See Table 1.3).

Table 1.3 Median age of 11 selected European Union countries: 1960, 1970, 1980, 1990, 2000, 2008 and 2015 (Years Old).

	1960	1970	1980	1990	2000	2008	2015
Germany	34,8	34,0	36,6	37,6	39,8	43,2	45,9
Italy	31,2	32,7	33,8	36,9	40,1	42,7	45,1
Portugal	27,8	29,4	30,4	33,9	37,5	40,4	43,5
Greece	31,2	33,9	34,0	36,0	38,5	40,3	43,4
Austria	35,5	33,9	34,7	35,6	37,9	40,9	43,0
Finland	28,4	29,4	32,6	36,3	39,2	41,5	42,4
Spain	29,6	30,2	30,7	33,4	37,2	39,2	42,3
Netherlands	28,7	28,5	31,2	34,4	37,3	40,0	42,2
Denmark	33,0	32,5	34,1	37,0	38,2	40,2	41,5
Belgium	35,2	34,5	33,9	36,2	38,7	40,7	41,4
France	33,0	32,5	32,2	34,7	37,6	39,5	41,2

Note: Own presentation from Eurostat data.

Concerning both population dimensions, namely negative growth rates and ageing (which are expected to be positively correlated), in 2015 the European Union exhibited a -0.3% crude natural rate and 42.4 years old in median. In this regard, we find that there are different realities in Europe, some more worrying than others. Amongst the most serious cases we have Germany, Italy, Portugal and Greece (See Figure 1.14).

Figure 1.14 Scatter plot of the centralized median age (horizontal axis) and crude natural rates (vertical axes) of 11 selected European Union countries relative to the European Union values: 2015 (Years Old, p.p.).

Note: Own presentation from Eurostat data.

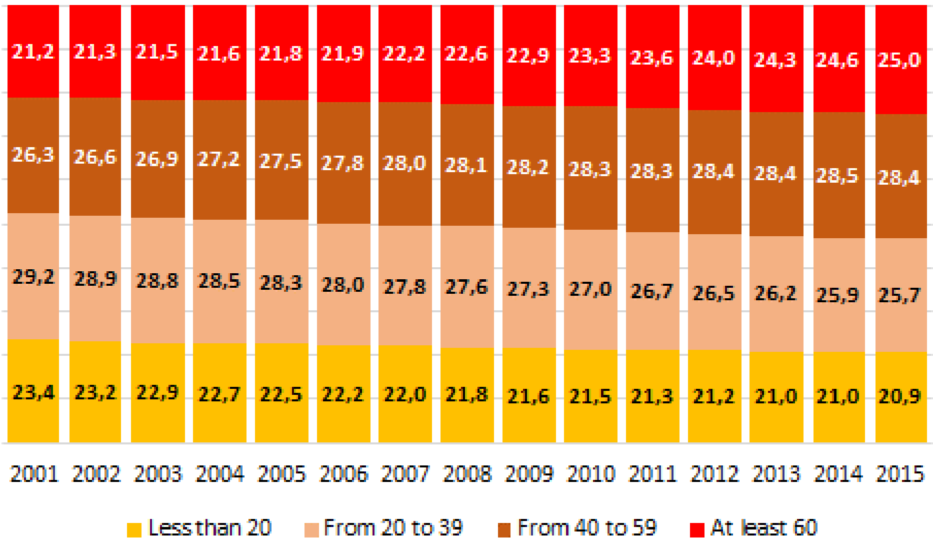
Age structures

The observed evolution of the age structure in the European Union adheres to our starting hypothesis of an ongoing ageing process.

From 2001 to 2015, the share of population from 60 years up has increased from 21.2% to 25% and the share of the population less than 20 years has decreased from 23.4% to 20.9%. In 2001 the youngest population was 2.2 percentage points ahead of the oldest population, whereas in 2015 the youngest became 4.1 percentage points behind (See Figure 1.15).

The labour force composition has also changed[21]. Considering the age classes between 20 and 39 years old and between 40 and 59 years old, in 2001 the workers less than 39 years old represented 52.6% of the workforce, whereas in 2015 they were just 47.5% (Figure 1.23). We must acknowledge that these evolutions do not involve a long period: they reflect the first 15 years of the new century (See Figure 1.15).

Figure 1.15 Age structure of the population in the European Union: 2001-2015 (Years).



Note: Own presentation from Eurostat data.

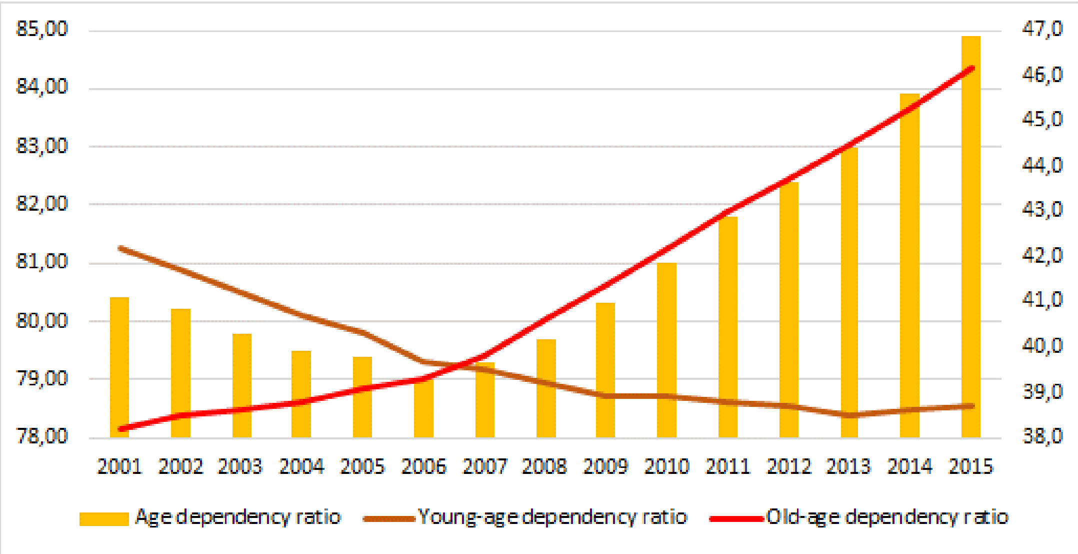
Dependency ratios

The identified changes in the age structure of the European Union population had to affect the evolution of the dependency ratios. Figure 1.16 exhibits their trends.

[21] We are approximating the labour force to population with ages between 20 and 60 years old.

Between 2001 and 2006, the total dependency ratio has fallen from 80.4 to 79.0 as a consequence of: (i) the decrease in the young-age dependency ratio from 42.2 to 39.7; and (ii) an increasing old-age dependency ratio from 38.2 to 39.3, down to values always beneath the first. During this period, the highest fraction of dependent population was that of the young people (See Figure 1.16).

Figure 1.16 Dependency ratio (principal axis), young-age dependency ratio (secondary axis) and old-age dependency ratio (secondary axis) for the European Union: 2001-2015 (%).



Note: Own presentation from Eurostat data.

In 2007, the old-age exceeds the young-age dependency ratio and the total dependency ratio began showing a continuously increasing trend. From 2007 to 2015, the total dependency ratio has risen from 79.3 to an unprecedented 84.9. The young-age dependency ratio has decreased from 39.5 to 38.7, while the old-age dependency ratio has increased from 39.8 to 46.2[22] (See Figure 1.16).

The tendencies identified for the European Union are registered for the 11 countries of the European countries of our sample. Except for Denmark with + 2.7 p.p. and France

[22] The age dependency ratios used correspond to the 2nd variant of the Eurostat, i.e., the population aged 0-19 and 60 and more to population aged 20-59 (for the total); the population aged 0-19 to population 20-59 year (for the young-age); and the population aged 60 and more to population 20-59 year (for the old-age).

with +0.5 p.p., all countries have experienced a decrease in the young-age dependency ratio in the period between 2000 and 2015, although much smaller in size than the overall increase in the old-age dependency ratio. Consequently, the total dependency ratio has also grown for all countries of the sample during this period (See Table 1.4 and Figure 1.17).

Table 1.4 Age-dependency ratio, young-age dependency ratio and old-age dependency ratio for 11 selected European countries: 2000-2015 (%, Var.: Percentage Points).

	Age dependency ratio			Young-age dependency ratio			Old-age dependency ratio		
	2000	2015	Var.	2000	2015	Var.	2000	2015	Var.
Belgium	83,6	87,1	3,5	43,4	42,3	-1,1	40,2	44,8	4,6
Denmark	76,7	91,5	14,8	41,8	44,5	2,7	34,8	47,0	12,2
Germany	79,6	83,7	4,1	38,3	33,4	-4,9	41,3	50,3	9,0
Greece	80,7	86,0	5,3	38,7	36,2	-2,5	42,0	49,8	7,8
Spain	74,4	77,6	3,2	37,4	35,2	-2,2	37,0	42,5	5,5
France	85,9	96,7	10,8	48,0	48,5	0,5	37,9	48,3	10,4
Italy	77,8	85,9	8,1	35,0	34,4	-0,6	42,8	51,5	8,7
Netherlands	74,0	87,4	13,4	42,5	42,4	-0,1	31,6	44,9	13,3
Austria	77,1	77,4	0,3	41,1	34,9	-6,2	36,0	42,6	6,6
Portugal	81,3	85,9	4,6	42,4	36,6	-5,8	38,9	49,3	10,4
Finland	80,2	95,3	15,1	44,4	42,9	-1,5	35,8	52,3	16,5

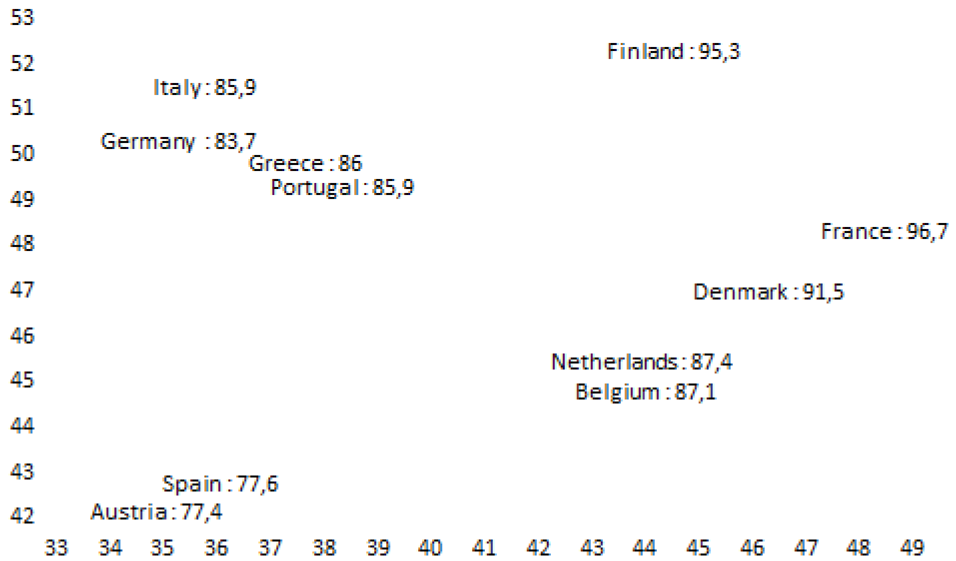
Note: Own presentation from Eurostat data.

The countries that exhibited the highest variations were Finland with 15.1 p.p., Denmark with 14.8 p.p. the Netherlands with 13.4 p.p. and France with 10.8 p.p. Most of such variation was the consequence of the increase in the old-dependency ratio:16.5 p.p. for Finland, 12.2 p.p. for Denmark, 13.3 p.p. for the Netherlands and 10.4 p.p. for France.

Portugal has also registered an increase in 10.4 p.p. in the old-age dependency ratio, although, the decrease of the young-dependency ratio was the second highest, -5.8 p.p., above Germany with -4.9 p.p. and under Austria with - 6.2 p.p.

As the old-dependency ratio increased in Austria for 6.6 p.p., in this country, total dependency ratio almost stagnated, form the second lowest value of 77.1 in 2000 (just above Spain with 74.4) to the lowest value of 77.4 in 2015.

Figure 1.17 Scatter-plot of the young-age dependency ratio (horizontal axis) and the old-age dependency ratio (vertical axis) for 11 selected countries of the European Union: 2015. (%).



Note: Own presentation from Eurostat data. Labels: “Country: total dependency ratio”. The values on the plane correspond to the total dependency ratio.

1.5.2 Policy Recommendations

The above succinctly described population processes in the European Union (Europe) gives us an unambiguous reading: Europe's population is declining and aging.

In our developed model, the European Union is in Case 3, $p_{10} < \beta p_{20}$, then, the model predicts then that there are resilient and increasing forces in the economy driving it to a collapsing growth path.

As analysed in the last section, considering our extended Solow model, savings/investment policies do not solve the problem of continuous economic downturn, they only delay it. Although the stimulus to save more (to invest more) has an impact on capital and output levels, it does not influence the long-run growth trajectories and ultimately is equivalent to a neutral demographic shock $\Delta p_1 = \Delta \beta p_2 < 0$.[23]

Savings policies cause short and medium-term improvements, because transitional growth rates of physical capital (and of output and consumption) are higher. The new steady state levels are higher; but the long-run growth trajectory does not reverse. This result is particularly important when the economy is in a collapse trajectory.

Political cycles may persuade governments to use savings/investment policies, however according to our results, next generations will endure the consequences of a systematic delay of long-term effective solutions. Amongst these delaying solutions, we find the generalized recommendation of more investments in ICT.

Investments in ICT are investments and, as such, will not solve the structural problem that with decaying labour, itself with fewer experience and smaller amounts of fresh skills, Services will experience decreasing productivity. Microchip technology adds nothing

[23] See Table 1.1.

when there is no one to work with it. In the end, this is a problem of both quantity and quality of human resources.

If the European Union has intrinsic demographic forces leading the region to decay, the long-term solution prescribed by the model consists in a demographic shock of the type $\Delta p_1 > \Delta\beta p_2$. As p_1 is significantly lower than p_2 the shock must be big enough to revert $p_{10} < \beta p_{20}$ into $p_{11} > \beta p_{21}$. As exposed in Table 1.1 the result of $\Delta p_1 > \Delta\beta p_2$ departing from $p_{10} < \beta p_{20}$ is inconclusive.

As $p_2 > 0$ is very high, it would be convenient to reduce the population of retirees. However, there is not much more that can be done to address this goal. In fact, the only way to reduce p_2 consists in increasing the age of retirement. Yet, in European Union, the age of retirement is already high. For instance, regarding the 11 European countries of our sample, 8 of them have retirement ages of 65 years old[24]. The retirement age in Portugal and in Italy is 66 years old and in Greece, 67 years old. Furthermore, many of them predict an increase in the retirement age to 67 years old during the 2020s and are, also, contemplating the indexation of the retirement age to life expectancy. So, much of the effort that can be done to decrease p_2 is being done, although, it does not seem to have a substantial impact. A United Nations' study concluded that to preserve the support ratios at the level they were in the beginning of the century for three decades – and they were much lower than they are today as we saw above – the retirement age would have to increase up to 75 years old (United Nations, 2001). Thus, there is not much to do concerning lowering the p_2.

[24] In Austria the retirement age for men is 65 y.o. and for women is 60 y.o.; though, in 2033 both retirement ages will equalize. In Finland we have considered 65 y.o. because it has a scheme that, in some cases, concedes retirement age at 62 y.o. and in others, at 68 y.o. In Germany, in the Netherlands and in Spain it is 3 months and 65 years.

If lowering p_2 does not produce a meaningful effect and is already being contemplated in social security measures, then the only solution consists, we believe, in the adoption of measures aimed at increasing p_1. Variable p_1 can increase by implementation of policies that encourage fertility and/or the attraction of young immigrants.

Fertility policies are, in our view, unpostponable and they should be structural and strategic. These policies contain however some shortcomings which render them as part of the solution, not as the solution.

A major limitation to fertility policies effectiveness consist in the difficulty in inducing changes in fertility behaviours. The low fertility that most advanced economies face is not a coincidence: it is the outcome of the whole socioeconomic organization, including the redefinition of its fundamental cell, the family. Consequently, fertility policies must be structural and, when implemented, they will be unequivocally structuring, because they must articulate with an entire reconfiguration of social systems.

Additionally, social security systems and public finances must be willing and ready to face more social spending because more births must be accompanied by adequate conditions to raise children. More nurseries, more availability of infant health resources, more time available for parenting without loss of their income and career expectations. This increase in social spending will bring high returns for society and the economy, hence must be looked upon as an important investment for the future. And thinking about the future, we address another shortcoming of fertility policies, in terms of their effectiveness in solving the problem. The concrete results of stimulating fertility take decades to rebound: fertility policy cannot thus be considered a solution by itself in a foreseeable future.

We believe that, along with fertility increasing policies, we must consider immigration. European governments can, in our view, implement policies aimed at attracting back their young diaspora as well as additional young immigrants, especially, young immigrants with children. Age seems to be a more correct benchmark than skills, therefore we believe that policies to attract skilled immigrants should be replaced by policies to attract young immigrants with children. In this regard, the next chapter will provide an explanation as to why skill-selective immigration policies are not necessary, on the contrary.

In short, a comprehensive policy to reverse the forces that push the European Union countries to a collapse trajectory, should place its effort to attract young immigrants with children for the short and medium term, while pursuing fertility policies for the long-term. The gradual increase in the age of retirement is already predicted by many countries and has no relevant impact.

Concluding, we have proposed our view that European countries should follow demographic policies of the type $\Delta p_1 > \Delta \beta p_2 > 0$, meaning that, since p_2 cannot be reduced, p_1 must increase significantly. Our model predicts that the adjustment to the new equilibrium will encompass short-term costs and losses of welfare (measuring naïvely the welfare by consumption per-capita).

Figures 1.18 to 1.20 illustrate the whole scenario.

Figure 1.18 Time evolution of the growth rate of physical capital per-capita, before and after demographic policy.

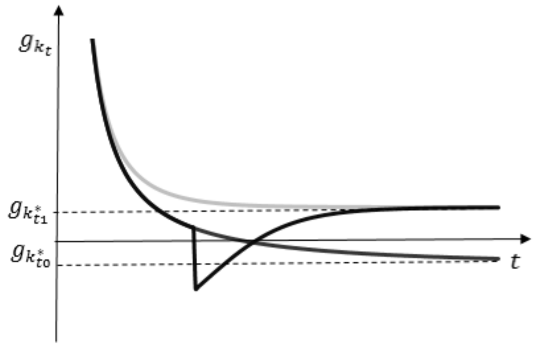


Figure 1.19 Time evolution of physical capital per-capita, before and after demographic policy.

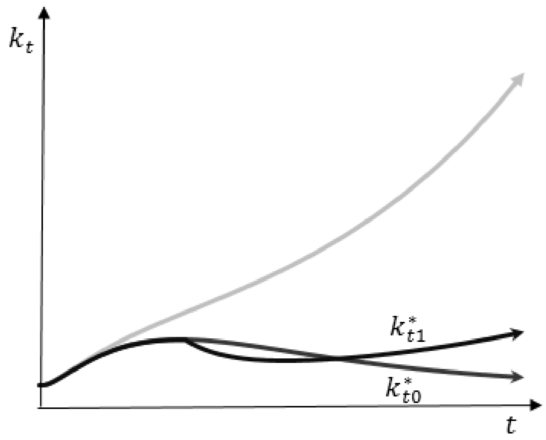
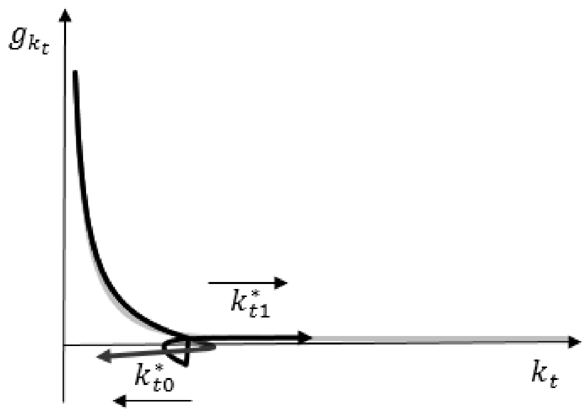


Figure 1.20 Evolution of the physical capital growth rate with its stock, before and after demographic policy.



The economy is initially in a collapse trajectory $p_{10} < \beta p_{20}$; i.e., on the growth curve that converges convexly to $g_{k^*_{t0}} < 0$ (Figure 1.18). While $g_{k_{t0}} > 0$, the stock of physical capital k_{t0} increases but, after a while, $g_{k_{t0}}$ turns negative, therefore k_{t0} starts decaying in time (Figure 1.19). This turning point is characterized by decreasing k_{t0} and $g_{k_{t0}}$ (Figure 1.20). As the underlying demographic forces that push the economy down are operating, this process ends up in ruin.

Accordingly, the government follows fertility policies together with age-selective immigration policies. They encourage natives to have more children and attract young working-aged immigrants with children, so that $\Delta p_1 > \beta \Delta p_2 > 0$. The depiction of these combined policies corresponds to the grey lines in Figures 1.18-1.20. They represent the force that will overcome the existing demographic forces to reverse the collapse trajectory.

There are costs in the short-term (requiring political courage to implement them). Those costs are mainly due to the impact of a higher $\Delta p_1 > 0$ on the effective depreciation. The growth rate (1.20) jumps to a lower value due to a more intensive use of infrastructures (Figure 1.18). This leads to an initial decrease of k_{t0} to k_{t1} curve (Figure 1.19). After a while, savings become high enough (due to the new demographic setting) to cause a non-stop increase in $g_{k_{t1}}$ up to the new growth curve, converging to $g_{k_{t1}^*} > 0$ (Figure 1.18). Accordingly, after a period, k_{t1} becomes higher than k_{t0} and grows incessantly, as the economy has now a demographic setting that allows for long-term positive growth (Figure 1.19). The best image of a reversing collapse trajectory into a positive growth trajectory is provided by Figure 1.20. Demographic and economic reversion become well illustrated.

1.6 Conclusions

The here developed model is not intended to describe an economy as a whole. It describes a set of partial forces pushing down the potential of growth of one stylized economy. Hence the real net growth effect can be different from the one predicted by the proposed model. Where it predicts technological regress and collapse, the net effect may be stagnation, as other sources of growth are acting in the opposite direction. Notwithstanding, the severity of the effects of an adverse demographic setting such as ageing together with decaying population, that many advanced economies are facing, is one prediction of the model which is not too far from reality. Such severe demographic effects on growth have been diluted by counter-acting effects of implicit migration, ICT investment, effective unemployment policies and other supply and demand side policy measures. We could have modelled these effects by introducing an exogenous technological parameter, however our goal has been to isolate and emphasize the implications of age structures on economic growth.

We have thus developed a framework to analyse the impact of age structures on growth by associating them with the technological evolution.

Technological regress is not a possible result in modern growth theory, however, the persistent low total factor productivity, initiated years before the 2008 crisis and predictions of an overall decay of most of advanced economies' potential growth led us to hypothesize that there is a subliminal technological regress in course and its origin is much deeper and structural than strictly economic decisions: it is rooted in demographics. Our question is whether a country that undergoes a severe ageing process can end up experiencing a long-term negative growth rate due to technological regress. Our theoretical model predicts that yes it can.

It has been our wish to convey the thought that ageing deteriorates technological evolution, reinforcing the diminishing returns of physical capital in production. Such dynamics originates a change in sign, from positive to negative, of growth rates of physical capital, leading to a progressive decapitalization of the economy. Any measure that does not address the alteration of the country's demographical characteristics will be delaying economic ruin, not preventing it from eventually occurring. The demographic setting is unstoppable and its eroding power through technological regress is a mechanism that, if not effectively reversed, will proceed its collapsing course.

The only way to solve structurally the problem is by reversing the ageing trend. That reversion can only be achieved by strategic fertility policies together with the attraction of the young diaspora as well as the young immigrants with children. The first, is a long-term policy, the second has immediate effects. The increase in the age of retirement is a limited policy, already in course, with few results expected.

The rejuvenation of the population would affect positively the technological evolution and it should be enough to compensate the eroding effect of an increase in the population over the output and of the retirement of the talent and know-how of the eldest. As these policies augment the population's size, the model predicts short-term losses concerning macroeconomic per-capita variables. Welfare may suffer some decay in the short-term but in the medium and long-term it will achieve higher standards. Also, the future of the next generations will be safeguarded, otherwise, our bequest will be a weakened economy tending to ruin.

If this is a symptom of a disease that is developing in silence, like many of the worst diseases do, it's devastating effects are being delayed by policies that hide the symptoms, not curing it. Accordingly, the possibility of technological regress and long-term collapse of economies, should not be overlooked. As time goes by, without an effective action counteracting those demographic forces, they will tend to become the predominant energies of growth.

