(Ethno)mathematical tasks in the context of proportional reasoning

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Abstract

Investigating mathematical knowledge used in everyday contexts of fishing communities has been our area of interest in mathematics education. Initially we investigated the fishing community of Câmara de Lobos (Madeira - Portugal) and later we also investigated the largest Portuguese fishing community (Caxinas - Vila do Conde in the North of Portugal). The initial purpose was to collect everyday mathematical knowledge of these fishing communities and then contextualize it within the curriculum documents that guide mathematics teaching. A second purpose was the development of mathematical tasks based on the information previously gathered from these fishing communities, for future use in the classroom.

Introduction

This research was developed in order to address some specific aspects which broadly fall into mathematics education, making it relevant to investigate the existing mathematical knowledge in specific cultural contexts, in this case, Portuguese fishing communities; their use in the classroom, seeking to improve the teaching and learning of mathematics in such specific contexts, and improving the educational success of children from these contexts.

The theoretical foundation of this research is centered in ethnomathematics, in its various aspects, but especially in the investigative and educational aspects. Investigative because it seeks to collect / recover mathematical knowledge of cultural daily life of the members of the fishing communities of Câmara de Lobos and Caxinas; educational because it seeks to recover and carry this mathematical knowledge to the classroom, trying to help teachers and improve the quality of mathematics teaching and learning by the consideration of these communities students mathematical reasoning in classroom.

As first step we gathered information in the fishing settings of Câmara de Lobos and Caxinas, with caulkers (in boats building) and with fishermen and their families in their daily lives. The information collected fits the mathematical topic related to proportionality and proportional reasoning as being the one fitting more significantly with the mathematics used in the daily life of members of these communities. In this

context, the methodology fits the qualitative benchmarks focusing on ethnography and multiple case studies. Two cases were considered, constituted by two sets of students: one is a set of students of a fishing community in Caxinas, and the other a set of students of a more urban location away from the shore. Data collection tools included: participant observation, document analysis (boats projects, tasks implemented in the classroom), video recordings, field notes and interviews. The use of these instruments has enabled the collection of information from the fishing communities of Câmara de Lobos and Caxinas. After transcription of caulkers interviews and of information taken from field notes, we selected data which was analyzed and organized descriptively and with a strong interpretive component, as there was an interest on particularizing and understanding the particular, instead of generalizing.

Based on data obtained after long periods of time in the fishing communities settings, a few tasks were developed (in the mathematical umbrella of proportional reasoning), framed in problem solving, validated both by the subjects themselves, but also by a panel of experts in mathematics education and ethnomathematics.

But the main motivation for this research is related to the possibility of passing the everyday context of fishing communities to the classroom teaching of mathematics also exploring the possible influence of school mathematics in students' performance. In a first phase we collected information in the ethno(mathematics) used within the fishing community. In a second phase, and based on the previous information, we constructed mathematical tasks validated both by the elements of the fishing community and by a panel of specialist in mathematics education.

As the data collection was more productive in the techniques used in the construction of boats, a very interesting topic from both a mathematics education and ethnomathematics point of view arose which was proportional reasoning and in special direct proportionality.

Another relevant aspect to the development of this research was the scarcity of ethnomathematics research on fishing communities in the context of proportional reasoning (direct proportionality).

The start point for the present research occurred in 2006, when we finished a comprehensive collection in the fishing community of Câmara de Lobos of mathematical knowledge used in day-to-day, verifying that due to the low education level of this community members, sometimes this everyday knowledge was clearly

beyond the knowledge they had acquired in schools (Author, 2015; Author, 2006; Author, 2008; Author, 2010; Pires, 2008).

Ethnomathematics

In this study we considered appropriate D'Ambrosio's definition of an etymological nature of the word "Ethnomathematics", as the "modes, styles, and techniques (*tics*) of explanation, of understanding, and of coping with the natural and cultural environment (*mathema*) in distinct cultural systems (*ethnos*)" (D'Ambrosio. (1999, p. 146).

Ethnomathematics supports a cultural approach to mathematics, because each culture develops and uses its mathematics. A mathematics education based on a cultural approach will necessarily imply curricular reorientation that includes different mathematical contributions of cultural groups. We are advocating a curriculum that works as a link between educational theory and teaching practice, necessarily considering cultural values, recognizing, valuing and integrating all existing mathematics.

Proportional reasoning

Proportionality is a mathematical theme used by people for several millennia. In nature we can find situations or artifacts involving proportionality, particularly measures and geometric shapes related to the golden ratio.

Proportionality and proportional reasoning are typically fundamental concepts of school mathematics. They allow students to establish connections with other mathematical topics, with other disciplines and with students' daily life (Ponte, 2006), presenting itself as the pinnacle of elementary mathematics and a foundation of advanced mathematics (Lesh, Post & Behr, 1988; Silvestre & Ponte, 2005).

