Exploring young children's reasoning and naming of fractions

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This study investigates the effects of a teaching intervention on children's reasoning and naming of fractions in quotient, part-whole and operator situations. A pre-test, intervention and post-test design was used with 37 six- to seven-year-olds from primary schools in Braga, Portugal. The children had not been taught about fractions in school. Reasoning and labelling questions were presented in the three situations in the pre- and post-test. During teaching, each intervention group learned about fractions in only one of the three situations. Children who were taught in the quotient situation made significant progress in the reasoning and naming fractions, but did not transfer this learning to the other situations. Children taught in the part-whole or in the operator situations only learned how to label fractions, showing no progress on reasoning items. However, they used the labels in both part-whole and operator items. Thus these situations affect differently children's understanding of fractions.

Keywords: fractions; part-whole, quotient and operator situations

Introduction

Fractions can be used to represent quantities in different types of situation. The aim of this study was to investigate the impact of the situation in which fractions are taught on children's learning. Three types of situation were included: quotient, part-whole and operator. In quotient situations, *a/b* represents the relation between *a* number of items shared equally among *b* number of recipients (e.g. 2/3 represents 2 chocolate bars shared fairly by 3 children); *a/b* also represents the quantity received by each recipient (e.g., 2/3 represents the amount of chocolate received by each child). In part-whole situations, *a/b* represents the relation between *b*, the number of equal parts in which the whole is divided, and *a*, the number of these parts taken (e.g. 2/3 of a chocolate bar means that the bar was divided into 3 equal parts and 2 of these parts were taken). In operator situations, which involve a set of discrete items taken as a whole, *b* indicates the number of equal groups into which the set was divided and *a* is the number of groups taken (Nunes & Bryant, 2008). The hypothesis in this study is that, although these situations may seem very similar to an adult, they are perceived as quite different by children. Thus it is predicted that, if children learn about fractions in one type of situation, they will not transfer easily what they have learned to the other two types of situation.

Framework

Previous research shows that children perform differently in similar problems presented in the context of quotient and part-whole situations. Nunes et al. (2007) asked 130 children from Year 4 (8 years-old approximately) and Year 5 (9 years-old approximately) to judge the equivalence of fractions in part-whole and in quotient situations. The proportion of correct responses on equivalence problems in the part-whole situations was .35 whereas in quotient situations it was .66, suggesting that

part-whole and quotient situations affect children's performance differently. Mamede, Nunes and Bryant (2005) asked Portuguese children, who had not received any formal instruction on fractions (aged 6 and 7), to solve ordering and equivalence problems of fractional quantities presented in quotient and in part-whole situations. In quotient situations, children got 42.5% and 60.8% of correct responses in equivalence and ordering problems, respectively; whereas in part-whole situations, the rates of correct responses were 13.8% and 22.1%, respectively. In another survey Nunes and Bryant (2008) asked to 318 Year 4 and 5 pupils to judge whether the fractions 1/3 and 2/6 were equivalent, or not. The items were presented simply as numbers, without a context, in the context of part-whole situations, and in the context of quotient situations. Pupils were most successful in quotient situations (68% correct), followed by part-whole situations (41% correct) followed by numerical problems without context (39% correct). Similar results were obtained in a study with 8- and 9-year-olds in England, who had been taught about fractions in part-whole situations and attained 40% (8-years-old) and 74% (9-years-old) correct responses in part-whole problems; their rates of correct responses to the quotient questions were 71% and 83% (Nunes & Bryant, 2011). Also Campos, Magina, Canova and Silva (2010), who compared the impact of intervention sessions focused on fractions in quotient, part-whole, operator and intensive quantities on 138 Brazilian third- and fourth-graders, refer that students of the quotient situation intervention group registered the higher improvement. More recently, Canova (2013) studied the effect of a teaching experiment, comprising reasoning and naming fractions tasks with part-whole and quotient intervention groups, involving 378 fourth- to sixth-graders from Brazilian primary schools. The quotient intervention group performed better on the reasoning fractions problems, and the part-whole intervention group performed better in the naming of fractions.

