A new perspective for robustness assessment of framed structures

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Abstract. Robustness has been recognized as interesting research topic due to several collapses that have been occurring over last years. Indeed, this subject is related with global failure or collapse. However, its definition is not consensual since several definitions have been proposed in the literature. This short-paper aims to present a framework for assessing bridge's robustness as a probabilistic performance indicator. In this study, a non-linear model of a clamped beam with two point loads using DIANA software was developed to validate the framework presented. By means of a probabilistic approach, the load carrying capacity and structural safety were evaluated. In this regard, special focus is placed on an adaptive Monte Carlo simulation procedure to achieve a proper meta-model.

Keywords: Robustness, Probabilistic Techniques, Non-linear Analysis, Performance Indicator, Structural Safety.

1 Introduction

The concept of structural robustness received significant attention around 40 years ago due to the partial collapse of Ronan Point building in London. This subject began to be seriously studied after the massive disaster of World Trade Centre collapse. In addition, several structural failures triggered by unexpected loads, severe human errors during design or execution and lack of maintenance contributed to this increased interest in this topic(Canisius et al., 2007). In this context, a workshop carried by JCSS in collaboration with IABSE at the Building Research Establishment in London, UK (December 2005) gathered 50 experts, from research institutions, companies and government, to discuss issues related with robustness. The conclusions led to a general consensus that the present situation with regard to ensuring sufficient structural robustness through codes and standards was highly unsatisfactorily. As a consequence, a joint European project in Robustness was created, namely the COST action TU06010 – Robustness of Structures.

The present work aims to develop a reliability-based robustness assessment framework to evaluate bridge's safety. In this way, a non-linear finite element model (FEM) combined with advanced reliability methods was used in order to validate the proposed framework.

2 Robustness

In general, robustness can be defined as the ability of a certain structure to resist without disproportionate damage to either abnormal events or given damage. However, it is well known that there are several definitions of robustness proposed by several authors over the literature. Starossek and Haberland (Starossek & Haberland, 2010) in their work present several definitions of robustness in civil engineering domain. The same authors also discuss several terms related with robustness, such as:

- *Exposure* – possibility of a structure to be affected by a threat during its life-cycle;
- *Vulnerability* – susceptibility of a structure to be damaged by an exposure;
- *Damage tolerance* – ability of a structure to survive once it is damaged;
- *Redundancy* – availability of alternative paths for a load to be transferred from a point of application to a point of resistance;

- *Ductility* – ability of a structure to suffer plastic deformations without occurring rupture;
- *Reliability* – ability of a structure to perform its intended function for a specific period of time under certain conditions.

Regarding the quantification of robustness, they have been proposed several approaches by different researchers that evaluates the robustness in a deterministic, probabilistic and risk-based way. Concerning the deterministic approach, the most relevant works are presented by Frangopol and Curley,1987 (Frangopol & Curley, 1987), Biondini and Restelli, 2008 (Biondini & Restelli, 2008), Starossek and Haberland,2011 (Starossek & Haberland, 2011) and Cavaco,2013 (Cavaco, 2013). In what concerns the probabilistic approach, the most relevant works are presented by Frangopol and Curley,1987 (Frangopol & Curley, 1987), Fu and Frangopol,1990 (Fu & Frangopol, 1990) Lind,1995 (Lind, 1995) and Goshn and Moses,1998 (Ghosn & Moses, 1998). Lastly, in risk-based approach the most relevant work can be consulted in Baker et al.,2008 (Baker et al., 2008).

3 Proposed framework

Despite this intense effort of the research community, both structural reliability analysis and robustness assessment require a comprehensive understanding of crucial topics, hindering their practical application in real situations. Indeed, the most complete approach, namely, the risk-based robustness, usually overtakes the structural engineers scope. Besides that, ranges of existing robustness indexes still need to be normalized from 0 to 1, facilitating comprehension and comparison. In this sense, herein, a reliability-based robustness assessment framework is introduced, seeking to combine the existing knowledge, in order to obtain a new robustness index to be applied at two performance levels: structural behavior at ultimate or service limit states.

