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On inflation and money demand in a portfolio model with shopping costs

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Abstract:

In this paper, we investigate the conditions under which expected inflation might influence the money demand, using a microeconomic model where the transactions of the representative agent are facilitated by its holdings of money. We assume that the agent holds a real asset, along with a range of nominal assets, that may include domestic money, foreign money, domestic bonds and foreign bonds. In this model, the optimal choice between money and bonds is embedded in a portfolio choice between the real asset and risky assets (the Merton problem). We show that, as long as the agent is not constrained in her holdings of bonds, the demand for domestic money will not, in general, depend on expected inflation. The demand for money may however become a *positive* function of the inflation rate in case the agent is constrained in her holdings of foreign bonds. The only case in which the demand for domestic money may depend negatively on the inflation rate is when the agent faces a binding constraint in her holdings of domestic bonds.

JEL Classification: E41, F41, G11.

Keywords: Money Demand, Currency Substitution, Portfolio Theory.

1. Introduction

A usual procedure in empirical models of money demand is to specify the inflation rate in the set of explanatory variables. This procedure is not controversial, when the inflation rate appears instead of the nominal interest rate in the money demand equation. This will be the natural thing to do, for instance, when estimating the money demand in economic environments characterized by financial underdevelopment or by financial repression: if individuals are not given the opportunity to buy interest-bearing bonds, or in case interest rates in domestic securities are administratively set at below-market levels, then the relevant opportunity cost of money may turn out to be a real asset. The same applies to episodes of hyperinflation, when the inflation rate becomes so high that dwarfs the real interest rate inside the Fisher relationship. In both cases, expected inflation replaces the nominal interest rate as an argument in the demand for domestic money.

More controversial is when both the inflation rate and the nominal interest rate are included as arguments in the money demand function. This procedure characterizes the so-called portfolio-balance approach to money demand, which roots lie in the works of Milton Friedman and James Tobin (see, for instance, Friedman, 1956, Tobin, 1958, 1969). The portfolio approach focuses on the store of value role of money. In light of this approach, money is modelled as an asset, without any particular feature that makes it distinguishable from other assets. In many applications of the portfolio model, money is postulated to be gross substitute of all other assets, giving rise to money demand functions that depend positively on income and wealth, and negatively on the return of each alternative asset. This includes the nominal interest rate (capturing substitutability between money and bonds) and the inflation rate (capturing substitutability between money and real assets). A recent article in this tradition, that has inspired various empirical studies focusing on the euro area money demand, is Ericsson, 1998)[1].

A problem with the Portfolio Balance Approach is that it is not capable of explaining why money is held in the portfolio despite being dominated by assets that, in the words of Barro and Fisher (1976, p. 139), "have precisely the same risk characteristics as money and yield higher returns". This criticism underlies a number of theoretical models that attempted to account for the means of payment role of money and integrate it into the theory of asset demands.

Attempts to account for the means of payment role of money include models where real money balances are specified as an argument in the consumer utility function (Sidrausky, 1967), and models assuming that holding money allows consumers to save in transaction (or "shopping") costs (Saving , 1971)[2] [3]. Both models give rise to optimal money demands that obey to a trade off between the benefits of holding a means of payment and the cost of a foregone interest, typically

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[1] In the literature trying to identify a stable money demand relationship in the euro area, authors that accounted for a possible role for inflation as opportunity cost of holding money include Fase and Winder (1998), Coenen and Vega (2001), and more recently, Dreger and Wolters (2010).

[2] A related approach is to postulate "cash in advance constraints", whereby individual' purchases each period cannot exceed the quantity of money being held (Clower, 1961). As pointed out by McCallum and Goodfriend (1988), the deterministic version of the cash-in-advance model can be interpreted as a special case of the "shopping-costs" model, with the relationship between money and transactions being linear, in contrast to the more general formulation where any volume of transactions can be undertaken with a given amount of money, though at increasing transaction costs.

[3] A completely different avenue is to address the essence of money, modelling the matching game between buyers and sellers (Kiyotaki and Right, 1989). In this paper, we abstract from the fundamentals of transaction services, to focus on the simpler case in which the transactions demand for money is implied by an ad hoc "shopping costs" function.

on a domestic bond (see, for instance, Barnett, 1978, McCallum and Goodfriend, 1988)[4].

Attempts to integrate the shopping costs model into the theory of asset demands include Branson and Henderson (1985) and Thomas (1985). These authors demonstrated that, as long as individuals have unrestricted access to interest-bearing nominal assets (or liabilities), they will be able to hedge the risk implied by their holdings of like-denominated monetary assets. In that case, money demands will be independent of portfolio decisions. In Branson and Henderson (1985), domestic money is the sole means of payment, so there is a unique opportunity cost of holding money, which is the nominal interest rate in the domestic bond. In Thomas (1985) both domestic and foreign money provide liquidity services, so the choice between these two means of payment involves a comparison between the respective marginal productivities in the production of liquidity services and holding costs (the domestic and the foreign interest rates, respectively). In any case, money demands are independent of portfolio decisions.

The assumption of complete bond markets is obviously a strong one. In many countries, common citizens have access to dollar banknotes, or even to bank deposits denominated in a foreign currency, but they hardly consider long term bonds denominated in foreign currency in the range of possible applications. Along this reasoning, Cuddington (1989) argued that, in case perfect capital mobility does not hold, the demand for money should reflect both a transactions and a portfolio component. Lebre de Freitas and Veiga (2006) explored this avenue, extending the Thomas (1985) model to the case in which the agent faces a binding constraint in her holdings of foreign bonds. The authors found that in this case the demand for domestic money may be indeed influenced by portfolio decisions, but only in case foreign money competes with the domestic money as means of payment.

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[4] As demonstrated by Feenstra (1986), under very general conditions, specifying real money balances as an argument in the utility function or as an argument of a transaction costs function appearing in the budget constraint leads to money demand functions that are functionally equivalent.

A limitation in Lebre de Freitas and Veiga (2006) is that the authors only accounted for the possibility of the representative agent investing in nominal assets. In the real world, however, people are given the opportunity to allocate part of their wealth to assets that offer some protection against the inflation risk. This includes, for instance, real state and bonds with interest rates being adjusted on a regular basis according to some specified index. In episodes of very high inflation, people are often given the opportunity to invest in assets that are fully indexed to the inflation rate[5]. To the extent that agents have the opportunity to hold assets that hedge the inflation risk, a question arises as to whether, in case the demand for money becomes influenced by portfolio considerations, it becomes influenced by the inflation rate too.