As most of the decline in TFP growth was attributed to low levels of investment in Information and Communications Technology (ICT) in the services sector (Van Ark et al., 2008; Dabla-Norris et al., 2015), the emphasis has been so far put on fostering this kind of investment. Although, some authors consider that the ICT revolution had already started to encounter diminishing returns years ago (Gordon, 2015), we agree that ICT investment policies are important. However, we also perceive the decay of TFP mainly in services, because this is a labour-intensive sector, that is, the qualitative and quantitative characteristics of the inputs are decaying with the ageing of the population, which is reflected on the TFP. Therefore, we believe that not only policies fostering microchip technology developments and adoption are required; a higher urgency must be put on demographic tendencies and respective counter-acting policies.

Chapter 2

Educational Heterogeneity and Economic Growth

2.1 Introduction

In this Chapter, we examine the impact of educational heterogeneity on economic growth. The concept of education that we adopt is that of formal education. The formal education sector produces human capital, as in Lucas (1988)[25] model, upon which our developed framework is built.

Lucas' (1988) model predicts that economic growth increases with the effectiveness of investments in human capital, that is, a higher efficiency in the formal education sector delivers a higher balanced growth path (BGP) growth rate. Despite its theoretical prominence and its widespread influence in policy making, empirical assessments of Lucas's (1988) prediction are not in agreement.

According to Durlauf and Johnson (1995), in empirical analyses, the impact of formal education on economic growth depends on the sample of countries used. Also, Durlauf et al. (2001) find a significant positive effect of education on growth, whereas, with a different database and a different estimation method, Durlauf et al. (2008) find little significance of such relationship.

Kalaitzidakis et al. (2001), too, conclude that the use of semiparametric instead of parametric models changes the impact of education on growth from insignificant to significant.

Sala-i-Martin (2004) discovers that higher education is insignificant for growth, but primary education exhibits positive significance, while Barro (2001) finds the opposite result for men and insignificant results for women, at any level of education. However, Barro and Sala-i-Martin (1995) had years before established empirically a positive

[25] Lucas (1988) also presents a model of on-the-job accumulation of human capital, although, the most important model of the paper; that became one of the most important models of new growth theory, is about human capital as the outcome of formal education.

significant relationship between male secondary and higher education and economic growth.

The lack of consensus concerning the economic growth outcomes of education is such that different conclusions are reached by one author or within one paper. In general, some studies conclude for a positive significant impact of education on growth[26], while others find its insignificance[27] or even the opposite effect[28]. This endemic characteristic of empirical research on education and economic growth constitutes our main motivation for the analysis that we present in this Chapter.

The different results achieved regarding the relationship between education and growth have been explained using different data sources, samples, proxies of human capital, functional forms and estimation methods. It is true that some economic realities are harder to access empirically, and this seems to be the case. Especially, the difficulty of measurement, the likely micro founded non-linearities and the modelling of heterogeneity are compelling arguments. However, in general, when economic variables are significantly related, they exhibit a significant econometric relationship in most cases, regardless of the chosen vector of methodological options. Coherently, if there was, in fact, a strong correlation between education and economic growth, the findings of empirical research should not reveal so much volatility.

We wish to argue that the above referred lack of empirical consensus is rooted in the absence of a meaningful relationship between education and economic growth. We propose an extension of Lucas's (1988) model with which to support our argument.

[26] Romer (1989), Barro (1991, 1998), Mankiw et al. (1992), Barro and Lee (1994), Barro and Sala-i-Martin (1995), Gemmell (1996), Temple (2001) and Minier (2007).
[27] Benhabib and Spiegel (1994), Pritchett (1995, 2001), Judson (1996), Nonneman and Vanhoudt (1996), Liu and Stengos (1999), and Bils and Klenow (2000), Maasoumi et al. (2007), Henderson (2010).
[28] Islam (1995) and Caselli et al. (1996).

Lucas (1988) model has been extended in many directions. The richness of Lucas (1988) on Uzawa (1965) goes beyond the paper itself. It provides a unique analytical framework device to explore many other possibilities. For instance, Rebelo (1991) generalize it for physical capital and Caballe and Santos (1993) study the case of a concave relationship between human capital production and the time allocated to production. Mauro and Carmeci (2003) analyze the opposing relationship between unemployment and human capital accumulation and Gupta and Chakraborty (2006) study the mechanism of human capital accumulation in an economy with rich and poor individuals. Sequeira and Ferreira-Lopes (2008) extend for social capital while Neustroev (2014) extends for the unsuspected natural resources. In line with Robertson (2002), we extend Lucas (1988) for unskilled labour.

Lucas (1988) considers that skilled and unskilled labour are perfect substitutes, their only difference then being their productivity. We wish to argue that they are imperfect substitutes, and provide distinct services.

Indeed, we believe that some jobs do not require high academic degree workers; and reciprocally high academic degree workers do not have the ability, the vocation and the motivation to accomplish them well. In general, skilled workers are relatively more productive at skilled tasks whereas unskilled workers are relatively more productive at unskilled tasks. For instance, except for a short term desperate time spell, doctors will only work in the grape harvest as a touristic activity, and lawyers will only become nannies to their own children. Likewise, teachers will only become waiters in a desperate situation and they will, most likely, be less productive waiters than individuals that wish to be waiters. One individual without the inclination and the motivation to perform a certain task, will reveal limited ability for learning-by-doing or on-the-job learning.

Each task requires a specific talent, vocation and motivation, a specific skill. Some skills require schooling while others are better acquired outside the formal education system. In other words, the concepts of occupation, specialization and division of labour ought to be considered when analyzing the effects of formal education on economic growth. We believe that these concepts are behind the inexistent linear bond between formal education and economic growth.

The argument we wish to convey analytically is that an expansion of formal education represents an increase in the share of skilled labour relative to the share of unskilled labour. More skilled labour originates the production of more skilled services. Imperfect substitutability implies some complementarity; hence the production of more skilled services is associated with the production of more unskilled labour. But there is less unskilled labour. Then, as imperfect substitutability implies some substitutability, skilled labour is allocated to unskilled jobs, in which it is less productive. Consequently, the relative variation in aggregate output due to an increase in formal education has an unpredicted net effect[29].

Like Robertson (2002), we model this situation by introducing Mankiw et al.'s (1992) production function in Lucas' (1988) framework. Our conclusions are different from Robertson's (2002), because we assume civilizational development to be intrinsically related with more education. In our model, the passage of time or civilizational development enlarges the share of skilled labour naturally. The quantity of skilled labour evolves with time (as a strategic development policy of compulsory education coverage). Skilled labour quality is an individual decision.

[29] Not considering the decreased time that skilled labour must frequent the educational system, which leads to lower growth rates of human capital and the relative decrease in skilled wages, which reduces the incentives to go on studying.

The remaining of this Chapter is organized as follows. Section 2.2 presents and discusses the assumptions of the model. Section 2.3 presents the derivation of the balanced growth path results. Section 2.4 proceeds with the transitional dynamics mechanics. Section 2.5 discusses results and infers implications. Section 2.6 concludes.

2.2 The Model

With the purpose of analysing the relationship between educational heterogeneity and economic growth, we follow Robertson (2002) by using Mankiw et al.'s (1992) production function in Lucas' (1988) model.

We introduce educational heterogeneity through consideration of skilled and unskilled workers, who provide distinct essential services for aggregate production. We assume that skilled workers and unskilled workers are imperfect substitutes in aggregate production.

Our premise is that a country undergoes civilizational development with time and that formal education evolves positively with a country's civilizational development. Hence, we expect there to be a growing ratio of skilled labour over unskilled labour throughout time.

We wish to convey theoretically that the above assumed labour dynamics have economic implications. Firstly, compulsory formal education can become a statistic bubble for the lack of individual incentives to acquire further (more specific) skills. Secondly, migration flows can be the solution to stabilize one economy in a balanced growth trajectory characterized by higher levels of wealth and higher growth rates.

2.2.1 Individuals

Individuals are workers and consumers. As workers they are heterogeneous: they are skilled or unskilled workers. As consumers they are homogeneous, with equal preferences and discount rates.

The Labour Force

As mentioned above, a crucial assumption for our model is that education and civilizational development are intimately related. They are strongly and positively correlated and cannot be dissociated.

In our model, an infinitely lived dynasty exhibits higher levels of education over time. The dynasty is standardized to one, becoming an infinitely lived individual that acquires more and more skills as time goes by, partially just because of the passage of time itself.

So, let us consider a fixed number of $L_t = L$ dynasties. This means that we consider the growth rate of L_t as being zero; yet, as time passes, the skill structure of the dynasty/individual modifies. Their unskilled fraction η_t shrinks while/then the skilled (complementary) fraction $1 - \eta_t$ augments. We take this evolution as given, without any interference of individual decisions, as a typical momentum of civilizational development. It can be viewed as the population coverage of the compulsory education established by law.

In this Chapter, population and labour coincide. The L_t individuals of the economy are workers, such that:

$$L_t = L_{ut} + L_{st} \tag{2.01}$$

where:

$$L_{u,t} = \eta_t L_t \tag{2.02}$$

and:

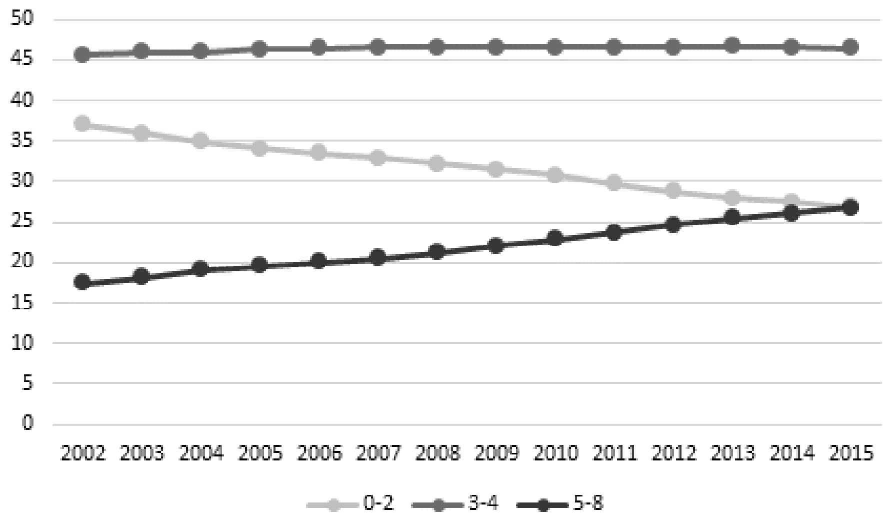
$$L_{s,t} = (1 - \eta_t) L_t \tag{2.03}$$

where indexes u and s stand for unskilled and skilled, respectively.

Then, referring to the growth rate of variable L_{jt} as $g_{L_{jt}}$, we assume that $g_{L_{ut}} < 0$ and $g_{L_{st}} > 0$. Also, as the growth rate of L_t is zero, the growth rate of the shares on the unskilled and the skilled is equal to the growth rate of the aggregates L_{jt}, with $j = u, s$.

Supporting this assumption, we present Figure 2.1 for the European Union.

Figure 2.1 Share of the population by educational attainment in the European Union: 2002-2015 (Unit: %).



Notes: Own presentation from Eurostat data. 0-2: Less than primary up to lower secondary education.3-4: Upper secondary and post-secondary non-tertiary education. 5-8: Tertiary education.

Between 2002 and 2015, the share of the unskilled (considered as the proportion of individuals with 0 to 2 level of education) has diminished successively from 37.0% to 26.9%, while the fraction of the most skilled (considered as the proportion of individuals with 5 to 8 level of education) has increased from 17.4% to 26.7%. The average yearly growth rate of the unskilled was -2% and that of the most skilled was +4%. The intermediate skill level (between the levels 3 and 4) has been stagnant (45.6% in 2002 to 46.4% in 2015).

The typical agent

The economy's labour force consists of skilled and unskilled workers; although we embody the educational heterogeneity of the labour market in the educational heterogeneity of the standardized dynasty. Dynasties/individuals are heterogeneous in their structure but homogeneous amongst them. They absorb the heterogeneity and allow the typical agent to be born as the average worker. With this methodological design, we are able to avoid analytical complications of aggregation, which would not be relevant to the objectives of the model. Nevertheless, we will refer indistinctly to a composite skilled-unskilled individual or to skilled and unskilled workers, according to the requirements of the exposition.

Consumption and Investment

The typical agent preferences are described by an isoelastic risk-aversion instantaneous utility:

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} \qquad (2.04)$$

with $0 < \sigma < 1$ representing the inverse of the instantaneous elasticity of substitution.

Given the discount rate $\rho > 0$, the typical agent intertemporal utility function becomes:

$$U = \int_0^{+\infty} e^{-\rho t} \frac{c_t^{1-\sigma}}{1-\sigma} dt \qquad (2.05)$$

and his budget constraint is:

$$\dot{k}_t = y_t - c_t \tag{2.06}$$

where $\dot{k}_t$ represents investment per capita, y_t is output per capita and c_t is consumption per capita.

2.2.2 Production

Lucas' (1988) model has two sectors: the final goods sector and the human capital sector. We introduce a third sector: the labour services sector.

The Labour Services Sector

The total amount of skilled services in the economy is:

$$H_{yst} = L_{st} u_t h_{st} \tag{2.07}$$

Variable L_{st} stands for the number of skilled workers, h_{st} is their level of human capital and u_t represents the fraction of their human capital that they employ in the production of final services.

Here u_t is not necessarily time: it is a percentage over units, a fraction of human capital.

It is assumed that $L_{st} h_{st} = L_t h_t$, where h_t represents the average level of human capital in the economy. Hence equation (2.07) can be rewritten in terms of the average worker:

$$H_{yst} = L_t u_t h_t \tag{2.08}$$

The total number of workers in the economy is L_t but the effective stream of labour services they provide amounts to L_{et} so that:

$$L_{et} = H_{yst}^{b} L_{ut}^{1-b} \tag{2.09}$$

with $0 \leq b \leq 1$.

Inserting (2.08) in (2.09), the amount of effective labour services employed in production is:

$$L_{et} = (L_t u_t h_t)^{b} L_{ut}^{1-b} \tag{2.10}$$

The liaison between this sector and the other two sectors is the following.

Concerning the final goods sector, each worker provides an amount of η_t unskilled services and allocates a proportion u_t of the human capital that he owns h_t to provide H_{yst} skilled services.

The worker decides upon u_t, while civilizational development dictates η_t.

Variable u_t is then defined as the fraction of human capital used in the final output production:

$$u_t = \frac{H_{yst}}{H_t} = \frac{H_{yt}}{H_t}$$

The remaining $1 - u_t$ of the human capital stock is used to produce more human capital.

The Human Capital Sector

Like in Lucas (1988) model, the human capital sector produces human capital using human capital as its unique input. Each worker has an average level of human capital, h_t, of which she employs fraction u_t in the production of final goods. The remaining fraction $1 - u_t$ is invested in her education/production of human capital.

The accumulation of the human capital in per capita terms is given by:

$$\dot{h}_t = [\phi(1 - u_t) - a]h_t \tag{2.11}$$

where ϕ represents the efficiency of the formal education system and a is the rate of depreciation of human capital. In the present context, we find it especially appropriate to consider parameter a as ageing, although it does not play a major role in our developed model.

The Final Goods Sector

The economy produces Y_t units of final goods using physical capital K_t and effective labour services L_{et}, so that:

$$Y_t = K_t^{\alpha} L_{et}^{1-\alpha} \tag{2.12}$$

where $0 < \alpha < 1$ and K_t is physical capital.

The amount of effective labour services is given by equation (2.09), then:

$$Y_t = K_t^{\alpha}[(L_t u_t h_t)^b L_{ut}^{1-b}]^{1-\alpha}$$

If $\beta = b(1 - \alpha)$ and $\gamma = (1 - b)(1 - \alpha)$, we have:

$$Y_t = K_t^{\alpha}(L_t u_t h_t)^{\beta} L_{ut}^{\gamma} \tag{2.13}$$

Equation (2.13) is Mankiw et al.'s (1992) production function, meaning that standard neoclassical assumptions apply.

Output per capita is given by:

$$y_t = k_t^{\alpha}(u_t h_t)^{\beta} \eta_t^{\gamma} \tag{2.14}$$

where the small caps refer to per capita variables, from now on.

2.3 Equilibrium

Lucas (1988) developed two models: One model where human capital accumulation occurs in the educational sector and where there are internal and external returns of the average stock of human capital in the production of final goods; and another model that allows for on-the-job accumulation of human capital. When we refer to Lucas' (1988) model, we are referring to the first one, although without considering the human capital externality.

As we do not consider external effects, the decentralized and the social planner solution coincide, hence this variant allows us to achieve first-best solutions when solving for the decentralized economy. This is a corollary of the first welfare theorem.

Also, the second welfare theorem allow us to solve directly for the optimal allocation if there is a backing framework of prices. Having these requirements settled, we next solve our model for its first-best closed-form solution.

In line with the precedent Chapter, we abdicate of the constant exogenous technological level parameter, as well as that of physical capital depreciation.

2.3.1 The Maximization Problem

The typical agent chooses a plan of c_t and u_t with $t \in [0, \infty[$ that maximizes:

$$U = \int_0^{+\infty} e^{-\rho t} \frac{c_t^{1-\sigma}}{1-\sigma} dt \qquad s.t.$$

$$c_t > 0, 0 \leq u_t \leq 1,\ 0 \leq \eta_t \leq 1$$

$$y_t = k_t^{\alpha}(u_t h_t)^{\beta} \eta_t^{\gamma}$$

$$\dot{k}_t = y_t - c_t$$

$$\dot{h}_t = \phi(1-u_t)h_t - a h_t$$

$$k_0 > 0, h_0 > 0 \text{ given}$$

$$k_t > 0, h_t > 0, \forall_t$$

Taking θ_{1t} and θ_{2t} as the shadow prices of, respectively, physical capital and human capital per capita along the optimal path, the current-value Hamiltonian becomes:

$$H_t = \frac{c_t^{1-\sigma}}{1-\sigma} + \theta_{1t}(y_t - c_t) + \theta_{2t}[\phi(1-u_t) - a]h_t \tag{2.15}$$

The first-order conditions:

$$\frac{\partial H_t}{\partial c_t} = 0$$

equivalent to:

$$c_t^{-\sigma} = \theta_{1t} \quad (2.16)$$

$$\frac{\partial H_t}{\partial u_t} = 0 \Leftrightarrow \theta_{1t}\frac{\partial y_t}{\partial u_t} - \theta_{2t}\phi h_t = 0$$

equivalent to:

$$\frac{\theta_{2t}}{\theta_{1t}} = \frac{1}{\phi h_t}\frac{\partial y_t}{\partial u_t} \quad (2.17)$$

$$\frac{\partial H_t}{\partial k_t} = \rho\theta_{1t} - \dot{\theta}_{1t} \Leftrightarrow \theta_{1t}\frac{\partial y_t}{\partial k_t} = \rho\theta_{1t} - \dot{\theta}_{1t}$$

equivalent to:

$$\frac{\dot{\theta}_{1t}}{\theta_{1t}} = \rho - \frac{\partial y_t}{\partial k_t} \quad (2.18)$$

$$\frac{\partial H_t}{\partial h_t} = \rho\theta_{2t} - \dot{\theta}_{2t} \Leftrightarrow \theta_{1t}\frac{\partial y_t}{\partial u_t} + \theta_{2t}[\phi(1-u_t) - a] = \rho\theta_{2t} - \dot{\theta}_{2t}$$

equivalent to:

$$\frac{\dot{\theta}_{2t}}{\theta_{2t}} = -\frac{\theta_{1t}}{\theta_{2t}}\frac{\partial y_t}{\partial h_t} - \phi(1-u_t) + a + \rho \quad (2.19)$$

The transversality conditions:

$$\lim_{t\to\infty} e^{-\rho t}\,\theta_{1t}k_t = 0 \tag{2.20}$$

$$\lim_{t\to\infty} e^{-\rho t}\,\theta_{2t}h_t = 0 \tag{2.21}$$

Equation (2.20) establishes that, in the margin, income will be equally valuable in consumption and saving and equation (2.21) establishes that the distribution of human capital between sectors will be such that its uses for manufacturing and education will be equally valuable (2.21). The transversality conditions, together, ensure that there will be no overaccumulation of physical nor human capital.

2.3.2 Balanced Growth Path

BGP growth rates

Log-time-differentiating equation (2.16) and using (2.18) we derive the growth rate of per capita consumption:

$$g_{c_t} = \frac{1}{\sigma}\left(\frac{\partial y_t}{\partial k_t} - \rho\right) \tag{2.22}$$

The marginal product of physical capital is:

$$\frac{\partial y_t}{\partial k_t} = \alpha k_t^{\alpha-1}(u_t h_t)^{\beta}\eta_t^{\gamma}$$

expressing in terms of physical capital per unit of effective labour $\tilde{k}_t$, we have:

$$\frac{\partial y_t}{\partial k_t} = \alpha \tilde{k}_t^{\alpha-1} \tag{2.23}$$

with:

$$\tilde{k}_t = \frac{k_t}{(u_t h_t)^b \eta_t^{1-b}} \tag{2.24}$$

In the balanced growth path:

$$g_{c_t} = \frac{1}{\sigma}\left(\alpha \tilde{k}_t^{\alpha-1} - \rho\right)$$

must be constant, then $\tilde{k}_t$ must be constant.

Solving equation (2.17) for $\tilde{k}_t$:

$$\alpha k_t^{\alpha-1}(u_t h_t)^{\beta} \eta_t^{\gamma}$$

$$\frac{\theta_{2t}}{\theta_{1t}} = \frac{1}{\phi h_t}\frac{\partial y_t}{\partial u_t} \Leftrightarrow \frac{\theta_{2t}}{\theta_{1t}} = \frac{\beta h_t k_t^{\alpha}(u_t h_t)^{\beta-1}\eta_t^{\gamma}}{\phi h_t} \Leftrightarrow \frac{\theta_{2t}}{\theta_{1t}} = \frac{\beta k_t^{\alpha}[(u_t h_t)^b \eta_t^{1-b}]^{1-\alpha}}{\phi u_t h_t} \Leftrightarrow$$

$$\Leftrightarrow \frac{\theta_{2t}}{\theta_{1t}} = \frac{\beta}{\phi} k_t^{\alpha}[(u_t h_t)^b \eta_t^{1-b}]^{-\alpha}(u_t h_t)^{b-1}\eta_t^{1-b}$$

then:

$$\frac{\theta_{2t}}{\theta_{1t}} = \frac{\beta}{\phi}\tilde{k}_t^{\alpha}\left(\frac{\eta_t}{u_t h_t}\right)^{1-b} \tag{2.25}$$

Log-differentiating (2.25):

$$\frac{\dot{\theta}_{2t}}{\theta_{2t}} - \frac{\dot{\theta}_{1t}}{\theta_{1t}} = (1-b)\left(g_{L_{ut}} - g_{u_t} - g_{h_t}\right) \tag{2.26}$$

From equations (2.17), (2.18) and (2.19):

$$\frac{\dot{\theta}_{2t}}{\theta_{2t}} - \frac{\dot{\theta}_{1t}}{\theta_{1t}} = -\left(\frac{1}{\phi h_t}\frac{\partial y_t}{\partial u_t}\right)^{-1}\frac{\partial y_t}{\partial h_t} - \phi(1-u_t) + a + \rho + \frac{\partial y_t}{\partial k_t} - \rho$$

then:

$$\frac{\dot{\theta}_{2t}}{\theta_{2t}} - \frac{\dot{\theta}_{1t}}{\theta_{1t}} = \frac{\partial y_t}{\partial k_t} - \phi + a \tag{2.27}$$

Combining (2.26) with (2.27):

$$\frac{\partial y_t}{\partial k_t} = \phi + (1-b)\left(g_{L_{ut}} - g_{u_t} - g_{h_t}\right) - a \tag{2.28}$$

Substituting (2.28) in (2.22):

$$g_{c_t} = \frac{1}{\sigma}\left[\phi + (1-b)\left(g_{L_{ut}} - g_{u_t} - g_{h_t}\right) - a - \rho\right]$$

From (2.11) we have $g_{h_t} = \phi(1-u_t) - a$. The growth rate g_{h_t} must be constant in the steady state, hence $u_t = u_t^*$ meaning that $g_{u_t} = 0$. Then g_{L_u} must be also constant.