These results strongly support the significance of the distinction between quotient and part-whole situations for educational practices. However, previous studies did not investigate the consequences of teaching and learning about fractions in these different situations; teaching had been done in schools without the researchers' interference. The present study analyses the effects of teaching children about fractions in each of these types of situation in comparison to the others. This study analyses the effects of teaching children about fractions in quotient, part-whole and operator situations in comparison to the others. It addresses two questions: 1) what can children learn about reasoning and naming in these situations? And 2) do children transfer learning across these situations?

Methods

Participants

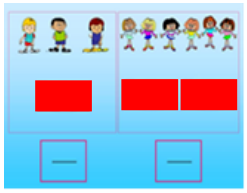
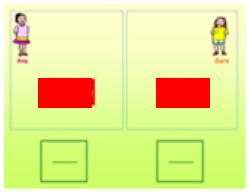
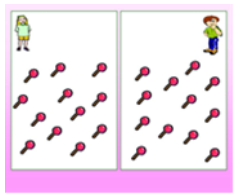
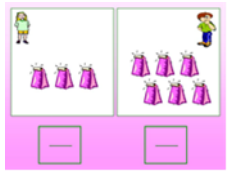
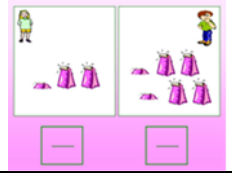
Participants were 40 six and seven-year-olds (mean age 6.6 years) from two state supported primary schools, in Braga, Portugal. The children had not received formal instruction in fractions at school. During the intervention, 3 children from the control group did not attend all sessions and were excluded from the analyses.

Measures

Pre- and post-tests, administered individually, were used to assess whether there was progress after the intervention. These tests comprised 12 reasoning items, involving equivalence or ordering fractions, presented in each type of situation - quotient, part-

whole and operator - without the use of fraction labels. After solving the reasoning questions, the children were also asked to name the 12 pairs of fractions in each of these situations. Figure 1 gives examples of an equivalence problem presented, respectively, in the quotient, part-whole and operator situations, in the pre- and post-tests.

Figure 1. Examples of an equivalence problem presented in the Pre- and Post-tests.

Type of situation	Example
Quotient 	Three boys are going to share 1 chocolate bar fairly. Six girls are going to share 2 chocolate bars fairly. Does each boy eat more chocolate than each girl? Does each girl eat more chocolate than each boy? Or do the boys and girls eat the same amount of chocolate? Explain your answer.
Part-whole 	Betty and Ruth have each a chocolate bar. But as they are not very hungry, they decide not to eat all the chocolate bar at once. Betty divides hers into 3 equal parts and eats 1 part; Ruth divides hers into 6 equal parts and eats 2 part. Does Betty eat more chocolate than Ruth? Does Ruth eat more chocolate than Betty, or are they eating the same amount of chocolate? Explain your answer.
Operator   	1. Anna and Phil have each 12 sweets (first slide). 2. Anna decided to share hers into 3 equal bags, with the same number of sweets in each; Phil shares his into 6 equal bags, all with the same number of sweets (second slide). 3. Anna eats 1 bag of sweets and Phil eats 2 bags (third slide). Does Ann eat more sweets than Phil, does Phil eat more sweets than Ann, or do they eat the same number of sweets? Circle the one that you think that ate more or both if they ate the same. Explain your answer.