In this paper, we extend Thomas (1985) and Lebre de Freitas and Veiga (2006), by investigating the properties of the optimal demand for money in the presence of a an asset offering a certain real return. We use an optimizing model where money reduces the frictional losses from transacting in the goods market. In this model, the inflation rate is random, so holdings nominal assets involves a risk. The model accounts for both domestic and foreign money, as well as for domestic and foreign bonds. The optimal demand for money is therefore embedded in a portfolio choice between the safe asset and risky assets. In this framework, we are able to distinguish three types of decisions concerning the asset composition of the agent's real wealth: speculation, which refers to the allocation of part of an agent' wealth away from the safe asset towards nominal (monetary and non-monetary) assets, in exchange for higher returns (the Merton problem)[6]; Asset Substitution, which refers to the switching from nominal assets denominated in domestic currency to nominal assets denominated in foreign currency[7]; and Currency Substitution,

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[5] A well known example is Brazil during the high inflation episodes. At that time, different forms of indexation spread across the economy, including in wages, rents and financial securities. Government bonds indexed to the inflation rate were instituted along 1964-1991 (see, for instance, Goldfajn, 1998).

[6] Merton (1969).

[7] The international investor portfolio choice, in Branson and Henderson (1985). Sahay and Végh (1996) label this as "dollarization".

which refers to the substitution of domestic money by foreign money as means of payment. The paper compares alternative scenarios regarding the availability of bonds, but in all scenarios the individual is allowed to hold an asset paying a certain real return.

In the more general case where all assets are available, the separation between money demands and portfolio decisions applies. In that case, the demand for domestic money does not depend on the inflation rate. When, in alternative, the agent faces a binding constraint in its holdings of foreign bonds, foreign money gets a store of value role, in addition to its eventual means of payment role (Lebre de Freitas and Veiga, 2006). In this case, means of payment substitutability opens a channel through which the demand for domestic money may be influenced by the relative yields of the different assets, including the inflation rate. The surprising result in this case, is that the eventual impact of expected inflation in the demand for domestic money will *positive*, rather than negative, as usually assumed. The intuition is as follows: suppose the individual holds a bank account denominated in foreign currency, along with a bank account denominated in domestic currency, a domestic bond paying a certain nominal return (say, a long term government bond) and an asset paying a certain real return (say, real state). If, everything else constant, the expected inflation decreases, the individual will reallocate wealth away from the real asset to nominal bonds and foreign currency deposits. In case foreign currency deposits are liquid enough to substitute for domestic money in the provision of liquidity services, the fact that the individual holds more of these deposits allows her to save on domestic currency deposits, which holdings involve an opportunity cost. All in all, the fall in inflation rate caused a decline in the demand for domestic money - hence, the positive relationship. Of course, the arguments presumes that the inflation rate declines while the expected exchange rate depreciation remains constant. In case prices and the exchange rate move exactly together - as it tends be the case in episodes of very high inflation – then the inflation rate does not influence the demands for foreign and domestic money. This is demonstrated in the analysis below.

As a second exercise, we restrict further the range of available assets, by imposing a binding constraint on the holdings of domestic bonds. Since in this case domestic money is not dominated by an interest-bearing asset, its' demand will be influenced by risk-return considerations, as well as by transaction motives. In this setup, the inflation rate arises as the relevant opportunity cost of holding domestic money. Strictly speaking, this does not assure, however, a negative relationship between money demand and inflation: as long as the return on foreign money is not perfectly correlated with inflation, the mechanism described above through which the demand for domestic money may increase with the inflation rate is still in operation. In this case, however, this mechanism is mitigated by the fact that inflation is the opportunity cost of holding money. Therefore, on balance, the sign of the inflation-money demand relationship is uncertain. In order to obtain an unambiguous negative relationship between money demand and inflation in the context in which the agent is constrained in the holdings of domestic and foreign bonds, one has to impose further restrictions in the model' parameters.

The paper proceeds as follows: The general model with 5 assets is presented in Section 2. In Section 3, we solve for the optimal money demand in the case with complete bond markets. The case in which the agent faces a binding restriction in her holdings of foreign bonds is examined in Section 4. In Section 5, we further restrict the agent' options, by imposing a binding constraint on her holdings of domestic bonds. Section 6 concludes.

2. The basic model

Consider an infinitely lived consumer, living in a small open economy. There is one consumption good only, which domestic price is equal to P. The consumer is endowed with a constant flow of the good, denoted by y. She maximises the expected value of a discounted sum of instantaneous utility functions of the form:

$$\mathrm{E}\int_{o}^{\infty} e^{-\beta t}\, \frac{c_t^{1-\phi}}{1-\phi} dt \,, \tag{1}$$

where c_t denotes real consumption at time t, β is a positive and constant subjective discount rate, and $\phi > 0$ is the Arrow-Pratt measure of relative risk aversion.

The individual has unrestricted access to domestic money (M), foreign money (F) and a real, safe asset (S). Bonds denominated in domestic currency (A) and in foreign currency (B) may or may not be freely available, depending on the institutional framework under consideration. Among these assets, only domestic money and foreign money are assumed to be liquid enough to provide transaction services.

The individual's real wealth is defined as:

$$w = m + f + a + b + s, \tag{2}$$

where $m = M/P$, $f = EF/P$, $a = A/P$, $b = EB/P$, $s = S/P$, P is the domestic price level, and E is the exchange rate.

Money holdings earn zero nominal returns. Domestic and foreign bonds have certain nominal returns, represented by i and j, respectively.

$$\frac{dA}{A} = idt$$

$$\frac{dB}{B} = jdt$$

Holding nominal assets is risky because prices and the exchange rate evolve stochastically, altering their real value. We postulate the following stochastic processes for prices and for the exchange rate:

$$\frac{dP}{P} = \pi dt + \sigma dZ, \tag{3}$$

and

$$\frac{dE}{E} = \varepsilon dt + \gamma dX, \tag{4}$$

where dZ and dX are standard Wiener processes. The instantaneous correlation between the two stochastic processes is given by $R = (dZ.dX)/dt = \rho/\sigma\gamma$, where ρ is the covariance.

In light with the theory of purchasing power parity, the exchange rate depreciation is expected to be positively correlated with the inflation rate. However, in the real world, this correlation is not in general perfect due to real shocks. Thus, in our baseline scenario, we assume that 0<R<1. Notwithstanding, in the discussion that follows we will also consider the extreme cases in which R=0 and R=1[8].

