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Young Children Solving Multiplicative Reasoning Problems

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Abstract. This paper describes a study focused on kindergarten children multiplicative reasoning. It addresses two questions: 1) how do children perform when solving multiplication, partitive and quotitive division problems? And 2) what arguments do children present to justify their resolutions? An intervention program comprising 4 sessions was conducted with 12 kindergarten children (5-6-years-old), from a state supported kindergarten, in Viseu, Portugal. Similar Pre- and Post-tests were used to identify changes on children's understanding during the intervention program. In each test children solved 28 problems (18 additive structure problems; 6 multiplicative structure problems; 4 control problems on geometry) in two different consecutive moments. The intervention comprised 4 partitive division problems, 4 multiplication problems, and 4 quotitive division problems. The problems were presented to the children by the means of a story, and material was available to represent each problem. After each resolution, each child was asked "Why do you think so?". Results suggest that young children can reach success levels when solving multiplication and division problems, relying on their informal knowledge, presenting arguments that show that they are able to establish the correct reasoning when solving the tasks, articulating properly all the quantities involved in the given problems. Results also suggest that children's multiplicative reasoning can be enhanced when they can experience multiplicative structures problem solving, being able to interact with peers and discuss their ideas, after receiving some prompts from teacher. This study also suggests that both additive and multiplicative reasoning, in their simplistic forms, seem to be simultaneously reachable to kindergarten children, making sense to them. Educational implications of these findings will be discussed.

Résumé. Cet article décrit une étude centrée dans le raisonnement multiplicatif des enfants de l'école maternelle. L'étude aborde deux questions: 1) comment les enfants résolvent des problèmes de multiplication et division, partitive et quotitive? Et 2) quels arguments les enfants présentent-ils pour justifier leurs résolutions? Un programme d'intervention comprenant 4 sessions a été organisé avec 12 enfants (5-6 ans), d'une école maternelle publique à Viseu, au Portugal. Des Pré- et Posttests similaires ont été utilisés pour identifier les changements sur la compréhension des enfants pendant le programme d'intervention. Dans chaque test, les enfants ont résolu 28 problèmes (18 problèmes de structure additive, 6 problèmes de structure multiplicative, 4 problèmes de contrôle sur la géométrie) en deux moments consécutifs différents. L'intervention comprenait 4 problèmes de division partitive, 4 problèmes de multiplication et 4 problèmes de division quotitive. Les problèmes ont été présentés aux enfants au moyen d'une histoire, et le matériel était disponible pour représenter chaque problème. Après chaque résolution, chaque enfant a été interrogé "Pourquoi tu as fait comme ça?". Les résultats suggèrent que les jeunes enfants peuvent réussir à résoudre des problèmes de multiplication et de division, en s'appuyant sur leurs connaissances informelles, en présentant des arguments qui montrent qu'ils sont en mesure d'établir le raisonnement correct lors de la résolution des tâches, en articulant correctement toutes les quantités impliquées dans les problèmes donnés. Les résultats suggèrent également que le raisonnement multiplicatif des enfants peut être amélioré lorsqu'ils peuvent résoudre des problèmes multiplicatifs, interagir avec des pairs et discuter leurs idées, après avoir reçu des instructions du professeur. Cette étude suggère encore que le raisonnement additif et multiplicatif, sous leurs formes simplistes, semble être simultanément accessible aux enfants de l’école maternelle, ce qui leur permet d'avoir un sens. Les implications éducatives de ces résultats seront discutées.

1. Framework

Children possess informal knowledge relevant for the learning of mathematical concepts. The mathematical ideas children acquire in kindergarten constitute de basis of future mathematical learning. Thus, the development of the mathematical skills in early age is crucial to the success for future learning (NCTM, 2008). Children can use their informal knowledge to analyze and solve simple addition and subtraction problems before they receive any formal instruction on addition and subtraction operations (Nunes & Bryant, 1996). But they also can know quite a lot about multiplicative reasoning when they start school (Nunes & Bryant, 2010b).

Piagetian theory supported the idea that children first quantify additive relations and can only quantify multiplicative relations much later (see Piaget, 1952). In spite of his undoubted contribution to research, more recently research has been giving evidence of a different position. Thompson (1994), Vergnaud (1983) and Nunes and Bryant (2010a) support the idea that additive and multiplicative reasoning have different origins. Vergnaud (1983), in his theory of conceptual fields, distinguishes the field of additive structures and the field of multiplicative structures, considering them as sets of problems involving operations of the additive or the multiplicative type. Vergnaud (1983) argues that “multiplicative structures rely partly on additive structures; but they also have their own intrinsic organization which is not reducible to additive aspects” (p.128). Nunes and Bryant (2010a) also consider that additive and multiplicative reasoning have different origins, arguing that “Additive reasoning stems from the actions of joining, separating and placing sets in one-to-one correspondence. Multiplicative reasoning stems from the action of putting two variables in one-to-many correspondence (one-to-one is just a particular case), an action that keeps the ratio between the variables constant.” (p.11).

Multiplicative reasoning involves two (or more) variables in a fixed ratio. Thus, problems such as: “Joe bought 5 sweets. Each sweet costs 3p. How much did he spent?” Or “Joe bought some sweets; each sweet costs 3p. He spent 30p. How many sweets did he buy?” are examples of problems involving multiplicative reasoning. The former can be solved by a multiplication to determine the unknown total cost; the later would be solved by means of a division to determine an unknown quantity, the number of sweets (Nunes & Bryant, 2010a).