The concept of proportionality seems an easy concept, but in reality proves to be quite difficult and it is estimated that over 90% of adults do not reason proportionally (Lamon, 2007). Still, we must distinguish between proportional reasoning and proportionality. Proportionality involves relationships between magnitudes, may refer to a situation in which there is an invariable relationship between two covariate quantities. Proportional reasoning is used to understand contexts and applications based on proportionality (Lamon, 2005). It implies both an understanding of a constant

relationship between two quantities as well as the perception that these same quantities are related and vary jointly.

In proportionality it is important to understand the concepts of ratio, proportion, as well as their meanings, applications and properties.

The type of tasks used is also an important issue because it defines the approach in the context of proportional reasoning, but also the strategies to be used. Vergnaud (1983, 2014) proposed the theory of conceptual fields based on multiplication and division. He proposes three multiplicative classes: measures isomorphism, product of measures and multiple proportion. Other types of tasks can be offered to students such as: problems of missing value, comparison problems, transformation problems, average value problems, problems involving the conversion between ratio, rate and fractions, problems involving measurement units, problems of conversion between representation systems and pseudo proportional problems.

In this work we have investigated the mathematics used in the daily life of the members of fishing communities. We have verified that they regularly use mathematical knowledge involving proportional reasoning. Given this information, it is pertinent to understand how children of one of these communities would react in a classroom situation, to proportional reasoning problems.

Methodology

Research has been of qualitative type, as we tried to investigate the problem without isolating it from its cultural context, seeking to understand using impartial, inductive and descriptive processes (Amado, 2013).

Our view is based on the assumptions of the interpretative paradigm, defending the subjective study of human experience and in this study trying to approach it from the standpoint of the members of the fishing community (Sarmento, 2003; Cohen & Manion, 2002; Ramos & Naranjo, 2014), in which the interest is to particularize instead of generalizing (Vilela, 2009).

The research designs used in this study are ethnography and case study. Ethnography is used in the first phase of data collection where the researcher remained within the fishing communities of Câmara de Lobos and Caxinas for considerable periods of time, collecting faithfully information of the everyday life of these communities, especially in the context of building boats. In the second phase we used multiple case studies where the researcher, in the context of the classroom implemented

mathematical tasks, intentionally and carefully prepared as to being similar to real experiences in a fishing community. The set of tasks was introduced in classrooms of two schools in different cultural settings. In one school all the tasks were introduced to a group of students all from a fishing community, while in the other the students were from a more urban and far from the shore.

In the fishing community of Câmara de Lobos the entrance on the ground has proved difficult, as it is a community somewhat closed in itself. In the fishing community of Caxinas entry into the community was easier and faster. Two important rules were followed: a) the consent of the people being researched; b) their protection from any damages (Bodgan & Biklen, 1994).

The fishing community of Câmara de Lobos and its territory is a landmark of the county, for its picturesque aspect, people's strong personality and the beautiful bay where fishermen repair their boats and prepare fishing material. They live and interact "isolated" from the surrounding society. They have acquired over time a way of thinking and acting, a way of life and a culture that characterizes as unique. Although disparaged by the remaining society, the fishermen neighborhood dominates the village and fishermen are the majority. They are characterized by their shyness, they are not very expansive but sure are very supportive one to the other.

Caxinas is a major population center to the north of Vila do Conde, which concentrates the largest fishing community in Portugal. The place where now stands Caxinas began to be occupied by some fishermen in 1840. Until the 70s they lived in very precarious situation, and like other fishing communities, were for decades socially discriminated, despised and stigmatized. They have developed a cultural identity, marked by a particular slang and behavior patterns, showing however some approximation to the other folks of the town, the result of schooling.

This research emphasizes the socio-cultural component of the fishing communities which meets the essence of ethnography. Based on Ethnomathematics, we tried (in the first phase) to collect accurate information to describe in detail the mathematical knowledge of the participants.

Data collection involves a close contact between researcher and researched and the use of various techniques that enable obtaining relevant information. We used participant observation, unstructured interviews, document analysis and video recording. It was participant observation in the strict sense that in the classroom the

researcher was having contact with the groups working, questioning their reasoning and ways of thinking.

Regarding data analysis, it was carried out to compare information provided by various sources, presented descriptively and with a strong interpretive component.

We have constructed some tasks, two of them will be presented below.

Task 1:

Task Type: Problem of inverse proportionality.

The "God provides" travels the distance between the fishing port of Sesimbra and the fishing port of Matosinhos in 8 hours at 25 miles per hour. How long will it take to make the same trip if the average speed was 16 miles per hour?

Description and development of the task

The task was presented to students through a short statement, and all necessary information is presented in the statement itself. There is no image, scheme or tip that could "help" students to interpret and induce a resolution strategy. Portrays a fishermen's common situation, which in addition to requiring mathematical calculations, also imposes a proportional reasoning somewhat different in its resolution.