Using Ito's lemma, the real returns to domestic bonds, domestic money, foreign bonds and foreign money are as follows:

$$\frac{da}{a} = \left(i + \sigma^2 - \pi\right)dt - \sigma dZ = r_a dt - \sigma dZ\,, \tag{5}$$

$$\frac{dm}{m} = \left(\sigma^2 - \pi\right)dt - \sigma dZ = r_m dt - \sigma dZ\ , \tag{6}$$

$$\frac{db}{b} = \left(j + \varepsilon + \sigma^2 - \pi - \rho\right)dt - \sigma dZ + \gamma dX = r_b dt - \sigma dZ + \gamma dX\,, \tag{7}$$

$$\frac{df}{f} = \left(\varepsilon + \sigma^2 - \pi - \rho\right)dt - \sigma dZ + \gamma dX = r_f dt - \sigma dZ + \gamma dX\,. \tag{8}$$

The real return on the safe asset is:

$$\frac{ds}{s} = rdt \tag{9}$$

Purchases of the consumption good are assumed to imply a transaction cost (τ), that depends positively on consumption expenditures (c) and negatively on real money holdings, according to the following functional form:

$$\tau = cv\left[\frac{m}{c}, \frac{f}{c}\right], \tag{10}$$

with $v(.) > 0$, $v_k < 0$, $v_{kk} > v_{12} \geq 0$, *and* $\Delta = v_{11}v_{22} - v_{12}^2 > 0$, *k=1,2*. In (10), τ refers to the amount of real resources spent in transacting, and a subscript *k (k=1,2)* to the function *v(.)* denotes partial differentiation with respect to the *k* argument.

The fact that foreign money provides liquidity services does not imply that it can substitute the domestic currency as means of payment. Means of payment substitutability occurs when some fraction of the consumption bundle can be

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[8] In case the exchange rate and the inflation rate are perfectly correlated, the foreign bond *B* and the real asset become perfect substitutes. Under such specification, foreign money, *F*, can be interpreted as an indexed means of payment (for instance, overnight deposits paying an interest rate that is fully indexed to the inflation rate).

purchased with money denominated in either currency, so that v_{12} is strictly positive. In this paper, we postulate a weak form of means of payment substitutability, whereby the marginal productivity of each money is more impacted by changes in the holdings of that money than by changes in the holdings of the competing money[9].

The consumer's flow budget constraint is determined by real returns and saving decisions:

$$dw = dm + df + da + db + dr + \left(y - c\left[1 + v(.)\right]\right)dt$$

Using (5)-(9), this becomes:

$$dw = \Phi dt + \left(b + f\right)\gamma dX - \sigma\left(w - s\right)dZ, \tag{11}$$

with $\Phi = r_m m + r_f f + r_a a + r_b b + sr + y - c\left[1 + v(.)\right]$

The consumer maximises (1), subject to (11). To account for restrictions on nominal bond holdings, we formulate the problem assuming that *a* and *b* are confined to the following control sets:

$$\bar{b} - b \geq 0 \tag{12}$$

$$\bar{a} - a \geq 0 \tag{13}$$

These constraints will be assumed to be binding or not, depending on the institutional framework under consideration.

The Hamilton-Jacobi-Bellman equation of the corresponding quasi-stationary problem is:

$$\beta V(w) = \max_{c,m,f,a\leq\bar{a},b\leq\bar{b}} \left\{ \frac{c^{1-\phi}}{1-\phi} + V'(w)\Phi + \frac{1}{2}V''(w)\left[\gamma^2\left(b+f\right)^2 + \sigma^2\left(w-s\right)^2 - 2\sigma\rho\left(w-s\right)\left(b+f\right)\right] \right\}$$

where *V(w)* is the optimal value function. The first order conditions in respect to *b, f*, *a* and *m* imply:

$$V'(w)\left[r_b - r\right] + V''(w)\left[\left(\gamma^2 - \rho\right)\left(b + f\right) + \left(\sigma^2 - \rho\right)\left(w - s\right)\right] = \lambda \tag{14}$$

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9 Apart from that assumption, the transactions technology follows Carlos Végh (1989). The model only deals with imperfect currency substitutability The equilibrium implications of perfect means of payment substitutability are discussed in Kareken and Wallace (1981), for the case in which agents face no binding restrictions on money holdings, and in Lebre de Freitas (2004), for the "asymmetric" case, in which foreign residents cannot hold domestic money

$$V'(w)\left[r_f - v_2 - r\right] + V''(w)\left[\left(\gamma^2 - \rho\right)\left(b + f\right) + \left(\sigma^2 - \rho\right)\left(w - s\right)\right] = 0 \qquad (15)$$

$$V'(w)\left[r_a - r\right] + V''(w)\left[\sigma^2\left(w - s\right) - \rho\left(b + f\right)\right] = \mu \qquad (16)$$

$$V'(w)\left[r_m - v_1 - r\right] + V''(w)\left[\sigma^2\left(w - s\right) - \rho\left(b + f\right)\right] = 0 \qquad (17)$$

where $\lambda \geq 0$ and $\mu \geq 0$ are the Lagrangian multipliers associated to the constraint (12) and (13), respectively. Conditions (14) and (16) accounts for both interior and boundary solutions: according to the Khun-Tucker complementary slackness conditions, if for instance constraint (12) is not binding, then $\lambda=0$. If, instead, constraint (12) is binding, then $\lambda>0$, meaning that lessening the constraint would have a positive impact on the optimal value function. The same holds for the Lagrangian multiplier μ.

3. The case with no restriction on nominal bond holdings

In this section, we briefly revisit the case in which nominal bonds in both currencies are freely available. In terms of the formulation above, this case is accounted for by postulating a large enough values for $\bar{a}$ and $\bar{b}$, so as to ensure that restrictions (12) and (13) are not binding.

Substituting $\mu=0$ and $\lambda=0$ in (16) and (14) and subtracting, respectively, from (15) and (17), one obtains[10]:

$$i + v_1\left(\frac{m}{c}, \frac{f}{c}\right) = 0, \qquad (18)$$

$$j + v_2\left(\frac{m}{c}, \frac{f}{c}\right) = 0. \qquad (19)$$

Equations (18) and (19) implicitly define the money demand functions, as obeying to a trade-off between transaction services and user costs.

