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Financial Market Contagion and the Sovereign Debt Crisis: A Smooth Transition Approach[∗]

Susana Martins † Cristina Amado ‡

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Abstract

In this paper, we investigate the timing and extent of sovereign debt contagion across nine Eurozone countries using daily returns on 10-year government bonds from 2007 until 2017. The novelty lies in modelling bond return correlations using a multivariate GARCH model with a multiplicative decomposition of the variance and time-varying conditional correlations. The model introduces flexibility by allowing the individual unconditional variances to be time-dependent and the correlations to change smoothly between two extreme states according to time and observable financial variables. The main results provide no evidence of asymmetric response of bond return comovements to negative shocks, as opposed to the size of innovations from the periphery which is expected to affect the dynamics of correlations. Our findings further indicate the presence of long-run contagion effects across peripheral countries following the more acute phase of the sovereign crisis. Interestingly, periods of high turbulence in the European stock market do not seem to drive financial contagion.

JEL classification codes: C32; C58; G01; G15.

Key words: Financial contagion; European sovereign debt crisis; Multivariate GARCH model; Dynamic correlations; Multiplicative decomposition of volatility.

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1 Introduction

Financial globalisation and the ensuing increased levels of financial interdependence across countries have contributed greatly in the manner country-specific shocks affect other markets. This means that a financial crisis occurring in one market can adversely affect other markets and, in extreme cases, result in the disruption of the stability of the entire financial system. The recent global financial crisis of 2007-2009 and the European sovereign debt crisis are prime examples of these phenomena.

The extent of financial crises are partly linked to the degree of financial market integration, how quickly shocks are spread from one market to another, and by what means shocks change the transmission mechanism itself. Besides, identifying contagion during periods of financial distress and understanding the transmission mechanisms of shocks across markets is of the utmost importance to policy makers and investors.

Motivated by these issues, a large body of research on market interdependence and contagion has spurred in recent years to gain new insights into understanding the dynamics of propagation of shocks. However, the literature focussing on the effects of transmission of financial shocks during the sovereign debt crisis remains scarce. Empirical evidence on the effects of the sovereign transmission of shocks can been found, among others, in Giordano et al. (2013), De Santis and Stein (2015), Bacchiocchi (2017), and Caporin et al. (2018).

Despite the vast literature on financial contagion, there is not yet consensus on the definition of contagion and it remains unclear the underlying transmission mechanism of shocks across markets. For an overview of the existing definitions and methods for measuring contagion, we refer to surveys of Pericoli and Sbracia (2003), Dungey et al. (2005), and Forbes (2013). The focus of this work lies on the correlation-based analysis approach popularised by Forbes and Rigobon (2002) and contagion shall be defined as a significant increase in cross-market linkages after a shock to one country.

We identify two main limitations in the existing literature of utilising the correlationbased approach. The first is related to the distinction between an abnormal increase in market interactions at times of crisis, and normal market interactions in tranquil times. In periods of market turbulence, cross-country correlations tend to be upwardly biased due to presence of heteroskedasticity, which can misguidedly lead to the presence contagion. Thus, an increase in cross-market comovements during a period of financial turmoil cannot be perceived per se as evidence of contagion, but merely a continued high level of market interdependence. Contagion, in turn, is identified as a structural change in the level of market interdependence in periods of distress. This problem has been addressed in Forbes and Rigobon (2002), Bae et al. (2003), Corsetti et al. (2005), and Dungey and Renault (2018). The second limitation is associated to the timing of crisis and the country originating the crisis. This is often not a data-driven process, but it is usually defined

beforehand by the modeller. Examples of this are the attempts of identifying Greece as the source of contagion on the sovereign crisis as in Missio and Watzka (2011), Mink and de Haan (2013), and Buchholz and Tonzer (2015), and October, 2009 as the beginning of the crisis when Greece revealed its distressed debt position.

We contribute to the literature by developing a novel approach based on the smooth transition conditional correlation GARCH model with multiplicative decomposition to investigate the presence of contagion during the sovereign debt crisis. We shall name the proposed model the multivariate Multiplicative Time-Varying Smooth Transition Conditional Correlation (MTV-STCC-) GARCH. A special case of this model is the specification with deterministically time-varying correlations introduced by Silvennoinen and Teräsvirta (2017). The new model introduces flexibility by allowing the individual unconditional variances to be time-dependent and the correlations to change smoothly between two extreme states according to time and observable financial variables.

Our modelling strategy addresses several pitfalls identified in the contagion literature. First, it has the advantage of adjusting the correlations to long-term and short-term volatility, and therefore avoiding the bias in cross-market correlations. As showed by Mikosch and Stărică (2004), the standard GARCH model is not suitable for fitting data with a long observation period because of deterministic changes in the long-run volatility. One way of dealing with this nonstationarity is to explicitly allow for a time-varying unconditional variance and model it accordingly as in Amado and Teräsvirta (2013, 2017). Second, we are able to control for time-variation in correlations using time and financial indicators. We can thus capture long-term and short-term movements in correlations depending on which variable is being used as driver of the regime-changes in correlations. Third, the timing of changes in volatility is estimated endogenously instead of being pre-defined exogenously. The identification of the crisis phases will be purely determined from the data and we shall rely on the estimates of the location parameters to distinguish those phases. For a similar procedure on different modelling frameworks, we refer to Kasch and Caporin (2013), Blatt et al. (2015), and Dungey et al. (2015).

The new model is applied to daily returns on 10-year government bond yields for nine member states of the Eurozone from 2007 until 2017. First we choose time as the transition variable controlling the time-varying correlations. Because conditional correlations may respond to country-specific (or idiosyncratic) and common (or systemic) shocks to sovereign yields, we also consider financial variables as transition variables. An appealing feature of the new model is the discrimination between long-run and short-run contagion effects on the basis of the variable used as indicator of the changes in correlations. Later, in this article, we will relate these concepts with pure and "wake-up-call" contagion effects. The main results provide no evidence of asymmetric response of bond return comovements to negative shocks, as opposed to the size of innovations from the periphery which is expected to affect the dynamics of correlations. Overall, the model provides to be a useful tool for

studying market contagion by explicitly focussing on the effects of mechanisms of shocks propagation in periods of tranquility and turbulence.

This paper is organized as follows. Section 2 sets up the modelling framework used to examine the presence of contagion. Section 3 provides a description of the data. Section 4 contains the empirical results of the new MTV-STCC-GARCH model applied to the daily returns on 10-year government bonds for nine Eurozone countries and the results of the correlation-based tests for contagion are discussed. Finally, Section 5 concludes the paper.

2 Modelling framework

2.1 Model set-up

In this section, we shall briefly describe the modelling framework used to study the presence of contagion in the European sovereign bond markets. More formally, consider an Ndimensional vector of bond returns with the representation:

$$
\mathbf{y}_t = \mathsf{E}[\mathbf{y}_t|\mathcal{F}_{t-1}] + \boldsymbol{\varepsilon}_t, \qquad t = 1, \dots, T,
$$
\n(1)

where \mathcal{F}_{t-1} is the sigma-algebra containing the historical information available at time $t-1$. To filter out any linear dependence in the data, we model the conditional mean for each series as an autoregressive process of order r:

$$
y_{it} = \psi_{i0} + \sum_{j=1}^{r} \psi_{ij} y_{i,t-j} + \varepsilon_{it}, \qquad i = 1, ..., N,
$$
 (2)

where the innovation sequence ε_{it} has a conditional mean $E(\varepsilon_{it}|\mathcal{F}_{t-1}) = 0$ and a potentially time-varying conditional variance $\text{Var}(\varepsilon_{it}|\mathcal{F}_{t-1}) = \sigma_{it}^2$, for $i = 1, ..., N$. Each univariate error process is decomposed as follows:

$$
\varepsilon_{it} = z_{it}\sigma_{it},
$$

where z_{it} forms a sequence of independent random variables with mean zero and variance one, and the variance σ_{it}^2 is further multiplicatively decomposed as:

$$
\sigma_{it}^2 = h_{it}g_{it},\tag{3}
$$

where h_{it} is a stationary component describing the short-run dynamics of volatility and g_{it} is a positive-valued deterministic component capturing long-run movements in volatility. We explicitly introduce nonstationarity in the variance to account for long-run movements and model volatility by the multiplicative time-varying (MTV-) GARCH model of Amado and Teräsvirta (2013, 2017). Specifically, the h_{it} component is modelled as the standard

 $GARCH(p, q)$ representation:

$$
h_{it} = \alpha_{i0} + \sum_{j=1}^{q} \alpha_{ij} \phi_{i,t-j}^2 + \sum_{j=1}^{p} \beta_{ij} h_{i,t-j}
$$
(4)

where $\alpha_{i0} > 0$, $\alpha_{ij} \geq 0$, $j = 1, ..., q - 1$, $\alpha_{iq} > 0$, $\beta_{ij} \geq 0$, $j = 1, ..., p$, and $\phi_{it} = \varepsilon_{it}/g_{it}^{1/2}$. For the process to be covariance-stationary it is required that $\sum_{j=1}^{q} \alpha_{ij} + \sum_{j=1}^{p} \beta_{ij} < 1, i =$ $1, ..., N$. The slowly time-varying trend g_{it} functions as a proxy for all factors that affect the unconditional variance and it is defined as:

$$
g_{it} = \delta_{i0} + \sum_{l=1}^{m_i} \delta_{il} G_{il}(t/T; \gamma_{il}, \mathbf{c}_{il}), \qquad \gamma_{il} > 0
$$
 (5)

where δ_{il} , $l = 0, \ldots, m_i$, are parameters such that $m_i = 1, \ldots, M$, with M being a finite integer. For identification reasons, the intercept δ_{i0} is assumed known and fixed to the value obtained in the first iteration when estimating (5) and setting $h_{it} = 1$; see Amado and Teräsvirta (2013) for further details. The function $G_{il}(s_i; \gamma_{il}, \mathbf{c}_{il})$ is the generalised logistic transition function:

$$
G_{il}(s_t; \gamma_{il}, \mathbf{c}_{il}) = (1 + \exp\{-\gamma_{il} \prod_{k=1}^{K_{il}} (s_t - c_{ilk})\})^{-1}
$$
(6)

where $\gamma_{il} > 0$ and $\mathbf{c}_{il} = (c_{il1}, \dots, c_{ilK_{il}})'$ such that $c_{il1} \leq \dots \leq c_{ilK_{il}}$. In (5) we choose the transition variable s_t as time t/T defined on the interval [0, 1]. The parameters c_{ilk} and γ_{il} determine, respectively, the location of the change and the smoothness of the transition between one regime to another. As $\gamma_{il} \rightarrow \infty$, g_{it} approaches a step function with a switch at c_{ilk} . For small values of γ_{il} , the transition between regimes is smooth. When γ_{il} is large, it is numerically convenient to use the transformation $\gamma_{il} = \exp(\eta_{il})$ and estimating η_{il} instead of γ_{il} ; see Goodwin et al. (2011) and Hurn et al. (2016) for further details.