Accordingly, the Keynes-Ramsey rule becomes:

$$g_{c_t} = \frac{1}{\sigma}\left[\phi + (1-b)\left(g_{L_{ut}} - g_{h_t}\right) - a - \rho\right] \tag{2.29}$$

We have seen that the marginal productivity of physical capital must be constant in the balanced growth path. Then, recalling $y_t = k_t^{\alpha}(u_t h_t)^{\beta}\eta_t^{\gamma}$ we derive:

$$\frac{\partial y_t}{\partial k_t} = \alpha k_t^{\alpha-1}(u_t h_t)^{\beta}\eta_t^{\gamma} \Leftrightarrow \frac{\partial y_t}{\partial k_t} = \alpha\frac{k_t^{\alpha}(u_t h_t)^{\beta}\eta_t^{\gamma}}{k_t}$$

then:

$$\frac{\partial y_t}{\partial k_t} = \alpha \frac{y_t}{k_t} \qquad (2.30)$$

meaning that $g_{y_t} = g_{k_t}$.

Dividing both members of equation (2.06):

$$g_{k_t} = \frac{y_t}{k_t} - \frac{c_t}{k_t}$$

Then, as $g_{y_t} = g_{k_t}$, for g_{k_t} to be constant it must be that $g_{k_t} = g_{c_t}$. Therefore:

$$g_{y_t} = g_{k_t} = g_{y_t} = g_t^* \qquad (2.31)$$

Log-time-differentiating $y_t = k_t^\alpha (u_t h_t)^\beta \eta_t^\gamma$, we get:

$$g_{y_t} = \alpha g_{k_t} + \beta g_{h_t} + \gamma g_{L_{ut}}$$

then:

$$g_{h_t} = \frac{1-\alpha}{\beta} g_t^* - \frac{\gamma}{\beta} g_{L_{ut}}$$

i.e.:

$$g_{h_t} = \frac{1}{b} g_t^* - \frac{1-b}{b} g_{L_{ut}} \qquad (2.32)$$

Replacing (2.32) in (2.29), and after some algebra, we obtain:

$$\sigma g_t^* + \frac{1-b}{b} g_t^* = \phi + \frac{1-b}{b} g_{L_{ut}} - a - \rho \Leftrightarrow$$

$$\Leftrightarrow g_t^* = \frac{b}{1-b(1-\sigma)}\left(\phi + \frac{1-b}{b} g_{L_{ut}} - a - \rho\right)$$

then:

$$g_t^* = \frac{1}{1-b(1-\sigma)}\left[b(\phi - a - \rho) + (1-b) g_{L_{ut}}\right] \tag{2.33}$$

Replacing (2.33) in (2.32):

$$g_{h_t} = \frac{1}{1-b(1-\sigma)}\left(\phi + \frac{1-b}{b} g_{L_{ut}} - a - \rho\right) - \frac{1-b}{b} g_{L_{ut}}$$

meaning that the growth rate of human capital is:

$$g_{h_t} = \frac{1}{1-b(1-\sigma)}\left[\phi - a - \rho + (1-b)(1-\sigma) g_{L_{ut}}\right] \tag{2.34}$$

BGP levels

Let the consumption-physical capital ratio be:

$$\chi_t = \frac{c_t}{k_t} \tag{2.35}$$

and the output-physical capital ratio be:

$$z_t = \frac{y_t}{k_t} \tag{2.36}$$

Then, equations (2.11) and (2.34) together give:

$$\phi(1-u_t) - a = \frac{1}{1-b(1-\sigma)}\left[\phi - a - \rho + (1-b)(1-\sigma) g_{L_{ut}}\right] \Leftrightarrow$$

$$\Leftrightarrow \phi(1-u_t) = \frac{1}{1-b(1-\sigma)}\left[\phi - b(1-\sigma)a - \rho + (1-b)(1-\sigma)g_{L_{ut}}\right] \Leftrightarrow$$

$$\Leftrightarrow u_t^* = 1 - \frac{1}{\phi[1-b(1-\sigma)]}\left[\phi - b(1-\sigma)a - \rho + (1-b)(1-\sigma)g_{L_{ut}}\right] \Leftrightarrow$$

$$\Leftrightarrow u_t^* = \frac{\phi[1-b(1-\sigma)] - \phi + b(1-\sigma)a + \rho - (1-b)(1-\sigma)g_{L_{ut}}}{\phi[1-b(1-\sigma)]} \Leftrightarrow$$

$$\Leftrightarrow u_t^* = \frac{\phi[1-b(1-\sigma)-1] + b(1-\sigma)a + \rho - (1-b)(1-\sigma)g_{L_{ut}}}{\phi[1-b(1-\sigma)]}$$

i.e.:

$$u_t^* = \frac{b(\sigma-1)(\phi-a) + \rho + (1-b)(\sigma-1)g_{L_{ut}}}{\phi[1-b(1-\sigma)]} \qquad (2.37)$$

Since, from (2.27), we know that:

$$\frac{\dot{\theta}_{2t}}{\theta_{2t}} - \frac{\dot{\theta}_{1t}}{\theta_{1t}} = \frac{\partial y_t}{\partial k_t} - \phi + a$$

And, from (2.18), we have that:

$$\frac{\dot{\theta}_{1t}}{\theta_{1t}} = \rho - \frac{\partial y_t}{\partial k_t}$$

we know that:

$$\frac{\dot{\theta}_{2t}}{\theta_{2t}} = -\phi + a + \rho$$

We also know that, from (2.21):

$$\lim_{t\to\infty} e^{-\rho t}\,\theta_{2t} h_t = 0$$

Consequently, in the balanced growth path, the following inequality holds:

$$-\rho + \frac{\dot{\theta}_{2t}}{\theta_{2t}} + g_{h_t} < 0$$

Inserting (2.34) in the inequality above:

$$-\rho - \phi + a + \rho + \frac{1}{1 - b(1-\sigma)}\left[\phi - a - \rho + (1-b)(1-\sigma)g_{L_{ut}}\right] < 0 \Leftrightarrow$$

$$\Leftrightarrow \frac{[-1 + b(1-\sigma)](\phi - a) + (\phi - a) - \rho + (1-b)(1-\sigma)g_{L_{ut}}}{1 - b(1-\sigma)} < 0 \Leftrightarrow$$

$$\Leftrightarrow \frac{b(1-\sigma)(\phi - a) - \rho + (1-b)(1-\sigma)g_{L_{ut}}}{1 - b(1-\sigma)} < 0$$

Then, multiplying the above expression by $-\phi^{-1}$, we get:

$$\frac{b(\sigma - 1)(\phi - a) + \rho + (1-b)(\sigma - 1)g_{L_{ut}}}{\phi[1 - b(1-\sigma)]} = u_t^* > 0$$

So, it is proven that $u_t^* > 0$, as no other solution would be feasible.

The Keynes-Ramsey rule (2.22) together with (2.30) and (2.36), give us:

$$\frac{1}{\sigma}\left(\frac{\partial y_t}{\partial k_t} - \rho\right) = \frac{1}{\sigma}(\alpha z_t - \rho)$$

Using result (2.33):

$$\frac{1}{\sigma}(\alpha z_t - \rho) = \frac{1}{1 - b(1-\sigma)}\left[b(\phi - a - \rho) + (1-b)g_{L_{ut}}\right]$$

and solving for the output-physical capital ratio:

$$z_t = \frac{\sigma}{\alpha[1 - b(1 - \sigma)]}\left[b(\phi - a - \rho) + (1 - b)g_{L_{ut}}\right] + \frac{\rho}{\alpha} \Leftrightarrow$$

$$\Leftrightarrow z_t = \frac{\sigma}{\alpha[1 - b(1 - \sigma)]}\left[b(\phi - a) - b\rho + \frac{\alpha - \alpha b + \alpha b\sigma}{\alpha\sigma}\rho + (1 - b)g_{L_{ut}}\right]$$

then:

$$z_t^* = \frac{\sigma}{\alpha[1 - b(1 - \sigma)]}\left[b(\phi - a) + \frac{1 - b}{\sigma}\rho + (1 - b)g_{L_{ut}}\right] \qquad (2.38)$$

The physical capital accumulation equation (2.06):

$$\dot{k}_t = y_t - c_t$$

tells us that:

$$g_{k_t} = z_t - \chi_t$$

Using the Keynes-Ramsey rule we conclude that:

$$z_t^* - \chi_t^* = \frac{1}{\sigma}(\alpha z_t^* - \rho)$$

then:

$$\chi_t^* = \frac{1}{\sigma}[(\sigma - \alpha)z_t^* + \rho]$$

Inserting (2.38) in the above expression

$$\chi_t^* = \frac{1}{\sigma}\left\{(\sigma - \alpha)\frac{\sigma}{\alpha[1 - b(1 - \sigma)]}\left[b(\phi - a) + \frac{1 - b}{\sigma}\rho + (1 - b)g_{L_{ut}}\right] + \rho\right\} \Leftrightarrow$$

$$\Leftrightarrow \chi_t^* = \frac{\sigma - \alpha}{\alpha[1 - b(1 - \sigma)]}\left[b(\phi - a) + \frac{1 - b}{\sigma}\rho + (1 - b)g_{L_{ut}}\right] + \frac{\rho}{\sigma} \Leftrightarrow$$

$$\Leftrightarrow \chi_t^* = \frac{\sigma - \alpha}{\alpha[1 - b(1 - \sigma)]}\left[b(\phi - a) + (1 - b)\frac{\rho}{\sigma} + \frac{\alpha - \alpha b + \alpha b \sigma}{\sigma - \alpha}\frac{\rho}{\sigma} + (1 - b)g_{L_{ut}}\right]$$

It follows that:

$$\chi_t^* = \frac{\sigma - \alpha}{\alpha[1 - b(1 - \sigma)]}\left[b(\phi - a) + \frac{1 - b(1 - \alpha)}{\sigma - \alpha}\rho + (1 - b)g_{L_{ut}}\right] \qquad (2.39)$$

Regarding the savings rate:

$$s_t^* = \frac{z_t^* - \chi_t^*}{z_t^*}$$

Inserting (2.38) and (2.39), we deduce that s_t^* is:

$$\frac{\sigma\left[b(\phi - a) + \frac{1 - b}{\sigma}\rho + (1 - b)g_{L_{ut}}\right] - (\sigma - \alpha)\left[b(\phi - a) + \frac{1 - b(1 - \alpha)}{\sigma - \alpha}\rho + (1 - b)g_{L_{ut}}\right]}{\sigma\left[b(\phi - a) + \frac{1 - b}{\sigma}\rho + (1 - b)g_{L_{ut}}\right]}$$

After some simplifications, equivalent to:

$$s_t^* = \frac{\alpha\left[b(\phi - a - \rho) + (1 - b)g_{L_{ut}}\right]}{\sigma\left[b(\phi - a) + \frac{1 - b}{\sigma}\rho + (1 - b)g_{L_{ut}}\right]} \qquad (2.40)$$

2.4 Transitional Dynamics

The assessment of how one economy adjusts after shifts in its parameters and/or explanatory variables provides crucial information for policy decision. A complete discussion of the model for an eventual policy outline involves the analyses of both long run and short run effects. In fact, short and medium-term consequences play a significant role in policy making. Sometimes, despite its long run positive growth effects, one policy's short and the medium term negative effects can be so damaging that such policy is deemed undesirable.

Then, let us assess how our stylized economy adjusts to its balanced growth path (long run effects) after suffering an exogenous shock.

2.4.1 The System in/of Equations

Manipulating the first-order condition (2.17):

$$\frac{\theta_{2t}}{\theta_{1t}} = \frac{1}{\phi h_t}\frac{\partial y_t}{\partial u_t} \Leftrightarrow \frac{\theta_{2t}}{\theta_{1t}} = \frac{\beta k_t^{\alpha}(u_t h_t)^{\beta-1}\eta_t^{\gamma}}{\phi} \Leftrightarrow \frac{\theta_{2t}}{\theta_{1t}} = \frac{\beta k_t^{\alpha} u_t^{\beta} h_t^{\beta-1}\eta_t^{\gamma}}{u_t\phi} \Leftrightarrow$$

$$\Leftrightarrow \frac{\theta_{2t}}{\theta_{1t}} = \frac{\beta k_t^{\alpha} u_t^{\beta}\left(h_t^{b}\eta_t^{1-b}\right)^{1-\alpha} h_t^{-1}}{u_t\phi} \Leftrightarrow$$

$$\Leftrightarrow \frac{\theta_{2t}}{\theta_{1t}} = \frac{\beta k_t^{\alpha} u_t^{\beta}\left(h_t^{b}\eta_t^{1-b}\right)^{-\alpha}\left(h_t^{b-1}\eta_t^{1-b}\right)}{u_t\phi} \Leftrightarrow$$

$$\Leftrightarrow \frac{\theta_{2t}}{\theta_{1t}} = \frac{1}{u_t}\frac{\beta}{\phi}\frac{y_t}{h_t^{b}\eta_t^{1-b}} h_t^{b-1}\eta_t^{1-b} \Leftrightarrow \frac{\theta_{2t}}{\theta_{1t}} = \frac{1}{u}\frac{\beta}{\phi}\frac{y_t}{k_t}\frac{k_t}{h_t^{b}\eta_t^{1-b}} h_t^{b-1}\eta_t^{1-b} \Leftrightarrow$$

$$\Leftrightarrow u_t = \frac{\theta_{1t}}{\theta_{2t}}\frac{\beta}{\phi} z_t \frac{k_t}{h_t^{b}\eta_t^{1-b}} h_t^{b-1}\eta_t^{1-b}$$

we get that:

$$u_t = \frac{\theta_{1t}}{\theta_{2t}}\frac{\beta}{\phi} z_t \omega_t h_t^{b-1}\eta_t^{1-b} \tag{2.41}$$

where:

$$\omega_t = \frac{k_t}{h_t^b \eta_t^{1-b}} \tag{2.42}$$

Then, log-time-differentiating (2.41), we get:

$$\frac{\dot{u}_t}{u_t} = \frac{\dot{\theta_{1t}}}{\theta_{1t}} - \frac{\dot{\theta_{2t}}}{\theta_{2t}} + \frac{\dot{z}_t}{z_t} + \frac{\dot{\omega}_t}{\omega_t} + (b-1)\frac{\dot{h}_t}{h_t} + (1-b)\frac{\dot{\eta}_t}{\eta_t} \tag{2.43}$$

From (2.27), (2.30) and (2.36) we deduct that:

$$\frac{\dot{\theta}_{1t}}{\theta_{1t}} - \frac{\dot{\theta}_{2t}}{\theta_{2t}} = \phi - a - \alpha z_t \tag{2.44}$$

Now, recalling that the production function per capita (2.14) is:

$$y_t = k_t^\alpha (u_t h_t)^\beta \eta_t^\gamma \Leftrightarrow y = k_t^\alpha u_t^\beta \left(h_t^b \eta_t^{1-b}\right)^{1-\alpha} \Leftrightarrow z_t = k_t^{\alpha-1} u_t^\beta \left(h_t^b \eta_t^{1-b}\right)^{1-\alpha}$$

we can express z_t as a function of ω_t and u_t, so that:

$$z_t = \omega_t^{\alpha-1} u_t^\beta \tag{2.45}$$

Log-time-differentiating (2.45), we obtain:

$$\frac{\dot{z}_t}{z_t} = (\alpha - 1)\frac{\dot{\omega}_t}{\omega_t} + \beta\frac{\dot{u}_t}{u_t} \tag{2.46}$$

Also, log-differentiating (2.42) in order to time:

$$\frac{\dot{\omega}_t}{\omega_t} = \frac{\dot{k}_t}{k_t} - b\frac{\dot{h}_t}{h_t} - (1-b)\frac{\dot{\eta}_t}{\eta_t} \tag{2.47}$$

Inserting in (2.47) equations (2.06) for the accumulation of physical capital, and (2.11) for the growth rate of human capital, we get:

$$\frac{\dot{\omega}_t}{\omega_t} = \frac{y_t}{k_t} - \frac{c_t}{k_t} - b\phi(1 - u_t) + ba - (1 - b)g_{L_{ut}}$$

then:

$$\frac{\dot{\omega}_t}{\omega_t} = z_t - \chi_t - b\phi(1 - u_t) + ba - (1 - b)g_{L_{ut}} \tag{2.48}$$

Incorporating (2.48) in (2.46):

$$\frac{\dot{z}_t}{z_t} = (\alpha - 1)\left[z_t - \chi_t - b\phi(1 - u_t) + ba - (1 - b)g_{L_{ut}}\right] + \beta\frac{\dot{u}_t}{u_t} \tag{2.49}$$

Plugging (2.44) and (2.49) in (2.43):

$$\frac{\dot{u}_t}{u_t} = \phi - a - \alpha z_t - (1 - \alpha)\left[z_t - \chi_t - b\phi(1 - u_t) + ba - (1 - b)g_{L_{ut}}\right] + \beta\frac{\dot{u}_t}{u_t} + z_t$$
$$- \chi_t - b\phi(1 - u_t) + ba - (1 - b)g_{L_{ut}} + (b - 1)\phi(1 - u_t)$$
$$- (b - 1)a + (1 - b)g_{L_{ut}} \Leftrightarrow$$

$$\Leftrightarrow (1 - \beta)\frac{\dot{u}_t}{u_t} = \phi - a - \alpha\chi_t - \alpha b\phi(1 - u_t) + \alpha ba - \alpha(1 - b)g_{L_{ut}}$$
$$+ (b - 1)\phi(1 - u_t) - (b - 1)a + (1 - b)g_{L_{ut}} \Leftrightarrow$$

$$\Leftrightarrow (1 - \beta)\frac{\dot{u}_t}{u_t} = \phi - a - \alpha\chi_t - \alpha b\phi + \alpha b\phi u_t + \alpha ba - \alpha(1 - b)g_{L_{ut}} + (b - 1)\phi$$
$$- (b - 1)\phi u_t - (b - 1)a + (1 - b)g_{L_{ut}}$$

we get:

$$\frac{\dot{u}_t}{u_t} = \frac{b(1 - \alpha)}{1 - \beta}(\phi - a) + \frac{(1 - \alpha)(1 - b)}{1 - \beta}g_{L_{ut}} - \frac{\alpha}{1 - \beta}\chi_t + \phi u_t$$

and recalling that $\beta = b(1-\alpha)$ and $\gamma = (1-b)(1-\alpha)$ it becomes:

$$\frac{\dot{u}_t}{u_t} = \frac{\beta}{1-\beta}(\phi - a) + \frac{\gamma}{1-\beta} g_{L_{ut}} - \frac{\alpha}{1-\beta}\chi_t + \phi u_t \tag{2.50}$$

Introducing (2.50) in (2.49):

$$\frac{\dot{z}_t}{z_t} = (\alpha - 1)\left[z_t - \chi_t - b\phi(1-u_t) + ba - (1-b)g_{L_{ut}}\right]$$
$$+ \beta\left[\frac{\beta}{1-\beta}(\phi - a) + \frac{\gamma}{1-\beta} g_{L_{ut}} - \frac{\alpha}{1-\beta}\chi_t + \phi u_t\right] \Leftrightarrow$$

$$\Leftrightarrow \frac{\dot{z}_t}{z_t} = \beta(\phi - a) + \gamma g_{L_{ut}} + \frac{\beta^2}{1-\beta}(\phi - a) + \frac{\beta\gamma}{1-\beta} g_{L_{ut}} - \frac{\beta\alpha}{1-\beta}\chi_t - (1-\alpha)z_t$$
$$+ (1-\alpha)\chi_t$$

That is:

$$\frac{\dot{z}_t}{z_t} = \frac{\beta}{1-\beta}(\phi - a) + \frac{\gamma}{1-\beta} g_{L_{ut}} + \frac{\gamma}{1-\beta}\chi_t - (1-\alpha)z_t \tag{2.51}$$

As the χ_t and the z_t correspond to the consumption-physical capital ratio and the output-physical capital ratio, remembering once again the equation of the accumulation of physical capital (2.06), the Keynes-Ramsey rule (2.22) and (2.30):

$$\frac{\dot{\chi}_t}{\chi_t} = \frac{\dot{c}_t}{c_t} - \frac{\dot{k}_t}{k_t} \Leftrightarrow \frac{\dot{\chi}_t}{\chi_t} = \frac{1}{\sigma}(\alpha z_t - \rho) - (z_t - \chi_t)$$

That is:

$$\frac{\dot{\chi}_t}{\chi_t} = -\frac{\rho}{\sigma} + \frac{\alpha - \sigma}{\sigma} z_t + \chi_t \tag{2.52}$$

Then, the dynamics of the economy is described by the system of differential equations (2.50), (2.51) and (2.52):

$$\begin{cases} g_{z_t} = \frac{\beta}{1-\beta}(\phi - a) + \frac{\gamma}{1-\beta} g_{L_{ut}} + \frac{\gamma}{1-\beta}\chi_t - (1-\alpha)z_t \\ g_{\chi_t} = -\frac{\rho}{\sigma} + \chi_t - \frac{\sigma - \alpha}{\sigma} z_t \\ g_{u_t} = \frac{\beta}{1-\beta}(\phi - a) + \frac{\gamma}{1-\beta} g_{L_{ut}} + \phi u_t - \frac{\alpha}{1-\beta}\chi_t \end{cases} \tag{2.53}$$

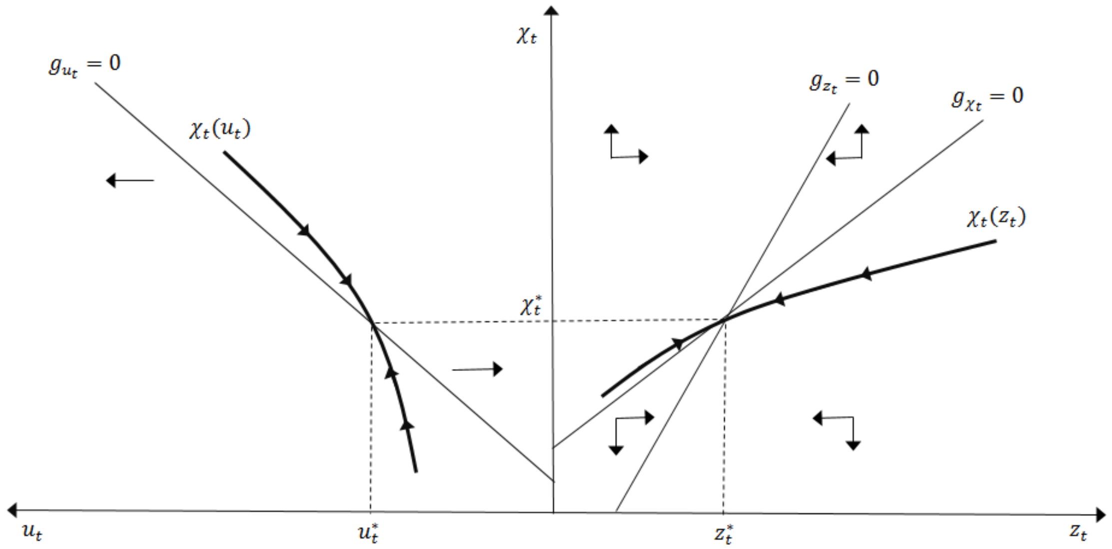
where, for any variable x_t, we use notation:

$$g_{x_t} = \frac{\dot{x}_t}{x_t}$$

2.4.2 The Phase Diagram

The differential equations system (2.53) is the framework with which we will examine the dynamics of our economy. We will use the phase diagram depicted in Figure 2.2.

Figure 2.2 The phase diagram.



Noticing in (2.53) that g_{z_t} and g_{χ_t} are dependent solely on z_t and χ_t, and inserting (2.52) in (2.51):

$$g_{z_t} = \frac{\beta}{1-\beta}(\phi - a) + \frac{\gamma}{1-\beta} g_{L_{ut}} + \frac{\gamma}{1-\beta}\left(\frac{\rho}{\sigma} + \frac{\sigma-\alpha}{\sigma} z_t + \frac{\dot{\chi}_t}{\chi_t}\right) - (1-\alpha) z_t$$

we obtain:

$$g_{z_t} = \frac{\beta}{1-\beta}(\phi - a) + \frac{\gamma}{1-\beta} g_{L_{ut}} + \frac{\gamma}{1-\beta}\frac{\rho}{\sigma} + \left[\frac{\gamma}{1-\beta}\frac{\sigma-\alpha}{\sigma} - (1-\alpha)\right] z_t + \frac{\gamma}{1-\beta}\frac{\dot{\chi}_t}{\chi_t}$$

For saddle-path stability, the coefficient of z_t must be negative, otherwise z_t and g_{z_t} would feed themselves mutually, becoming explosive. This is equivalent to the condition that ensures saddle-path stability as long as we have a negative value for the determinant of Jacobian J_1:

$$J_1 = \begin{bmatrix} -(1-\alpha) & \frac{\gamma}{1-\beta} \\ -\frac{\sigma-\alpha}{\sigma} & 1 \end{bmatrix}$$

In our model, such condition $|J_1| < 0$ is universal. Indeed:

$$|J_1| < 0 \Leftrightarrow \frac{\gamma}{1-\beta}\frac{\sigma-\alpha}{\sigma} - (1-\alpha) < 0$$

implying that:

$$\sigma > -\frac{\gamma}{\beta}$$

which, as $0 < \sigma, \beta, \gamma < 1$, is always true.