Using $\lambda=0$ and $\mu=0$ in (14) and (16) and the envelope condition $\phi = -V''(w)w/V'(w)$, one obtains:

[10] Thomas (1985).

$$\frac{r_b - r}{\phi} = (\sigma^2 - \rho)\left(1 - \frac{s}{w}\right) + (\gamma^2 - \rho)\left(\frac{b+f}{w}\right) \tag{14a}$$

$$\frac{r_a - r}{\phi} = \sigma^2\left(1 - \frac{s}{w}\right) - \rho\left(\frac{b+f}{w}\right) \tag{16a}$$

Subtracting (16a) from (14a) and after some manipulation, the following two conditions are obtained:

$$\frac{b+f}{w} = \left(\frac{r_b - r_a}{\phi\gamma^2}\right) + \left(1 - \frac{s}{w}\right)\left(\frac{\rho}{\gamma^2}\right) \tag{20}$$

$$1 - \frac{s}{w} = \frac{(r_a - r)\gamma^2 + (r_b - r_a)\rho}{\phi\Sigma} \tag{21}$$

Where $\Sigma = \sigma^2\gamma^2 - \rho^2 > 0$. This parameter is positive, because R<1.

Equation (21) is the reincarnation of the Merton formula for this particular context, and captures the speculative demand for nominal (risky) assets: it states that the agent is induced to allocate part of her wealth away from the safe asset towards nominal assets, depending on her degree of risk aversion, the expected return differential and uncertainty (in this case, with the later two adjusted for the presence of a foreign bond[11]).

Equation (20) recovers the international investor portfolio rule (Branson and Henderson, 1985) in this specific context of asset availability (the case with s=0 is addressed in Lebre de Freitas and Veiga, 2006). It states that the optimal level of Asset Substitution, depends on a speculative component (first term) and on an hedging component (second term). The term ρ/γ^2 gives the proportion of assets denominated in foreign currency (bonds plus money) that minimises the purchasing power risk of the nominal component of the portfolio. According to (20), the consumer is induced to move away from that proportion by the expected return differential (first term on the right hand side), and the extent to which she moves depends on her degree of risk aversion, ϕ.

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[11] In case $\rho=0$, the demands for domestic denominated assets and for foreign denominated assets simplify to the conventional Merton formula.

In this version of the model, because domestic and foreign money are dominated by interest-bearing assets, their demands are driven by transaction purposes, only (equation 18 and 19): after deciding the optimal money balances in each currency, taking into account the respective liquidity services and opportunity costs, the consumer can borrow or lend in both currencies so as to achieve the optimal denomination structure of its portfolio (20), and then the optimal partition between risky assets and the safe asset (21). These two choices are independent of money holdings (Thomas, 1985).

As an example, consider the extreme case in which the degree of risk aversion is infinity, so that the agent wants all its wealth to be held in the form of the safe asset (*s=w*). In that case, she will hire liabilities in domestic and foreign currency so as to exactly match its holdings in like-denominated moneys (that is, *a+m=0* and *b+f=0*). Thus, money demands are determined by interest rates and transaction services, only, and the optimal structure of the portfolio in terms of real assets and nominal assets does not depend on money holdings.

Using (18), (19), and (10), the demand for domestic money takes the following form:

$$\frac{m}{c} = L^m(i,j) \text{ with } L_i^m = -\frac{v_{22}}{\Delta} < 0 \text{ and } L_j^m = -\frac{v_{12}}{\Delta} \geq 0. \quad (22)$$

$$\frac{f}{c} = L^f(i,j) \text{ with } L_i^f = \frac{v_{12}}{\Delta} \geq 0 \text{ and } L_j^f = \frac{v_{11}}{\Delta} < 0. \quad (23)$$

In the particular case in which there is no currency substitutability ($v_{12} = 0$), each money demand will depend only on the respective opportunity cost.

4. The case with a binding constraint on foreign bond holdings

We now turn to the case in which the agent faces a binding restriction on foreign bond holdings. This case captures the context of many developing and emerging market economies, where private agents have no easy access to bonds denominated in foreign currency[12].

Since the individual cannot use foreign bonds to hedge the risk exposure implied by foreign money balances, unless inflation and exchange rate depreciation are perfectly correlated, the demand for foreign money will obey to risk-return considerations. In that case, foreign money will compete with the real asset in the store of value function.

When condition (12) is binding, the lagrangian multiplier λ in (14) is positive. Subtracting (14) from (15) with $\lambda>0$, one obtains:

$$j + v_2\left(\frac{m}{c}, \frac{f}{c}\right) > 0 \tag{19b}$$

Comparing to (19), equation (19b) reveals that, in this case, the consumer holds a higher amount of foreign money than if there was no restriction on foreign bond holdings. This captures the existence of a portfolio demand for foreign money.

This case solves similarly to the one before, except that equation (14a) is now replaced by

$$\frac{r_f - v_2 - r}{\phi} = \left(\sigma^2 - \rho\right)\left(1 - \frac{s}{w}\right) + \left(\gamma^2 - \rho\right)\left(\frac{\bar{b} + f}{w}\right) \tag{14b}$$

Subtracting (14b) from (16a), and using (12) in equality, one obtains the optimal level of "asset substitution" in this particular context:

$$\frac{f + \bar{b}}{w} = \left(\frac{1}{\phi}\right)\left(\frac{r_f - v_2 - r_a}{\gamma^2}\right) + \left(1 - \frac{s}{w}\right)\left(\frac{\rho}{\gamma^2}\right) \tag{20b}$$

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[12] Sahay and Végh (1996) adapted the model in Section 3 to the context of developing countries, by interpreting foreign money f as denoting for foreign banknotes held by the public and the foreign bond *b* as denoting for bank deposits denominated in foreign currency, which are available to common citizens in many developing countries. In light of that interpretation, the proposition that there is no portfolio demand for money applies. Note however that this interpretation presumes that foreign currency deposits provide no transaction services at all, which is not likely to be a general case. The model in this sections proposes and alternative framework, in which foreign money (broad sense) plays simultaneously a store of value and a means of payment role.

The novelty in (20b) relative to (20) is that the marginal productivity of foreign money ($v_2 < 0$) replaces j in the expected return differential. This reflects the fact that the demand for foreign money is driven by both transaction motives and risk hedging considerations.

Because the properties of the money demand in this setup depend critically on the assumption regarding the covariance between the exchange rate and the inflation rate, in the following we solve the model for three cases regarding the size of that covariance.