The task is part of the theme proportionality, however involves a proportional reasoning that requires from the students a careful analysis of the information provided, also involves knowledge of inverse proportionality. Proportionality is worked on in schools, but the focus is completely in direct proportionality. Still and since proportional reasoning is not synonymous with proportionality (Lamon, 2005), we think that students may be able to develop strategies to answer this problem. It may also be a way to ascertain students' informal knowledge especially of those coming from the fishing communities.

Students, organized in groups of three to four elements, enabling discussion of ideas and reasoning, would have to make a careful reading so that they could correctly interpret the information.

This task does not require only the use of relations and reasoning. In other tasks there was a direct proportionality relation, in that as a magnitude increases, the other magnitude increases in proportion. In this case also there is a proportional relation, but as the speed increases, the time decreases. Apparently it seems a problem that goes beyond the mathematical knowledge of students and therefore they will not have enough tools to solve it. However, if students develop proportional reasoning in order to seek an intermediate magnitude (distance between the two sea ports), the problem

becomes a problem of direct proportionality. Students must reason as follows based on the information provided in the statement: the boat travels in one hour 25 miles. How many miles will travel in 8 hours? Using the strategy that they deem most appropriate, they can conclude that in eight hours it will travel 200 miles, corresponding to the distance between the two ports. Once this intermediate magnitude is found, this is a direct proportion problem with which students are more familiar. Applying the cross product strategy or the "rule of three", they can calculate easily and quickly the time needed to travel the same distance at an average speed of 16 miles per hour. Of course, students may choose other strategies with which they identify better or in alternative they think to be more effective. Still, it is expected that students show their proportional reasoning to establish a proportional relationship between the magnitudes involved mainly multiplicative relationships within and between variables. The estimated time for the resolution of this task was 50 minutes.

Task 2:

In everyday life, with some frequency, come up situations that seem to be one thing, but, well analyzed, are not what at first seemed to us. In the context of direct proportionality, there are real-life situations that appear to involve direct proportionality but in reality this is not the case.

The task presented below is one such case, and may be suitable for students distinguish real situations involving relations of direct proportionality from other situations that are not of direct proportionality, and are hereby designated as pseudo proportional problems.

Task type: pseudo proportional problem.

When a sardine is born, its length is 1.3 cm. With 12 months of age its length is 14 cm. What will be the length of a sardine with 48 months?

Description and development of the task

The scenario presented in this task is a usual situation of everyday fishing, in which the results, denounce the situation and brings immediately the perception that it is not a situation of direct proportionality. Especially for the students of the fishing community of Caxinas it is expected that this happens, as it is not reasonable to have a sardine 56 cm long when 4 years old. Besides being rather strange to see such a large sardine, students of the fishing community should be aware that a sardine has an age limit not exceeding four years and a measure of length not exceeding 21 centimeters (Coelho, 2009; Castello, 2015).

The task is presented in a simple way, with a small statement, little information and numbers which are easy to operate with. In fact, they are small numbers and with all the conditions to be operated through mental arithmetic.

The most important part of the task is supposed to be the justification of the answer.

This task has been purposely prepared to determine if students are able to distinguish situations of direct proportionality from situations in which there is no such relationship.

Students in small groups of three or four elements, should resolve this problem and justify the answer they give in an approximate time of 40 minutes. Each group will have to analyze the problem, select strategies and discuss ideas related to the answer and respective justification.

It is expected that students foresee that the situation proposed does not fit in a direct proportionality situation.

It is also expected that they are capable of presenting valid justifications with strong arguments. It will be a good sign that the knowledge they have on direct proportionality, are properly mobilized and adjusted to the degree of difficulty of the task.

As referred before, these tasks, focusing on proportional reasoning used in the daily activities of the fishing community, display some of the mathematics the fishermen and the remaining members of the community use (chores, boat construction, daily fishing on land, etc.). As the students of one of the schools are from a fishing community we expected that perhaps their performance in these tasks matched their adult family members. If true, then before being taught proportionality, one could reasonably expect that these students would perform better than the students of the other school away from shore.

Conclusion

Throughout the research it was important to collect information relating to some fishing communities' cultural contexts, trying to realize what mathematical skills are used in the everyday life of these communities. The comprehensive collection of information on the ground, gave us ground to realize that proportional reasoning is used in both fishing communities and is very important in their day-to-day.

The primary interest of this research was focused on mathematics education, particularly in classroom work (There are no strong evidences of the connections of

ethnomathematics to classroom work). Thus, research was directed accordingly, producing significant mathematical tasks for students of the fishing communities, but significant also from a mathematical point of view. We consider them relevant tasks because: (a) their construction took into account the cultural background of students; (b) they fit into curriculum documents, involving a mathematical topic considered of immense importance in the mathematical development of students; (c) these are tasks that by their nature (problem solving) promote the development of reasoning and the ability to develop mathematical skills necessary for students' daily life; d) these are tasks validated by the scientific community and by those investigated and therefore can be considered good tasks for teachers to use in classroom considering they were constructed in accordance with the common mathematical knowledge of the members of the fishing community, that they use on their daily activity.

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