The transition function (6) allows the unconditional variance to change deterministically as a function of time. By construction, function (6) is continuous for $\gamma_{il} < \infty$, $l = 1, ..., m_i$, and bounded between zero and unity. The order K_{il} determines the shape of the transition function. In practice, we usually select $K_{il} = 1$ and $K_{il} = 2$. Typical shapes of the transition function are illustrated in Figure 1. The transition with $K_{il} = 1$ is well-suited for describing processes whose dynamics is different before and after the change. When the series is expected to return to its original level after the change, then $K_{il} = 2$ is the suitable choice. In some occasions is necessary to choose $K_{il} = 3$ to allow for a more complex, but flexible and possibly non-monotonic change. In practice, the parametric structure of the g_{it} component is determined from the data, which involves determining the number of m_i transitions and choosing the integer K_{il} using statistical inference; see Amado and Teräsvirta (2017) for more details. Visual inspection may be also useful for

choosing K_{il} .

It follows that the N-dimensional vector of innovations ε_t is defined as

$$
\varepsilon_t = \mathbf{H}_t^{1/2} \mathbf{z}_t = \mathbf{S}_t \mathbf{G}_t \mathbf{z}_t, \tag{7}
$$

where H_t is an $N \times N$ positive definite matrix, and the error vector z_t form a sequence of independent and identically distributed variables with $E(z_t|\mathcal{F}_{t-1}) = 0$ and positive definite covariance matrix $\mathsf{E}(\mathbf{z}_t \mathbf{z}_t' | \mathcal{F}_{t-1}) = \mathbf{P}_t$. This implies that $\mathbf{P}_t^{-1/2} \mathbf{z}_t \sim iid(\mathbf{0}, \mathbf{I}_N)$. The stochastic diagonal matrix $\mathbf{S}_t = \mathsf{diag}(h_{1t}^{1/2})$ $h_1^{1/2}, \ldots, h_{Nt}^{1/2}$ contains the conditional standard deviations of ϕ_{it} , $i = 1, ..., N$, as defined in (4) and $\mathbf{G}_t = \text{diag}(g_{1t}^{1/2})$ $t_1^{1/2}, \ldots, t_{Nt}^{1/2}$ is a deterministic diagonal matrix containing positive-valued time-dependent elements $g_{it}^{1/2}$. Under these assumptions, the time-varying conditional covariance matrix of ε_t has the representation introduced by Amado and Teräsvirta (2014):

$$
\mathsf{E}[\varepsilon_t \varepsilon'_t | \mathcal{F}_{t-1}] = \mathbf{H}_t = \mathbf{S}_t \mathbf{G}_t \mathbf{P}_t \mathbf{G}_t \mathbf{S}_t, \tag{8}
$$

where $\mathbf{P}_t = [\rho_{ij,t}]_{i,j=1,\dots,N}$ is a positive definite conditional correlation matrix for ε_t to be defined in the next section whose elements can be time-varying for $i \neq j$. It follows that when $G_t = I_N$, the model belongs to the family of Conditional Correlation (CC-) GARCH models.

2.2 Dynamics of correlation structure

In this work, we assume the correlations of bond returns to change smoothly between two extreme states as in the smooth transition conditional correlation model of Silvennoinen and Teräsvirta (2005, 2015). More specifically, let the correlation matrix P_t be defined as:

$$
\mathbf{P}_t(s_t) = \{1 - G(s_t, \gamma, \mathbf{c})\} \mathbf{P}_1 + G(s_t, \gamma, \mathbf{c}) \mathbf{P}_2
$$
\n(9)

where P_1 and P_2 are positive definite correlation matrices that describe the two extreme states of correlations driven by the transition variable s_t . For each point in time t, the conditional correlations are computed as an average between the two extreme correlation states weighted by the logistic transition function $G(s_t, \gamma, c)$ defined in (6) (omitting subscripts i and l). This is a bounded function of an observable and continuous variable $s_t \in$ \mathcal{F}_{t-1} . The extended multivariate multiplicative GARCH model with a correlation matrix defined as (9) shall be called the MTV-STCC-GARCH model. Recently, Silvennoinen and Teräsvirta (2017) derive the asymptotic properties for the MTV-STCC-GARCH model with deterministically time-varying correlations when the transition variable is rescaled time, i.e., $P_t \equiv P(t/T)$. Their model is known as the multivariate multiplicative GARCH model with time-varying correlations or the MTV-TVC-GARCH model.

The observable indicator s_t is generally chosen by the investigator to suit the research problem at hand. Possible choices include a function of lagged returns, exogenous variables or calendar time as in Berben and Jansen (2005). Economic theory may provide insight into the appropriate choice of the transition variable. However, if there is uncertainty about which alternative to use as transition variable, testing the constancy of correlations may be a useful tool for selecting a particular transition variable; see Silvennoinen and Teräsvirta (2009). The strongest rejection of the null hypothesis of constant conditional correlations using a specific transition variable provides empirical evidence for time variation in the correlation dynamics according to that particular variable. We shall further discuss the choice of this variable in Section 4.2.

A popular parameterization for the correlation structure assumes the conditional correlation matrix follows the Dynamic Conditional Correlation (DCC-) GARCH model of Engle (2002). Consider the process

$$
\mathbf{Q}_t = (1 - \alpha - \beta)\overline{\mathbf{Q}} + \alpha \mathbf{z}_{t-1} \mathbf{z}'_{t-1} + \beta \mathbf{Q}_{t-1}
$$
(10)

where $\mathbf{z}_t = (z_{1t}, \ldots, z_{Nt})'$ with $z_{it} = \varepsilon_{it}/(h_{it}^{1/2} g_{it}^{1/2}),$ and $\overline{\mathbf{Q}}$ is the unconditional correlation matrix of the standardised errors z_t . The parameters satisfy $\alpha > 0$, $\beta > 0$ and $\alpha + \beta < 1$ to ensure positive definiteness of \mathbf{Q}_t . To produce valid correlation coefficients, the symmetric matrix \mathbf{Q}_t is rescaled as follows:

$$
\mathbf{P}_t = \text{diag}(\mathbf{Q}_t)^{-1/2} \mathbf{Q}_t \text{diag}(\mathbf{Q}_t)^{-1/2}.
$$
 (11)

If we assume time-dependence in the baseline volatilities, the conditional covariance matrix is decomposed as in (8) with P_t defined as (11). This model shall be named as MTV-DCC-GARCH model. It follows that, when $g_{it} \equiv 1$, the model becomes the DCC-GARCH model.

In the simplest multivariate correlation model, the decomposition (8) assumes $G_t \equiv I_N$ and a time-invariant correlation matrix $P_t \equiv P$, where $P = [\rho_{ij}], i, j = 1, ..., N$ and $i \neq j$. This model becomes the CCC-GARCH model of Bollerslev (1990). When the conditional covariance matrix is defined as in (8) with $G_t \neq I_N$, we obtain the MTV-CCC-GARCH model.

2.3 Estimation of parameters

Assuming joint conditional normality of the errors, $\varepsilon_t | \mathcal{F}_{t-1} \sim N(\mathbf{0}, \mathbf{H}_t)$, the parameters of the MTV-STCC-GARCH model can be estimated by maximum likelihood (ML). Let $\boldsymbol{\theta} = (\boldsymbol{\theta}_d^{\prime})$ $'_{g},\boldsymbol{\theta}'_{l}$ $_{h}^{\prime},\boldsymbol{\theta}_{\rho}^{\prime}$ ^{ℓ}_ρ)' be the vector of all parameters of the model, where $\theta_g = (\theta_g^{\ell})$ $({\boldsymbol{\theta}}'_{g1},\ldots,{\boldsymbol{\theta}}'_{gN})'$ is the parameter vector of g_{it} , $\boldsymbol{\theta}_h = (\boldsymbol{\theta}_h^t)$ $h_1, \ldots, \theta_{hN}'$ is the parameter vector of $h_{it}, i = 1, \ldots, N$, and θ_{ρ} contains the parameters of the correlation matrix P_t . For notational simplicity, we

Figure 1: The logistic transition functions for $K_{il} = 1$ (upper left plot), $K_{il} = 2$ (upper right plot) and $K_{il} = 3$ (lower plot). The transition variable is the calendar time $t/T \in [0, 1]$, and the speed parameter $\gamma = 5, 10, 50,$ and 100. The lowest γ corresponds to the smoothest transition function. The vertical blue lines indicate the location of the change.

use $S_t = S_t(\theta_q, \theta_h)$, $G_t = G_t(\theta_q)$, and $P_t = P_t(\theta_q)$.