4.1. Positive but imperfect correlation between prices and the exchange rate (0<R<1)

To investigate the determinants of money demand in this case, we first solve together (14b) and (20b) as functions of the exogenous parameters, only, obtaining:

$$1 - \frac{s}{w} = \frac{\rho\left[r_f - v_2 - r_a\right] + \left[r_a - r\right]\gamma^2}{\phi\Sigma} \tag{21b}$$

$$\frac{f + \bar{b}}{w} = \frac{\sigma^2\left[r_f - r_a - v_2\right] + \rho\left[r_a - r\right]}{\phi\Sigma} \tag{24}$$

Taking differences in (18) and (24) and solving for dm and df as functions of the exogenous parameters, the following partial derivatives are obtained:

$$\frac{dm}{di} = \frac{-c}{\Omega}\left[\frac{c\phi\Sigma}{w} + \sigma^2\left(v_{22} - v_{12}\right) + v_{12}\rho\right] < 0 \tag{25}$$

$$\frac{dm}{d\varepsilon} = \frac{-c}{\Omega}\sigma^2 v_{12} \leq 0 \tag{26}$$

$$\frac{dm}{d\pi} = \frac{dm}{dr} = \frac{c}{\Omega}\rho v_{12} \geq 0 \tag{27}$$

$$\frac{dm}{d\sigma} = \frac{-cv_{12}}{\Omega}\left(2\sigma\rho\right)\left(1 - \phi\frac{w - s}{w}\right) \tag{28}$$

$$\frac{dm}{d\gamma} = \frac{cv_{12}}{\Omega}\left(2\sigma^2\gamma\right)\left(\phi\frac{\bar{b} + f}{w}\right) \geq 0 \tag{29}$$

$$\frac{dm}{dw} = \frac{-cv_{12}}{\Omega}\left(\frac{\phi\Sigma}{w}\right)\left(\frac{\bar{b} + f}{w}\right) < 0 \tag{30}$$

$$\frac{df}{di} = \frac{c}{\Omega}\left[\rho v_{11} - \sigma^2\left(v_{11} - v_{12}\right)\right] \tag{31}$$

$$\frac{df}{d\varepsilon} = \frac{c}{\Omega}\sigma^2 v_{11} > 0 \quad (32)$$

$$\frac{df}{d\pi} = \frac{df}{dr} = \frac{-c}{\Omega}\rho v_{11} \leq 0 \quad (33)$$

$$\frac{df}{d\sigma} = \frac{cv_{11}}{\Omega}(2\sigma\rho)\left(1 - \phi\frac{w-s}{w}\right) \quad (34)$$

$$\frac{df}{d\gamma} = \frac{-cv_{11}}{\Omega}(2\sigma^2\gamma)\left(\phi\frac{\bar{b}+f}{w}\right) < 0 \quad (35)$$

$$\frac{df}{dw} = \frac{cv_{11}}{\Omega}\left(\frac{\phi\Sigma}{w}\right)\left(\frac{\bar{b}+f}{w}\right) > 0 \quad (36)$$

With $\Omega = \sigma^2\Delta + \frac{c\phi\Sigma}{w}v_{11} > 0$.

In this version of the model, there is no portfolio role for domestic money: since domestic money is dominated by an interest-bearing bond, its demand is driven by transaction purposes, only (eq. 18). The demand for domestic money may however be influenced by portfolio considerations through the currency substitution channel: as long as $v_{12} > 0$, then any change in the demand for foreign money by speculative or risk hedging reasons will impact on the demand for domestic money, even if the later is dominated by an interest-bearing asset (equations 26-29). In case of no currency substitutability ($v_{12} = 0$), the demand for domestic money assumes the conventional form:

$$\frac{m}{c} = L^m(i), \text{ with } L_i^m = \frac{-c}{\Omega}\left[\frac{c\varphi\Sigma}{w} + \sigma^2 v_{22}\right] < 0 \quad (37)$$

From (27) and (33), we see that expected inflation influences the demand for foreign money negatively and the demand for domestic money positively, at most. The reason is that foreign money is imperfect substitute of the real asset in the store of value function. Hence, when the inflation rate increases, the agent will reallocate wealth away from foreign money to the real asset (eq. 33). If, in plus, foreign money competes with domestic money in the means of payment function, then the higher inflation rate will give rise to a Currency Substitution effect through which the higher inflation rate translates into a higher demand for domestic money[13].

Note however that the positive relationship between money demand and inflation only holds for the definition of money comprehending monetary assets denominated in domestic currency. A broad definition of money, including monetary assets denominated in both currencies (the sum m+f), is expected to depend negatively on the inflation rate, because the sum of the partial derivatives (27) and (33) is positive. The implication is that the expected sign of a coefficient capturing the influence of the inflation rate in a money demand equation depends critically on the type of money aggregate we are handling with: when one estimates the demand for a monetary aggregate that includes assets denominated in domestic currency only, then the expected sign of the inflation coefficient, after controlling for the exchange rate depreciation, is – at most – positive. If however the monetary aggregate includes foreign currency deposits, which – in the absence of foreign bonds - are likely to be held for both transaction motives and portfolio reasons, then the relationship between inflation and money demand is expected to be negative.

Similar comments hold for the relationship between money and wealth. The fact that foreign money gets a portfolio role implies that it will depend positively on real wealth (equation 36). In case of currency substitutability, an increase in wealth that leads to an increasing demand for foreign currency translates into a lower demand for domestic currency (equation 30). On balance, the demand for total money (m+f) increases with real wealth.

As for the expected exchange rate depreciation, it acts in the model as the yield on foreign currency: whenever the expected exchange rate depreciation increases, everything else constant, people will hold more of foreign money (equation 32). In case the two monies compete as means of payment, this causes a fall in the demand

[13] If, in alternative, prices and the exchange rate were negatively correlated, the agent would optimally respond to an increase in expected inflation with a diversification move, increasing simultaneously her holdings of the real asset and of foreign money. In that case, the sign of the partial derivative (27) would be negative. The assumption of a means of payment with a return that correlates negatively with the inflation rate is not however realistic.

for domestic money (equation 26). Because a higher expected depreciation implies a higher return on average money, it causes the demand for total money to increase (the sum of the derivatives 25 and 31 is positive).

4.2. Purchasing power parity holding instantaneously ($\rho = \sigma^2 = \gamma^2$)

In episodes with very high inflation, citizens often replace domestic currency by a foreign currency (usually the US dollar) in the unit of account role of money. When this is so, agents first set prices in units of foreign currency, and then they use the current exchange rate to calculate the corresponding prices in units of domestic currency, for invoicing and settlement purposes. When this is so, prices and the exchange rate correlate almost perfectly.