Research has been giving evidence that children can solve multiplication and division problems of these kinds even before receiving formal instruction about multiplication and division in school. For that they use the schema of one-to-many correspondence. Carpenter, Ansell, Franke, Fennema and Weisbeck (1993), reported high percentages of success when observing kindergarten children solving multiplicative reasoning problems involving correspondence 2:1, 3:1 and 4:1. Nunes et al. (2005) analysed primary Brazilian school children performance when solving multiplicative reasoning problems. When children were shown a picture with 4 houses and then were asked to solve the problem: “In each house are living 3 puppies. How many puppies are living in the 4 houses altogether?”, 60% of the 1st-graders and above 80% of the children of the other grades succeeded. When children were asked to solve a division problem, such as: “There are 27 sweets to share among three children. The children want to get all the same amount of sweets. How many sweets will each one get?”, the levels of success for 1st-graders was 80% and above that for the other graders (2nd to 4th-graders).

In Portugal, there is still not much information about kindergarten children understanding of multiplicative reasoning, relying on their informal knowledge.

2. Methods

An intervention program was conducted with 12 kindergarten children (5-6-years-old), from a state supported kindergarten in Viseu, Portugal. Pre- and Post-tests were used to identify changes on children's understanding during the intervention program.

2.1 The intervention program

In the intervention program, the participants were divided into three groups of four elements each, having each the same age and pre-test results conditions. The intervention took place in the pre-test following week and lasts for 3 weeks. Four sessions were planned, organized by level of difficulty, equal to all the groups. In each session children solved 3 problems, and the same kind of problems was explored twice a week. Each group had the opportunity to discuss and solve the same type of problem 4 times, in a total of 12 problems. The tasks presented to the children, during the intervention comprised 4 partitive division problems (for example, "Sara has 10 candies to give to 5 children. She is doing it fairly. How many candies is each child receiving?"), 4 multiplication problems (for example, "Bill has 3 boxes with pencils. Each box has 4 pencils. How many pencils does Bill have in total?"), and 4 quotitive division problems (for example, "The teacher Anna has 12 children in her group. She wants to seat the children in groups in the tables. Each group must have 4 children. How many tables does teacher Anna need?"). The problems presented to the children in the intervention program were similar to those of the multiplicative structure problems given in the tests. All the problems were presented to the group of children by the means of a story, and material was available to represent them. After each answer, each child was asked "Why do you think so?" in order to reach a better understanding of his/her reasoning. No judgments were conducted, and group discussion was stimulated. Video recorder and field notes were used in data collection.

2.2 Pre- and Post-tests

Individual interviews were used in the Pre- and Post-tests, in which children solved a battery of 28 problems (18 additive structure problems; 6 multiplicative structure problems; 4 control problems) in two different moments. The problems presented to the children were selected and adapted from the Vergnaud's classification (see Vergnaud, 1982, 1983).

The problems of both tests were similar. The additive structure problems presented to children in the tests comprised: i) composition of two measures, (for example, "Mary has 8 dolls but only 2 are in the box. How many dolls are outside the box?"); ii) transformation liking two measures, with the starting and element of transformation omitted, (2 for addition, 2 for subtraction), (for example, "There are 5 frogs in the lake. Some more join the group. Now there are 8 frogs. How many frogs came to join the group?"); iii) static relation linking two measures, (2 involving "more than", 2 for "less than"), (for example, "Anna has 4 puppies. John has 2 more than Anna. How many puppies does John have?"). The multiplicative structure problems in the tests comprised: iv) Isomorphism of Measures, selecting the problems of Multiplication, Partitive Division, and Quotitive Division. The control problems included only geometry tasks (geometric regularities and shape with tangram).

The problems were presented to the children by the means of a story, and material was available to represent the problems. After each resolution, each child was asked "Why do you think so?" in order to reach a better understanding of his/her reasoning. Data was registered in video and researcher's field notes.

3. Final remarks

This study explores the effects of a short intervention program focused on multiplicative reasoning on young children solving additive and multiplicative structure problems. Results will be presented in the conference. The intervention was effective as children improved their understanding of multiplicative reasoning problems. Multiplication problems revealed to be easier for children than division ones. Also children's arguments revealed improvements. Young children provided arguments and explanations that sustain the idea that their successful resolutions were not obtained randomly, as they were supported by valid or partially valid explanations.

Previous research on kindergarten children solving multiplicative reasoning problems (see Carpenter, et al., 1993) reports levels of success, but does not refer children's explanations or arguments to give a better insight of children's way of thinking. Also Nunes et al. (2005) reports remarkable success levels when 1st-graders solve multiplication and division problems, but give no reference to their explanations. The study reported here gives evidence that young children can reach success levels when solving multiplication and division problems, relying on their informal knowledge, presenting arguments that show that they are able to establish the correct reasoning when solving the tasks, articulating properly all the quantities involved in the given problems.

This study suggests that children's multiplicative reasoning can be enhanced when they can experience problem solving being able to interact with peers and discuss their ideas, after receiving some prompts from teacher. In agreement with Soutinho's (2016) ideas, this study suggests that both additive and multiplicative reasoning, in their simplistic forms, seem to be simultaneously reachable to kindergarten children, making sense to them. Thus, perhaps kindergarten mathematics should include more of these experiences in order to develop children's mathematisation. Educational implications will be discussed.

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