The log-likelihood function for each observation t can be expressed as

$$
\ell_t(\boldsymbol{\theta}) = -(N/2)\ln(2\pi) - (1/2)\ln|\mathbf{S}_t\mathbf{G}_t\mathbf{P}_t\mathbf{G}_t\mathbf{S}_t| - (1/2)\varepsilon'_t\mathbf{S}_t^{-1}\mathbf{G}_t^{-1}\mathbf{P}_t^{-1}\mathbf{G}_t^{-1}\mathbf{S}_t^{-1}\varepsilon_t
$$

\n
$$
= -(N/2)\ln(2\pi) - \ln|\mathbf{G}_t| - (1/2)\tilde{\varepsilon}'_t\mathbf{G}_t^{-2}\tilde{\varepsilon}_t - \ln|\mathbf{S}_t| - (1/2)\phi'_t\mathbf{S}_t^{-2}\phi_t
$$

\n
$$
-(1/2)\ln|\mathbf{P}_t| - (1/2)\mathbf{z}'_t\mathbf{P}_t^{-1}\mathbf{z}_t + \mathbf{z}'_t\mathbf{z}_t
$$
 (12)

where $\tilde{\boldsymbol{\varepsilon}}_t = \mathbf{S}_t^{-1} \boldsymbol{\varepsilon}_t$, $\boldsymbol{\phi}_t = \mathbf{G}_t^{-1} \boldsymbol{\varepsilon}_t$, and $\mathbf{z}_t = \mathbf{G}_t^{-1} \mathbf{S}_t^{-1} \boldsymbol{\varepsilon}_t$. Maximising $\sum_{l=1}^T \ell_l(\boldsymbol{\theta})$ over the parameters of the model yields the maximum likelihood estimator θ_T . Maximising the joint log-likelihood function with respect to all parameters is numerically very difficult, usually converging to local maxima and leading to computational problems in the standard errors. To facilitate parameter estimation, we use the fact that the log-likelihood (12) can be decomposed (ignoring the constant) as the sum of an unconditional and conditional volatility part, and a correlation part:

$$
\ell_t(\boldsymbol{\theta}) = \ell_t^{UV}(\boldsymbol{\theta}_g) + \ell_t^{CV}(\boldsymbol{\theta}_g, \boldsymbol{\theta}_h) + \ell_t^{CC}(\boldsymbol{\theta}_g, \boldsymbol{\theta}_h, \boldsymbol{\theta}_\rho)
$$

and use the two-step approach as suggested by Engle (2002) for estimating the parameters. The unconditional variance term is

$$
\ell_t^{UV}(\boldsymbol{\theta}_g) = -(1/2) \sum_{i=1}^N \{ \ln g_{it}(\boldsymbol{\theta}_{gi}) + \tilde{\varepsilon}_{it}^2 / g_{it}(\boldsymbol{\theta}_{gi}) \}
$$
(13)

with $\tilde{\varepsilon}_{it} = \varepsilon_{it}/h_{it}^{1/2}(\theta_{gi}, \theta_{hi}), i = 1, \ldots, N$, and the conditional volatility component is

$$
\ell_t^{CV}(\boldsymbol{\theta}_g, \boldsymbol{\theta}_h) = -(1/2) \sum_{i=1}^N \{ \ln h_{it}(\boldsymbol{\theta}_{gi}, \boldsymbol{\theta}_{hi}) + \phi_{it}^2 / h_{it}(\boldsymbol{\theta}_{gi}, \boldsymbol{\theta}_{hi}) \}
$$
(14)

where $\phi_{it} = \varepsilon_{it}/g_{it}^{1/2}(\theta_{gi})$, for each $i = 1, ..., N$. In the second step, the correlation parameters are estimated conditionally on the volatility parameters estimated in the first step by maximising the correlation component:

$$
\ell_t^{CC}(\widehat{\boldsymbol{\theta}}_g, \widehat{\boldsymbol{\theta}}_h, \boldsymbol{\theta}_\rho) = -(1/2) \{ \ln |\mathbf{P}_t(\boldsymbol{\theta}_\rho)| + \mathbf{z}_t^{\prime} \; \mathbf{P}_t^{-1}(\boldsymbol{\theta}_\rho) \mathbf{z}_t - 2 \mathbf{z}_t^{\prime} \mathbf{z}_t \}.
$$
 (15)

where $\mathbf{z}_t = (z_{1t}, \ldots, z_{Nt})'$ and $z_{it} = \varepsilon_{it}/(h_{it}^{1/2}(\boldsymbol{\theta}_{gi}, \boldsymbol{\theta}_{hi})g_{it}^{1/2}(\boldsymbol{\theta}_{gi}))$, for each $i = 1, \ldots, N$. Maximum likelihood estimates of the variance equations are obtained by splitting the maximisation problem into (13) and (14) and iterate between them until convergence. This method is called maximisation by parts by Song et al. (2005). Amado and Teräsvirta (2013) applied it to the estimation of the univariate MTV-GARCH model and proved consistency and asymptotic normality for the ML estimator. Under regularity conditions, consistency on the first step will ensure consistent estimators on the second step.

Our scheme differs from that of Silvennoinen and Teräsvirta (2017) who generalise maximisation by parts to the estimation of the multivariate MTV-GARCH model with deterministically time-varying correlations. Under standard regularity conditions, they established consistency and asymptotic normality of the ML estimator of the MTV-TVC-GARCH model. Compared to the two-step estimates, parameter estimators of their maximisation algorithm are fully efficient. Asymptotic properties of the ML estimator when the transition variable is stochastic are not yet known and deriving the asymptotic properties is beyond the scope of this paper.

3 Data

In order to study the presence of cross-market contagion in the European sovereign debt crisis, we use daily data on 10-year government benchmark bond yields. The sample period extends from January 1, 2007 to April 18, 2017 which amounts to 2684 observations. The data was collected from the Thomson Reuters Datastream and it covers nine Eurozone countries that we group as core (Belgium, Finland, France and Germany) and periphery (Greece, Ireland, Italy, Portugal and Spain). This division has become standard due to the similarity in yields and the debt positions of countries in each group. Core countries also have different levels of risk and liquidity from the peripheral countries.

Figure 2 shows the daily 10-year government benchmark bond yields. We observe that after the end of 2009, the yields on the Greek bonds peaked sharply, followed foremost

Figure 2: Daily data on 10-year government bond yields for nine Eurozone countries from January 1, 2007 and April 18, 2017. The vertical red lines show the estimated start of the crisis phases and post-crisis period for the periphery countries and the black vertical line corresponds to the Greek deficit revision in October 22, 2009.

by Ireland and Portugal. Over time, the diverging behaviour between core and periphery countries gradually disappears, leaning to stronger comovements across markets. Yet, the Greek bonds drifts away again around 2015, before moving closer to the other countries' yields. Results from the MTV-TVC-GARCH model (to be further discussed in section 4.3) identify three changes in comovements for periphery countries, represented by the red vertical lines in Figure 2. According to these results, dominant changes in correlations occurred in May 2009, June 2014 and March 2016. We shall discuss these dates in further detail in section 4.3.

To examine bond market linkages, we compute the simple returns as $r_{i,t} = (p_{i,t}/p_{i,t-1} - p_{i,t})$ 1) \times 100, where $p_{i,t}$ is the bond yield at time t for country i. To prevent problems in estimation, the returns are truncated such that extremely large positive/negative values are limited to $+/-5$ times the standard deviation of the series. The truncated returns are depicted in Figure 3. Returns above or below three times their standard deviation (represented in blue) occur mostly after October 2009. For most series, large returns are the indication of unusually large shocks to sovereign bond yields and high levels of volatility and these are especially noticeable at the end of the sample.

Summary statistics for each series are provided in Table 1. The skewness and kurtosis show that the bond returns have a right-skewed and significant fat-tailed distribution. Results of the robust $Q(10)$ statistic show linear time-dependence for the Greek, Italian and Portuguese returns. There is also evidence of time-dependence in the second moment for the series of returns, but no remaining ARCH effects are found on the standardised

Figure 3: Daily returns on 10-year government bonds after truncation. The horizontal blue lines correspond to $+/-3$ times the sample standard deviation of the truncated series.

residuals. Results of these tests are not shown for space reasons.

4 Empirical results

4.1 Modelling long-term bond market volatility

In this section we investigate the evidence of financial contagion across markets and characterize the extent of such market linkages during the sovereign debt crisis. This is done by using an approach to modelling financial contagion based on the MTV-STCC-GARCH model. We begin by modelling the conditional mean to filter out the linear dependence in the bond returns. This is done by fitting an autoregressive model where the number of lags is determined by the robust portmanteau $Q(10)$ test until no evidence of remaining autocorrelation is found in the residuals. Results are not shown to conserve space, but the autocorrelation is taken into account by fitting an $AR(2)$ to the Italian returns, and an AR(1) to the Greek and Portuguese returns.

To account for conditional heteroskedasticity, we first estimate a GARCH(1,1) model to the series. The estimation results after fitting a standard GARCH model to the residuals $\widehat{\varepsilon}_{it}$ are presented in Table 3. In most cases, the persistence of volatility measured by $\widehat{\alpha}_1 + \widehat{\beta}_1$ exceeds unity, implying that the conditional variance is nonstationary. For the

Table 1: Descriptive statistics and diagnostic tests for the daily bond returns after truncating the extreme returns. The threshold is $+/-5$ standard deviations above/below the extreme returns. SK. and KR. denote, respectively, the skewness and excess kurtosis. ROB. $Q(10)$ is the corrected portmanteau test statistic for serial correlation up to order 10 in the presence of ARCH effects of Francq and Zakoïan (2009). ARCH(5) is the Engle's (1982) test for ARCH effects up to order 5. The reported values of the tests are *p*-values.