The proposed robustness index aims to depict the structural performance by assessing a selection of four key attributes traditionally related with robustness. In this approach, robustness is computed as equal to the area of a quadrilateral, whose sides' lengths represent a performance indicator according to Table 1. In order to obtain these indicator, deterministic analysis on design points are carried out.

Table 1 – Adopted performance indicators.

Attribute	Performance Indicator	Reasoning
Reliability	$P_\beta = \dfrac{\beta_{dam}}{\beta_{\text{int}}}$	Reliability indexes
Damage tolerance	$P_{Dt} = \dfrac{LF_{dam}}{LF_{\text{int}}}$	Load factors
Redundancy	$P_R = \dfrac{\int M(\phi)_{dam}}{\int M(\phi)_{\text{int}}}$	Moment curvature areas
Ductility	$P_\phi = \dfrac{\phi_u/\phi_y\big\vert_{dam}}{\phi_u/\phi_y\big\vert_{\text{int}}}$	Flexural curvature ductility factor

With regard to structural reliability, since the expected probability of failure is low, crude Monte Carlo requires a large number of numerical simulations in order to solve the convolution integral. To tackle this, the performance limit function is approximated by the so-called meta-models, namely, quadratic response surfaces, polynomial chaos, and so on. Herein, quadratic response surfaces (RS), which are able to efficiently cope with highly non-linear relations between inputs and outputs, are used.

To do so, an adaptive procedure based on Monte Carlo realizations inspired on schemes proposed by Bucher and Bourgund (Bucher & Bourgund, 1990) and also Rajashekhar and Ellingwood (Rajashekhar & Ellingwood, 1993) is accomplished. In this approach, a stepwise regression, which combines forward and backward regression methods to select the most important terms according to their statistical significance, is used to minimize the approximation error. This RS is built based on an initial experimental design (ED), a Monte Carlo sample, whose realizations are dispersed around the mean value according to their bias. Both design point coordinates and probability of failure are computed through the first reliability method (FORM). Regarding the following steps, new sampling points are added to enrich the ED around the design point. The procedure is stopped when a convergence criterion is satisfied, which is based on reliability index relative error tolerance between consecutives iterations. In this procedure, the limit state function can be defined according to problem definition. Herein, a performance limit function based on the difference of resisting and acting loads, $G(X) = R(X) - S(X)$, is highlighted.

4 Case Study

The present case study aims to assess the safety of a clamped beam as it can be seen in figure 1a, longitudinal view, and figure 1b, cross section. This beam was designed according with Eurocode 2 for an F_{sd} of 27kN. The reinforcing was performed in order that, in yielding state, the bending moment in support could redistribute the loads to the mid-span in order to equalize the bending moments in an ultimate limit state.

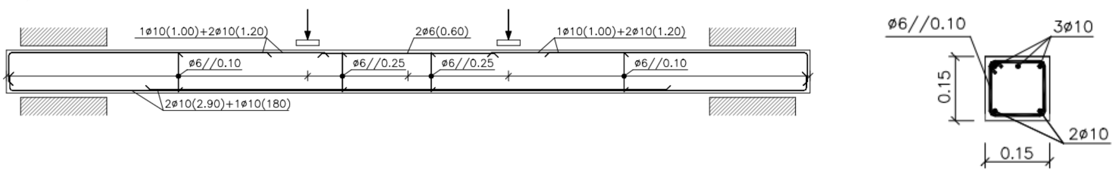

Fig. 1 – Clamped Beam:
a – longitudinal view; b – Cross section

Concerning its analysis, a non-linear finite element analysis were made through DIANA software. About the type of analysis, a 2D non-linear structural analysis was performed with class III beam elements based on Mindlin-Reissner theory with incremental load steps until its failure. The adopted method to solve the non-linear problem was the Modified Newton-Raphson method.

Regarding the definition of the constitutive laws for the materials, for concrete, a total strain fixed crack model was adopted in which for tensile behavior a linear ultimate strain based was used and an ideal behavior for compression. For the reinforcing steel, a tri-linear diagram was carried out.