We denote the saddle-path on plane (z_t, χ_t) as $\chi_t(z_t)$ in Figure (2.1). The saddle-path $\chi_t(z_t)$ is positively sloped, meaning that z_t and χ_t increase/decrease to the steady state values when they are above/under them.

In equilibrium $g_{z_t} = 0$:

$$\frac{\beta}{1-\beta}(\phi - a) + \frac{\gamma}{1-\beta} g_{L_{ut}} + \frac{\gamma}{1-\beta}\chi_t - (1-\alpha) z_t = 0 \Leftrightarrow$$

$$\Leftrightarrow -\frac{\gamma}{1-\beta}\chi_t = \frac{\beta}{1-\beta}(\phi - a) + \frac{\gamma}{1-\beta} g_{L_{ut}} - (1-\alpha) z_t$$

that is:

$$\chi_t = -\frac{\beta}{\gamma}(\phi - a) - g_{L_{ut}} + \frac{(1-\alpha)(1-\beta)}{\gamma} z_t \tag{2.54}$$

Curve $g_{z_t} = 0$ is positively sloped, with an inclination of:

$$\frac{(1-\alpha)(1-\beta)}{\gamma}$$

and crosses the z_t axis at value:

$$z_t(\chi_t = 0) = \frac{\beta(\phi - a) + \gamma g_{L_{ut}}}{(1-\alpha)(1-\beta)} > 0 \tag{2.55}$$

For all values above curve $g_{z_t} = 0$, that is, for all the points (z_t, χ_t) such that:

$$\chi_t = -\frac{\beta}{\gamma}(\phi - a) - g_{L_{ut}} + \frac{(1-\alpha)(1-\beta)}{\gamma} z_t + \varepsilon$$

with $\varepsilon > 0$.

We have that:

$$g_{z_t} = \frac{\beta}{1-\beta}(\phi - a) + \frac{\gamma}{1-\beta} g_{L_{ut}} + \frac{\gamma}{1-\beta}\left[-\frac{\beta}{\gamma}(\phi - a) - g_{L_{ut}} + \frac{(1-\alpha)(1-\beta)}{\gamma} z_t + \varepsilon\right] - (1-\alpha) z_t$$

that is: $g_{z_t} = \varepsilon > 0$; and the opposite is also true. Therefore, above the $g_{z_t} = 0$ curve, we have that $g_{z_t} > 0$ and below the $g_{z_t} = 0$ curve, we have that $g_{z_t} < 0$.

In equilibrium $g_{\chi_t} = 0$. Hence:

$$0 = -\frac{\rho}{\sigma} + \chi_t - \frac{\sigma - \alpha}{\sigma} z_t$$

that is:

$$\chi_t = \frac{\rho}{\sigma} + \frac{\sigma - \alpha}{\sigma} z_t \qquad (2.56)$$

Assuming that $\sigma > \alpha$ (a restriction, from now on), the curve $g_{\chi_t} = 0$ is positively sloped, with an inclination of:

$$\frac{\sigma - \alpha}{\sigma}$$

and crosses the χ_t axis on the value:

$$\chi_t(z_t = 0) = \frac{\rho}{\sigma} > 0 \qquad (2.57)$$

For the existence of steady state, curves $g_{\chi_t} = 0$ and $g_{z_t} = 0$ must cross, meaning that curve $g_{\chi_t} = 0$ must be flatter than curve $g_{z_t} = 0$; i.e.:

$$\frac{\sigma - \alpha}{\sigma} < \frac{(1-\alpha)(1-\beta)}{\gamma}$$

which is equivalent to:

$$\frac{\gamma}{1-\beta}\frac{\sigma-\alpha}{\sigma} - (1-\alpha) < 0$$

which coincides with the $|J_1| < 0$ condition for saddle-path stability.

Also, when $\varepsilon > 0$ and:

$$\chi_t = \frac{\rho}{\sigma} + \frac{\sigma-\alpha}{\sigma} z_t + \varepsilon$$

$$g_{\chi_t} = -\frac{\rho}{\sigma} + \left(\frac{\rho}{\sigma} + \frac{\sigma-\alpha}{\sigma} z_t + \varepsilon\right) - \frac{\sigma-\alpha}{\sigma} z_t$$

implying that:

$$g_{\chi_t} > 0$$

Then, above the $g_{\chi_t} = 0$ curve, we have $g_{\chi_t} > 0$ and beneath the $g_{\chi_t} = 0$ curve, we have $g_{\chi_t} < 0$. This describes the dynamics depicted on the plane (z_t, χ_t) of the phase diagram in Figure 2.1.

Then, through (2.50), recall:

$$g_{u_t} = \frac{\beta}{1-\beta}(\phi - a) + \frac{\gamma}{1-\beta} g_{L_{ut}} - \frac{\alpha}{1-\beta}\chi_t + \phi u_t$$

we find that:

$$\frac{\partial g_{u_t}}{\partial u_t} = \phi > 0$$

then, for saddle path stability, i.e. for a non-explosive growth rate, when $\chi_t > \chi_t^*$, g_{u_t} must be decreasing: and when $\chi_t < \chi_t^*$, g_{u_t} must be increasing. This is described by curve $\chi_t(u_t)$ in plane (u_t, χ_t).

Making $g_{u_t} = 0$:

$$\frac{\beta}{1-\beta}(\phi - a) + \frac{\gamma}{1-\beta} g_{L_{ut}} + \phi u_t - \frac{\alpha}{1-\beta}\chi_t = 0$$

we obtain:

$$\chi_t = \frac{\beta}{\alpha}(\phi - a) + \frac{\gamma}{\alpha} g_{L_{ut}} + \frac{(1-\beta)\phi}{\alpha} u_t \tag{2.58}$$

Then, curve $g_{u_t} = 0$ is positively sloped, with an inclination of:

$$\frac{(1-\beta)\phi}{\alpha}$$

and crosses the χ_t axis on the value:

$$\chi_t(u_t = 0) = \frac{\beta}{\alpha}(\phi - a) + \frac{\gamma}{\alpha} g_{L_{ut}} > 0 \tag{2.59}$$

If $\varepsilon > 0$ and:

$$\chi_t = \frac{\beta}{\alpha}(\phi - a) + \frac{\gamma}{\alpha} g_{L_{ut}} + \frac{(1-\beta)\phi}{\alpha} u_t + \varepsilon$$

it follows that:

$$g_{u_t} = \frac{\beta}{1-\beta}(\phi - a) + \frac{\gamma}{1-\beta} g_{L_{ut}} - \frac{\alpha}{1-\beta}\left[\frac{\beta}{\alpha}(\phi - a) + \frac{\gamma}{\alpha} g_{L_{ut}} + \frac{(1-\beta)\phi}{\alpha} u_t + \varepsilon\right] + \phi u_t$$

then:

$$g_{u_t} < 0$$

Consequently, above the $g_{u_t} = 0$ curve, we have that $g_{u_t} < 0$ and beneath the $g_{u_t} = 0$ curve, we have $g_{u_t} > 0$. This describes the dynamics exhibited in plane (u_t, χ_t) of Figure 2.1.

2.5 Discussion

2.5.1 Comparative Statics

The balanced growth path of the model is characterized by equations (2.33)-(2.34) and (2.37)-(2.40).

According to these, when $b = 1$ – that is., when the labour services correspond totally to skilled labour services – we are in the presence of Lucas (1988) model.

When $b = 1$, equation (2.33) for the growth rate of the economy, recall:

$$g_t^* = \frac{1}{1 - b(1-\sigma)}\left[b(\phi - a - \rho) + (1-b)g_{L_{ut}}\right]$$

delivers growth rate:

$$g_t^* = \frac{1}{\sigma}(\phi - a - \rho)$$

Then, when $0 < b < 1$, we have:

$$\frac{\partial g_t^*}{\partial(\phi - a)} = \frac{b}{1 - b(1 - \sigma)}$$

whereas, when $b = 1, we\ have$:

$$\frac{\partial g_t^*}{\partial(\phi - a)} = \frac{1}{\sigma}$$

Noticing that:

$$\frac{b}{1 - b(1 - \sigma)} < \frac{1}{\sigma}$$

we can conclude that once we assume that, in addition to skilled labour, unskilled labour is also essential for aggregate production, then the net marginal productivity of formal education has a lower impact on the BGP growth rates. This result is significant as it adheres to the empirical studies according to which the estimated impacts of formal education on growth are not as significant as they are expected to be. The lack of conclusive empirical confirmation of the positive effects of human capital on economic growth may possibly be a result of theoretical negligence regarding the division of labour in the economies. In a country with a low b, formal education has a low impact on the economic growth rate.

Our assumption of imperfect substitutability is critical for the results of the proposed model. Firstly, there is a degree of complementarity that leads to our results regarding g_t^*. Secondly, there is a degree of substitutability that explains the results concerning g_{h_t}. In fact, recalling (2.34):

$$g_{h_t} = \frac{1}{1 - b(1 - \sigma)} \left[\phi - a - \rho + (1 - b)(1 - \sigma) g_{L_{ut}} \right]$$

we conclude that the growth rate of human capital is positively related with the growth rate of unskilled labour. Evidently, as expected, the marginal impact of $g_{L_{ut}}$ in g_{h_t} is higher, the higher is b:

$$\frac{\partial g_{h_t}}{\partial g_{L_{ut}}} = \frac{(1 - b)(1 - \sigma)}{1 - b(1 - \sigma)} > 0$$

This result could actually lead us to some bizarre but theoretically possible conclusions, namely, that increasing $g_{L_{ut}}$ up to infinity would cause g_{h_t} and g_t^* to rise as well. When $g_{L_{ut}}$ tends to infinity, η_t converges asymptotically to 1 and the infinitesimal skilled individual in the economy would embody infinite amounts of human capital. But this would imply that such individual could not be as cognitively limited as humans are. Hence, one of the central assumptions of our model is that, in one economy, the share of the population becoming more educated increases continuously with time (i.e. $g_{L_{ut}} < 0$), as a reflection of civilizational development (Figure 2.1).

Assumption $g_{L_{ut}} < 0$ is also critical as it implies that, although $g_{L_{ut}}$ together with $\phi - a$ explain g_t^*, the engine of growth is still human capital production effectiveness, because for $g_t^* > 0$, it must be that:

$$\phi - a > \frac{b - 1}{b} g_{L_{ut}} + \rho$$

Nevertheless, a higher $g_{L_{ut}} < 0$ is equivalent to improvements of educational effectiveness. According to the BGP characterization, by (2.33)-(2.34) and (2.37)-(2.40), we summarize in Table 2.1 the main results concerning comparative statics.

Table 2.1 First order partial derivatives of g_t^*, g_{h_t}, u_t^*, z_t^*, χ_t^* and s_t^* in order to ϕ, a, ρ and $g_{L_{ut}}$.

	$\boldsymbol{\phi}$	$\boldsymbol{a}$	$\boldsymbol{\rho}$	$\boldsymbol{g_{L_{ut}}}$
$\boldsymbol{g_t^*}$	$\frac{b}{1-b(1-\sigma)} > 0$	$-\frac{b}{1-b(1-\sigma)} < 0$	$-\frac{b}{1-b(1-\sigma)} < 0$	$\frac{1-b}{1-b(1-\sigma)} > 0$
$\boldsymbol{g_{h_t}}$	$\frac{1}{1-b(1-\sigma)} > 0$ +	$-\frac{1}{1-b(1-\sigma)} < 0$	$-\frac{1}{1-b(1-\sigma)} < 0$	$\frac{(1-b)(1-\sigma)}{1-b(1-\sigma)} > 0$
$\boldsymbol{u_t^*}$	$\frac{-b(1-\sigma)a+\rho-(1-b)(1-\sigma)g_{L_{ut}}}{\phi^2[1-b(1-\sigma)]} \lesseqgtr 0$	$\frac{b(1-\sigma)}{\phi[1-b(1-\sigma)]} > 0$	$\frac{1}{\phi[1-b(1-\sigma)]} > 0$	$-\frac{(1-b)(1-\sigma)}{\phi[1-b(1-\sigma)]} < 0$
$\boldsymbol{z_t^*}$	$\frac{\sigma b}{\alpha[1-b(1-\sigma)]} > 0$	$-\frac{\sigma b}{\alpha[1-b(1-\sigma)]} < 0$	$\frac{1-b}{\alpha[1-b(1-\sigma)]} > 0$	$\frac{\sigma(1-b)}{\alpha[1-b(1-\sigma)]} > 0$
$\boldsymbol{\chi_t^*}$	$\frac{(\sigma-\alpha)b}{\alpha[1-b(1-\sigma)]} > 0$	$-\frac{(\sigma-\alpha)b}{\alpha[1-b(1-\sigma)]} < 0$	$\frac{1-b(1-\alpha)}{\alpha[1-b(1-\sigma)]} > 0$	$\frac{(\sigma-\alpha)(1-b)}{\alpha[1-b(1-\sigma)]} > 0$
$\boldsymbol{s_t^*}$	$\frac{\alpha b[1-b(1-\sigma)]\rho}{\sigma^2\left[b(\phi-a)+\frac{1-b}{\sigma}\rho+(1-b)g_{L_{ut}}\right]^2} > 0$	$-\frac{\alpha b[1-b(1-\sigma)]\rho}{\sigma^2\left[b(\phi-a)+\frac{1-b}{\sigma}\rho+(1-b)g_{L_{ut}}\right]^2} < 0$	$\frac{[\alpha b^2(1-\sigma)-\alpha b]\left[(\phi-a)+(1-b)g_{L_{ut}}\right]}{\sigma^2\left[b(\phi-a)+\frac{1-b}{\sigma}\rho+(1-b)g_{L_{ut}}\right]^2} < 0$	$\frac{\alpha(1-b)[1-b(1-\sigma)]\rho}{\sigma^2\left[b(\phi-a)+\frac{1-b}{\sigma}\rho+(1-b)g_{L_{ut}}\right]^2} > 0$

The impacts of the effectiveness of the educational system

An increase in the effectiveness of formal education ϕ increases the steady state growth rates for all macroeconomic variables g_t^* and g_{h_t}. It also leads to higher steady state levels of the output-physical capital ratio z_t^*, the consumption-physical capital ratio, χ_t^*, and of the savings rate s_t^*. Regarding the fraction of human capital employed in the production of final goods u_t^*, the effect is inconclusive in sign, although its size diminishes with ϕ.

The impacts of ageing

The effects of ageing a are symmetrical to those of ϕ as a is the negative component of productivity in the human capital sector.

Hence both effects of ϕ and a must be considered. In ageing countries, increases in ϕ can deliver negative results concerning the net effectiveness of education if a grows more than ϕ. The only variable relative to which a does not deliver a symmetrical marginal effect equal to ϕ is u_t^*. In fact, ageing has an unequivocal positive effect on u_t^*. As ageing erodes human capital, keeping the state of the nature, the fraction of human capital that must be allocated to production must be higher.

The impacts of the discount rate

An increase in the discount rate ρ has a lowering effect on the steady state growth rates g_t^* and g_{h_t} as well as on the savings rate s_t^*. It enhances the steady state levels of the consumption-physical capital ratio χ_t^* and of the output-physical capital ratio z_t^*, as a result of relatively more consumption and/or relatively less physical capital. The fraction of human capital employed in the production of final goods u_t^* also increases, eventually, to reimburse the relatively lower levels of physical capital allocated to production and the lower human capital growth.

The impacts of unskilled labour

Unskilled labour is the focus of this Chapter. An increase in $g_{L_{ut}}$ leads to increases in the steady state growth rates of output per capita, physical capital per capita, consumption per capita g_t^* and human capital per capita g_{h_t}. The same happens with the output-physical capital ratio z_t^* and the consumption-physical capital ratio χ_t^*. The only variable that decreases with $g_{L_{ut}}$ is u_t^*, meaning that skilled workers can invest more of their human capital on the human capital sector; i.e., the unskilled workers free skilled workers resources so that they can acquire more education.

2.5.2 Immigration of Unskilled Labour

The incentives to acquire human capital

The role of incentives cannot, we believe, be neglected, though. Compulsory education explains the statistics, the enlargement of the share, but it does not explain the accomplishments. That is, the average level of human capital in an economy involves quantity and quality; both, the horizontal (enlargement of the basis) and the vertical (the grades achieved) achievements account for this equation.

In our model, we can view formal education as a result of two decisions: the individual(s) decide(s) upon how much human capital to invest in human capital accumulation; the exogenous government (represented by time, civilizational development) decides upon the coverage of the population.

In what concerns the individual decision, the wage rate of skilled and unskilled labour services must be considered.

From production function (2.12):

$$Y_t = K_t^{\alpha} L_{et}^{1-\alpha}$$

total wage rate is:

$$w_t = \frac{\partial Y_t}{\partial L_{et}} = \frac{\partial Y_t}{\partial L_{et}} \frac{\partial L_{et}}{\partial L_{ut}} + \frac{\partial Y_t}{\partial L_{et}} \frac{\partial L_{et}}{\partial H_{syt}} \Leftrightarrow$$

$$\Leftrightarrow w_t = (1-\alpha) K_t^{\alpha} L_{et}^{-\alpha} . (1-b)(L_t u_t h_t)^b L_{ut}^{-b} + (1-\alpha) K_t^{\alpha} L_{et}^{-\alpha} . b(L_t u_t h_t)^{b-1} L_{ut}^{1-b} \Leftrightarrow$$

$$\Leftrightarrow w_t = (1-\alpha) K_t^{\alpha} L_{et}^{-\alpha} [(1-b)(L_t u_t h_t)^b L_{ut}^{-b} + b(L_t u_t h_t)^{b-1} L_{ut}^{1-b}]$$

Physical capital per unit of effective labour $\tilde{k}_t$ is given by (2.24), below:

$$\tilde{k}_t = \frac{k_t}{(u_t h_t)^b \eta_t^{1-b}}$$

then:

$$K_t^{\alpha} L_{et}^{-\alpha} = \left(\frac{K}{L_{et}}\right)^{\alpha} = \left[\frac{k_t}{(u_t h_t)^b \eta_t^{1-b}}\right]^{\alpha} = \tilde{k}_t^{\alpha}$$

thus:

$$w_t = (1-\alpha)\tilde{k}_t^{\alpha} \left[(1-b)\left(\frac{u_t h_t}{\eta_t}\right)^b + b\left(\frac{\eta_t}{u_t h_t}\right)^{1-b}\right] \quad (2.60)$$

meaning that total wage rate w_t has two components, the skilled work component w_{st} and the unskilled work component w_{ut},; i.e.:

$$w_t = w_{st} + w_{ut} \tag{2.61}$$

with:

$$w_{ut} = (1-\alpha)(1-b)\tilde{k}_t^{\alpha}\left(\frac{u_t h_t}{\eta_t}\right)^b \tag{2.62}$$

and:

$$w_{st} = (1-\alpha)b\tilde{k}_t^{\alpha}\left(\frac{\eta_t}{u_t h_t}\right)^{1-b} \tag{2.63}$$

Considering, separately, the skilled and the unskilled workers, we conclude that the unskilled worker receives w_{ut} while the skilled worker receives $w_{st}u_t h_{st}$, where h_{st} stands for her human capital level.

Remembering that $L_{st}h_{st} = L_t h_t$:

$$h_{st} = \frac{h_t}{1-\eta_t}$$

then, the skilled worker earns:

$$w_{st}u_t h_{ts} = (1-\alpha)b\tilde{k}_t^{\alpha}\left(\frac{\eta_t}{u_t h_t}\right)^{1-b}\frac{u_t h_t}{1-\eta_t} = (1-\alpha)b\tilde{k}_t^{\alpha}\frac{\eta_t^{1-b}u_t^b h_t^b}{1-\eta_t}$$

thus, one individual will have an incentive to invest in formal education as long as:

$$(1-\alpha)b\tilde{k}_t^{\alpha}\frac{\eta_t^{1-b}u_t^b h_t^b}{1-\eta_t} > (1-\alpha)(1-b)\tilde{k}_t^{\alpha}\left(\frac{u_t h_t}{\eta_t}\right)^b$$

which happens when the ratio of the share of skilled services to the share of skilled workers is higher than the ratio of the share of unskilled services to the share of unskilled workers; i.e.:

$$\frac{b}{1-\eta_t} > \frac{1-b}{\eta_t}$$

that is, when the share of unskilled labour on labour force is still higher than the share of unskilled services on the production of labour services:

$$\eta_t > 1 - b \tag{2.64}$$

If condition (2.64) holds, the skilled workers wage is higher than the unskilled workers wage. Hence, from the individual's point of view, formal education is a good investment.

While it is possible to assume a $g_{L_{ut}}$ low enough to be in conformity with (2.64) – also because we are looking at countries where b is high (i.e., where the weight of human capital services in the effective labour supply is high) – a general solution that guarantees (2.64) is preferable.

Given that the technological parameters are fixed, we can only ensure the holding of (2.64) by retarding the decay of η_t, which can be pursued either through lowering the expansion of formal education coverage, or through opening frontiers to unskilled immigration and/or skilled emigration.

The reduction in public investment on education implies delaying civilizational development, hence this via does not seem strategically plausible for a modern nation. Regarding the second via, welcoming unskilled immigrants seems to be a better strategy than the brain drain implied by skilled emigration.

Immigration of unskilled labour

Unskilled immigration will depend on the individual incentives to immigrate. Many policies can be pursued to attract people to a country, although, the most powerful consists of expected earnings. Unskilled workers will migrate to another country if:

$$\frac{w_{utd}}{w_{uto}} > 1$$

where the index d stands for destiny country and o for origin.

Considering that all countries have the same interest rate – i.e., that the international markets are perfectly competitive and that the economies are small open economies – the profit maximizing rule leads to:

$$\frac{\partial Y_t}{\partial K_t} = r \Leftrightarrow \alpha K_t^{\alpha-1} L_{et}^{1-\alpha} = r \Leftrightarrow \alpha \left[\frac{k_t}{(u_t h_t)^b \eta_t^{1-b}}\right]^{\alpha-1} = r \Leftrightarrow \alpha \tilde{k}_t^{\alpha-1} = r$$

i.e.:

$$\tilde{k}_t = \left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}} \qquad (2.65)$$

If the technological and preference parameters that characterize both economies are identical:

$$\frac{w_{utd}}{w_{uto}} = \frac{(1-\alpha)(1-b)\left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}}\left(\frac{u_{td}h_{td}}{\eta_{td}}\right)^b}{(1-\alpha)(1-b)\left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}}\left(\frac{u_{to}h_{to}}{\eta_{to}}\right)^b}$$

meaning that

$$\frac{w_{utd}}{w_{uto}} = \left(\frac{u_{td}h_{td}}{u_{to}h_{to}}\right)^b \left(\frac{\eta_{to}}{\eta_{td}}\right)^b > 1$$

that is:

$$w_{utd} > w_{uto} \tag{2.66}$$

If country d is richer than country o, then by definition, $u_{td}h_{td} > u_{to}h_{to}$ and $\eta_{to} > \eta_{td}$. Then, as b is positive, we have that $w_{utd} > w_{uto}$, meaning that unskilled workers have an incentive to migrate from o to d.