			CORE					PERIPHERY	
	BELGIUM	FINLAND	FRANCE	GERMANY	GREECE	IRELAND	ITALY	PORTUGAL	SPAIN
NO. TRUNC.	24	20	23	29	11	21		9	10
OBS.	2684	2684	2684	2684	2684	2684	2684	2684	2684
MIN.	-24.44	-52.13 -25.94		-76.48		-14.15 -14.27 -10.24		-10.98 -10.85	
MEAN	-0.016	0.025	-0.018	-0.014	0.078	-0.029	-0.008	0.020	-0.014
MAX.	24.44	52.13	25.94	76.48	14.15	14.27	10.24	10.98	10.85
STD.DEV.	4.005	6.789	4.131	11.38	2.437	2.723	1.999	2.102	2.105
SK.	0.831	0.880	0.766	0.468	0.244	0.506	0.461	0.518	0.213
KR.	15.36	31.15	16.94	26.36	8.844	7.945	4.609	4.683	4.106
ROB. $Q(10)$ TEST	0.903	0.496	0.711	0.909	0.016	0.314	0.029	0.011	0.206
ARCH(5) TEST	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

French series we estimate a GARCH(2,1) model as it requires a higher order process to guarantee no remaining ARCH effects. We then test for changes in the unconditional variance and constancy is rejected for all series; for details of the test, see Amado and Teräsvirta (2017). The shape of the deterministic component g_{it} function is specified by the sequential testing proposed by Amado and Teräsvirta (2017). The test results are not provided for space reasons. After specifying the shape of the g_{it} function, we estimate the MTV-GARCH model by maximisation by parts as in Amado and Teräsvirta (2013). As mentioned before, to achieve identification, δ_{i0} is kept fixed and equal to the first estimate of this parameter. The estimation algorithm is carried out also without iterating γ_{il} after the first iteration. Therefore, their standard errors are not available because the parameters δ_{il} , $i = 1, ..., m_i$, and c_{il} are estimated conditionally on those parameters. The estimation results of the g_{it} function are shown in Table 2 and its dynamics is displayed in Figure 4. For comparison, the conditional standard deviation from the GARCH model is also plotted. A common pattern is visible from the graphs. There is a clear distinction between core and periphery countries. The long-run volatility of the core countries looks rather flat in the beginning of the sample. Around 2015 starts a more volatile period that continues an upward trajectory until the end of the sample period. With respect to the periphery countries, the unconditional variance show different patterns, but they have the common feature of showing an increasing trend over time. The estimation results from the MTV-GARCH model for the rescaled residuals $\hat{\epsilon}_{it}/\hat{g}_{it}^{1/2}$ can be found in Table 3. The persistence is now smaller for all series and it has decreased remarkably for Belgium and Portugal. Persistence remains, however, high for Finland and Germany because the values

				BELGIUM FINLAND FRANCE GERMANY GREECE IRELAND				ITALY PORTUGAL	SPAIN
$\hat{\delta}_{i0}$	1.266	4.575	2.356	4.323	10.40		5.580 1.154	20.94	10.92
	$(-)$	$(-)$	$(-)$	$(-)$	$(-)$	$(-)$	$(-)$	$(-)$	$(-)$
$\hat{\delta}_{i1}$	2.256	179.1	51.72	509.8	-9.813		23.83 3.332	3.837	-1.710
	(0.184)	(14.39)	(3.685)	(23.43)	(0.061)	(1.641)	(0.168)	(0.258)	(0.144)
$\hat{\gamma}_{i1}$	10.95	2.149	2.244	2.442	1.760		2.706 9.707	5.838	2.670
	$(-)$	$(-)$	$(-)$	$(-)$	$(-)$	$(-)$	$(-)$	$(-)$	$(-)$
\hat{c}_{i11}	0.439 (0.000)	0.833 (0.005)	0.796 (0.005)	0.823 (0.003)	0.339 (0.010)		0.784 0.468 (0.004) (0.000)	0.036 (0.001)	0.080 (0.013)
\hat{c}_{i12}					0.893			0.298	0.518
					(0.009)			(0.032)	
$\hat{\delta}_{i2}$	55.20					-4.022		-4.934	-8.977
	(3.844)					(0.057)		(0.258)	(0.124)
$\hat{\gamma}_{i2}$	2.687					11.65		4.531	3.783
	$(-)$					$(-)$		$(-)$	$(-)$
\hat{c}_{i21}	0.794					0.320		0.039	0.539
	(0.004)					(0.000)		(0.001)	(0.005)
\hat{c}_{i22}						0.466 (0.000)		0.650 (0.003)	0.539 (0.005)
$\hat{\delta}_{i3}$								-19.48	9.972
								(0.081)	(0.588)
$\hat{\gamma}_{i3}$								3.446	2.945
								$(-)$	$(-)$
\hat{c}_{i31}								0.843	0.740
								(0.003)	(0.004)
\hat{c}_{i32}								0.843	
								(0.003)	

Table 2: Estimation results (robust standard errors in parentheses) for the deterministic component g_{it} .

close to the end of the sample are too extreme for the long-run component to accommodate. We further observe an improvement in the optimised log-likelihood value for all series when using the MTV-GARCH model over the GARCH model.

4.2 Model specification

Modelling the conditional correlations must begin by testing the adequacy of constant correlations. This is an important statistical tool because there is empirical evidence that correlations are time-dependent and neglecting variation of parameters leads to invalid asymptotic inference. For model selection, we use the Lagrange Multiplier statistic for testing constancy of correlations proposed in Silvennoinen and Teräsvirta (2005, 2015). For details on the test statistic and analytical expressions for the partial derivatives we refer to Silvennoinen and Teräsvirta (2005, 2015). Failure to reject the null hypothesis can be interpreted that the transition variable is not enough informative about time-variation in the correlation structure. Rejection of the null hypothesis of parameter constancy suggests time-varying correlations driven by an indicator variable or evidence in favour of other types of misspecification. Thus, rejecting the null hypothesis is a necessary, but not a

Table 3: Estimation results (robust standard errors in parentheses) for the conditional variance from the GARCH (upper panel) and MTV-GARCH (lower panel) models.

Figure 4: Estimated long-run volatility from the MTV-GARCH model (blue curve) and the conditional standard deviation from the GARCH model (black curve) for the nine Eurozone countries.

sufficient condition for the presence of financial contagion.

As discussed by Silvennoinen and Teräsvirta (2005), valuable information for the correlation dynamics can be obtained by studying submodels instead of higher-dimensional models. For this reason, we shall study lower dimensional models of bond returns instead of considering the nine-dimensional case. The choice of the model dimension depends on how far we wish to go into detail about the dynamics of comovements across countries. Therefore, we combine bond returns into three higher-dimensional models comprising of core, periphery, and core and each periphery country. Bivariate test results are also provided for comparison.

To investigate if the comovements of bond returns are linked to the behaviour of observable indicators, we shall use three potential transition variables. First we use s_t as the calendar time for which we are able to observe long-run movements or dominant trends in the correlations. Results of the tests when time is the transition variable are shown in Table 4. The left panel contains the p-values for the bivariate MTV-CCC-GARCH models for core and periphery countries. The strongest rejections occur for the periphery countries and when Greece is included in the model. The right panel contains the p -values for higher dimensional models. This includes the results from the four-variate model across core countries, and the five-variate models for peripheral countries, and between core and each periphery country. The null hypothesis of constant correlations is strongly rejected in

Table 4: Results from the test of constant conditional correlations against the alternative of time-varying correlations when the transition variable is time. The reported values are p-values. The bivariate test results for core and periphery countries are shown in the left panel. The test results for the higher dimensional case are shown in the right panel for core countries, periphery countries (GIIPS), and between core and each periphery country.

all cases. Yet again, the strongest evidence against the null of constant correlations occurs when Greece is considered: the p-values equal 4×10^{-63} and 6×10^{-93} for the five-variate models for the GIIPS and between core and Greece, respectively. In what follows, we shall use higher dimensional models to account possibly for common long-term movements in the correlations.

Another possibility is to consider the direction and size of the price movements as the time-variation indicator in correlations. In that case, a function of lagged returns that preserves the sign is an appropriate choice for the transition variable. Specifically, besides using the contemporaneous returns, we consider lagged returns up to five days for each periphery country. To smooth out some high-frequency noise, we also use a rolling mean of up to ten days of lagged returns as the transition variable, that is, $s_t = \frac{1}{n}$ $\frac{1}{n} \sum_{j=0}^{n-1} r_{i,t-j}$ for $n = 2, 3, 4, 5, 10$, where n represents the number of days of the rolling window. These windows are rolled through the whole sample a single observation at a time. The constancy of correlations is tested for each of these transition variables and using each periphery country as the source country. The strongest rejection of the constancy of correlations occurs by a large extent when the bond returns is chosen as the transition variable, suggesting that comovements are strongly linked to the behaviour of country-specific returns for the periphery. The left panel of Table 5 contains the results of the constancy test based on this transition variable.

The third choice for the transition variable is motivated by the empirical finding that comovements in the returns are stronger during volatile periods than during tranquil times; see Andersen et al. (2001). To capture this phenomenon, one option is to consider the general market turbulence as the time-variation indicator in correlations. In that

Table 5: Results from the test of constant conditional correlations against the alternative of time-varying correlations for the periphery countries. The reported values are p -values. The transition variables are the bond returns for each periphery country (left panel), the lagged VSTOXX index averaged over two weeks (right panel), and the lagged VIX index averaged over two days (right panel).

	GIIPS		
GREEK BOND RETURNS IRISH BOND RETURNS ITALIAN BOND RETURNS PORTUGUESE BOND RETURNS SPANISH BOND RETURNS	9×10^{-07} 2×10^{-11} 2×10^{-22} 2×10^{-26} 1×10^{-10}	2-W AVL VSTOXX 2-D AVL VIX	3×10^{-19} 2×10^{-17}

case, the changes in returns, absolute changes in returns and squared changes in returns would be obvious choices. Alternatively, one can use functions of the Chicago Board Options Exchange volatility index (VIX) or the Euro Stoxx 50 volatility index (VSTOXX) to account for uncertainty in the US and European markets, respectively. The tests of constant correlations reject constancy for each model, but the strongest rejections occurred when the transition variable were the lagged 2-week average of VSTOXX and the lagged 2-day average of VIX. These results may be explained by the following. The 2-day average of lagged VIX can been seen as the adjustment for time zone differences between the two markets, where the impact of the US market on the European market is delayed until the next day. The reasoning behind the lagged 2-week average VSTOXX is that it controls for any within week-variations. The test results are presented in the right panel of Table 5. The volatility indexes seem to be informative about the time-variation in correlations, but the market trend measured by time appears to carry more information. These results are in line with those of Longin and Solnik (2001) in which they conclude that the market trend affects international market correlations more than volatility.

4.3 Long-run movements in correlations and crisis dating

In this section we investigate long-term comovements across bond markets and characterize the extent of these long-run linkages during the sovereign debt crisis. We begin by estimating the MTV-TVC-GARCH model discussed in subsection 2.2 when time is chosen as the transition variable. For selecting K_l in (6) , we use model selection criteria after estimating the MTV-TVC-GARCH model with $K_l = 1, 2, 3$ to choose the best model. The Bayesian information criterion (BIC) of Schwarz (1978) selects three location parameters for the five-variate model among peripheral countries and two location parameters for the five-variate model between each periphery and core countries. Results are not shown due to space limitation. As expected, the model that performs poorly according to information

criteria is the multivariate MTV-GARCH model with constant correlations.