To capture this case, we solve the model above assuming that R=1. It is also assumed that the standard deviations of the stochastic processes (3) and (4) are the same[14]:

$$\rho = \sigma^2 = \gamma^2 \tag{38}$$

Since expected inflation and expected exchange rate depreciation correlate perfectly, in this setup foreign money provides a perfect hedge against the inflation risk, just like the real asset. The main difference between foreign money and the real asset is that the later is not liquid enough to provide transaction services.

Using (38) in (14b), one obtains[15]:

$$\varepsilon - v_2\left(\frac{m}{c}, \frac{f}{c}\right) - \pi = r \tag{39}$$

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[14] One may interpret f in this version of the model as standing for an overnight bank deposit denominated in domestic currency paying a nominal interest rate equal to the daily inflation rate. In terms of the model above, these two interpretations are equivalent.

[15] We stick with the interior solution postulating $r + \pi > \varepsilon$.

This condition implicitly defines the demand for foreign money in this particular setup. The condition is similar to (19), except that in this case foreign money is dominated by the real asset, instead as by a nominal bond.

Because in this version of the model both monies are dominated, the proposition that there is no portfolio demand for money is recovered: each money demand depends only on the respective productivity in the provision of transaction services and of opportunity costs (equations 18 and 39). Variables that are relevant for portfolio decisions such as total wealth and inflation volatility fail to influence the money demands.

The partial derivatives of the money demands in respect to the relevant parameters are obtained totally differentiating (18) and in (39) and solving together:

$$\frac{dm}{di} = \frac{-cv_{22}}{\Delta} < 0 \qquad (40)$$

$$\frac{dm}{d\pi}\Big|_{d\pi = d\varepsilon} = \frac{cv_{12} - cv_{12}}{\Delta} = 0 \qquad (41)$$

$$\frac{dm}{dr} = \frac{cv_{12}}{\Delta} \geq 0 \qquad (42)$$

$$\frac{df}{di} = \frac{cv_{12}}{\Delta} \geq 0 \qquad (43)$$

$$\frac{df}{d\pi}\Big|_{d\pi = d\varepsilon} = \frac{cv_{11} - cv_{11}}{\Delta} = 0 \qquad (44)$$

$$\frac{df}{dr} = \frac{-cv_{11}}{\Delta} < 0 \qquad (45)$$

Because in this version of the model the exchange rate and the inflation rate as collinear, it makes no sense to calculate the two partial derivatives separately. As shown in (41) and (44), changes in expected inflation and on expected exchange rate depreciation cancel out, so they fail to influence the money demands.

In this version of the model, the elasticity of money demand in respect to the real interest rate is expected to differ from that of expected inflation. Because the real interest rate is the relevant opportunity cost of holding foreign money, whenever it rises, the demand for foreign money will decline. In case domestic and foreign money compete as means of payment, the lower demand for foreign money will translate into a higher demand for domestic money.

4.3. Foreign money delivering a certain nominal return ($\gamma = \rho = 0$)

We now examine another extreme case, in which the correlation between expected inflation and the expected exchange rate depreciation is zero[16].

$$\gamma = \rho = 0 \qquad (46)$$

Because in this version of the model there is no uncertainty regarding the exchange rate, one may interpret f as standing for a time deposit denominated in domestic currency paying a certain nominal return that is lower than that in the domestic bond ($\varepsilon <$ i), but that at the same time is liquid enough to complement narrow money (m) in the means of payment role[17].

In this version of the model, the real return on f is:

$$\frac{df}{f} = \left(\varepsilon + \sigma^2 - \pi\right)dt - \sigma dZ = r_f dt - \sigma dZ \qquad (8d)$$

The real returns on narrow money (*m*), the domestic bond (*a*), and the real asset (s) are given, respectively, by (5), (6), and (9). Because both moneys are now dominated by the same nominal asset, conditions (18) and (19) are replaced by:

$$-v_1\left(\frac{m}{c}, \frac{f}{c}\right) = -v_2\left(\frac{m}{c}, \frac{f}{c}\right) + \varepsilon \qquad (47)$$

That is, at the optimum, the agent will hold the two moneys such that the difference in productivities in the provision of liquidity services is exactly matched by

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[16] The case with $\gamma > 0$ does not differ qualitatively from the one analysed below.

[17] In alternative, one may think a credibly fixed exchange rate regime, with the domestic inflation rate drifting up and down around some level consistent with the peg, and with ε denoting for a nominal interest rate in foreign currency demand deposits (without loss of generality, this parameter can be set equal to zero).

the nominal return on the time deposit. As long as the time deposit pays a positive interest rate (ε>0), narrow money will be at the margin more productive as means of payment than quasi money.

The signs of the partial derivatives can be obtained substituting (46) in (25)-(36), which implies:

$$\frac{\partial m}{\partial i} = \frac{-c}{\Delta}\left(v_{22} - v_{12}\right) < 0 \qquad (48)$$

$$\frac{\partial m}{\partial \varepsilon} = \frac{-c}{\Delta} v_{12} \leq 0 \qquad (49)$$

$$\frac{\partial f}{\partial i} = \frac{-c}{\Delta}\left(v_{11} - v_{12}\right) < 0 \qquad (50)$$

$$\frac{\partial f}{\partial \varepsilon} = \frac{c}{\Delta} v_{11} > 0 \qquad (51)$$

Once again, because both moneys are dominated (in this case, by the domestic bond), there is no portfolio demand for money. The expected inflation rate and inflation volatility influence the optimal demands for the real assets and for the domestic bond, but fail to influence the money demands. The later are only driven by transaction services and opportunity costs (i and i-ε). The inflation rate and inflation volatility will influence the optimal demand for the real asset and for the domestic bond, but will not impact on money demands.

In sum, splitting money into a narrow component, more productive in transaction services, and quasi money, less liquid but paying a positive interest does not change the main conclusion that, as long as both are dominated by an interest bearing bond there should be no portfolio demand for domestic money.

5. The case with binding constraints on domestic and foreign bond holdings

We now turn to the case in which constraints (12) and (13) are both binding. In this case, unless prices and the exchange rate are perfectly correlated, no money will be dominated as store of value.

When condition (13) is binding, the Lagrangian multiplier μ in (16) is positive. Subtracting (16) from (17), with $\mu>0$, one gets:

$$i + v_1\left(\frac{m}{c}, \frac{f}{c}\right) > 0 \qquad (18c)$$

Similarly to (19b), equation (18c) implies that the consumer will hold a higher amount of domestic money than in the case in which domestic money is purely held for transaction purposes.