As for the skilled workers, this is not so clear. The ratio:

$$\frac{w_{std}}{w_{sto}} = \frac{(1-\alpha)b\left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}} \frac{\eta_{td}^{1-b}u_{td}^b h_{td}^b}{1-\eta_{td}}}{(1-\alpha)b\left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}} \frac{\eta_{to}^{1-b}u_{to}^b h_{to}^b}{1-\eta_{to}}} \Leftrightarrow$$

$$\Leftrightarrow \frac{w_{std}}{w_{sto}} = \frac{\eta_{td}^{1-b}u_{td}^b h_{td}^b}{\eta_{to}^{1-b}u_{to}^b h_{to}^b} \frac{1-\eta_{to}}{1-\eta_{td}}$$

delivers:

$$\frac{w_{std}}{w_{sto}} = \left(\frac{u_{td}h_{td}}{u_{to}h_{to}}\right)^b \left(\frac{\eta_{td}}{\eta_{to}}\right)^{1-b} \left(\frac{1-\eta_{to}}{1-\eta_{td}}\right)$$

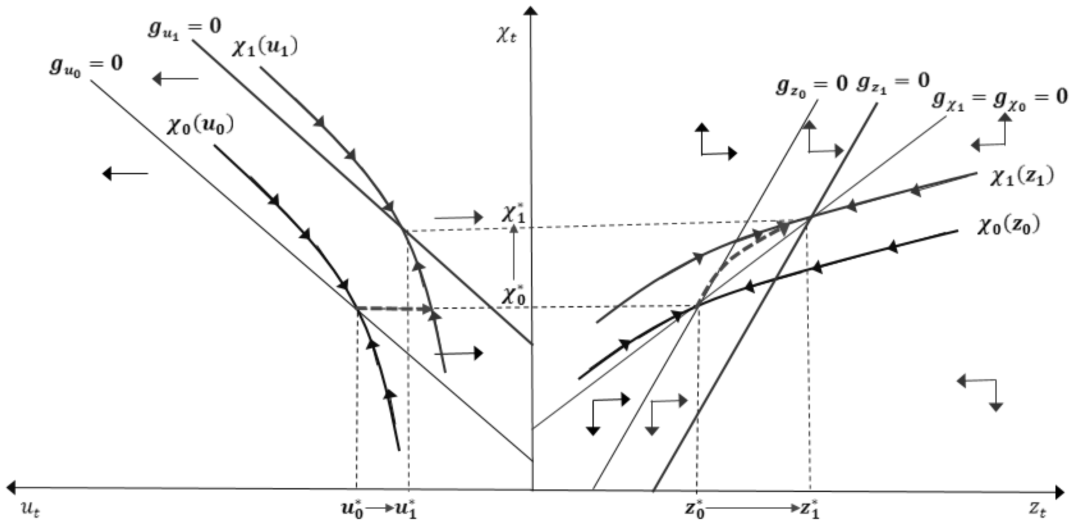
that can either be or not higher than 1. In fact, we have $u_{td}h_{td} > u_{to}h_{to}$, but it is also true that $\eta_{td} < \eta_{to}$ and that $1-\eta_{td} > 1-\eta_{to}$. As b and $1-b$ are positive values under unity, we do not know if, in the end, w_{std} is higher or lower than w_{otd}.

While in rich countries the skilled workers earn naturally a high payment; in poor countries they earn a high wage because of scarcity. It may occur the case of skilled migration from a rich to a poor country or to a richer country and, this being the case, it also causes $g_{L_{ut}}$ to grow.

Still, we can only be certain that unskilled international migrants have incentives and will travel from poor to rich countries in the absence of legal impediments to international labour mobility[30].

Figure 2.3 depicts the results of a permanent immigration flow of unskilled workers into a country.

Figure 2.3 Phase diagram for the case of a permanent immigration flow of unskilled workers.



Allowing for the case of net unskilled immigration flows leading to an increase of the growth rate of unskilled workers $g_{L_{ut}}$, the steady state ratios of output and consumption to physical capital will grow from (z_0^*, χ_0^*) to (z_1^*, χ_1^*) so that $z_1^* > z_0^*$ and $\chi_1^* > \chi_0^*$.

From Table 2.1 we know the variation size of z_t^* :

[30] Next, we will use the notation of x_0 and x_1 for the initial and final values of any variable x_t and Δx_t for the operation $x_1 - x_0$.

$$\Delta z_t^* = \frac{\sigma(1-b)}{\alpha[1-b(1-\sigma)]}\Delta g_{L_{ut}}$$

The variation size of χ_t^* is:

$$\Delta \chi_t^* = \frac{(\sigma-\alpha)(1-b)}{\alpha[1-b(1-\sigma)]}\Delta g_{L_{ut}}$$

which will be positive if $\sigma > \alpha$, as we have assumed above.

The initial steady state values (z_0^*, χ_0^*) are, by definition, the combination of z_t and χ_t for which the growth rate of both ratios is zero. This is equivalent to saying that (z_0^*, χ_0^*) is the solution of the system of equations $g_{z_0} = 0$ and $g_{\chi_0} = 0$, given by equations (2.54) and (2.56).

From (2.54), curve $g_{z_0} = 0$ (that corresponds to $g_{z_t} = 0$, considering $g_{L_{ut}} = g_{L_{u0}}$) is:

$$\chi_t = -\frac{\beta}{\gamma}(\phi - a) - g_{L_{u0}} + \frac{(1-\alpha)(1-\beta)}{\gamma} z_t$$

and curve $g_{\chi_0} = 0$, given (2.56), is:

$$\chi_t = \frac{\rho}{\sigma} + \frac{\sigma-\alpha}{\sigma} z_t$$

Then, we conclude that the combination of (z_t, χ_t) for which $g_{\chi_0} = 0$ is only reactive to the discount rate, the intertemporal rate of substitution and the share of physical capital in the production of goods and services. The increase in $g_{L_{ut}}$ from $g_{L_{u0}}$ to $g_{L_{u1}}$ does not affect $g_{\chi_0} = 0$.

In turn, when $g_{L_{ut}}$ increases from $g_{L_{u0}}$ to $g_{L_{u1}}$, curve $g_{z_0} = 0$ shifts to a higher $g_{z_1} = 0$ so that, for each level of χ_t, the correspondent z_t will be equal to:

$$0 = -\Delta g_{L_{ut}} + \frac{(1-\alpha)(1-\beta)}{\gamma}\Delta z_t$$

i.e.:

$$\Delta z_t = \frac{1-b}{1-\beta}\Delta g_{L_{ut}}$$

meaning that, for each χ_t, a percentage point more of $g_{L_{ut}}$ causes an increase in z_t equal to the ratio of the share of unskilled labour in the production of labour services to the complementary of the share of the skilled labour service in the production of goods and services. In other words, the effect of $\Delta g_{L_{ut}}$ on Δz_t is higher, the higher is the share of unskilled labour and the lower is the share of physical capital. In any case, an increase in $g_{L_{ut}}$ causes an expansion to the right of curve $g_{z_t} = 0$, meaning that the new steady state value of z_t is above the initial value. It also means that the new steady state value of χ_t is higher than the initial. Then, for majority of reason, as the dynamic equations (2.53) apply in general, there will exist a new saddle-path $\chi_1(z_1)$ above the initial one $\chi_0(z_0)$ that exhibits the same conjoint evolutions of z_t and χ_t such that above (z_1^*, χ_1^*), they decrease whereas below (z_1^*, χ_1^*), they increase.

Regarding the transition from (z_0^*, χ_0^*) to (z_1^*, χ_1^*), immediately after the increase in $g_{L_{ut}}$ the growth rate of z_t increases by[31]:

[31] This result is easily proved using equation (3.51) for $g_{L_{u1}}$and for $g_{L_{u0}}$.

$$\Delta g_{z_t} = \frac{\gamma}{1-\beta} \Delta g_{L_{ut}} > 0$$

Initially, $g_{z_0} = 0$, hence we conclude that the growth rate g_{z_t} becomes positive, originating increases in z_t. As z_t increases, from (2.52):

$$g_{\chi_t} = -\frac{\rho}{\sigma} + \frac{\alpha - \sigma}{\sigma} z_t + \chi_t$$

we conclude that g_{χ_t} turns positive, originating increases in χ_t. Then, both, z_t and χ_t increase up to the new saddle-path $\chi_1(z_1)$, converging naturally, from that point on, to the new (z_1^*, χ_1^*).

In turn, the saddle path $\chi_0(u_o)$ shifts to a saddle-path $\chi_1(u_1)$ above. The final saddle-path is characterized by a higher χ_t for each level of u_t . That is, each χ_t of the new saddle-path will be associated with lower requirements concerning the allocation of human capital to production, because there is more unskilled labour to provide labour services (within the logic of imperfect substitutability).

We assume that, after the shock, control variable u_t jumps to the new saddle-path, at the point corresponding to χ_0^*. From (2.58) we conclude that the initial variation of u_t will be from u_0^* to u_1' so that:

$$\frac{\gamma}{\alpha} g_{L_{u1}} + \frac{(1-\beta)\phi}{\alpha} u_1' = \frac{\gamma}{\alpha} g_{L_{u0}} + \frac{(1-\beta)\phi}{\alpha} u_0^*$$

hence:

$$(u_1' - u_0^*) = \frac{\gamma}{\phi(1-\beta)} \left(g_{L_{u0}} - g_{L_{u1}}\right) < 0$$

This decrease will lead the economy to the saddle-path $\chi_1(u_1)$ and, as $u_1' < u_1^*$, then u_t will converge to the new steady state (u_1^*, χ_1^*) as described above. Both variables will increase to their new steady state.

The net variation of u_t during the process will be of:

$$\Delta u_t^* = -\frac{(1-b)(1-\sigma)}{\phi[1-b(1-\sigma)]}\Delta g_{L_{ut}} < 0$$

All the remaining variables of the model will be enhanced by the permanent increase of unskilled labour immigration, as shown in Table 2.1.

2.5.3 Policy Recommendations

The developed model demonstrates that an increase in $g_{L_{ut}}$ can improve the global indicators of the system. However, condition $g_{L_{ut}} < 0$ captures a country's development strategy, hence a policy to increase $g_{L_{ut}}$ in a closed economy would signify the abandonment of the strategic nation's investment in education. If this abandonment would not be fomented by the government, it would end up being fomented by wages, because once $\eta_t = 1 - b$, unskilled labour wages would become higher than skilled wages, therefore education would become empty of economic value.

The abandonment of the education-for-all strategy is not necessarily negative from the economic point of view. As the proposed model shows, an increase in $g_{L_{ut}}$ can even improve the economy's growth rate, from a comparative statics point of view, and keeping all the parameters fixed.

Nevertheless, education's benefits extend beyond the economic results and cannot be fully captured or quantified in economic terms. As Nelson Mandela emphasized "*education* is the most powerful weapon which you can use to change the world"; i.e., education adds to economic growth the wider virtues of development. Democracy, freedom, social cohesion, ecology and other ethical principles and values. As Pritchett (2001) writes, at the end of a paper where he verifies the minor significance of education to economic growth, "none of the arguments in this paper suggest that governments should invest less in basic schooling, for many reasons. First, most, if not all, societies believe that at least basic education is a merit good so that its provision is not, and need not be, justified on economic grounds. To deny education to a child on grounds of a small expected economic growth impact would be a moral travesty".

Wishing to be in perfect harmony with Pritchett (2001), the abandonment of the strategic option of increasing continuously the share of educated residents is not a policy we recommend. We believe that the only solution consists in increasing $g_{L_{ut}}$ through migration flows. Then countries have, in our view, two possibilities: To implement policies of attraction of unskilled international migrants; or to allow for a massive brain drain to occur; or a mix of the former two.

We wish to defend the attraction of unskilled immigrants for several reasons. Firstly, it avoids the brain drain. Secondly, it allows for the formation of more resident brains (including the immigrants and their descendants). Thirdly, it compensates for the expected unskilled labour shortages in industrialized countries. Fourthly, it gives a life chance to the most vulnerable amongst the vulnerable (with future returns for the country). Finally, it corrects the skilled-selective immigration policy, that has been followed by many advanced countries, based on the fallacy that formal education enhances growth. Under labour market criteria, this widespread, not empirically supported creed has led to inadequate migration policies. And constitutes our main motivation for writing the present Chapter.

Many European countries are adopting skill-selective immigration policies (Chaloff and Lemaitre, 2009), leaving behind the unskilled to their luck, submitted to all kinds of abuse. Thousands die in the "Mediterraneans". If there were, at least, a strong economic reason to justify that, it would be acceptable, but there is not. Europe does not want unskilled immigration, although, European immigrants with equal qualifications to those of natives work in segments where they earn much less than natives. Even those who have been in the country for decades have jobs that require lower qualifications (Steinhardt 2011). For instance, in the United Kingdom, immigrants downgrade right upon arrival. They take on occupations that are well below the occupations they would perform based on their skills. Evidence shows that 26% of the highly educated recent immigrants are employed in the lowest paid occupation categories (Goos et al. 2009, Dustmann et al. 2009). The pursuit of a skilled immigration policy to employ skilled immigrants in unskilled jobs is, we believe, economically irrational and inefficient, therefore, it is twice immoral, as it discriminates twice: (i) between immigrants by the skills criteria; and (ii) between immigrants and natives.

Accordingly, our policy recommendation is the promotion of young unskilled immigrant's attraction policies and the abandonment of skilled-selective immigration policies. We write young immigrants to reinforce the positive effects of lowering $g_{L_{ut}}$ with a lower a, to also enhance the net effectiveness of formal education. Then skilled and unskilled labour would be both reinforced, and would unequivocally, produce more economic growth.

2.6 Conclusions

Lucas (1988) model predicts that economic growth increases with the effectiveness of formal education, although there is no empirical certainty supporting such prediction. The existing studies suggest divergent, even opposite, relationships between formal education and economic growth.

In addressing this literary divergence, researchers in this field have been focusing on methodological aspects. With this Chapter, we have argued that the absence of a definitive empirical conclusion is rooted in the absence of an expressive association between education and economic growth.

To substantiate our premise, we have offered to contribute to growth theory with one extension of Lucas' (1988) model to include unskilled labour.

Our proposed introduction of unskilled labour as a specific input modifies the implicit assumption of Lucas (1988) model that the only difference between skilled and unskilled labour is their productivity. In our developed model, skilled and unskilled labour are imperfect substitutes and provide distinct streams of services. Each type of labour is more productive according to the skill's level required by the service it provides. That is schooling benefits production through skilled labour and no-schooling benefits production through unskilled labour. Consequently, the variation in aggregate production due to variations in formal education have an unpredicted net effect in sign. Additionally, we have assumed exogenous continuous civilizational development in the form of continuous increases in Education-for-All.

We have found that, under the assumed circumstances, the balanced growth path of the model is a generalization of Lucas' (1988) balanced growth path, and that the higher the weight of the unskilled labour, the lower the effect of formal education on growth. We have also found that the growth rate of unskilled labour has a positive impact on the growth rate of human capital. The first result reflects complementarity between labour types while the second reflects substitution between them. In general, the growth rate of unskilled labour improves all the macroeconomic variables considered.

We have further concluded that an increase of the growth rate of unskilled labour leads to higher BGP growth rates and higher wealth levels, with no transition costs. The output-physical capital ratio and the consumption-physical capital ratio will increase from after the shock to a higher steady state value. This means that the output and consumption per capita growth rates will grow faster than the accumulation of human capital. However, the growth rate of physical capital will also be increasing as the savings rate too increases. Concerning the new steady state fraction of human capital devoted to production, the first reaction to an expansion of the growth rate of unskilled labour will consist in a jump to a lower level, which occurs due to the substitution effect, achieving the new saddle-path, where it then increases to the new steady state value which is under the initial. This means that, in the new BGP, skilled workers have more human capital resources to invest in human capital.

This can only happen where there is a diaspora of skilled native workers or immigration of unskilled workers. If this migration flows do not occur, then the skilled wages will fall below the unskilled wages and compulsory education becomes a meaningless statistic. Therefore, we have suggested that, as opposed to what has been done in Europe, the skill-selective policies must target the unskilled people rather than the skilled ones. The unskilled workers have the skills for the unskilled jobs. Unskilled labour shortages are predicted to occur due to the globalization of education. Furthermore, empirical studies have found that the skilled immigrants are usually allocated to unskilled jobs. Disregarding unskilled immigrants while employing former skilled immigrants in unskilled occupations does not make sense economically and is morally objectionable, with negative social implications.

Finally, we have recommended the attraction of young unskilled immigrants, as such inflow can lower the depreciation rate of human capital and reinforce the positive impact of a higher growth rate of unskilled labour through an increase in the net effectiveness of formal education.

Chapter 3

Ethnic Diversity and Economic Growth

3.1 Introduction

Literature on the relationship between diversity and economic growth is vast, with Jacobs (1969), Glaeser et al. (1992), Henderson et al. (1995), Feldman and Audretsch (1999) and Duranton and Puga (2001) constituting good examples. The quoted studies encompass local industry and urban diversity, that is, they contemplate economic diversity.

While the effects of economic diversity on growth have long been studied, the theoretical consideration of ethnic diversity is quite recent in economic growth literature. Still, most empirical studies on the subject find a positive impact of ethnic diversity on innovation; and innovation is the engine of growth of the most advanced economies, as framed by the R&D-based endogenous theory (e.g., Romer, 1990; Grossman and Helpmann, 1991; Aghion and Howitt, 1992).

The findings of Fujita and Weber (2004), Alesina and La Ferrara (2005) and Berliant and Fujita (2008), to quote a few, are rather enlightening regarding the importance of ethnic diversity to innovation outcomes. For Fujita and Weber (2004), the production of knowledge and ideas is positively associated with the combined skills and abilities of ethnically diverse workers. R&D activities benefit substantially from the interaction between culturally different workers. Alesina and La Ferrara (2005), too, conclude that ethnic diversity promotes innovation through the combination of a variety of knowledge and skills. In addition, ethnic diversity implies responsiveness to new preferences, revision of production processes and adjustments of firms' behaviours. Berliant and Fujita (2008) highlight cultural diversification as exceptionally important to produce new ideas and the facilitation of knowledge transfer.

In an analysis involving 12 Western European countries, Ozgen et al. (2011) find that the rate of innovation is higher in the presence of a high index of ethnic diversity. Nathan and Lee (2013) study the relationship between cultural diversity and innovation, the team spirit and sales strategies in London firms, concluding that firms with a diversified management are better prepared to introduce innovative products and to undertake internationalization strategies. For the authors, ethnic diversity is an economic asset as well as social capital.

Immigration generally implies increased ethnic diversity, hence is expected to increase innovation. In fact, in a study for the United States, Hunt and Gauthier-Loiselle (2009) find that skilled immigration contributes to more than double the number of patents when compared with natives only. Also for the United States, Kerr (2010) analyzes the rate of innovation in centres of breakthrough innovations, concluding that immigrants contribute to a faster spatial reallocation, and that patenting transfers to places with breakthrough technologies are faster for technologies that employ immigrant inventors. For New Zeland, Maré et al. (2010) find that the presence of immigrant workers has a positive impact on innovation in terms of products and of production processes. For Germany, Niebuhr (2010) concludes that the diversification of knowledge and skills in workers with different cultural origins improves the economic performance of regional R&D sectors.

In this Chapter, we develop a model to analyze the impact of ethnic diversity on economic growth through the channels of innovation and social capital. Firstly, ethnic diversity produces bridging social capital, which has a significant augmenting effect on labour productivity. Secondly, ethnic diversity facilitates the production of new ideas via the interaction of different knowledge, experiences and points of view. Finally, ethnic diversity is also associated with the appearance of new demand preferences that must be satisfied. This fact, together with a larger pool of ideas, reduces the redundancy of research projects.

Our proposed framework builds on Jones' (1995) variant of Romer's (1990) model, that corrects the latter for scale effects. Romer's (1990) model focuses on economic diversity, more precisely, the variety of capital goods. We introduce ethnic diversity and analyze its relationship with economic growth.

The remaining of this Chapter is organized as follows. Section 3.2 we present and discusses the assumptions of the model. In Section 3.3 we solve the model for the equilibrium of society and of the markets and characterize the balanced growth path. Section 3.4 contains a discussion of our main results. Section 3.5 concludes.

3.2 The Model

To analyse the relationship between ethnic diversity and economic growth, we develop a model that builds on Jones' (1995) variant of Romer's (1990) model. Ethnic diversity is introduced by assuming that it affects the innovation process through the channels of knowledge spillovers and redundancy of research projects, as well as the effectiveness of the network through which workers cooperate in production.

Ethnic diversity corresponds to a situation where people with different racial, religious and cultural backgrounds live together, hence institutions play a decisive role in the socioeconomic processes and results. Then, we expect that the institutional frame of one society influences the impact of ethnic diversity on economic growth.

3.2.1 Society

In our setup, social interactions are described by social capital S_t and social capital is a function of ethnic diversity ε_t. Based on the empirical findings above referred, our starting premisse is that there is a level of ethnic diversity $\varepsilon_t = \varepsilon_t^*$ that maximizes social capital $S_t = S_t^*$. Institutions reflect such outcome.

Let us use ε_t as the average ethnic composition of workers. It can be equal to or different from the desired level ε_t^*. It determines the labour-augmenting coefficient of social capital S_t in the aggregate production function, and determines the average effect of ethnic diversity on knowledge spillovers and redundancy of projects on innovation β_t.

Next, we explain in more detail the conceptualization of ethnic diversity ε_t and social capital S_t. Variable β_t will be better understood right after, in the context of the R&D production function.

Ethnic Diversity

Ethnic diversity is a gradation of racial, religious and cultural differences in a society. Each country has its own gradation. It is a concept that is more precisely addressed in relative terms: Some societies are more multiethnic than others. Then, ethnic diversity can correspond to an index like the Atlas Narodov Mira fractionalization index, which has been used by Easterly and Levine (1997), Arcand et al. (2000) and Alesina and La Ferrara (2005) in their studies on the economic results of ethnic diversity. The index calculates ethnic diversity ε_t as:

$$\varepsilon_t = 1 - h_t \qquad (3.01)$$

where h_t is an Herfindhal-type index:

$$h_t = \int_1^{j_t} \eta_{j_t}^2 \, dj_t \tag{3.02}$$

so that the j_t represents the number of ethnic groups in the population at time t and the η_{j_t} corresponds to the share of the j_t-th ethnic group of the population. Then:

$$\int_1^{j_t} \eta_{j_t} \, dj_t = 1 \tag{3.03}$$

Consequently, a society with only one ethnic group exhibits $\varepsilon_t = 0$ (or $h_t = 1$), and a society in which everyone is an ethnic group has $\varepsilon_t = 1$ (or $h_t = 0$).

Social Capital

The individuals of the model are consumers and workers. Workers belong to the same social class. They belong to the working-class. From a social capital standpoint, horizontal relations produce either bonding or bridging social capital. Hence, linking social capital is out of our equation because it is produced through vertical interactions.

The, so-called, “bonding social capital” and “bridging social capital” introduced by Gittell and Vidal (1998), correspond to an upgrade of the Granovetter’s (1973) concepts of “strong ties” and “weak ties”. Both terminologies are quite enlightening concerning the contents of the concepts.

Bonding social capital bonds people with strong ties. It is produced within associations of individuals of the same ethnical group (Wakefield and Blake, 2005); thus, exhibiting a high degree of homogeneity (Field, 2003). Then, we assume that bonding social capital is positively related with ethnic homogeneity h_t.

Bridging social capital creates bridges between people of diverse ethnical origins. It is related with weaker ties than bonding; although uniting people from both sides of natural and artificial frontiers. While bonding is circumscribed to primary associations, bridging represents the affiliation in secondary ones, where ethnically different people meet (Putnam, 2000; Woolcock, 2001). Then, we assume that bridging social capital is positively related with ethnic diversity ε_t.

The above exposed leads us to specify component S_{ε_t} of our social capital function as:

$$S_{\varepsilon_t} = S\varepsilon_t^{\theta} h_t^{1-\theta} \qquad (3.04)$$

with $0 \leq \theta \leq 1$, and $S > 1$ is a constant introduced so that S_{ε_t} is always above unity (i.e. to be labour-augmenting).

Then S_{ε_t} is a weighted average of ethnic diversity ε_t and ethnic homogeneity h_t, aiming to capture the bridging and the bonding dimensions of social capital. Special attention must be payed to θ, to what it represents. Parameter θ is the weight that society assigns to ethnic diversity. A high θ means that society values ethnic diversity highly. It means that high levels of ethnic diversity improve the quality of social interactions. Then it seems fair to give θ an institutional meaning.

When θ is high, the construct of mentality, sensibility and perceptions towards ε_t, favors the development of inclusive daily attitudes and behaviors (informal institutions), as well as inclusive legislation (formal institutions) – pro-ethnic diversity Institutions. When θ is low, institutions are less inclusive of ethnical minorities. Figures 3.1 and 3.2 depict S_{ε_t} produced with ε_t for a more conservative ($\theta = 0.1$) society, and for a more multiethnic ($\theta = 0.8$) society, respectively.

Both Figure 3.1 and Figure 3.2 illustrate the mechanism behind the global evolution of S_{ε_t}.

Figure 3.1 Evolution of the social capital component S_{ε_t} with ethnic diversity ε_t according to the evolutions of bonding and bridging social capital in a conservative society: $\theta = 0.1$.