The dynamics of the estimated unconditional correlations from the MTV-TVC-GARCH model for the periphery countries is depicted in Figure 5. For comparison, we also plot the estimated short-run correlations obtained from the MTV-DCC-GARCH model and the constant correlations from the MTV-CCC-GARCH model. It is interesting to note how the short-run dynamic linkages (grey curve) tend to fluctuate around the time-varying unconditional correlations (black curve). We observe a declining trend in the unconditional correlations in the early phase of the crisis, starting in 2009 and reaching its lowest level around 2011, followed by an upward trend in 2014. This increase is offset by another decrease that is particularly noticeable for the pairwise correlations involving Greece. The drop in correlations during the last phase is especially pronounced for the pairs Greece-Italy, Portugal-Greece and Spain-Greece which may be due to the political instability in those countries during that period. These results suggest that the transmission of shocks across markets is far from being immediate. A possible explanation is that investors do not entirely recognise crisis signals and they interpret news as being country-specific. Over time, investors slowly incorporate negative news into prices, which in turn leads to a gradual convergence in sovereign yields, resulting in higher comovements across markets. Our conclusions are consistent to those of Chiang et al. (2007) for the Asian financial crisis.

We shall use as proxy the estimated constant correlations from the MTV-CCC-GARCH model for normal comovements across markets. The interdependence threshold in normal times for peripheral countries is then represented by the horizontal line (blue colour) in Figure 5. Correlations are generally moderate, ranging from 0.339 to 0.808, with the strongest correlation for the pair Italy-Spain, followed by the combination Portugal-Spain with a correlation of 0.615. It follows that a significant increase in market interactions beyond the normal level of interdependence may be regarded as financial contagion.

The estimated transition parameters of the five-variate model are reported in Table 6. The estimated slope parameter is relatively small ($\hat{\gamma} = 63.15$) yielding a fairly smooth change between the correlation states. The speed and the location parameters determine how level shifts occur in the unconditional correlations. One challenging task when testing for contagion effects is the demarcation of crisis from non-crisis periods. In this work, the identification of the crisis phases will be purely determined from the data and we shall rely on the estimates of the location parameters to distinguish those phases. Table 6 presents the estimated periods. We identify four distinct phases in our sample period. A transition from the pre-crisis phase to the first crisis phase is identified in early May 2009 ($\hat{c}_1 = 0.225$) marked by a decline in correlations a few months before the Greek deficit revision. The next period is identified as the first crisis phase ending in June 2014 ($\hat{c}_2 = 0.724$), a month after Portugal announced its exit from the bailout mechanism. This period includes the most acute phase of the sovereign debt crisis with the bailouts programmes for Greece,

	\hat{c}_k	BEGINNING DATE	
		18-01-2007	PRE-CRISIS
	0.225 (0.006)	07-05-2009	CRISIS PHASE I
(0.260)	63.15 0.724 (0.026)	20-06-2014	CRISIS PHASE II
	0.893 (0.016)	15-03-2016	POST-CRISIS

Table 6: Estimated transition parameters (standard errors in parentheses) for the MTV-TVC-GARCH model across the periphery countries and crisis dating.

Ireland and Portugal, and the rating downgrades of Spain and Italy, further supporting our crisis dating. A second phase of the crisis, characterised by the third Greek bailout package and triggered by the rise of Greek bond yields, ends by mid-March 2016 ($\hat{c}_3 = 0.893$). The last phase, from March 2016 until the end of the sample, is identified as possibly the post-crisis phase in the light of historical events pointing out to signs of recovery.

Next we examine if the unconditional correlations behave differently over time when core countries are considered in the analysis. Now, the best selected model is when $K_l = 2$ and crisis dating is done as before. Figure 6 shows the long-term correlations between core countries and each periphery country. The corresponding estimation results are presented in Table 7. We observe a downward trend in the long-term correlations mostly around 2009-2012 before rising up about 2014-2016. The only exception is for the pair Germany-Finland whose unconditional correlations increase during the critical phase of the crisis. We find that level shifts in comovements are fairly smooth across core (grey colour) and sharper between core and periphery countries (blue colour). Furthermore, the long-term correlations in the former countries preserved their high levels, contrary to the correlations across the latter countries which became negative at the peak of the crisis. Results also suggest a faster adjustment for Ireland and Italy. towards core yield levels. It is interesting to note that, at the end of the sample, other peripheral countries than Ireland and Italy yields have not converged entirely back to pre-crisis levels.

The decline in the long-term correlations may be explained by the flight-to-quality phenomenon from bad to good bond markets. This concept derives from the flight-toquality from stocks to bonds when investors reallocate their portfolio to reduce the risk of loss during turbulent times; see Baur and Lucey (2009) for further details. This idea may also be extended to the bond-bond case, where flight-to-quality occurs when investors move from falling to safer bond markets, causing them to move in opposite directions. We shall thus conclude for the presence of flight-to-quality effects if correlations across markets decline to negative levels. In order to do this, we compute the average weekly correlations for the estimated MTV-TVC-GARCH model in times of distress. Our findings indicate flight-to-quality from Greece to all core countries, from Ireland to Finland, France and

Figure 5: Estimated time-varying (un)conditional correlations for the periphery countries.

Figure 6: Estimated unconditional correlations for core (grey curves) and between core and periphery countries (blue curves). Each panel corresponds to the five-variate MTV-TVC-GARCH model for each periphery and core countries.

Germany, from Italy and Portugal to Finland and Germany, and from Spain to Germany. Therefore, our results indicate that flight-to-quality flows from all peripheral countries are mostly pronounced to the German bond market.

4.4 Effects of shocks on bond return comovements

There is widely accepted evidence of the asymmetric phenomenon in the correlation dynamics of equities, yet the literature is scarce about this effect for bond comovements. In order to fill this gap, we now proceed by examining how correlations across periphery countries respond to country-specific shocks. This is done by using the bond returns from each peripheral country as proxy of shocks and studying the time-variation in correlations given this market indicator. This choice is explained by the fact, as discussed in Section 4.2, that bond returns of peripheral countries play an important role on changes in correlations, albeit weaker than time. By doing this, the model will be able to accommodate, if present, an asymmetric response in correlations to negative shocks.

As before, we start the model-building cycle with the specification of the model. The best model is selected using the BIC criterion after fitting MTV-STCC-GARCH models to the data with alternative shapes for the transition function. Specifically, we estimate five-variate models with $K_l = 1, 2, 3$ with the best specification pointing towards $K_l = 2$. The estimation results from the MTV-STCC-GARCH model are reported in Table 8. We observe that transitions in correlations are close to the regime-switching behaviour as the smoothness parameter is fairly abrupt for all models (except the one assuming Irish returns as the transition variable). Visual inspection of the estimated transition functions is also depicted in Figure 7 where each panel corresponds to the indicator used as transition variable. It is evident that for returns of the periphery located between \hat{c}_1 and \hat{c}_2 , the logistic function becomes close to zero and the conditional correlations approach the high extreme state P_1 . On the contrary, large returns, positive or negative, result in correlations close to the low extreme state P_2 .

An interesting pattern can be perceived from these results. Small absolute shocks or periods of lower uncertainty are associated with higher correlations in bond markets. Conversely, smaller correlations are linked to large absolute shocks or higher uncertainty in these markets. On a different note, we find bond return comovements to be unaffected by the direction, positive or negative, of shock. In other words, while there is compelling evidence that bond correlations across periphery are strongly affected by the size of shock (small or large), they are not linked to the sign (negative or positive) of innovations from each periphery country. One can therefore expect higher correlations for smaller changes in yields rather than for larger movements in bond yields. A plausible explanation is because during calm periods, even if the market is hit by positive or negative news, investors do not reallocate their investments from high-risk yield into safe-haven bond

Table 7: Estimated results from the five-variate MTV-TVC-GARCH model (standard errors are in parentheses) for core and each periphery country (in boldface). The transition variable is the calendar time. Each panel shows the estimated correlation matrices $\hat{\mathbf{P}}_1$ (left panel) and \hat{P}_2 (right panel).