Design

Children were randomly assigned to learning in one of the three situations - quotient, part-whole, or operator intervention – or to a comparison group, who solved multiplication and division problems with whole numbers. Each small group (with five children) participated in two teaching sessions of about 35 minutes. During the intervention, the researcher presented questions to the children, asked them to try to

answer the questions, and then discussed the answers with the children. All problems were presented using an approach similar to the test items (exemplified in figure 1): the researcher showed the children an illustration, and the children had a booklet with the same illustration. The researcher then explained the question and the children answered in their own booklets. Finally, the researcher discussed with the children their answers. In each session intervention, the children worked on 2 problems about ordering fractions and 2 about equivalence; the children were then taught how to name the fractions in the problem. After being taught how to label the fractions in the first problem, they were asked to name the fractions in the subsequent problems and their answers were discussed by the researcher.

Results

Table 1 presents the means and standard deviations for accuracy on reasoning items in each situation by testing occasion. The means are separated by intervention group.

	Testing Occasion and Type of Situation Used in the Test Items					
Situation in the Intervention Group	Pre-test			Post-test		
	Reasoning problems (Maximum score = 12)					
	Quotient	Part-whole	Operator	Quotient	Part-whole	Operator
Quotient (n=10)	5,6 (3,3)	0	0	8,6 (3,13)	0	0
Part-whole (n=10)	2,7 (3,38)	0,1 (0,32)	0	3,0 (3,71)	0,6 (1,9)	0
Operator (n=10)	2,5 (2,55)	0	0	3,8 (3,65)	0	0
Control (n=7)	3,0 (3,92)	0,29 (0,76)	0,43 (1,13)	3,0 (4,51)	1,57 (4,16)	1,71 (4,53)

Table 1: Mean accuracy (standard deviations in brackets) by testing occasion (pre- vs post-tests) on reasoning problems in each situation by intervention group.

At pre-test (Table 1), all children performed better on reasoning problems presented in quotient situations, irrespective of the group to which they were later assigned. There were almost no correct responses to reasoning problems presented in part-whole or operator situations. At post-test, children in the quotient intervention group improved in accuracy in the quotient reasoning items, but no other improvement in reasoning is noticeable.

Table 2 presents the means and standard deviations for accuracy on naming items in each situation by testing occasion. The means are separated by intervention group.

	Testing Occasion and Type of Situation Used in the Test Items					
Situation in the Intervention Group	Pre-test			Post-test		
	Naming problems (Maximum score = 12)					
	Quotient	Part-whole	Operator	Quotient	Part-whole	Operator
Quotient (n=10)	0	0	0	10,8 (1,62)	0,9 (2,51)	1,9 (4,18)
Part-whole	0	0	0	2,6	9,7	9,4

(n=10)				(4,14)	(2,78)	(4,97)
Operator (n=10)	0	0	0	1,2 (3,8)	10,3 (1,42)	11,6 (0,7)
Control (n=7)	0	0	0	0	0	0

Table 2: Mean accuracy (standard deviations in brackets) by testing occasion (pre- vs post-tests) on naming problems in each situation by intervention group.

At pre-test, no child was able to label a fraction correctly (Table 2) but there are improvements in the children's accuracy in labelling items from pre- to post-test. These improvements are selective: children in the quotient group improve their performance in naming fractions in quotient situations only, whereas children in the part-whole and operator intervention groups improve their accuracy in naming fractions in both types of situation.

In view of the floor effects in pre- and post-test accuracy scores in reasoning items in part-whole and operator situations, it was only possible to analyse the effect of the intervention on reasoning items in quotient situations. In order to analyse whether one type of intervention led to greater improvement than the other on quotient reasoning items, an ANCOVA was carried out, controlling for the pre-test. The score for the pre-test Quotient reasoning problems was a factor and type of intervention session (quotient, part-whole, operator, control) was a between–participants factor. The dependent variable was the score for post-test quotient reasoning problems. The results showed that the covariate predicts significantly the children's performance in solving the quotient reasoning items ($F(1,32)=86.74$, $p<.001$). There was also an interaction of quotient reasoning items by Session intervention group ($F(3,32)=4.48$, $p<.05$). Post-hoc (Bonferroni) tests revealed that the intervention sessions on quotient situations significantly increased children's performance compared to both the part-whole intervention session group, $t(32)=-3.15$, $p< .05$), and the control intervention sessions group ($t(32)=-319$, $p<.05$), but not with the operator intervention sessions group ($t(32)=-2.07$, n.s). Thus, the type of situation affects differently children's reasoning of fractions.