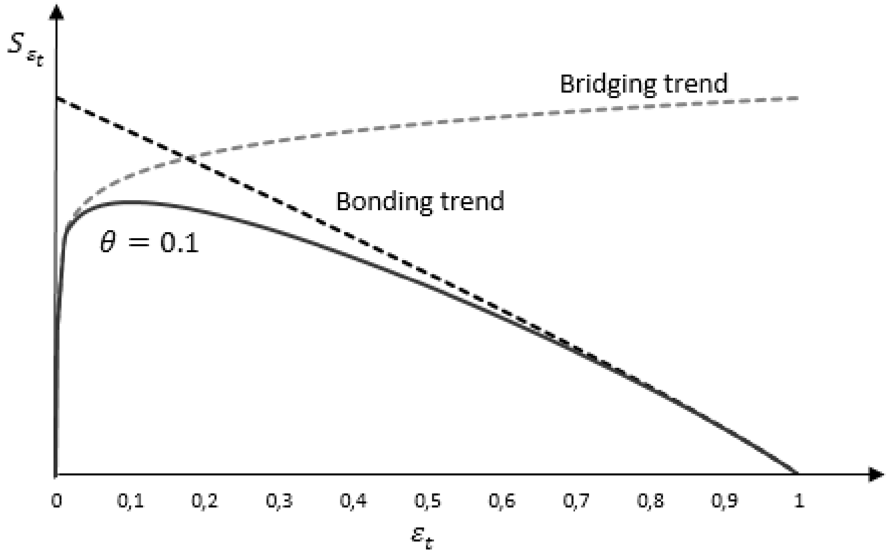
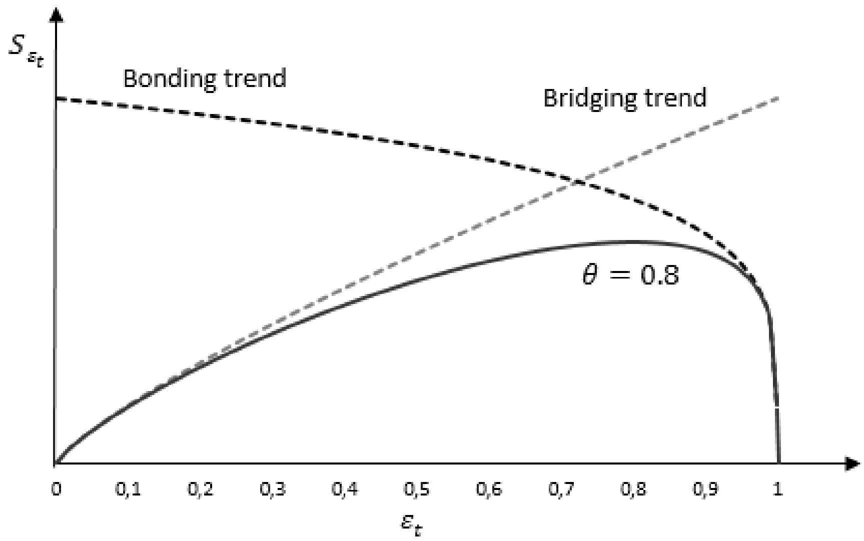


Figure 3.2 Evolution of the social capital component S_{ε_t} with ethnic diversity ε_t according to the evolutions of bonding and bridging social capital in a multiethnic society: $\theta = 0.8$.



When ε_t grows, the bonding social capital decreases and the bridging social capital increases. The weight θ given to ε_t is determinant for the relative impacts of both social capital components (bonding and bridging) in S_{ε_t}. The higher θ is, the higher is the impact of bridging on S_{ε_t} . Conversely, the lower θ is, the higher the impact of bonding. Hence, when θ is high, the maximum of S_{ε_t} is achieved for a high-value ε_t; and when θ is low, the maximum of S_{ε_t} is achieved for a low-value ε_t.

Bonding social capital is not necessarily contrary to bridging social capital. Indeed, it also represents its foundations. Bridging can only happen when bonding is already present (Putnam, 2000; Woolcock, 2001). Family, close friends, neighbours and primary associations are the starting point of social relationships. Nevertheless, it is also true that, while bonding has some major identified faults, bridging has almost always positive outcomes (Putnam, 2000, 2002; Field, 2003)[32].

Bonding social capital reflects the mobilization capacity of "the equal" around causes (Putnam, 2000; Grant, 2001). The occasions in which such causes are against "the different" are quite often (Portes, 1998). Many radical anti-social organizations are rooted into extreme forms of bonding (Putnam, 2000, 2002; Field, 2003). In contrast, bridging is inclusive, reflects openness towards different races, cultures and religions (Narayana and Pritchett, 1999). In one sentence: bridging generalizes trust (Murphy, 2002).

The proposed final social capital function is:

$$S_t = S_{\varepsilon_t} e^{\varphi \theta t} \tag{3.05}$$

with $0 < \varphi < 1$ (and considerably low for realistic purposes) and where $e^{\varphi \theta t}$ explains the time-evolution of social capital.

[32] We use the terms bonding and bridging to designate bonding social capital and bridging social capital, indistinctly.

The $e^{\varphi\theta t}$ signifies that the growth rate of social capital is a positive function of institutions' degree of inclusiveness. With such specification, we are conferring, aprioristically, a more significant role of bridging relative to bonding social capital.

Bonding social capital is limited, it preserves the network of cooperation within the boundaries of one (ethnic) group: it is an island. In contrast, by definition, the borders of bridging social capital are the earth itself. Then, we believe that it is reasonable to consider that the expansion capacity of bridging is higher than that of bonding. Bridging social capital is very important to expand the pool of resources and opportunities (Putnam, 2000; Levitte, 2003). As Putnam (2002) puts it, while bonding helps us to "get by", bridging is decisive to "get ahead".

3.2.2 Production

The stylized economy has three sectors: The R&D sector, the physical capital goods sector and the final good sector. The R&D sector and the final goods sector hire workers. Workers have an average degree of ε_t units of ethnic diversity, which is the registered in society.

The R&D Sector

The R&D sector produces blueprints of new varieties of capital goods $\dot{A}_t$.

The production function of the sector is:

$$\dot{A}_t = \phi(A_t S_t L_{At})^{\beta_t} \tag{3.06}$$

where $\phi > 0$ represents the efficiency of blueprints invention.

Variable A_t represents the existing varieties of blueprints. Equation (3.06) means that the capacity to create new blueprints $\dot{A}_t$ is enhanced by the stock of existing varieties. This is a critical assumption in both Romer's (1990) and Jones' (1995) models, although, in Romer (1990) $\beta_t = 1$, meaning that there are pure positive knowledge spillovers in the production of new ideas, while in Jones (1995) $\beta_t < 1$, meaning that the discovery of new ideas becomes increasingly difficult (once easier discoveries have been already made).

Employees in the R&D sector are L_{At}. Assuming $\beta_t = 1$, Romer (1990) considers that the effective labour in the R&D sector is equal to the number of workers L_{At}. In Jones (1995) $\beta_t < 1$, meaning that the L_{At} workers correspond to an effective labour of $L_{At}^{\beta_t}$ due to redundancy of research projects. Following Jones (1995), we assume that $0 < \beta_t < 1$.

We depart from Jones (1995) in several other aspects. Firstly, we introduce social capital S_t in the production function for blueprints (and for output). Workers cooperate with each other and these interactions increase their productivity.

Secondly, we consider a joint effect of: (i) increasing difficulty in producing new ideas from past ideas; and (ii) redundancy in research projects. In Jones' (1995) specification, the power of A_t is different from the power of L_{At}. We propose a specification in which the A_t and the L_{At} are both raised to the same power β_t; i.e., β_t captures an average effect. We consider an average effect under the intuition that redundancy in research projects and knowledge spillovers are very much associated. It also contributes to analytical simplicity of the developed model.

The findings of Gauthier-Loiselle (2009) and Kerr (2010) of the duplication of the number of patents due to immigration; Maré et al.'s (2010) conclusion that the presence of immigrant workers has a positive impact on the innovation of products and production processes and Nathan and Lee's (2013) finding that ethnically diverse managements are better prepared to introduce new products and to pursue successful internationalization are cases where we can put together new ideas and new research projects.

Then, and thirdly, our β_t is a function of ethnic diversity, so that:

$$\beta_t = \beta + (1-\beta)\varepsilon_t - \Delta_t \qquad (3.07)$$

where:

$$\Delta_t = \begin{cases} 0 & \varepsilon_t \leq \varepsilon_t^* \\ \gamma(\varepsilon_t - \varepsilon_t^*) & \varepsilon_t > \varepsilon_t^* \end{cases} \qquad (3.08)$$

with $0 < 1-\beta < \gamma < 1$ if $0 \leq \beta_t \leq 1$ holds.

Function β_t exhibits a discontinuity point at $\varepsilon_t = \varepsilon_t^*$ because at that point more ethnic diversity produces social disruption. When $\varepsilon_t < \varepsilon_t^*$, increases of ethnic diversity produce improvements in the joint knowledge spillovers and research redundancy of $1-\beta > 0$, on the margin. This marginal effect changes to $1-\beta-\gamma < 0$ for $\varepsilon_t > \varepsilon_t^*$.

Profit maximization in the R&D sector is obtained by choosing the number of workers to hire. With fraction:

$$\delta = \phi(A_t S_t)^{\beta_t} L_{At}^{\beta_t - 1} \qquad (3.09)$$

out of maximizing control, the R&D sector decides upon:

$$\dot{A}_t = \delta L_{At} \qquad (3.10)$$

The Capital Goods Sector

First, the capital goods producers enter in the market. To enter the market, they must invest in the blueprint patent whose price is P_{At}. Next to buying the patent, the capital good firm has property rights on the blueprint, hence it exhibits monopolistic profits thereafter.

This sector is in monopolistic competition. There is a degree of monopoly power arising from the exclusivity in the production of a variety of capital goods but there is also some competition amongst varieties. The market power implies that the producer is a price setter, because the producer must have positive a stream of instantaneous positive profits to refund his initial investment on the patent. The free-entry condition implies intertemporal zero profits.

All the capital goods depreciate fully after one period and the production of one unit of a capital variety requires an investment of r_t units of final goods.

The Final Goods Sector

The final goods' producers operate in competitive markets upstream and downstream. Their strategic variables are the number of workers to hire L_{Yt} and the amount of each variety of capital good x_{it} to use in production for given wages w_{Yt} and prices of the physical capital goods p_{it}, with $i \in [0, A_t]$.

The aggregate production function is:

$$Y_t = \left(S_t L_{yt}\right)^{1-\alpha} \int_0^{A_t} x_{it}^{\alpha} di \qquad (3.11)$$

with $0 < \alpha < 1$ and where S_t represents the augmenting effect of social capital.

Jumping some steps in the model's solving, we wish to show that the model exhibits symmetry across the different types of capital goods. In fact, final goods producers maximize their profits:

$$\max_{L_{Yt},x_{it}} \pi_{Yt} = (S_t L_{Yt})^{1-\alpha} \int_0^{A_t} x_{it}^{\alpha}\, di - \int_0^{A_t} p_{it} x_{it}\, di - w_{Yt} L_{Yt}$$

and the demand for capital goods varieties corresponds to the solution of:

$$\frac{\partial \pi_{Yt}}{\partial x_{it}} = 0$$

delivering the demand function:

$$p_{it} = \alpha \left(\frac{S_t L_{Yt}}{x_{it}}\right)^{1-\alpha}$$

which is the restriction subject to which the capital goods' producers maximize their profits:

$$\max_{p_{it},x_{it}} \pi_{xit} = p_{it} x_{it} - r_t x_{it}$$

i.e.:

$$\max_{x_{it}} \pi_{xit} = \alpha (S_t L_{Yt})^{1-\alpha} x_{it}^{\alpha} - r_t x_{it}$$

so, the optimal rule is:

$$\frac{\partial \pi_{xit}}{\partial x_{it}} = 0$$

which delivers:

$$x_{it} = \left(\frac{\alpha^2}{r_t}\right)^{\frac{1}{1-\alpha}} S_t L_{Yt}$$

As the right-hand side of the equation has no variety index, symmetry is proven.

With symmetry, we can drop the variety index and use x_t and p_t because all physical capital varieties have the same price and are produced in the same quantities. This implies that:

$$\int_0^{A_t} x_{it}^{\alpha}\, di = \int_0^{A_t} x_t^{\alpha}\, di = A_t x_t^{\alpha} \tag{3.12}$$

and the final goods production function (3.11) becomes:

$$Y_t = \left(S_t L_{yt}\right)^{1-\alpha} A_t x_t^{\alpha} \tag{3.13}$$

Aggregate physical capital K_t also simplifies to:

$$K_t = \int_0^{A_t} x_{it}\, di = \int_0^{A_t} x_t\, di = A_t x_t \tag{3.14}$$

meaning that the aggregate production function can be rewritten as:

$$Y_t = \left(A_t S_t L_{yt}\right)^{1-\alpha} K_t^{\alpha} \tag{3.15}$$

without loss of generality.

3.2.3 Individuals

The individuals are workers and consumers. They work in the final goods and R&D sectors. As consumers they maximize their intertemporal utility according to the assets they detain.

The Labour Force

Total labour force L_t divide into L_{At} workers allocated to the R&D sector and L_{Yt} workers employed in the final goods sector:

$$L_t = L_{At} + L_{Yt} \tag{3.16}$$

The growth rate of L_t is n, constant and the growth rates of L_{At} and L_{Yt} are n_{At} and n_{Yt} so that:

$$n_{At} = n - g_{\eta_t} \tag{3.17}$$

and:

$$n_{Yt} = n - \frac{1}{1-\eta_t} g_{\eta_t} \tag{3.18}$$

if:

$$\eta_t = \frac{L_t}{L_{At}} \tag{3.19}$$

Result (3.18) is derived from:

$$\frac{L_t}{L_{Yt}} = \frac{L_t}{L_t - L_{At}} = \frac{\eta_t}{\eta_t - 1}$$

and noticing that:

$$\frac{\left(\dot{\frac{L_t}{L_{Yt}}}\right)}{\frac{L_t}{L_{Yt}}} = n - n_{Yt}$$

and that:

$$\frac{\left(\dot{\frac{\eta_t}{\eta_t - 1}}\right)}{\frac{\eta_t}{\eta_t - 1}} = -\frac{\dot{\eta}_t}{(\eta_t - 1)^2}\frac{\eta_t - 1}{\eta_t} = \frac{1}{1 - \eta_t} g_{\eta_t}$$

Equation (3.18) represents an important analytical step in determining the model's balanced growth path (BGP).

Consumers

Consumers optimize their utility by choosing consumption C_t and next period assets that maximize their intertemporal utility function:

$$U = \int_0^{+\infty} \frac{C_t^{1-\sigma}}{1 - \sigma} e^{-\rho t} \tag{3.20}$$

where $0 < \sigma < 1$ is the inverse of the elasticity of substitution, and $0 < \rho < 1$ is the discount rate.

Their budget constraint is:

$$\dot{B}_t = r_t B_t + w_t L_t - C_t \tag{3.21}$$

where r_t is the interest rate and B_t represents assets. The $r_t B_t$ represents profits, rents and interest income, while w_t represents labour income.

An important intermediate step of the model solving consists in the demonstration that (3.21) is equivalent to the familiar budget constraint of a closed economy without government:

$$\dot{K}_t = Y_t - C_t \tag{3.22}$$

3.3 Equilibrium

In this Section we solve for the BGP of the economy.

3.3.1 Social Equilibrium

As advanced earlier, there is one level of ethnic diversity $\varepsilon_t = \varepsilon_t^*$ that maximizes the economy's stock of social capital $S_t = S_t^*$. That is, with S_t given by (3.04) and (3.05); i.e.:

$$S_t = S\varepsilon_t^{\theta} h_t^{1-\theta} e^{\varphi\theta t}$$

then:

$$\frac{\partial S_t}{\partial \varepsilon_t} = 0$$

gives us:

$$\theta S\varepsilon_t^{\theta-1}h_t^{1-\theta}e^{\varphi\theta t} - (1-\theta)S\varepsilon_t^{\theta}h_t^{-\theta}e^{\varphi\theta t} = 0 \Leftrightarrow$$

$$\Leftrightarrow S\varepsilon_t^{\theta-1}h_t^{-\theta}e^{\varphi\theta t}[\theta h_t - (1-\theta)\varepsilon_t] = 0 \Leftrightarrow$$

$$\Leftrightarrow \ \theta - \varepsilon_t = 0$$

that is:

$$\varepsilon_t^* = \theta \tag{3.23}$$

According to (3.23), our desired level of ethnic diversity ε_t^*, is constant and equal to θ. Parameter θ represents institutions, specifically, their promptness for inclusion of diversity. The corresponding desired level of social capital at time t is given by:

$$S_t^* = S\theta^{\theta}(1-\theta)^{1-\theta}e^{\varphi\theta t} \tag{3.24}$$

Societies may deviate from such level whenever the existing ethnic diversity is $\varepsilon_t \neq \varepsilon_t^*$.

The general level of social capital at time t is:

$$S_t = S\varepsilon_t^{\theta}(1-\varepsilon_t)^{1-\theta}e^{\varphi\theta t} \tag{3.25}$$

In any case, the growth rate of social capital is:

$$g_{S_t} = g_{S_t^*} = \varphi\theta \tag{3.26}$$

meaning that regardless of the level of ethnic diversity, the installed network has the potential to grow at rate $\varphi\theta$ over time.

Under the same rationale, our proposed desired level of β_t is:

$$\beta_t^* = \beta + (1-\beta)\theta \tag{3.27}$$

but in general terms, if $\varepsilon_t \leq \varepsilon_t^*$, it is equal to:

$$\beta_t = \beta + (1-\beta)\varepsilon_t \tag{3.28}$$

whereas if $\varepsilon_t > \varepsilon_t^*$, it is equal to:

$$\beta_t = \beta + (1-\beta)\varepsilon_t - \gamma(\varepsilon_t - \theta) \tag{3.29}$$

In what follows, we present the results with notations S_t, S_t^*, g_{S_t}, $g_{S_t^*}$, β_t and β_t^* as a compact form of the right-hand side of equations (3.24) to (3.29).

3.3.2 Market Equilibrium

The Final Goods Sector

The final goods' producers maximize their profits π_{Yt}. Being price takers, they choose the best combination of L_{Yt} and x_t given w_{Yt} and p_t, so that:

$$\max_{L_{Yt},x_t} \pi_{Yt} = \left(S_t L_{yt}\right)^{1-\alpha} A_t x_t^{\alpha} - A_t p_t x_t - w_{Yt} L_{Yt} \tag{3.31}$$

Hence, they will hire workers and purchase capital goods up to:

$$\frac{\partial \pi_{Yt}}{\partial L_{Yt}} = 0$$

and:

$$\frac{\partial \pi_{Yt}}{\partial x_t} = 0$$

respectively.

The optimal condition:

$$\frac{\partial \pi_{Yt}}{\partial L_{Yt}} = 0 \Leftrightarrow (1-\alpha)S_t\big(S_t L_{yt}\big)^{-\alpha} A_t x_t^{\alpha} - w_{Yt} = 0 \Leftrightarrow w_{Yt} = \frac{(1-\alpha)S_t^{1-\alpha} A_t x_t^{\alpha}}{L_{Yt}^{\alpha}} \Leftrightarrow$$

$$\Leftrightarrow w_{Yt} = \frac{(1-\alpha)(A_t S_t)^{1-\alpha}(A_t x_t)^{\alpha}}{L_{Yt}^{\alpha}} \Leftrightarrow w_{Yt} = \frac{(1-\alpha)(A_t S_t L_{Yt})^{1-\alpha} K^{\alpha}}{L_{Yt}}$$

delivers the demand of labour as:

$$w_{Yt} = (1-\alpha)\frac{Y_t}{L_{Yt}} \tag{3.32}$$

The demand for capital goods is computed as follows:

$$\frac{\partial \pi_{Yt}}{\partial x_t} = 0 \Leftrightarrow \alpha\big(S_t L_{yt}\big)^{1-\alpha} A_t x_t^{\alpha-1} - A_t p_t = 0 \Leftrightarrow \alpha\big(S_t L_{yt}\big)^{1-\alpha} A_t x_t^{\alpha-1} = A_t p_t$$

then:

$$p_t = \alpha\left(\frac{S_t L_{Yt}}{x_t}\right)^{1-\alpha} \tag{3.33}$$

The Capital Goods Sector

Capital good firms maximize their profits. They face a negatively sloped demand (3.33), meaning that they have some market power. Thus, they choose the combination of x_t and p_t that maximize their profits, given the cost of producing one unit of x_t, which is r_t units of final goods. They solve:

$$\max_{p_t, x_t} \pi_{xt} = p_t x_t - r_t x_t \tag{3.34}$$

subject to (3.33).

Then, their maximization problem is:

$$\max_{x_t} \pi_{xt} = \alpha(S_t L_{Yt})^{1-\alpha} x_t^{\alpha} - r_t x_t$$

and the optimal rule becomes:

$$\frac{\partial \pi_{xt}}{\partial x_t} = 0 \Leftrightarrow \alpha^2 (S_t L_{Yt})^{1-\alpha} x_t^{\alpha-1} - r_t = 0$$

Solving for x_t, we get the supply of each capital good:

$$x_t = \left(\frac{\alpha^2}{r_t}\right)^{\frac{1}{1-\alpha}} S_t L_{Yt} \tag{3.35}$$

Inserting (3.35) into (3.33):

$$p_t = \alpha \left(\frac{S_t L_{Yt}}{x_t}\right)^{1-\alpha} \Leftrightarrow p_t = \alpha \left[\frac{S_t L_{Yt}}{\left(\frac{\alpha^2}{r_t}\right)^{\frac{1}{1-\alpha}} S_t L_{Yt}}\right]^{1-\alpha} \Leftrightarrow p_t = \alpha \left[\left(\frac{\alpha^2}{r_t}\right)^{-\frac{1}{1-\alpha}}\right]^{1-\alpha}$$

we obtain the equilibrium markup price:

$$p_t = \frac{r_t}{\alpha} \tag{3.36}$$

Rearranging (3.35):

$$x_t^{1-\alpha} = \frac{\alpha^2}{r_t} (S_t L_{Yt})^{1-\alpha} \Leftrightarrow r_t = \alpha^2 (S_t L_{Yt})^{1-\alpha} x_t^{\alpha-1} \Leftrightarrow$$

$$\Leftrightarrow r_t = \alpha^2 (A_t S_t L_{Yt})^{1-\alpha} (A_t x_t)^{\alpha} \Leftrightarrow r_t = \alpha^2 (A_t S_t L_{Yt})^{1-\alpha} \frac{K_t^{\alpha}}{K_t}$$

then:

$$r_t = \alpha^2 \frac{Y_t}{K_t} \tag{3.37}$$

Capital good firms have profits each period of:

$$\pi_{xt} = p_t x_t - r_t x_t \Leftrightarrow \pi_{xt} = (p_t - r_t)x_t \Leftrightarrow \pi_{xt} = \left(\frac{r_t}{\alpha} - r_t\right) x_t$$

i.e.:

$$\pi_{xt} = \frac{1-\alpha}{\alpha} r_t x_t \tag{3.38}$$

While, in equilibrium final goods producers have zero profits, each period; capital goods producers must have positive profits, each period. To enter the capital goods market, firms must acquire a blueprint patent for the value of P_{At}. After this initial investment, each producer has an infinite-horizon property rights over the blueprint. As there is free entry in the market, each capital good firm will buy the patent for the value of the discounted stream of its profits over an infinite horizon, that is:

$$P_{At} = \int_t^{\infty} \pi_{xu}\, e^{-\int_t^u r_v dv} du \tag{3.39}$$

Equation (3.39) is equivalent to the standard Hamilton-Jacobi-Bellman equation:

$$r_t P_{At} = \pi_{x_\tau} + \dot{P}_{At} \tag{3.40}$$

As:

$$\dot{P}_{At} = \left(\int_t^{\infty} \pi_{x_u} e^{-\int_t^u r_v dv} du\right)_t' \Leftrightarrow \dot{P}_{At} = -\pi_{x_\tau} e^{-\int_t^t r_v dv} + \int_t^{\infty} \pi_{x_u} \left(e^{-\int_t^u r_v dv}\right)_t' du \Leftrightarrow$$

$$\Leftrightarrow \dot{P}_{At} = -\pi_{x_\tau} + \int_t^{\infty} \pi_{x_u} \left[\left(-\int_t^u r_v dv\right)_t' \left(e^{-\int_t^u r_v dv}\right)'_{-\int_t^u r_v dv}\right] du \Leftrightarrow \dot{P}_{At}$$

$$= -\pi_{x_\tau} + r_t P_{At}$$

The R&D Sector

The blueprints production function is given by (3.06), recall:

$$\dot{A}_t = \phi(A_t S_t L_{At})^{\beta_t}$$

although the R&D producers do not have control over (3.09):

$$\delta = \phi(A_t S_t)^{\beta_t} L_{At}^{\beta_t - 1}$$

Then, given wages w_{At}, they maximize their profit controlling for L_{At}, so that:

$$\max_{L_{At}} \pi_{At} = P_{At}\dot{A}_t - w_{At}L_{At} \tag{3.41}$$

with (3.10) giving:

$$\dot{A}_t = \delta L_{At}$$

Profit maximization:

$$\frac{\partial\, \pi_{x_{At}}}{\partial L_{At}} = 0$$

delivers the demand for labour in the R&D sector as:

$$w_{At} = P_{At}\delta \tag{3.42}$$

The Labour Market

The wage in the final goods sector is given by (3.32):

$$w_{Yt} = (1-\alpha)\frac{Y_t}{L_{Yt}}$$

Expression (3.32) is obtained by inserting x_t given by (3.35):

$$x_t = \left(\frac{\alpha^2}{r_t}\right)^{\frac{1}{1-\alpha}} S_t^* L_{Yt}$$

into aggregate production function:

$$Y_t = \left(S_t L_{yt}\right)^{1-\alpha} A_t x_t^\alpha$$

that is:

$$Y_t = \left(S_t L_{yt}\right)^{1-\alpha} A_t x_t^\alpha \Leftrightarrow Y_t = \left(S_t L_{yt}\right)^{1-\alpha} A_t \left[\left(\frac{\alpha^2}{r_t}\right)^{\frac{1}{1-\alpha}} S_t L_{Yt}\right]^\alpha$$

hence:

$$Y_t = S_t L_{yt} A_t \left(\frac{\alpha^2}{r_t}\right)^{\frac{\alpha}{1-\alpha}} \tag{3.43}$$

equivalent to:

$$\frac{Y_t}{L_{Yt}} = S_t A_t \left(\frac{\alpha^2}{r_t}\right)^{\frac{\alpha}{1-\alpha}}$$

Accordingly, the wage rate in the final goods sector is:

$$w_{Yt} = (1-\alpha) S_t A_t \left(\frac{\alpha^2}{r_t}\right)^{\frac{\alpha}{1-\alpha}} \tag{3.44}$$

In equilibrium $w_{Yt} = w_{At}$, i.e.:

$$P_{At}\delta = (1-\alpha) S_t A_t \left(\frac{\alpha^2}{r_t}\right)^{\frac{\alpha}{1-\alpha}}$$

therefore, the price of the blueprint or the value of each capital goods firm at time t, is:

$$P_{At} = \frac{(1-\alpha) S_t A_t}{\delta} \left(\frac{\alpha^2}{r_t}\right)^{\frac{\alpha}{1-\alpha}}$$

or:

$$P_{At} = \frac{1-\alpha}{\phi} (A_t S_t L_{At})^{1-\beta_t} \left(\frac{\alpha^2}{r_t}\right)^{\frac{\alpha}{1-\alpha}} \tag{3.45}$$

3.3.3 Consumption Decisions

The Maximization Problem

Consumers maximize utility U given by (3.20):

$$U = \int_0^{+\infty} \frac{C_t^{1-\sigma}}{1-\sigma} e^{-\rho t} dt$$

subject to constraint $\dot{B}_t$, given by (3.21):

$$\dot{B}_t = r_t B_t + w_t L_t - C_t$$

The current-value Hamiltonian is:

$$H_t = \frac{C_t^{1-\sigma}}{1-\sigma} + \lambda_t (r_t B_t + w_t L_t - C_t)$$

where λ_t is the shadow price of income.