	BELGIUM	FINLAND	FRANCE	GERMANY	BELGIUM	FINLAND	FRANCE	GERMANY
FINLAND	0.406				0.776			
FRANCE	(0.038) 0.565	0.708			(0.016) 0.943	0.718		
	(0.031) 0.269	(0.020)			(0.007) 0.902	(0.016) 0.644	0.802	
GERMANY	(0.049)	0.957 (0.020)	0.645 (0.021)		(0.010)	(0.017)	(0.013)	
GREECE	-0.419 (0.109)	-0.546 (0.100)	-0.667 (0.126)	-0.689 (0.104)	0.775 (0.039)	0.599 (0.036)	0.823 (0.039)	0.631 (0.033)
			$\hat{c}_1 = 0.360$ (0.022)	$\hat{c}_2 = 0.838$ (0.024)	$\hat{\gamma} = 14.78$	(0.416)		
	BELGIUM	FINLAND	FRANCE	GERMANY	BELGIUM	FINLAND	FRANCE	GERMANY
FINLAND	0.116				0.785			
FRANCE	(0.057) 0.391 (0.047)	0.490 (0.036)			(0.009) 0.914 (0.004)	0.780 (0.009)		
GERMANY	-0.019 (0.069)	0.895 (0.024)	0.446 (0.037)		0.843 (0.006)	0.736 (0.010)	0.817 (0.007)	
IRELAND	-0.409 (0.095)	-0.626 (0.093)	-0.650 (0.103)	-0.808 (0.102)	0.758 (0.011)	0.637 (0.015)	0.733 (0.012)	0.690 (0.014)
			$\hat{c}_1 = 0.359$ (0.018)	$\hat{c}_2 = 0.570$ (0.017)	$\hat{\gamma} = 41.37$	(0.411)		
	BELGIUM	FINLAND	FRANCE	GERMANY	BELGIUM	FINLAND	FRANCE	GERMANY
FINLAND	-0.163 (0.057)				0.755 (0.009)			
FRANCE	0.127 (0.054)	0.404 (0.045)			0.913 (0.003)	0.756 (0.009)		
GERMANY	-0.456 (0.074)	0.897 (0.024)	0.273 (0.048)		0.826 (0.007)	0.758 (0.008)	0.825 (0.006)	
ITALY	0.278 (0.049)	-0.822 (0.067)	-0.354 (0.059)	-0.951 (0.073)	0.634 (0.015)	0.470 (0.018)	0.667 (0.014)	0.445 (0.019)
			$\hat{c}_1 = 0.426$ (0.017)	$\hat{c}_2 = 0.532$ (0.018)	$\hat{\gamma} = 55.06$	(0.299)		
	BELGIUM	FINLAND	FRANCE	GERMANY	BELGIUM	FINLAND	FRANCE	GERMANY
FINLAND	0.570				0.709			
FRANCE	(0.016) 0.711 (0.011)	0.737 (0.010)			(0.019) 0.932 (0.006)	0.662 (0.020)		
GERMANY	0.488 (0.018)	0.874 (0.008)	0.695 (0.011)		0.896 (0.008)	0.589 (0.022)	0.794 (0.012)	
PORTUGAL	0.086 (0.031)	-0.047 (0.033)	0.009 (0.032)	-0.176 (0.032)	0.924 (0.007)	0.669 (0.022)	0.924 (0.007)	0.780 (0.016)
			$\hat{c}_1 = 0.264$ (0.006)	$\hat{c}_2 = 0.941$ (0.010)	$\hat{\gamma} = 24.42$	(0.586)		
	BELGIUM	FINLAND	FRANCE	GERMANY	BELGIUM	FINLAND	FRANCE	GERMANY
FINLAND	0.519 (0.021)				0.736 (0.016)			
FRANCE	0.676 (0.014)	0.724 (0.013)			0.930 (0.006)	0.690 (0.017)		
GERMANY	0.414 (0.024)	0.894 (0.009)	0.670 (0.014)		0.900 (0.008)	0.624 (0.018)	0.801 (0.011)	
SPAIN	0.256 (0.030)	-0.045 (0.038)	0.097 (0.037)	-0.221 (0.040)	0.928 (0.006)	0.713 (0.016)	0.964 (0.004)	0.800 (0.013)
			$\hat{c}_1 = 0.294$ (0.008)	$\hat{c}_2 = 0.890$ (0.010)	$\hat{\gamma} = 22.24$	(0.360)		
				ററ				

Figure 7: Estimated logistic functions for the five-variate MTV-STCC-GARCH models for the periphery countries as a function of s_t . The transition variable s_t is the bond returns for each periphery country (in each panel). The lower and upper regime of correlations are represented, respectively, in blue and red colour.

markets, possibly leading to an increase in bond return correlations. By contrast, relying on the estimation results, we find no evidence of asymmetric response of bond return comovements to negative shocks. Thus, contrary to equity markets, the results reveal that bond correlations are not sensitive to the sign of shocks. This conclusion accords with Cappiello et al. (2006) who argue that bond correlations exhibit no leverage effect and therefore its presence is implausible in these markets.

4.5 Bond return correlations and financial market uncertainty

It is a widespread phenomenon that the correlations between asset returns often increase during periods of turbulence, while in tranquil times the returns are expected to behave more independently. However, it is also of interest to investigate how bond return correlations respond to changes in market distress. We now turn our attention to bond return comovements and study how their dynamics is affected by general market turbulence. Recent empirical evidence for the relationship between sovereign yield correlations and market volatility has been provided in De Santis and Stein (2015) and Xu (2017), among

Table 8: Estimated results from the five-variate MTV-STCC-GARCH model (standard errors in parentheses) for periphery countries. The transition variable are the bond returns for each periphery country (in boldface). Each panel shows the estimated correlation matrices \hat{P}_1 (left panel) and \hat{P}_2 (right panel).

	GREECE	IRELAND	ITALY	PORTUGAL	GREECE	IRELAND	ITALY	PORTUGAL
IRELAND	0.445 (0.028)				0.252 (0.021)			
ITALY	0.599 (0.020)	0.562 (0.021)			0.284 (0.024)	0.450 (0.028)		
PORTUGAL	0.580 (0.025)	0.525 (0.023)	0.628 (0.021)		0.338 (0.023)	0.429 (0.041)	0.562 (0.031)	
SPAIN	0.594 (0.020)	0.571 (0.020)	0.828 (0.009)	0.653 (0.021)	0.294 (0.024)	0.466 (0.030)	0.760 (0.019)	0.543 (0.032)
		$\hat{c}_1 = -2.486$	(0.002)	$\hat{c}_2 = 1.551$ (0.006)		$\hat{\gamma} = 100.0$ $(-)$		
	GREECE	IRELAND	ITALY	PORTUGAL	GREECE	IRELAND	ITALY	PORTUGAL
IRELAND	0.462 (0.027)				0.227 (0.027)			
ITALY	0.526 (0.023)	0.558 (0.020)			0.204 (0.035)	0.476 (0.028)		
PORTUGAL	0.522 (0.024)	$\,0.555\,$ (0.026)	0.612 (0.023)		0.308 (0.041)	0.410 (0.031)	0.574 (0.047)	
SPAIN	0.520 (0.024)	0.579 (0.019)	0.813 (0.010)	0.640 (0.023)	0.246 (0.037)	0.478 (0.030)	0.792 (0.017)	0.546 (0.047)
		$\hat{c}_1 = -\frac{2.306}{(0.239)}$		$\hat{c}_2 = 2.650$ (0.074)		$\hat{\gamma} = 4.655$ (0.741)		
	GREECE	IRELAND	ITALY	PORTUGAL	GREECE	IRELAND	ITALY	PORTUGAL
IRELAND	0.476 (0.021)				0.144 (0.027)			
ITALY	0.587 (0.025)	0.589 (0.023)			0.268 (0.024)	0.437 (0.021)		
PORTUGAL	0.551 (0.022)	0.540 (0.023)	0.665 (0.023)		0.321 (0.029)	0.415 (0.036)	0.496 (0.026)	
SPAIN	0.573 (0.022)	0.587 (0.020)	0.844 (0.007)	0.670 (0.021)	0.259 (0.031)	0.457 (0.026)	0.728 (0.019)	0.508 (0.029)
		$\hat{c}_1 = -2.523$	(0.003)	$\hat{c}_2 = 1.533$ (0.009)	$\hat{\gamma} = 100.0$	$(-)$		
	GREECE	IRELAND	ITALY	PORTUGAL	GREECE	IRELAND	ITALY	PORTUGAL
IRELAND	0.427 (0.023)				0.215 (0.030)			
ITALY	0.517 (0.025)	0.570 (0.021)			0.311 (0.028)	0.450 (0.026)		
PORTUGAL	0.586 (0.026)	0.568 (0.028)	0.725 (0.014)		0.316 (0.023)	0.397 (0.025)	0.443 (0.027)	
SPAIN	0.517 (0.025)	0.580 (0.020)	0.823 (0.009)	0.740 (0.013)	0.331 (0.028)	0.465 (0.028)	0.777 (0.016)	0.451 (0.027)
			$\hat{c}_1 = -\frac{2.643}{(0.003)}$	$\hat{c}_2 = 1.644$ (0.004)		$\hat{\gamma} = 100.0$ $(-)$		
	GREECE	IRELAND	ITALY	PORTUGAL	GREECE	IRELAND	ITALY	PORTUGAL
IRELAND	0.450 (0.023)				0.173 (0.027)			
ITALY	0.547 (0.025)	0.569 (0.022)			0.267 (0.028)	0.451 (0.024)		
PORTUGAL	0.538 (0.023)	0.535 (0.022)	0.638 (0.021)		0.322 (0.032)	0.416 (0.038)	0.536 (0.031)	
SPAIN	0.565 (0.027)	0.589 (0.021)	0.845 (0.008)	0.681 (0.021)	0.285 (0.025)	0.458 (0.023)	0.719 (0.017)	0.496 (0.026)
		$\hat{c}_1 = -2.467$	(0.002)	$\hat{c}_2 = 1.986$ (0.004)	$\hat{\gamma} = 100.0$	$(-)$		
				$\overline{01}$				

others.

We shall measure the level of uncertainty by using the VIX and VSTOXX indexes as indicators of expected stock market volatility for the US and the euro area over the next 30 days. These are calculated using the 30-day implied volatility of the S&P 500 and EURO STOXX 50 indexes, respectively. The VIX and VSTOXX are commonly perceived as "investor fear indexes" of future stock market volatility in the US and European market and thus viewed as measures for global and regional market risk aversion, respectively. Therefore, high values of the indexes are generally associated with high levels of volatility and hence to periods of high uncertainty. On the contrary, low values of the indexes are often related to tranquil times or less uncertainty.

We now proceed with the estimation of the MTV-STCC-GARCH model whose correlations are driven by functions of lagged VIX and VSTOXX as discussed in section 4.2. The correlation estimates of the model using either the lagged 2-week average of VSTOXX or lagged 2-day average of VIX as transition variable can be found in Table 9. The estimated transition functions for both indicator variables are displayed in Figure 9. Results clearly indicate that the bond return correlations tend to decline following an increase in the European index, which is certainly reflecting a period of increasing regional uncertainty. Correspondingly, bond comovements driven by the European volatility index tend to exhibit a pro-cyclical behavior. This is not surprising as Xu (2017) demonstrated that bond return correlations are weakly pro-cyclical, as opposed to the countercyclical behavior of equity return correlations. Our findings are also consistent with Longin and Solnik (2001) who demonstrated that the market trend is a major driver of the increase in correlations, instead of volatility. Furthermore, they showed that high volatility per se does not explain the rise in conditional correlations. As regards the effect of VIX on correlations, the results reveal an interesting pattern. There is empirical support of higher global uncertainty being associated with higher conditional correlations in bond returns, but this effect is essentially observed for the pairwise correlations involving either Greece or Portugal. This observation corroborates the countercyclical bond comovements behaviour for each of these countries as they tend to move in the opposite direction as the world economic cycle. This behaviour is, however, reversed for the remaining periphery countries. Such effects can be observed in Figure 9. From the results, we may therefore conclude that it is most likely to exist contagion effects from the global financial crisis to the Greek and Portuguese bond markets.