Because the consumer faces no binding constraints on money holdings, conditions (15) and (17) hold in equality. Rearranging, and using $\phi = -V''(w)w/V'(w)$, one obtains (14b) again and:

$$\frac{r_m - v_1 - r}{\phi} = \sigma^2\left(1 - \frac{s}{w}\right) - \rho\left(\frac{\bar{b} + f}{w}\right) \qquad (17c)$$

As before, we proceed investigating the properties of the money demand considering three different cases regarding the magnitude of the co-variance parameter.

5.1. Positive but imperfect correlation between prices and the exchange rate (0<R<1)

Solving together (14b) and (17c) for the exogenous parameters, one obtains:

$$1 - \frac{s}{w} = \frac{(r_m - v_1 - r)\gamma^2 + (r_f - v_2 - r_m + v_1)\rho}{\phi\Sigma} \qquad (21c)$$

$$\frac{f + \bar{b}}{w} = \frac{\sigma^2[r_f - r_m + v_1 - v_2] + \rho[r_m - v_1 - r]}{\phi\Sigma} \qquad (24c)$$

Subtracting (22c) from (21c), one obtains:

$$\frac{m + \bar{a}}{w} = \frac{(\rho - \sigma^2)[r_f - r_m + v_1 - v_2] + (\gamma^2 - \rho)[r_m - v_1 - r]}{\phi\Sigma} \qquad (52)$$

Equation (21c) reveals that the individual' optimal deviation from the safe asset depends now on the "yields" of domestic and foreign money, as well as on the degree of risk aversion. On the other hand, (24c) and (52) imply that the optimal demands for foreign money and for domestic money also obey to a balance between risk and return. This captures the portfolio role of moneys.

As long as domestic money is essential for transactions (that is, if v_1 tends to minus infinity as *m* approaches zero), it will be impossible for the consumer to completely get rid of the inflation risk. She may, however, optimally decide to accept higher costs in transacting in the good market against a lower risk exposure, in case the cost of holding money increases. Because this choice is complicated by the fact that the demands for both moneys are driven by risk-hedging considerations as well as by transaction services, in this version of the model, the signs of the different partial derivatives are not obvious.

To see this, let's totally differentiate the equations determining the money demands (24c) and (52), and solve for *dm* and *df*. After some manipulation, the partial derivative in respect to the inflation rate can be expressed as follows:

$$\frac{\partial m}{\partial \pi} = \frac{-c}{\Psi\Sigma}\left\{\gamma^2\left[\frac{c\phi\Sigma}{w} + \sigma^2(v_{22} - v_{12}) + v_{12}\rho\right] - \rho\left[\frac{c\phi\Sigma}{w} + \gamma^2 v_{12} + \rho(v_{22} - v_{12})\right]\right\}, \quad (53)$$

where $\Psi = \left(\frac{c\phi}{w}\right)^2\Sigma + \frac{c\phi}{w}\left[\sigma^2(v_{11} + v_{22} - 2v_{12}) + \gamma^2 v_{11} - 2\rho(v_{11} - v_{12})\right] + \Delta$.

Simplifying, this gives:

$$\frac{\partial m}{\partial \pi} = \frac{-c}{\Psi}\left[(\gamma^2 - \rho)\left(\frac{c\phi}{w}\right) + (v_{22} - v_{12})\right] \quad (53a)$$

The corresponding partial derivative in the demand for foreign money is:

$$\frac{\partial f}{\partial \pi} = \frac{-c}{\Psi}\left[\rho\frac{c\phi}{w} + (v_{11} - v_{12})\right] \quad (54)$$

Equation (53) reveals that the optimal response of the demand for domestic money to an increase in expected inflation involves a balance between two opposing effects:

- On one hand, to the extent that the inflation rate is the relevant opportunity cost of holding money (instead of the nominal interest rate), its influence on domestic money demand will be negative (note the similarity between the first term in equation 53 and equation 25);
- On the other hand, the same mechanism identified in Section 4.1 is in operation: a higher inflation rate, by inducing agents to hold less foreign money, causes the transactions demand for domestic money to increase (second term in 53).

On balance, the impact of the inflation rate in the demand for domestic money is more likely to be negative. Strictly speaking, however, the sign of the partial derivative is not certain. The presence of terms with opposite signs in equations (53a) and in the denominator Ψ implies that the signs of (53a) and of (54) as well as of the remaining partial derivatives are, in general, undetermined. The key parameter in this ambiguity is the co-variance between prices and the exchange rate, that underlies the substitutability between foreign money and the real asset in the store of value function[18].

5.2 Purchasing power party holding instantaneously ($\rho = \sigma^2 = \gamma^2$)

In this sub-section, we return to the case in which the stochastic processes of the exchange rate and of prices have equal variances and are perfectly correlated. As stated above, this case can be thought as describing an environment with very high inflation.

As argued in Section 4.2, in this case foreign money is dominated by the real asset, so its demand is driven by a trade off between transaction services and opportunity costs (equation 39). The novelty in respect to the model in 4.2 is that the agent can no longer use domestic bonds to hedge its exposure to monetary assets

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[18] Interesting enough, ruling out currency substitutability is not sufficient to obtain negative signs in (53a) and in (54). A sufficient condition to obtain negative signs when $v_{12} = 0$ is $\gamma > \sigma$, that is, when the exchange rate is more volatile than prices, as it is likely to be the case under float. This conclusion relies on the fact that $R = \rho / \sigma\gamma < 1$.

denominated in domestic currency. Hence, in this version of the model, the demand for domestic money is driven by risk-return considerations, in contrast to foreign money, which is purely held for transaction purposes.