The first-order conditions deliver:

$$\frac{\partial H_t}{\partial C_t} = 0 \Leftrightarrow C_t^{-\sigma} = \lambda_t \Leftrightarrow g_{C_t} = -\frac{1}{\sigma} g_{\lambda_t}$$

and:

$$\frac{\partial H_t}{\partial B_t} = \rho \lambda_t - \dot{\lambda}_t \Leftrightarrow \lambda_t r_t = \rho \lambda_t - \dot{\lambda}_t \Leftrightarrow g_{\lambda_t} = \rho - r_t$$

which together give:

$$g_C = \frac{1}{\sigma}(r_t - \rho) \qquad (3.46)$$

This condition, together with the transversality condition:

$$\lim_{t \to \infty} e^{-\rho t} \lambda_t B_t = 0$$

guarantee that the consumers do not over-save; i.e., that growth of the value of the assets does not exceed the discount rate.

The Physical Capital Dynamics

Consumers' budget constraint:

$$\dot{B}_t = r_t B_t + w_t L_t - C_t$$

where $r_t B_t$ represents profits, rents and interest income, while w_t represents labour income.

Individuals' assets B_t consist in physical capital K_t and ownership of shares on capital firms with an aggregate value of $P_{At} A_t$; i.e.:

$$B_t = K_t + P_{At} A_t$$

Substituting in (3.21):

$$(K_t + \dot{P}_{At} A_t) = r_t (K_t + P_{At} A_t) + w_t L_t - C_t \Leftrightarrow$$

$$\Leftrightarrow \dot{K}_t + \dot{P}_{At} A_t + P_{At} \dot{A}_t = r_t K_t + r_t P_{At} A_t + w_t L_t - C_t \Leftrightarrow$$

$$\Leftrightarrow \dot{K}_t = r_t K_t + \left(r_t P_{At} - \dot{P}_{At}\right) A_t - P_{At} \dot{A}_t + w_t L_t - C_t$$

Using Hamilton-Jacobi-Bellman-equation (3.40):

$$r_t P_{At} - \dot{P}_{At} = \pi_{x_\tau}$$

we get:

$$\dot{K}_t = r_t K_t + \pi_{x_\tau} A_t - P_{At} \dot{A}_t + w_t L_t - C_t \qquad (3.47)$$

Recalling (3.37)

$$Y_t = \frac{r_t}{\alpha^2} K_t$$

and as:

$$K_t = A_t x_t$$

it follows that:

$$Y_t = \frac{r_t}{\alpha^2} A_t x_t$$

implying that:

$$x_t = \frac{\alpha^2}{r_t} \frac{Y_t}{A_t}$$

Substituting x_t in (3.38) we get:

$$\pi_{xt} = \frac{1-\alpha}{\alpha} r_t \frac{\alpha^2}{r_t} \frac{Y_t}{A_t}$$

then:

$$\pi_{xt} = \alpha(1-\alpha) \frac{Y_t}{A_t} \quad (3.48)$$

Inserting (3.48) into (3.47):

$$\dot{K}_t = r_t K_t + \alpha(1-\alpha) \frac{Y_t}{A_t} A_t - P_{At} \dot{A}_t + w_t L_t - C_t$$

then:

$$\dot{K}_t = r_t K_t + \alpha(1-\alpha) Y_t - P_{At} \dot{A}_t + w_t L_t - C_t \quad (3.49)$$

Recalling (3.32):

$$w_{Yt} = (1-\alpha)\frac{Y_t}{L_{Yt}}$$

as $w_{Yt} = w_t = w_{At}$, from (3.49):

$$\dot{K}_t = r_t K_t + \alpha(1-\alpha)Y_t - P_{At}\dot{A}_t + (1-\alpha)Y_t\frac{L_t}{L_{Yt}} - C_t \tag{3.50}$$

The free entry condition in the capital good's market is equivalent to:

$$P_{At}\dot{A}_t = w_{At}L_{At}$$

then using once again (3.32) and $w_{Yt} = w_t = w_{At}$, we obtain:

$$P_{At}\dot{A}_t = (1-\alpha)Y_t\frac{L_{At}}{L_{Yt}}$$

which we can insert into (3.50) to get:

$$\dot{K}_t = r_t K_t + \alpha(1-\alpha)Y_t - (1-\alpha)Y_t\frac{L_{At}}{L_{Yt}} + (1-\alpha)Y_t\frac{L_t}{L_{Yt}} - C_t \Leftrightarrow$$

$$\Leftrightarrow \dot{K}_t = r_t K_t + \left(\alpha - \frac{L_{At}}{L_{Yt}} + \frac{L_t}{L_{Yt}}\right)(1-\alpha)Y_t - C_t = r_t K_t + (1+\alpha)(1-\alpha)Y_t - C_t$$

which is equivalent to:

$$\dot{K}_t = r_t K_t + (1-\alpha^2)Y_t - C_t \tag{3.51}$$

Recalling, once again (3.37), equivalent to:

$$r_t K_t = \alpha^2 Y_t$$

and introducing it in (3.51):

$$\dot{K}_t = \alpha^2 Y_t + (1 - \alpha^2) Y_t - C_t$$

We get the equation for the evolution of physical capital stock (3.21):

$$\dot{K}_t = Y_t - C_t$$

which provides us the equation for the computation of relationships between the BGP growth rates, whose deductions follow next.

3.3.4 Balanced Growth Path

Next, we will use notation g_{z_t} for the growth rate of variable z at time t.

As equation (3.46) says, on the BGP, g_{Ct} must be constant, which requires r_t to be constant.

With r_t constant, the output-physical capital ratio is also constant: Recall (3.37):

$$\frac{Y_t}{K_t} = \frac{r_t}{\alpha^2}$$

Log-time-differentiating:

$$\frac{\dot{K}_t}{K_t} - \frac{\dot{Y}_t}{Y_t} = 0$$

then:

$$g_{Y_t} = g_{K_t}$$

Dividing the consumers' budget constraint (3.21) by K_t, we get:

$$g_{K_t} = \frac{Y_t}{K_t} - \frac{C_t}{K_t}$$

As $g_{Y_t} = g_{K_t}$ are constant, the output-physical capital ratio is also constant, then the consumption-physical capital ratio must also be constant; i.e.:

$$\frac{\dot{C}_t}{C_t} - \frac{\dot{K}_t}{K_t} = 0$$

or:

$$g_{C_t} = g_{K_t}$$

then:

$$g_{Yt} = g_{Kt} = g_{Ct} \tag{3.53}$$

Log-time-differentiating equation (3.43):

$$Y_t = S_t L_{yt} A_t \left(\frac{\alpha^2}{r_t}\right)^{\frac{\alpha}{1-\alpha}}$$

we get:

$$g_{Y_t} = g_{A_t} + g_{S_t} + n_{Yt}$$

then:

$$g_{Yt} = g_{Kt} = g_{Ct} = g_{A_t} + g_{S_t} + n_{Yt} \tag{3.54}$$

As g_{S_t} is constant then so must n_{Yt} be. With n_{Yt} given by (3.18):

$$n_{Yt} = n - \frac{1}{1-\eta_t} g_{\eta_t}$$

the only value of g_{η_t} compatible with n_{Yt} constant, is $g_{\eta_t} = 0$. It then follows that:

$$n_{At} = n_{Yt} = n \tag{3.55}$$

Then we derive the growth rate of A_t by dividing both members of the R&D production function (3.06) by A_t. That is:

$$\dot{A}_t = \phi (A_t S_t L_{At})^{\beta_t}$$

is equivalent to:

$$g_{A_t} = \phi A_t^{\beta_t - 1} (S_t L_{At})^{\beta_t}$$

In steady state, g_{A_t} is constant. Thus log-time-differentiating g_{A_t} we obtain:

$$(\beta_t - 1) g_{A_t} + \beta_t \left(g_{S_t} + n_{At} \right) = 0$$

Remembering (3.55) and solving for g_{A_t}:

$$g_{A_t} = \frac{\beta_t}{1-\beta_t} \left(g_{S_t} + n \right) \tag{3.56}$$

Inserting (3.56) into (3.54) we get that:

$$g_{Yt} = g_{Kt} = g_{Ct} = \frac{\beta_t}{1-\beta_t} \left(g_{S_t} + n \right) + g_{S_t} + n$$

i.e. our economy's aggregate growth rate is given by:

$$g_{Yt} = g_{Kt} = g_{Ct} = \frac{n + g_{S_t}}{1-\beta_t} \tag{3.57}$$

Regarding the growth rates of profits/values of firms, remembering expressions for r_t given by (3.37) and for π_{xt} given by (3.38), we get that:

$$\pi_{xt} = \frac{1-\alpha}{\alpha}\left(\alpha^2 \frac{Y_t}{K_t}\right) x_t \Leftrightarrow \pi_{xt} = \alpha(1-\alpha)\left(\frac{Y_t}{A_t x_t}\right) x_t$$

i.e.:

$$\pi_{xt} = \alpha(1-\alpha)\frac{Y_t}{A_t}$$

whose log-time-differentiation gives us:

$$g_{\pi_{xt}} = g_{Y_t} - g_{A_t}$$

From the Hamilton-Jacobi-Bellman-equation (3.40):

$$r_t P_{At} = \pi_{x_\tau} + \dot{P}_{At}$$

we deduce that:

$$r_t - \frac{\dot{P}_{At}}{P_{At}} = \frac{\pi_{xt}}{P_{At}}$$

As in steady state $g_{P_{At}}$ is constant, the right side of the equation above must also be constant, therefore:

$$g_{\pi_{xt}} = g_{P_{At}} = g_{Y_t} - g_{A_t} \tag{3.58}$$

Then equations (3.57) and (3.58) together give:

$$g_{\pi_{xt}} = g_{P_{At}} = \frac{n + g_{S_t}}{1-\beta_t} - \frac{\beta_t}{1-\beta_t}\left(n + g_{S_t}\right)$$

then:

$$g_{\pi_{xt}} = g_{P_{At}} = g_{S_t} + n \qquad (3.59)$$

Ultimately, the value of each capital goods' firm evolves according to (3.59).

Economic growth means per capita output growth. To obtain our economy's economic growth rate, we need to extract the growth rate of the population, n, from the aggregate growth rates (3.57) to get:

$$g_t^* = \frac{g_{S_t} + \beta_t n}{1 - \beta_t} \qquad (3.60)$$

3.4 Discussion

We now discuss the model's results and propose some policy recommendations.

3.4.1 Ethnic Diversity and Economic Growth

A significant result concerning the sustainability of the economy is represented by the evolution of the value of the capital good firms, which coincides with the stream of positive profits required for there to be investment in new blueprints.

According to (3.59), the BGP growth rates $g_{P_{At}}$ and $g_{\pi_{xt}}$ are equal to the sum of the growth rate of social capital g_{S_t} with the growth rate of population n. Then, remembering (3.26):

$$g_{P_{At}} = g_{\pi_{xt}} = \varphi\theta + n \qquad (3.61)$$

Let us look at (3.61) from the perspective of a very conservative (θ low) country with a stagnant or decaying population ($n \leq 0$). Under these circumstances, $g_{P_{At}} = g_{\pi_{xt}}$ will also be very low or even negative. Consequently, there will be low or nil investment on capital goods. The capital goods' sector is a provider to the final goods' sector and a client of the R&D sector. Hence, no investment on new blueprints leads the economy to bankruptcy. Solving such problem requires increasing the growth of the population n and/or the degree of institutional inclusiveness θ. However, both solutions are difficult to implement.

Firstly, a low θ represents a collective aversion to ethnically different people. It is not flexible and does not change overnight. Secondly, as we have shown in Chapter 1, increases in n can only be achieved my means of immigration, there is no other solution in a foreseeable time horizon. In a low θ country, the immigrants socially accepted necessarily belong to a similar ethnic group than that of the country. Immigrants of a similar ethnic group come from countries with the same racial, religious and cultural background. Such countries exist – usually are the neighbouring countries – but the most likely to occur is that "equal countries" face "equal problems"; meaning that the desired type of immigration in the required amounts is most probably inexistent.

Given the above discussed, our first conclusion regarding low θ countries, is that the best solution to overcome economic growth shortages is to pursue structural policies that increase θ as it: (i) directly enhances profits of capital good firms through the growth rate of social capital; and (ii) indirectly enhances n because the most probable accessible international migrants are ethnically different.

In general, the chain of events linking positively ethnic diversity with the value of capital good firms is the following. There is a level of ethnic diversity that maximizes social capital. Such desired level of ethnic diversity $\varepsilon_t^* = \theta$ determines the configuration of the country's formal and informal institutions relative to their inclusiveness: Formal and informal institutions are the networks upon which social capital is produced. The higher is the tolerance towards ethnic diversity, the larger is the network and its expansion prospects: $g_{S_t} = \varphi\theta$. The growth rate of social capital is higher and so is the probability of pursuing effective demographic policies to increase the growth rate of the population, n. Then, both, the growth rate of social capital and the growth rate of the population reinforce the attractiveness of investing in new capital goods' blueprints, $g_{P_{At}} = \varphi\theta + n$, stimulating innovation and economic growth: $g_{y_t} = g_{P_{At}} + g_{A_t} - n$.

Still regarding the growth rate of the value of the physical capital firms, as equation (3.61) elucidates, it is insensitive to the registered levels of ethnic diversity ε_t, since on the right side of the equation we only have θ. This means that, the prospective behaviors of P_{At} and π_{xt} are anchored on one economy's institutions. Institutions are the basis of the formation of investment expectations as they rely on society values. Therefore, ethnic diversity affects $g_{P_{At}}$ and $g_{\pi_{xt}}$, through $\varepsilon_t^* = \theta$.

In terms of levels, P_{At} and π_{xt} are influenced by the registered ethnic diversity levels ε_t. According to (3.45), the value of the firm at t is:

$$P_{At} = \frac{1-\alpha}{\phi}(A_t S_t L_{At})^{1-\beta_t}\left(\frac{\alpha^2}{r_t}\right)^{\frac{\alpha}{1-\alpha}}$$

then, for instance, when $\varepsilon_t < \varepsilon_t^*$, β_t is given by (3.28) and S_t by (3.25), hence:

$$P_{At} = \frac{1-\alpha}{\phi}\left[\varepsilon_t^{\theta}(1-\varepsilon_t)^{1-\theta}e^{\varphi\theta t}A_t S L_{At}\right]^{(1-\beta)(1-\varepsilon_t)}\left(\frac{\alpha^2}{r_t}\right)^{\frac{\alpha}{1-\alpha}}$$

With the Hamilton-Jacobi-Bellman equation we relate π_{x_τ} to P_{At}, meaning that π_{x_τ} is also dependent on ε_t.

While the growth rates $g_{\pi_{xt}} = g_{P_{At}}$ are only dependent on the desired level of ethnic diversity (institutions), the growth rates of output, consumption and physical capital per capita g_t^* and of innovation g_{A_t} are dependent on both the current level ε_t and the desired level of ethnic diversity ε_t^*.

Considering equation (3.58) in the form $g_{y_t} = g_{P_{At}} + g_{A_t} - n$, presented above, we find that the motive for this behavior of g_{y_t} is the consequence of the effects of g_{A_t}. Contrarily to investments, innovation, by nature, relies more on real world dynamics than on institutions. The creativity to produce new things has a lot to do with the ensemble of ideas, market demands and social conditions, at the moment. Institutions count for their role on the network of cooperation between researchers ε_t^*, but also for their degree of as guarantees of social inclusion, $\varepsilon_t^* - \varepsilon_t \geq 0$, or exclusion, $\varepsilon_t^* - \varepsilon_t < 0$. An adverse social setting, $\varepsilon_t^* - \varepsilon_t < 0$, influences the innovation process through social instability and the valuation of the present-time multiethnic environment. It has an eroding effect over g_{A_t}, then over $g_t^* = g_{y_t}$. The only case in which only the institution counts for g_{A_t} corresponds to the benchmark case in which $\varepsilon_t = \varepsilon_t^*$. Although this is only apparent, that is, ε_t counts, yet, it coincides with ε_t^*.

When $\varepsilon_t = \varepsilon_t^*$, the economy has reached its desired ethnical diversity level. Every minority is integrated, and people live well together. If ε_t falls, then it would be convenient to raise it, and if ε_t rises, social exclusion starts to occur. Let us show this.

Case 1: $\boldsymbol{\varepsilon_t = \varepsilon_t^* = \theta}$

When $\varepsilon_t = \varepsilon_t^*$ the growth rate of innovation is given by (3.56) evaluated at ε_t^*, that is:

$$g_{A_t} = \frac{\beta_t^*}{1-\beta_t^*}\left(g_{S_t^*} + n\right)$$

Using equations (3.26) for $g_{S_t^*}$ and (3.27) for β_t^*, g_{A_t} becomes:

$$g_{A_t} = \frac{\beta + (1-\beta)\theta}{(1-\beta)(1-\theta)}(\varphi\theta + n) \tag{3.62}$$

Calculating the effect on g_{A_t} of a marginal variation in θ, we conclude that:

$$\frac{\partial g_{A_t}}{\partial \theta} = \frac{\varphi + n}{(1-\beta)(1-\theta)^2} - \varphi \tag{3.63}$$

because:

$$\frac{\partial g_{A_t}}{\partial \theta} = \frac{\{(1-\beta)(\varphi\theta + n) + [\beta + (1-\beta)\theta]\varphi\}(1-\theta) + [\beta + (1-\beta)\theta](\varphi\theta + n)}{(1-\beta)(1-\theta)^2} =$$

$$= \frac{(2\varphi\theta - 2\beta\varphi\theta + \beta\varphi - \beta n + n)(1-\theta) + (\beta\varphi\theta + \varphi\theta^2 - \beta\varphi\theta^2 + \beta n + \theta n - \beta\theta n)}{(1-\beta)(1-\theta)^2}$$

$$=$$

$$= \frac{(2\varphi\theta - 2\beta\varphi\theta + \beta\varphi - \beta n + n)(1-\theta) + (\beta\varphi\theta + \varphi\theta^2 - \beta\varphi\theta^2 + \beta n + \theta n - \beta\theta n)}{(1-\beta)(1-\theta)^2}$$

$$=$$

$$= \frac{\beta\varphi\theta^2 - \varphi\theta^2 + 2\varphi\theta - 2\beta\varphi\theta + \beta\varphi + n}{(1-\beta)(1-\theta)^2} = \frac{-(1-\beta)\varphi\theta^2 + 2(1-\beta)\varphi\theta + \beta\varphi + n}{(1-\beta)(1-\theta)^2}$$

$$=$$

$$= \frac{-(1-\beta)\varphi\theta^2 + 2(1-\beta)\varphi\theta + \beta\varphi - \varphi + \varphi + n}{(1-\beta)(1-\theta)^2} =$$

$$= \frac{-(1-\beta)\varphi + 2(1-\beta)\varphi\theta - (1-\beta)\varphi\theta^2 + \varphi + n}{(1-\beta)(1-\theta)^2} =$$

$$= \frac{-\varphi(1-\beta)(1-\theta)^2 + \varphi + n}{(1-\beta)(1-\theta)^2} = \frac{\varphi + n}{(1-\beta)(1-\theta)^2} - \varphi$$

Equation (3.63) is clearly positive, meaning that an increase (decrease) in the desired level of ethnic diversity leads to an increase (decrease) in g_{A_t}. Moreover, the evolution of the impact of θ on g_{A_t} is positive, as:

$$\frac{\partial^2 g_{A_t}}{\partial \theta^2} = \frac{2(\varphi + n)}{(1-\beta)(1-\theta)^3} \tag{3.64}$$

is positive.

This behaviour affects the equilibrium growth rate g_t^*, because $g_t^* = g_{P_{At}} + g_{A_t} - n$. The pattern of evolution of g_t^* is very similar to one above, as depicted in Figure 3.3. In this case, g_t^* is given by (3.60) evaluated at ε_t^*, that is:

$$g_t^* = \frac{g_{S_t^*} + \beta_t^* n}{1 - \beta_t^*}$$

then making use of (3.26) and (3.27), we get:

$$g_t^* = \frac{\varphi\theta + [\beta + (1-\beta)\theta]n}{(1-\beta)(1-\theta)} \tag{3.65}$$

Using (3.54), adjusted for per capita variables:

$$g_t^* = g_{A_t} + g_{S_t^*}$$

then:

$$\frac{\partial g_t^*}{\partial \theta} = \frac{\partial g_{A_t}}{\partial \theta} + \frac{\partial g_{S_t^*}}{\partial \theta}$$

i.e.:

$$\frac{\partial g_t^*}{\partial \theta} = \frac{\varphi + n}{(1-\beta)(1-\theta)^2} \tag{3.66}$$

and:

$$\frac{\partial^2 g_t^*}{\partial \theta^2} = \frac{\partial^2 g_{A_t}}{\partial \theta^2} \tag{3.67}$$

because $g_{S_t^*}$ is linear in θ.

Given that both (3.66) and (3.67) are positive, we conclude that increases (decreases) in θ lead to accelerated increases (decreases) in g_t^*, in the same pattern as that of g_{A_t}.

Figure 3.3 BGP Growth rates according to institutional conditions of countries. Case1: $\varepsilon_t^* = \theta$.

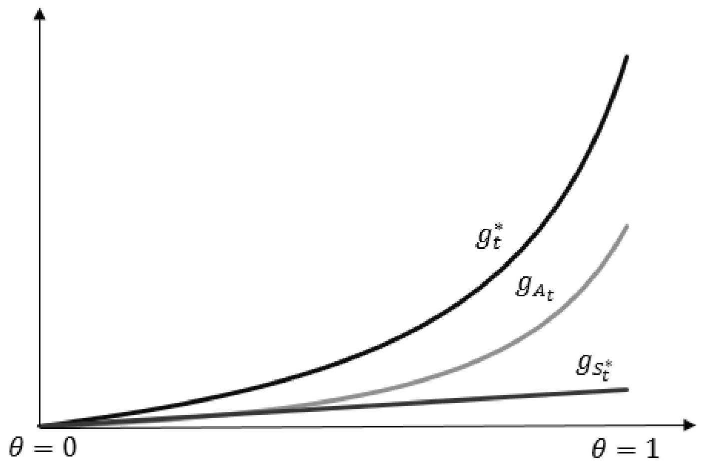


Figure 3.3 pictures the evolution of g_t^*, g_{A_t} and $g_{S_t^*}$ for different levels of $\varepsilon_t^* = \theta$. Assuming social harmony and full use of the bridging network; i.e., $\varepsilon_t = \varepsilon_t^* = \theta$. It

illustrates very clearly the impact of ethnic diversity on the economic growth rate. In this case, increases in $\varepsilon_t = \varepsilon_t^* = \theta$ lead to an accelerated increase in g_{A_t}, which is reflected in g_t^* in the same pattern. There is an increase in $g_{S_t^*}$, too, although in a linear rate, also contributing to the rise in g_t^*.