Despite the evidence that both volatility measures carry information about the dynamics of the correlations, a few differences emerge from them. First, the transmission between the extreme states of correlations is abrupt when using the VIX as indicator variable and quite smooth when using its European counterpart. The estimates of the location parameters are 20.25 for the VIX and about 25 for the VSTOXX. Therefore, values of the US and European volatility indexes larger than 20.25 or 25, depending on the index,

Figure 8: The lagged VSTOXX index averaged over two weeks (left panel) and the lagged VIX index averaged over two days (right panel).

Figure 9: Estimated transition functions for the five-variate MTV-STCC-GARCH models for periphery countries as a function of s_t . The transition variables are the lagged VSTOXX averaged over two weeks (upper panel) and the lagged VIX averaged over two days (bottom panel). The lower and upper regime of correlations are represented, respectively, in blue and red colour.

expect to lead to lower correlations across bond returns (with Greece or Portugal being an exception). The implication of these results is that bond return comovements in the eurozone are thus expected to react faster to higher levels of uncertainty in the US market than that of the euro area.

4.6 Testing for financial contagion

In this section we use the correlation-based statistical test for contagion of Forbes and Rigobon (2002) adjusted to our approach. As discussed before, we assume interdependence measured by the long-term level of comovements across markets, whereas contagion is identified as a significant increase in cross-market linkages after a shock. We shall interpret the correlations across markets in tranquil periods as the long-run equilibrium correlations, from where they can fluctuate in the short-run during times of distress. Financial contagion

Table 9: Estimated results for the five-variate MTV-STCC-GARCH model (robust standard errors in parentheses) for periphery countries. The transition variables are the lagged VSTOXX index averaged over two weeks (upper panel) and the lagged VIX index averaged over two days (bottom panel). Each panel shows the estimated correlation matrices $\hat{\mathbf{P}}_1$ (left panel) and \hat{P}_2 (right panel).

				GREECE IRELAND ITALY PORTUGAL GREECE IRELAND ITALY PORTUGAL				
IRELAND	0.299 (0.023)				0.408 (0.030)			
ITALY	0.357 (0.027)	0.573 (0.021)			0.529 (0.030)	0.441 (0.028)		
PORTUGAL	0.396 (0.026)	0.481 (0.027)	0.579 (0.024)		0.540 (0.031)	0.513 (0.029)	0.631 (0.031)	
SPAIN	0.388 (0.025)	0.568 (0.019)	0.818 (0.010)	0.609 (0.023)	0.495 (0.034)	0.484 (0.030)	0.792 (0.015)	0.625 (0.031)
			$\hat{c} = 20.25$	(0.028)	$\hat{\gamma} = 100.0$ $(-)$			

2-D AVL VIX

will be thus related to this excess of short-run comovements from its long-term level during periods of crisis. In what follows, the long-run correlations shall be measured by the estimated constant conditional correlations for the full observation period.

Let $\rho_{ij,ccc}$ be the long-run level of correlations proxied by the constant conditional correlation between countries i and j for the non-crisis period, and $\rho_{ij,n,stcc}$ be the timevarying conditional correlation from the MTV-STCC-GARCH model for the crisis period n adjusting for long-run movements in the volatility. In the case of deterministically time-varying correlations, the notation will be changed to $\rho_{ij,n,tvc}$. It follows that the pair of hypotheses for testing the null hypothesis of interdependence against the alternative of contagion are (suppressing subscripts for notational convenience):

$$
H_0: \bar{\rho}_{stcc} = \rho_{ccc}
$$

$$
H_1: \bar{\rho}_{stcc} > \rho_{ccc}
$$

where $\bar{\rho}_{stcc} = \mathsf{E}(\rho_{n,stcc})$ is the expected value of $\rho_{n,stcc}$ over the crisis period n. After the crisis phases have been identified (see section 4.3) and the correlations estimated for the crisis and non-crisis periods, the test statistic can be easily computed in a straightforward fashion as a standard t-test.

In their testing procedure, Forbes and Rigobon (2002) define the non-crisis period as the full observation period. Their test statistic is based on the underlying assumption of independence, which becomes inappropriate when using overlapping data due to the conservative property of the test. The solution lies in correcting the asymptotic variance of the test statistic to improve the asymptotic approximation to the normal distribution. Under the null hypothesis of no contagion, the adjusted test statistic is given by:

$$
FR_{\text{adj}} = \frac{\frac{1}{2} \ln \left(\frac{1 + \bar{\hat{\rho}}_{n,\text{stcc}}}{1 - \bar{\hat{\rho}}_{n,\text{stcc}}} \right) - \frac{1}{2} \ln \left(\frac{1 + \hat{\rho}_{ccc}}{1 - \hat{\rho}_{ccc}} \right)}{\sqrt{\frac{1}{T_{\text{stcc}} - 3} - \frac{1}{T_{ccc} - 3}}} \xrightarrow{d} N(0, 1),
$$
\n(16)

where T_{stcc} and T_{ccc} denote the number of contagious observations during the crisis period and the full observation period, respectively. As the sample counterpart of $\bar{\rho}_{stcc}$ we use $\bar{\hat{\rho}}_{n,stcc}$ which is defined as the sample mean of the estimated correlations from the MTV- $STCC-GARCH$ model over the crisis period n. The estimate of the long-term level of correlations in the full sample period is given by ρ_{ccc} . To compute the test statistic, besides the estimated constant correlation, we need to look at estimated time-varying correlations beyond the long-run level of interdependence in the crisis period and compute their average thereafter.

It may be useful to examine if unspecified changes affecting the overall level of correlations leads to contagion effects. We thus begin by testing for contagion when time is

Table 10: Results from the correlation-based tests of contagion within the periphery countries when the transition variable is time. The coefficients ρ_{ccc} and $\rho_{2,tvc}$ denote the estimated correlations from the MTV-CCC-GARCH model and the MTV-TVC-GARCH model above the interdependence threshold averaged over the second phase of the crisis, respectively. "C" and "I" denote, respectively, cross-market contagion and interdependence at the 5% significance level.

selected as the transition variable. As the crisis period, we select the second phase of the crisis because the long-term level of correlations increase during that period. The test results from the correlation-based test across the periphery countries using time as the indicator variable are reported in Table 10. Our test results suggest that there is evidence of contagion either from or to Greece, Italy, Spain and Portugal (with Italy-Spain being the exception). Interestingly, the rejection of the null hypothesis mostly occurs when either Greece or Portugal are involved in the model. Our results fail to reject the null hypothesis of interdependence either from or to Ireland, Italy and Spain, which is not surprising since the constancy correlation tests failed to reject the null for these countries in the bivariate case. Therefore, focussing on a shorter period of crisis, and not necessarily in the full crisis period, we find evidence of financial contagion across peripheral countries. This accords with the findings of Beirne and Fratzscher (2013) who found regional contagion to be fairly trivial during the most acute phase of the European sovereign debt crisis.

We next examine the strength of market interactions across core and each periphery country. As before, we estimate the time-varying correlations whose dynamics is driven by time. Results of the tests are shown in Table 11. The tests strongly reject the hypothesis of interdependence and thus supporting for the presence of contagion effects across core and periphery countries. Interestingly, the test results within core lead in general to higher p-values. To save space, these results are not shown. Opposite conclusions can be drawn when Belgium is involved. We find stronger contagious linkages for the combinations

Table 11: Results from the correlation-based tests of contagion between core countries and each periphery country when the transition variable is time. The coefficients ρ_{ccc} and $\hat{\rho}_{2,tvc}$ denote the estimated correlations from the MTV-CCC-GARCH model and the MTV-TVC-GARCH model above the interdependence threshold averaged over the second phase of the crisis, respectively. "C" and "I" denote, respectively, cross-market contagion and interdependence at the 5% significance level.

	ρ_{ccc}	$\rho_{2,tvc}$	P-VALUE			ρ_{ccc}	$\rho_{2,tvc}$	P-VALUE	
$IRELAND \leftrightarrow BELGIUM$	0.478	0.711	0.000		$ITALY \leftrightarrow BELGIUM$	0.557	0.623	0.000	\mathcal{C}
$IRELAND \leftrightarrow FINLAND$	0.323	0.584	0.000	C	$ITAI.Y \leftrightarrow FINLAND$	0.273	0.444	0.000	\mathcal{C}
$IRELAND \leftrightarrow FRANCE$	0.394	0.676	0.000	$\left(\cdot \right)$	$ITALY \leftrightarrow FRANCE$	0.488	0.643	0.000	$\left(\begin{array}{c} \cdot \end{array} \right)$
$IRELAND \leftrightarrow GERMANY$	0.313	0.626	0.000	$\mathbf C$	$ITAIY \leftrightarrow GERMANY$	0.228	0.416	0.000	\mathcal{C}
$PORTUGAL \leftrightarrow BELGIUM$	0.387	0.608	0.000	$\left(\begin{array}{c} \cdot \end{array} \right)$	$SPAIN \leftrightarrow BELGIUM$	0.560	0.717	0.000	\mathcal{C}
$PORTUGAL \leftrightarrow FINLAND$	0.205	0.399	0.003	$\left(\cdot \right)$	$SPAIN \leftrightarrow FINLAND$	0.293	0.474	0.000	\mathcal{C}
$PORTUGAL \leftrightarrow FRANCE$	0.335	0.579	0.000	$\left(\begin{array}{c} \cdot \\ \cdot \end{array} \right)$	$SPAIN \leftrightarrow FRANCE$	0.482	0.691	0.000	$\left(\begin{array}{c} \cdot \\ \cdot \end{array} \right)$
$PORTUGAL \leftrightarrow GERMANY$	0.158	0.419	0.000		$SPAIN \leftrightarrow GERMAN$	0.229	0.479	0.000	\mathfrak{C}
$GREECE \leftrightarrow BELGIUM$	0.248	0.409	0.000	\mathcal{C}					
$GREECE \leftrightarrow FINLAND$	0.093	0.248	0.001	\mathcal{C}					
$GREECE \leftrightarrow FRANCE$	0.174	0.371	0.000	\mathcal{C}					
$GREECE \leftrightarrow GERMANY$	(0.040)	0.222	0.000	\mathcal{C}					

Belgium-France and Belgium-Germany, implying there is a clear evidence of contagion effects among these pairs. However, we fail to reject the constancy of correlations for the pair Belgium-Germany and therefore the evidence of contagion for this pair may be seen as spurious. To summarize, when time is selected as the transition variable, we find strong evidence of shift contagion within the periphery countries, mostly either from or to Greece and Portugal, and between core and periphery countries following the more severe period of the crisis.