The probabilistic values for the mechanical properties of the materials and applied loads are presented in table 2 as well as their mean values, coefficient of variation (CoV) and distribution functions.

Table 2 – Material Properties and applied Loads.

Random Variable	Mean Value	CoV	Distribution Function	Reference
Material Properties				
Compressive strength (fc)	30 MPa	12%	Normal	(Wiśniewski, 2007)
Tensile strength (f_{ct})	2.9 MPa	20%	Log-Normal	(Wiśniewski, 2007, EN CEN 1992, 2010)
Young modulus (E_c)	32 GPa	8%	Normal	(Wiśniewski, 2007)
Steel yielding strength (f_{sy})	460 MPa	6.5 %	Normal	(JCSS, 2001)
Steel ultimate strength (f_{su})	530 MPa	7.5%	Normal	(JCSS, 2001)

Applied Loads				
Permanent load (G)	10 kN	9.5%	Normal	(Wiśniewski, 2007, JCSS, 2001)
Additional load (Q)	9 kN	15%	Gumbel	(JCSS, 2001)

4.1 Damage Scenarios

Both idealized damage scenarios are formulated assuming a degradation of reinforcing steel cross-section area. Knowing that beam is designed to redistribute bending moments between critical cross sections, the main goal is to analyze the ability of forming plastic hinges. Indeed, according to deterministic analysis, beam presents a ductile behavior since rupture is ruled by steel yielding. The restrained cross-sections evidence a moment-curvature diagram with well-defined losses of stiffness. Since the structure does not present fragile ruptures, namely, a single plastic hinge, two scenarios involving a reduction of steel cross section are assumed. The first appoints to general degradation phenomena with a percentage of loss near 25%. A localized reduction of steel cross section area up to 40% regarding top layers at beams ends is also considered.

4.2 Obtained Results

The adaptive Monte Carlo procedure used to achieve a quadratic response surface considered an initial sample N equal to $N = 3 \cdot M$ with M input random variables. For further iterations, the same sample size is added. A MATLAB built-in function, stepwiselm, is used to select potential model terms according to different criteria (eg. sum of squared errors, AIC, BIC,...). Finally, the best model is chosen based on log-likelihood value. Both simulations converged quite rapidly due to the existence of well-defined failure mode. In fact, after four iterations the RS presented interesting approximation errors in which engineering reasoning validate mathematical models.

In the following, deterministic analysis of design values for intact and damage scenarios are presented. Design points coordinates, reliability index, load factor are shown in Table 3. Displacement at mid-span is schematically presented in figure 2. Herein, three different phases regarding structural performance can be distinguished, namely, initial elastic phase, cracking phase and plastification of steel, i.e. yielding phase. Indeed, this behavior is well depicted in the moment-curvature diagram at beam end shown in figure 3.

Table 3 – Results for intact and damage scenarios.

	int.	dam. 1	dam. 2
fc	16.3	16.2	15.6
fct	1.7	1.7	1.6
Ec	23.3	23.2	22.8
fsy	371.7	398.0	403.5
fsu	399.7	436.8	444.0
G	11.2	11.2	11.2
Q	27.3	23.2	21.5
β	8.78	7.83	7.58
LF	38.5	34.4	32.5
ϕ_y	0.016	0.014	0.012