Concerning the contribution of social capital growth to economic growth, although its quantitative evolution with ethnic diversity is clearly lower than the channel of innovation, its qualitative role is crucial. The desired social interactions produce social capital. Then, all the socioeconomic results ahead, are the outcome of maximizing social capital through the choice of ε_t^*. Ethnic diversity contributes thus to a higher quantitative evolution through innovation, and to a higher qualitative evolution through social capital.

Next, we analyse the cases in which $\varepsilon_t \neq \varepsilon_t^*$; i.e., when the registered ethnic diversity does not coincide with its optimal level. These are likely to be more realistic cases.

Case 2: $\boldsymbol{\varepsilon_t < \varepsilon_t^* = \theta}$

When $\varepsilon_t < \varepsilon_t^*$, the growth rate of innovation g_{A_t} is given by:

$$g_{A_t} = \frac{\beta + (1-\beta)\varepsilon_t}{(1-\beta)(1-\varepsilon_t)}(\varphi\theta + n) \tag{3.68}$$

as the result of inserting (3.26) and (3.28) into (3.56).

Then, it is straightforward that increasing ethnic diversity will produce very good results on the evolution of innovation, in an increasing manner, because:

$$\frac{\partial g_{A_t}}{\partial \varepsilon_t} = \frac{1}{(1-\beta)(1-\varepsilon_t)^2} > 0 \tag{3.69}$$

and:

$$\frac{\partial^2 g_{A_t}}{\partial \varepsilon_t^2} = \frac{2}{(1-\beta)(1-\varepsilon_t)^3} > 0 \tag{3.70}$$

In this case, institutions are fully inclusive and are prepared/willing to have more ethnic diversity. Then, although the evolution of g_{A_t} with θ is still positive, it is quite lessened relative to the previous case. Nonetheless, when ε_t grows and approaches ε_t^*, the impact of θ in g_{A_t} tends to increase, as the network becomes increasingly saturated:

$$\frac{\partial g_{A_t}}{\partial \theta} = \frac{\beta + (1-\beta)\varepsilon_t}{(1-\beta)(1-\varepsilon_t)}\varphi > 0 \tag{3.71}$$

As:

$$\frac{\partial g_t^*}{\partial \varepsilon_t} = \frac{\partial g_{A_t}}{\partial \varepsilon_t} + \frac{\partial g_{S_t}}{\partial \varepsilon_t} = \frac{\partial g_{A_t}}{\partial \varepsilon_t} \tag{3.72}$$

the behaviours concerning the impact of ε_t on $g_{A_t}$are reflected on g_t^* that in this case is given by:

$$g_t^* = \frac{\varphi\theta + [\beta + (1-\beta)\varepsilon_t]n}{(1-\beta)(1-\varepsilon_t)} \tag{3.73}$$

after plugging (3.26) and (3.28) into (3.60). Then, under these circumstances, the impact of ethnic diversity ε_t on economic growth g_t^* will have a positive accelerating effect, through innovation.

Like g_{A_t}, g_t^* will also evolve positively with θ, although in a very slow pace, which tends to increase when ε_t approaches ε_t^*:

$$\frac{\partial g_t^*}{\partial \theta} = \frac{\varphi}{(1-\beta)(1-\varepsilon_t)} > 0 \tag{3.74}$$

Case 3: $\boldsymbol{\varepsilon_t > \varepsilon_t^* = \theta}$

If $\varepsilon_t > \varepsilon_t^*$, the growth rate of innovation g_{A_t} becomes:

$$g_{A_t} = \frac{\beta + (1-\beta)\varepsilon_t - \gamma(\varepsilon_t - \theta)}{(1-\beta)(1-\varepsilon_t) + \gamma(\varepsilon_t - \theta)}(\varphi\theta + n) \tag{3.75}$$

as the result of inserting (3.26) and (3.29) into (3.56).

We have (implicitly) assumed, recall, that social exclusion is present whenever institutions are not able to include all individuals, regardless of their race, creed, culture or origin. Social exclusion generates social instability, which causes a deterioration in social interactions and in innovation processes. More specifically, we have assumed an erosion term, $\gamma(\varepsilon_t - \theta)$, that increases with the widening of the gap between ethnic diversity and the installed institutional capacity for inclusion (with γ assumed higher than $1-\beta$).

Concerning the evolution of g_{A_t} with ε_t, we derive:

$$\frac{\partial g_{A_t}}{\partial \varepsilon_t} =$$

$$= \frac{[(1-\beta)-\gamma](\varphi\theta+n)[(1-\beta)(1-\varepsilon_t)+\gamma(\varepsilon_t-\theta)+\beta+(1-\beta)\varepsilon_t-\gamma(\varepsilon_t-\theta)]}{[(1-\beta)(1-\varepsilon_t)+\gamma(\varepsilon_t-\theta)]^2}$$

i.e.:

$$\frac{\partial g_{A_t}}{\partial \varepsilon_t} = \frac{[(1-\beta)-\gamma](\varphi\theta+n)}{[(1-\beta)(1-\varepsilon_t)+\gamma(\varepsilon_t-\theta)]^2} \tag{3.76}$$

which is negative because $(1-\beta)-\gamma < 0$, by assumption.

We also find that:

$$\frac{\partial^2 g_{A_t}}{\partial \varepsilon_t^2} = \frac{[(1-\beta)-\gamma]^2(\varphi\theta+n)}{[(1-\beta)(1-\varepsilon_t)+\gamma(\varepsilon_t-\theta)]^4} > 0 \qquad (3.77)$$

Derivative (3.76) together with (3.77) mean that when $\varepsilon_t > \varepsilon_t^* = \theta$, an increase in ε_t deteriorates innovation increasingly with ε_t, which is equivalent to saying that when ε_t is higher than society's wish, it is better for the innovation rate g_{A_t} to lower the levels of ethnic diversity.

At this point, we wish to notice that social inclusion should not be regarded solely through an economic perspective. We believe this to be a humanitarian question, concerning the case of migrants, not native minorities. If a society is not capable of accepting diversity, its invitation to immigration is perverse in the sense that ethnically different immigrants will face deprivation of all kinds of rights. Regarding such society's native minorities, the solution resides in a struggle for the same rights as the majority. Such struggle will improve immigrants' conditions as well. Still, we would not advise a sub-Saharan African to migrate to Germany in the 1930s or 1940s, for instance, when the Jewish minority were under severe deprivation of rights.

As expected, in this third case, the evolution of g_{A_t} with θ is clearly positive as:

$$\frac{\partial g_{A_t}}{\partial \theta} = \frac{\gamma(\varphi\theta+n)[(1-\beta)(1-\varepsilon_t)+\gamma(\varepsilon_t-\theta)]+\gamma[\beta+(1-\beta)\varepsilon_t-\gamma(\varepsilon_t-\theta)]}{[(1-\beta)(1-\varepsilon_t)+\gamma(\varepsilon_t-\theta)]^2} =$$

$$= \frac{\gamma^2(\varepsilon_t-\theta)(\varphi\theta+n-1)+\gamma(1-\varepsilon_t)(1-\beta)(\varphi\theta+n-1)+\gamma}{[(1-\beta)(1-\varepsilon_t)+\gamma(\varepsilon_t-\theta)]^2}$$

i.e.:

$$\frac{\partial g_{A_t}}{\partial \theta} = \gamma\left[\frac{\varphi\theta + n - 1}{(1-\beta)(1-\varepsilon_t) + \gamma(\varepsilon_t - \theta)} + \frac{1}{[(1-\beta)(1-\varepsilon_t) + \gamma(\varepsilon_t - \theta)]^2}\right] \quad (3.78)$$

and:

$$\frac{\varphi\theta + n - 1}{(1-\beta)(1-\varepsilon_t) + \gamma(\varepsilon_t - \theta)} < \frac{-1}{[(1-\beta)(1-\varepsilon_t) + \gamma(\varepsilon_t - \theta)]^2}$$

This means that an evolution in θ will progressively include the excluded people, leading to a normalization of social life, consequently boosting innovation. Once again:

$$\frac{\partial g_t^*}{\partial \varepsilon_t} = \frac{\partial g_{A_t}}{\partial \varepsilon_t} + \frac{\partial g_{S_t}}{\partial \varepsilon_t} = \frac{\partial g_{A_t}}{\partial \varepsilon_t} \quad (3.79)$$

then the behaviours concerning the impact of ε_t on $g_{A_t}$are reflected in g_t^*:

$$g_t^* = \frac{\varphi\theta + [\beta + (1-\beta)\varepsilon_t - \gamma(\varepsilon_t - \theta)]n}{(1-\beta)(1-\varepsilon_t) + \gamma(\varepsilon_t - \theta)} \quad (3.80)$$

from plugging (3.26) and (3.29) into (3.60).

Under such circumstances, the impact of ethnic diversity ε_t on economic growth g_t^* will have an increasingly damaging effect, through the innovation channel. Also, through that channel, g_t^* tends to improve with θ as it constitutes an approximation of ε_t^* to ε_t, the only difference from (3.78) being that the impact is higher than that on g_{A_t} by φ:

$$\frac{\partial g_t^*}{\partial \theta} = \frac{\partial g_{A_t}}{\partial \theta} + \frac{\partial g_{S_t}}{\partial \theta} = \frac{\partial g_{A_t}}{\partial \theta} + \varphi \quad (3.81)$$

3.4.2 Policy Recommendations

The model suggests that a higher degree of institutional ability for inclusion fosters the growth rate of the economy, as it allows ethnic diversity to increment innovation and social capital. Then, in general terms, policy measures that foster the inclusion capacity of institutions are beneficial, in both economic and humanistic criteria.

Let us now, and once more, look at the European Union, especially, to what we are witnessing in present times, a widespread increase of anti-immigration feelings in Europe, after decades of relative social peace during which democratic inclusive institutions were developed.

Portraying the European Union through means of the developed model, if the initial situation corresponds to $\varepsilon_t = \varepsilon_t^* = \theta$, then the recently arisen anti-immigration feelings can be captured by a decay of $\varepsilon_t^* = \theta$, meaning that Europe is now facing a $\varepsilon_t > \varepsilon_t^* = \theta$ situation. Such situation causes social exclusion of minorities. Minorities lose rights and become vulnerable. Therefore, the best policy concerning international migrants consists in dissuading them from moving into Europe. They are not welcome in Europe. As to former immigrants already living in Europe, they ought to find an alternative hosting region or return to their original country. However, many are prepared to take their chances and remain in host (rather hostile) countries. Poverty and tyranny may be even worse in their original countries.

The cancellation of further immigration flows together with the exodus of former immigrants from such a country lowers its ε_t, approximating it to the more stable situation $\varepsilon_t = \varepsilon_t^* = \theta$. Social injustice will be perpetuated, and native minorities will continue enduring social exclusion. Strictly speaking, in a very conservative country, with a low

ε_t^*, it is very probable that the peaceful situation of $\varepsilon_t = \varepsilon_t^* = \theta$ is never achieved, because there are always ethnic groups different from “the equal”.

But let us suppose that $\varepsilon_t = \varepsilon_t^* = \theta$ is achieved through ε_t outflows. We wish to argue that not only would native minorities continue undergoing social exclusion and loss of rights, but the vast majority would also suffer. The entire population would suffer from economic depression. Because the path to $\varepsilon_t = \varepsilon_t^* = \theta$ consists in the former immigrants’ acceptance of the unjust situation. Then θ remains low which reflects itself in a very low innovation growth rate g_{A_t}, and in an almost stagnant social capital formation $g_{S_t}^*$, leading to tacit economic stagnation g_t^*.

This situation together, with a stagnant/decaying population – that most European countries are practically experiencing, as we saw in Chapter 1 – can be catastrophic.

Recalling equations (3.68) and (3.73) and computing their partial derivatives in order to n, we conclude that:

$$\frac{\partial g_{A_t}}{\partial n} = \frac{\partial g_t^*}{\partial n} = \frac{\beta + (1-\beta)\theta}{(1-\beta)(1-\theta)} > 0 \qquad (3.82)$$

which means that, through the growth rate of innovation, the economic growth is positively related with the growth rate of the population. This positive relationship establishes that when n is decaying, g_{A_t} and g_t^* also decrease due to that effect. If g_{A_t} and g_t^* are already very low due to institutional constraints, we can imagine what happens when this demographic effect is added.

Solutions to such problem are not simple. As discussed above, one diversity-intolerant country may increase it’s n through one selective immigration policy directed at ethnically similar people. It is however very likely that countries with ethnically similar people are facing “similar” demographic problems. Our suggestion to ethnically diverse immigrants into an anti-diversity country was a humanistic concerned recommendation, of leaving such hostile country. If followed, it generates the perverse growth effects exposed.

Our policy recommendations are addressed to the immigration destiny countries, in particular to the European Union countries. According to the results of the model concerning economic, social and human outcomes, the best policy consists, we believe, in departing from the actual situation $\varepsilon_t > \varepsilon_t^* = \theta$ and moving into the desired situation $\varepsilon_t = \varepsilon_t^* = \theta$ by creating conditions for θ to grow. This is economically challenging, but it is also socially fairer, it does not abandon national minorities and fosters growth, as increases in θ are positively related with increases in g_t^* through the reinforced effect of increases in g_{A_t} together with rises in g_{S_t}.

Furthermore, the achievement of $\varepsilon_t = \varepsilon_t^* = \theta$ through increases of θ would allow for the inclusion of all, consequently a higher effective population growth rate, which, according to the model, produces an additional positive impact (adding to g_{S_t} and g_{A_t} while keeping the state of the nature) of an amount equal to:

$$\frac{\partial^2 g_{A_t}}{\partial\theta\partial n} = \frac{\partial^2 g_t^*}{\partial\theta\partial n} = \frac{1}{(1-\beta)^2(1-\theta)^2} > 0 \qquad (3.83)$$

which is obtained by differentiating (3.82) for θ.

The question now is: how can we stimulate increases in θ in the European Union?

Parameter θ corresponds to informal and formal democratic institutions that mirror the average citizen’s diversity acceptance. It is deeply rooted in collective perceptions towards human diversity. Consequently, the main structural policy to implement consists in Education. Educate the people. Not only for the traditional subjects: Educate for citizenship and for the ability to develop empathy with any other human being.

We believe that a common educational program for the European Union is as viable as common monetary policy and fiscal compression. Europe is a project grounded on values and principles of solidarity, justice, equal opportunities and democracy. Those values and principles can and should be taught, not to promote a uniform way of thinking, but to create coherence in matters that are eminently humanistic and civilizational. It is not about being right-wing or left-wing: it is about being higher our lower. It is about generalized acceptance or not of racism, xenophobia and religious persecution. It is about countries that naturalize racism, xenophobia and religious persecution having not the right to be European Union members.

The main outlines of policy arising from our proposed model are:

- When $\varepsilon_t < \varepsilon_t^* = \theta$, the promotion of immigration will benefit the economic growth of the nation, without loss of social cohesion. The adjustment to $\varepsilon_t = \varepsilon_t^* = \theta$ must be done through increases in ε_t, although, stimulus to θ increases are always very positive.

- When $\varepsilon_t > \varepsilon_t^* = \theta,$ the promotion of immigration will aggravate the social exclusion environment in such nation. There is inequality, injustice, social instabilities and lack of minorities rights. The adjustment to $\varepsilon_t = \varepsilon_t^* = \theta$ must be done through increases in θ. The vehicle for that must be incisive educational and informational policies on human and social rights and other humanistic values.

- The main source of ethnic diversity is present and past immigration, hence an increase in ethnic diversity implies immigration. However, it is not any type of immigration. For Ethnic diversity – and innovation – to be boosted by immigration, the community of immigrants must be composed by a significant variety of ethnicities. This is the type of immigration that we wish to recommend.

3.5 Conclusions

In this Chapter, we have provided a framework for the analysis of the effects of ethnic diversity on economic growth. In particular, we have explored the growth channels of innovation and social capital.

According to the proposed model, ethnic diversity increases innovation outcomes through the enhancement of knowledge spillovers and the detraction of redundancy in research projects. New ideas arise faster from the confrontation of different visions. Also, the increase in demand for new products originates new research projects. Moreover, ethnic diversity enhances bridging social capital, which is the type of social capital that reduces anti-social behaviour, and has higher prospects of expansion.

By assuming that each society is built upon democratic institutions, and that institutions are rigid, our model allows for each country to have their own configuration of ethnic diversity. We have examined analytically the economic implications of different configurations in terms of ethnic diversity.

The developed model predicts that when a country is very ethnic-diversity averse, its growth rate is low, due to a low growth of social capital, but also and mostly, since innovation is compressed by shortage of new ideas and abundancy of redundant research projects. Furthermore, there is no foreseeable way out of the situation, as the institutions do not allow for multiethnic migrants to come. In an ethnic-diversity averse country, more ethnic diversity is equivalent to more social exclusion, which has an eroding effect on social capital and the creation of new ideas.

In contrast, multiethnic countries will exhibit higher economic growth rates because of the labour-augmenting role of high levels of bridging social capital, and of the availability of a larger pool of creativity/innovation resources.

We have suggested former and future immigrants to opt out of an ethnic-diversity averse country (which would be that country's loss too). But this was just a suggestion. As a policy recommendation, we have proposed the stimulus of immigration into countries institutionally prepared for ethnic diversity, which would generate higher economic growth and benefit everyone, so the model predicts.

For ethnic-diversity averse countries, at least in the European Union, we have recommended a serious and obstinate effort to increase the levels of tolerance and inclusiveness of the democratic institutions. We are not referring solely to the regulatory building. Indeed, we are referring to such formal institutions but, also, to the informal ones. We are thinking of an institutional setup capable of shaping daily behaviours and attitudes regarding ethnic diversity. We are appealing to a collective perception on the matter, deeply rooted on the mentality of the average native citizen. Such an institutional all-inclusive setup is not produced overnight. It will arise from the implementation of an obstinate educational and informational policy that emphasizes the primordial foundational principles and values upon which the European Union has been built. We believe this to be a matter of coherence: The European Union is by principle non-racist, non-xenophobe and does not exclude people because of their religion. A country that does not accomplish these minimums is not European Union. Unless the European Union is not herself.

Conclusion

"*No one is born hating another person because of the colour of his skin, or his background, or his religion. People must learn to hate, and if they can learn to hate, they can be taught to love, for love comes more naturally to the human heart than its opposite.*"

Mandela, Nelson *in* "A Long Way to Freedom"

Conclusion

The analytical development of theoretical models involves simplifying assumptions. Simplifications are misrepresentations of reality, hence errors. We have inevitably made assumptions to allow for the functioning of our proposed analytical models. That is, our hopefully undistorted findings have been conveyed through models that require simplifying distortions. These distortions consist of the exogenous character of some processes. Still, it has not been our goal to explain the "why" or the "how". We have instead proposed to answer the "what if" question. That is, our goal has been to explain the consequences of assuming the occurrence of the alleged processes. Hence, in Chapter 1 we have introduced age structures, leaving out of the model fertility decisions and mortality expectations. In Chapter 2, we have assumed that the proportion of unskilled workers decays at a constant rate with time - civilizational development. In Chapter 3, ethnic diversity has been assumed as an index corresponding to one society's desired level of ethnic diversity according to its social capital. These simplifying assumptions have allowed us to achieve meaningful results and thus convey relevant conclusions. We also believe that the three developed models contribute to related literature, providing a significant range of opportunities for further theoretical developments.

The contextual goal of the present thesis has been the analysis of the consequences of currently emerging anti-immigration feelings, ideas and policy pressures amidst the European Union. In particular, we have wished to participate in the ongoing literary debate about the effects of immigration on economic growth. Our approach has consisted in the extension of foundational growth models, through the introduction of new assumptions. By building on the three pillar-frameworks sustaining growth theory, we have intended to remove any theoretical doubts regarding the three chosen setups, thus placing the debate one step ahead, on the new assumptions that we have brought into place. It has been our ultimate wish that if, by majority of reason, the introduced assumptions are deemed as reasonable, then our results can be taken in account in policy making.

In Chapter 1, we have introduced population age structure in Solow's (1957) model. One critical prediction is the possibility of technological regression. Lower entries of renewed human capital and higher withdrawals of know-how, in ageing countries, originates a decay in total factors' productivity. Such decay causes the long-term collapse of economies via mechanisms that fail to compensate through increased productivity for the erosion generated by population ageing and decline. The model predicts that, under severe aging, saving/investment policies can only delay, not solve, the economic collapsing process. We have argued then that an economic collapse trajectory can only be reversed through the implementation of fertility encouragement policies, the increase in the retirement age and the immigration of young people.

Regarding fertility policies, they have deferred outcomes and involve a whole societal reorganization. They must be implemented, although we believe that the solution should not rely solely on them. The increase in the retirement age has a minor impact, given a few more years allowed by life expectancy. And, after all, in a civilized nation, one is not meant to work until one dies. We argue therefore that immigration of young people with children appears to be an indispensable part of the solution to obtain sustained economic growth.

In Chapter 2, we have introduced skilled and unskilled labour in Lucas' (1988) model. The new features consist in assuming imperfect substitutability between the two types of labour, together with decaying unskilled labour due to civilizational development. With the proposed setup, we have been able to emphasize the importance of the unskilled labour for the growth of advanced economies. The developed model predicts that even when an education-for-all policy is in motion, without unskilled labour immigration, individual investment in human capital accumulation will decline. In this case, formal education will increasingly become a statistic devoid of meaning in terms of human capital.

We have hence inferred as economically inadequate policies that deny visas to unskilled workers, such as the skill-selective immigration policies recently adopted by many advanced countries. We find that a permanent immigration flow of unskilled labour into one country can simultaneously increase its economic growth rate and its average level of human capital.

In Chapter 3, we have introduced ethnic diversity in Jones' (1995) version of Romer's (1990) model. We have considered that ethnic diversity affects positively knowledge spillovers and redundancy of research projects. It also enhances labour productivity through social capital. The proposed model predicts that a country is better off when its level of ethnic diversity maximizes its social interactions. A conservative country maximizes its social interactions at low levels of ethnic diversity whereas a multicultural country achieves that with high ethnic diversity levels. The later' growth potential is higher than that of the first. However, ethnic diversity is only prescribed for both types of countries up to their respective desirable level of ethnic diversity, because, above that level, social exclusion and violence arise.

We have then suggested, as one strategic solution, an educational system integrating contents and practices inspired by the European Union's foundational principles and values.

Recapitulating in a contextual integrating perspective, with Chapter 1 we have found that the European Union needs young immigrants to avoid long-term economic collapse. A simple computation over the figures presented in the Introduction has led us to the conclusion that 53.4% of Europe's immigrants are European. As most European countries are facing the same ageing pyramids, the predominant intra-European migration flows are causing the ageing of the typical European immigrant. Between 2000 and 2015, the average age of the typical European immigrant has increased from 41 to 43 years old. In fact, the typical European immigrant has been ageing faster than the World's typical immigrant, from 38 to 39 years old.

In contrast, the African, the Asian, the Latin American and Caribbean typical immigrants exhibited, in 2015, ages of 29, 35 and 36 years old, respectively. Therefore, we find it plausible to recommend the migration policy of attracting immigrants from these continents. They are very young and at fertility ages. The attraction of young couples with children strikes us as a fruitful immigration policy to pursue. Furthermore, attracting such immigrants without selecting for their skills would, on one hand solve for unskilled labour shortages and, on the other hand, enrich R&D teams. In this policy adoption scenario, not only would economic collapse predicted in Chapter 1 be avoided, but also, according to our findings in Chapter 2 and Chapter 3, higher growth rates of consumption, physical and human capital, innovation and output per capita would be attained.

Alas, a major obstacle to economic growth arises. The contagious spread of anti-immigration spirits. Many European citizens are currently opting for more conservative settlements. More conservative settlements bring about social exclusion in the case of multi-ethnic immigration. Conservative institutions represent, from this point of view, a growth trap, as they hinder growth for the same reason that they hinder the solution to growth.

Ethnic diversity is currently not accepted as part of the economic growth solution; it is not welcome. Only an articulated educational system based on the core principles and values of the European Union can change it.

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