If one wishes to infer about the direction of the spillover effects, using time as the indicator variable will not be a suitable choice. Instead, by selecting the returns of each periphery as transition variable, we are able to identify the source of contagion from which shocks emanate. Two remarks are in order. First, using the returns of periphery as the market indicator, the correlations tend to be higher during the first phase of the crisis. Thus, we shall define the unstable period as the first phase of the crisis. Second, before computing the correlation level for the crisis period, we consider the weekly average of the time-varying correlations to smooth out high-frequency noise. The test results can be found in Table 12. The results reveal either unidirectional or bidirectional contagion effects between countries. For instance, there is evidence of unidirectional contagion from the Portuguese bond market to the Italian during the first phase of the crisis, but not the other way around. In fact, we find no significant increase in financial linkages between Italy and Portugal when the Italian returns are selected as the transition variable as opposed

Table 12: Results from the correlation-based tests of contagion between the periphery countries using as transition variable the returns of the source country (represented by the leftmost country) averaged over one week. The coefficients ρ_{ccc} and $\rho_{1,stcc}$ denote the estimated correlations from the MTV-CCC-GARCH model and the MTV-STCC-GARCH model above the interdependence threshold averaged over the first phase of the crisis, respectively. "C" and "I" denote, respectively, cross-market contagion and interdependence at the 5% significance level.

	$\hat{\rho}_{ccc}$		$\rho_{1,stcc}$ P-VALUE			$\hat{\rho}_{ccc}$		$\rho_{1,stcc}$ P-VALUE	
$GREECE \rightarrow IRELAND$ $\textsc{Gre} \rightarrow \textsc{ITALY}$		0.340 0.411 0.097	0.422 0.545 0.006 C	\mathbf{I}	$\operatorname{IRELAND} \!\to\! \operatorname{GREECE}$ IRELAND-HTALY 0.525 0.550 GREECE-PORTUGAL 0.450 0.538 0.036 C ireland-portugal 0.493		0.542	0.340 0.441 0.023 C 0.274 1 0.131	
$\text{GREECE} \rightarrow \text{SPAIN}$					0.429 0.542 0.011 C ireland Spain 0.537 0.570			0.219 I	
$ITALY \rightarrow GREECE$					0.422 0.547 0.004 C PORTUGAL \rightarrow GREECE 0.450 0.541			0.029 C	
$ITALY \rightarrow IRELAND$		0.525 0.569	0.147		I PORTUGAL ^{\rightarrowIRELAND 0.493 0.539}			0.157	\blacksquare
$ITALY \rightarrow PORTUGAL$					0.598 0.652 0.095 I PORTUGAL-HTALY 0.598 0.677			0.017 C	
$ITALY \rightarrow SPAIN$	0.808	0.835	0.110	\mathbf{L}	PORTUGAL \rightarrow SPAIN 0.615 0.692			0.017 C	
$SPAIN \rightarrow GREECE$			0.429 0.532 0.013 C						
$SPAIN \rightarrow IRELAND$	0.537 0.579		0.177 I						
$SPAIN \rightarrow ITALY$	0.808	0.836	0.096	- 1					
$SPAIN \rightarrow POPUGAL$	0.615	-0.667	$0.087 \quad I$						

to when we use the Portuguese returns. Surprisingly, the results suggest an increase in comovements from the Irish bond market to the Greek, but not from the reverse direction. Another interesting finding is that there is empirical evidence of bidirectional contagion effects between Greece and the remaining periphery countries (Ireland being an exception).

If the interest lies in finding out whether general market turbulence plays a role on contagion, one may use regional and global volatility indexes as drivers of the correlation dynamics. The tests of contagion when choosing these indicators are presented in Table 13. Our results indicate contagion effects for the pairs Ireland-Italy and Portugal-Spain during the first phase of the crisis for low values of VSTOXX. Conversely, higher levels of the regional stock market volatility do not seem to drive contagion. When it comes to global volatility, we find contagion effects for all pairwise correlations involving Greece during the first phase of the crisis. Therefore, cross-country linkages between Greece and other peripheral countries are strengthened when uncertainty is persistently high in the global financial market. An interesting finding can be perceived from the results. The correlations between Greece and other periphery countries appear to be strongly affected by global financial market volatility, whereas other pairwise correlations are mainly influenced by the euro area bond market. Our results corroborate the findings of Broto and Pérez-Quirós (2015) and Mink and de Haan (2013) who found Greece, Portugal and Ireland to be the

major sources of contagion. Another interesting result, also consistent with ours, is that the size of a shock is not proportional to its capacity for spreading and leading contagion. They argue that, in fact, Portugal showed smaller shocks than Greece and yet it was the main driver of contagion.

The findings discussed above can be explained by the alternative choices for the transition variable. When using the calendar time, the modeller is interested in identifying level shifts in the unconditional correlations and therefore how the overall level of correlations is changing over time. Thus, when choosing time as driver of the correlation dynamics, any significant increase in the bond return comovements may be seen as long-run contagion. Because it seems to occur beyond what is explained by country-specific and global financial indicators, the long-run contagion effect is closely related to pure contagion. Instead, by selecting a financial indicator, the correlations are expected to update quite frequently. When a significant increase in cross-market linkages occurs in such erratic behaviour, this may be perceived as short-run contagion. Such contagion effect is closely related to the wake-up-call definition and we find this form of contagion to prevail mostly during the first-phase of the crisis. Wake-up-call contagion occurs when additional information about a crisis in one market prompts investors to reassess the default risk of other markets with similar structural problems. It is then strongly related to country-specific factors. When using the indicator of global market uncertainty, our findings support Greece as the transmission channel of shocks from the global financial crisis to the sovereign debt crisis. The above results suggest two variants for the tests of contagion. Depending on the selection of the indicator variable, the tests of contagion will be testing the presence of either long-run or short-run contagion. The choice of either one will thus depend on the problem at hand.

5 Conclusions

In this paper, we study the presence and extent of contagion during the sovereign debt crisis for nine Eurozone countries. We address this problem by testing for financial contagion using a correlation-based analysis relying on the multiplicative time-varying STCC-GARCH model. The new model extends the multiplicative time-varying GARCH model of Amado and Teräsvirta (2013) to the multidimensional case where the timevariation in the conditional correlations is driven by time and observable financial indicators. This approach has the advantage of adjusting the correlations to long-term and shortterm time-varying volatility, and therefore avoiding the bias in cross-market correlations. Besides, when time is selected as the indicator, the identification of the crisis phases can be determined endogenously by the model and not pre-defined by the modeller.

The new model is applied to nine daily returns on 10-year government bond yields using time and financial variables to control the changes in correlations. We find a smooth decline

Table 13: Results from the correlation-based tests of contagion for the periphery countries using as transition variable the lagged VSTOXX index averaged over two weeks and the lagged VIX index averaged over two days. The coefficients ρ_{ccc} and $\rho_{1,stcc}$ denote the estimated correlations from the MTV-CCC-GARCH model and the MTV-STCC-GARCH model above the interdependence threshold averaged over the first phase of the crisis, respectively. "C" and "I" denote, respectively, cross-market contagion and interdependence at the 5% significance level.

		2-W AVG VSTOXX				2-D AVG VIX		
	ρ_{ccc}	$\rho_{1,stcc}$	P-VALUE		$\rho_{1,stcc}$	P-VALUE		
$GREECE \leftrightarrow IRELAND$	0.340	0.350	0.357	I	0.408	0.025	\mathcal{C}	
$GREECE \leftrightarrow ITALY$	0.422	0.424	0.474	T	0.529	0.000	\mathcal{C}	
$GREECE \leftrightarrow PORTUGAL$	0.450	0.457	0.393	T	0.539	0.002	С	
$GREECE \leftrightarrow SPAIN$	0.429	0.441	0.312	T	0.495	0.019	\mathcal{C}	
$IRELAND \leftrightarrow ITALY$	0.525	0.609	0.000	C	0.573	0.008	С	
IRELAND⇔PORTUGAL	0.493	0.528	0.059	T	0.513	0.252	I	
$IRELAND \leftrightarrow SPAIN$	0.537	0.606	0.001	T	0.568	0.059	I	
$ITALY \leftrightarrow PORTUGAL$	0.598	0.625	0.084	T	0.630	0.103	I	
$ITALY \leftrightarrow SPAIN$	0.808	0.823	0.075	T	0.818	0.165		
$PORTUGAL \leftrightarrow SPAIN$	0.615	0.653	0.019	\bigcap	0.625	0.350		

in the long-term trend in correlations between core and periphery, which may be explained by the flight-to-quality phenomenon from bad to good bond markets, when investors move their portfolio from falling to safer bond markets. When the transition variable is defined as a function of idiosyncratic shocks from peripheral markets, the transmission channel is viewed as country-specific. It is found that bond correlations across periphery are strongly affected by the size of shock (small or large), but they are not sensitive to the sign of shocks (negative or positive). When changes in conditional correlations are controlled by a regional market turbulence indicator, bond return comovements tend to exhibit a pro-cyclical behavior as opposed to the countercyclical behavior of equity return correlations. There is, however, empirical support of higher global uncertainty being associated with higher conditional correlations in bond returns, but this effect is essentially observed for the pairwise correlations involving either Greece or Portugal.

Finally, our results also suggest strong evidence of long-run contagion effects within the periphery countries, mostly either from or to Greece and Portugal during the second phase of the crisis. With respect to the presence of short-run contagion effects, these are mainly observed around 2009-2014. We observe that short-term contagion is essentially driven by country-specific shocks from peripheral countries and higher levels of global stock market volatility. On the contrary, regional stock market turbulence does not seem to play a role on financial contagion.

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