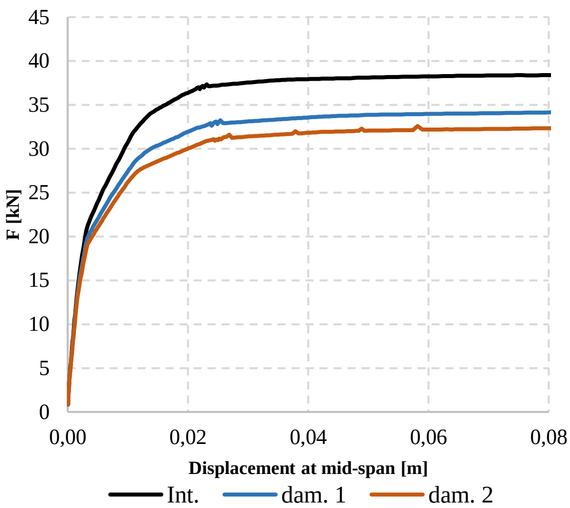

Fig. 2 – P-delta curve

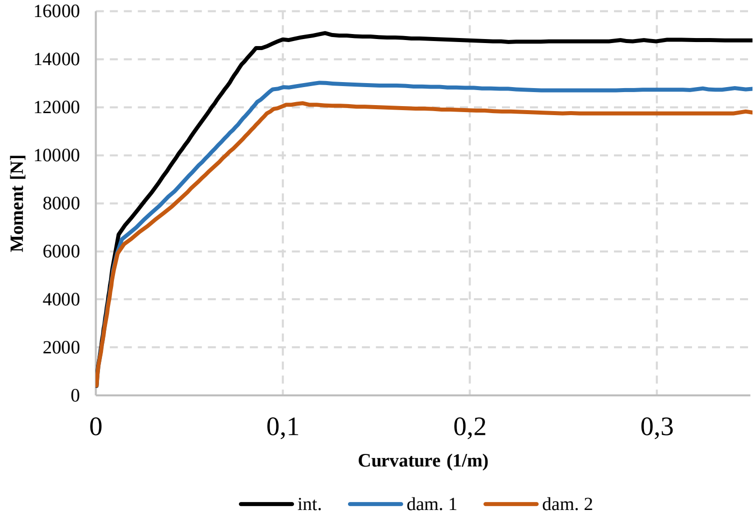

Fig. 3 – Moment curvature diagram

According to the proposed methodology, robustness index is given by the area of quadrilaterals which are schematically represented in figure 4. Althought both scenarios led to similar reliability indexes, the robustness indicator is worsen by the reduction of ductility and redundancy. However, a high robustness indicator is achived in both cases, since this structure has the ability of redistribuiting forces, specially due to small cross-section heigth and good ratio of steel/concrete area.

Table 4 – Robustness Assessment

P_β	0.892	0.863
P_{Dt}	0.892	0.845
P_R	0.775	0.627
P_ϕ	0.896	0.748
Robustness	0.74	0.58

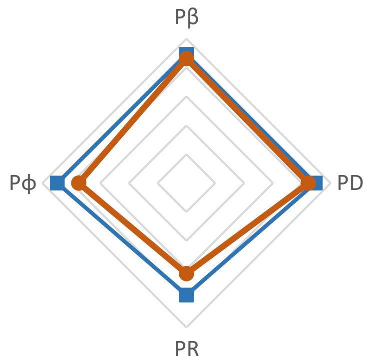

Fig. 4 – Performance Indicators in robustness assessment

5 Conclusions

A reliability-based robustness assessment framework to evaluate bridge's safety is introduced. Herein, a simple example concerning a clamped beam with two point loads is used to validate the proposed methodology in order to extent its application to a real bridge. Indeed, this paper presents some preliminary studies concerning reliability analysis and robustness assessments. The main goal is to facilitate the understanding of some attributes regarding robustness, aiming to propose a versatile framework to evaluate robustness according to a choice of key performance indicators. The methodology seeks not only to obtain a normalized robustness index but also to visualize the influence of different attributes. Regarding reliability analysis, used approach intends to reduce computational time and also to reproduce an explicit limit state function avoiding overfitting and diminishing approximation error. In fact, this methodology can be improved by introducing some features: i) use of pseudo random-generators to populate region of failure; ii) establishing cross-validation procedures; iii) considering model error as random variable; iv) bootstrap sampling to estimate boundaries of probability of failure. Finally, the application of these framework with additional improvements is to be applied in a near future.

Acknowledgements

The authors would like to thank ISISE – Institute for Sustainability and Innovation in Structural Engineering (PEst-C/ECI/UI4029/2011 FCOM-01-0124-FEDER-022681), FCT– Portuguese Scientific Foundation for the research grant PD/BD/113677/2015. This study also received funding from the European Union's Seventh Framework Programme for research, technological development and demonstration under grant agreement No. 606229. Also the collaboration and information provided by the professor António Abel Henriques is gratefully acknowledged.

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