As there was no variation in the children's accuracy in naming fractions in the pre-test, only post-test performance can be analysed. Three one-way ANOVAs were carried out on post-test scores, one for each type of situation; children in the control group were excluded as there was no variation in their post-test scores. There are differences on type of intervention group sessions when solving naming problems in quotient situations ($F(2, 27)=23.61$, $p<.001$). Children in the quotient intervention group performed significantly better than those in the part-whole and operator intervention groups when naming fractions in quotient situations; however, their performance was significantly weaker when naming fractions in part-whole or operator situations. There are also differences on type of intervention group sessions when solving naming problems in part-whole situations ($F(2, 27)=50.15$, $p<.001$) and in operator situations ($F(2,27)=18.20$, $p<.001$). The part-whole and the operator intervention groups performed equally well when naming fractions in part-whole and operator situations.

Final remarks

The findings of this study shows that some changes occurred with the teaching experiment in which the children were introduced to fractions, in each type of situation analysed. The children who were introduced to fractions in quotient

situations improved their performance on reasoning problems, involving equivalence and ordering, revealing some understanding of the inverse divisor-quotient relation. This understanding was also found previously in the literature (see Mamede, Nunes & Bryant, 2005), when fractions were introduced to young children, but also when comparing fractions problems were solved by older children in quotient situations (see Nunes & Bryant, 2008; Canova, 2013). Contrasting with these findings, the children who were introduced to fractions either in part-whole or operator situations did not show improvement with the instruction sessions when solving reasoning problems. These findings suggest that part-whole and operator situations are very difficult situations for the children to attend to all the dimensions involved in the problem.

It is concluded that learning in quotient situations was more effective, as the children progressed both in reasoning and naming items, but it was situated: there was no transfer. In contrast, learning in part-whole and operator situations was limited, as there was no progress in reasoning, but the use of fraction labels was generalized between the two situations.

Teaching about fractions in many countries is often done in part-whole and operator situations, with emphasis on learning to name fractions. Children easily learn to name fractions in specific situations, so it is easy to believe that they understand the reasoning underlying this new numerical form. This study underscores the limitations of teaching in these situations and the need to combine different situations in teaching fractions, as each of them has strengths and weaknesses.

References

Campos, T. M. M., Nunes, T., Costa, N. M. L. D., & Ceragioli, L. (2012). A Representação de Quantidades Menores do que uma Unidade. *Acta Scientiae*, 14, 363-373.

Canova, R. (2013). Um estudo das situações parte-todo e quociente no ensino e aprendizagem do conceito de fração. *Unpublished PhD Thesis*, Universidade Bandeirante Anhanguera. Brasil: São Paulo.

Mamede, E., Nunes, T. & Bryant, P. (2005). The equivalence and ordering of fractions in part-whole and quotient situations. In: H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 3, pp. 281–288). Melbourne, Australia: PME.

Nunes, T., & Bryant, P. (2008). Rational Numbers and Intensive Quantities: Challenges and Insights to pupils' Implicit Knowledge. *Anales de psicología*, 24 (2), 262-270.

Nunes, T., & Bryant, P. (2011). The development of mathematical thinking. In R. Gillibrand, V. Lam & V. O'Donnell (Eds.), *Developmental Psychology* (pp. 168-213). London: Pearson.

Nunes, T., Bryant, P., Pretzlik, U., Bell, D., Evans, D., & Wade, J. (2007). La compréhension des fractions chez les enfants. In M. Merri (Ed.), *Activité humaine et conceptualisation* (pp. 255-262). Toulouse: Presses Universitaires du Mirail.