To investigate the properties of the money demand in this context, we substitute (38) in (17c), obtaining

$$\frac{\bar{a}+m}{w}=\frac{\sigma^2-v_1-\pi-r}{\phi\sigma^2} \quad (55)$$

This expression determines the optimal demand for domestic money as a trade off between risk and return. Totally differentiating (55) and (39) and solving together, the following partial derivatives are obtained:

$$\frac{\partial m}{\partial r}=\frac{-c}{Z}\left(v_{22}-v_{12}\right)<0 \quad (56)$$

$$\frac{\partial m}{\partial \pi}\Big|d\pi=d\varepsilon=\frac{-c}{\mathrm{Z}}v_{22}<0 \quad (57)$$

$$\frac{\partial m}{\partial \sigma}=\frac{cv_{22}}{\mathrm{Z}}(2\sigma)\left(1-\phi\frac{m+\bar{a}}{w}\right) \quad (58)$$

$$\frac{\partial m}{\partial w}=\frac{cv_{22}}{\mathrm{Z}}\left(\frac{\phi\sigma^2}{w}\right)\left(\frac{m+\bar{a}}{w}\right)>0 \quad (59)$$

$$\frac{df}{d\pi}\Big|d\pi=d\varepsilon=\frac{c}{\mathrm{Z}}v_{12}\geq 0 \quad (60)$$

$$\frac{\partial f}{\partial r}=\frac{-c}{Z}\left(\sigma^2\frac{c\phi}{w}+v_{11}-v_{12}\right)<0 \quad (61)$$

$$\frac{\partial f}{\partial \sigma}=\frac{-cv_{12}}{\mathrm{Z}}(2\sigma)\left(1-\phi\frac{m+\bar{a}}{w}\right) \quad (62)$$

$$\frac{\partial f}{\partial w}=\frac{-cv_{12}}{\mathrm{Z}}\left(\frac{\phi\sigma^2}{w}\right)\left(\frac{m+\bar{a}}{w}\right)>0 \quad (63)$$

With $Z=\Delta+\frac{c\phi\sigma^2}{w}v_{22}>0$

The interesting novelty in this case is that the demand for foreign money becomes influenced by portfolio decisions through the currency substitution channel, just like the case of domestic money in Section 4. In case the two moneys do not compete in the means of payment role, then the demand for foreign money will be driven by transaction services and the real interest rate, only.

Because in this version of the model, the inflation rate plays no role in the cost of holding foreign money, an increase in the inflation rate primarily impacts negatively on the demand for domestic money as opportunity cost, and then positively on the demand for foreign money through the currency substitution channel. In this version of the model, an increase in the inflation rate unambiguously causes the demand for domestic money to decline.

5.3. Foreign money delivering a certain nominal return ($\gamma = \rho = 0$)

We now return to the setup in which there is no exchange rate risk. In this case, one may interpret *f* as standing for a time deposit denominated in domestic currency paying a certain nominal return (ε). In contrast to Section 4.3, however, in this case there is no domestic bond dominating both types of money in the store of value role. Thus, the demand for both monies will depend on the respective productivities in the provision of transaction services and on risk-taking considerations.

To solve this model, we turn again to (14b) and (17c). The optimal demand for risky assets is obtained substituting (46) in (17c), which gives:

$$1 - \frac{s}{w} = \frac{\bar{a} + \bar{b} + m + f}{w} = \frac{\sigma^2 - v_1 - \pi - r}{\phi\sigma^2} \qquad \text{(21d)}$$

Equation (21d) determines the demand for broad money, *m+f*.

Substituting (46) in (14b) and solving together with (21d), these two conditions deliver again condition (47), which states that the returns of the two moneys at the margin should be equal. Totally differentiating (47) and (21d), and solving together, one obtains:

$$\frac{\partial m}{\partial \pi} = \frac{\partial m}{\partial r} = \frac{-c}{\Pi}\left(v_{22} - v_{12}\right) < 0 \qquad (64)$$

$$\frac{\partial m}{\partial \varepsilon} = \frac{-c}{\Pi}\left(\frac{c\phi\sigma^2}{w} + v_{12}\right) < 0 \qquad (65)$$

$$\frac{\partial m}{\partial \sigma} = \frac{c}{\Pi}\left(v_{22} - v_{12}\right)\left(2\sigma\right)\left(1 - \phi\frac{w-s}{w}\right) \qquad (66)$$

$$\frac{\partial m}{\partial w} = \frac{c}{\Pi}\left(v_{22} - v_{12}\right)\left(\frac{w-s}{w}\right)\left(\frac{\sigma^2\phi}{w}\right) > 0 \qquad (67)$$

$$\frac{\partial f}{\partial \pi} = \frac{\partial f}{\partial r} = \frac{-c}{\Pi}\left(v_{11} - v_{12}\right) < 0 \qquad (68)$$

$$\frac{\partial f}{\partial \varepsilon} = \frac{c}{\Pi}\left(\frac{c\phi\sigma^2}{w} + v_{11}\right) > 0 \qquad (69)$$

$$\frac{\partial f}{\partial \sigma} = \frac{c}{\Pi}\left(v_{11} - v_{12}\right)\left(2\sigma\right)\left(1 - \phi\frac{w-s}{w}\right) \qquad (70)$$

$$\frac{\partial f}{\partial w} = \frac{c}{\Pi}\left(v_{11} - v_{12}\right)\left(\frac{w-s}{w}\right)\left(\frac{\sigma^2\phi}{w}\right) > 0 \qquad (71)$$

Where $\Pi = \frac{c\phi\sigma^2}{w}\left(v_{11} + v_{22} - 2v_{12}\right) + \Delta > 0$

Thus, both money demands depend on wealth as well as on risk considerations, reflecting their portfolio roles. In this case, expected inflation influences negatively the demand for both monies.

6. Summary of the results above

The exercises above illustrate the fact that the properties of the optimal demand for money depend critically on the institutional setup regarding asset availability. They also reveal that the signs of some elasticities may change when one moves from a narrow monetary aggregate to a broader aggregate that includes domestic and foreign monetary assets. In this section, we summarise these results.

As suggested in Section 3, when most agents in an economy have unrestricted access to domestic and foreign bonds, a money demand specification based on equations (22) looks appropriate:

$$m = L^m\left(c, i.j\right), \text{ with } L_i^m < 0,\ L_j^m \geq 0 \qquad (72)$$

In (72), the partial derivative in respect to the foreign interest rate becomes zero in case of no currency substitutability[19]. In this setup, a broad monetary aggregate including domestic and foreign monetary assets (*x*=*m*+*f*) will have the following properties:

$$x = L^x(c, i.j), \text{ with } L_i^x < 0,\ L_j^x < 0 \qquad (73)$$

In Section 4, we addressed the case in which agents cannot use foreign bonds to hedge the risk exposure implied by foreign money balances. In that case, foreign money gets a portfolio role, unless prices and the exchange rate are perfectly correlated.

In the more general case in which the correlation is positive but not one (Section 4.1), the properties of the demand for domestic money are as follows (equations 25-30):

$$m = L^m(c, i, \varepsilon, \pi, r, \gamma, \sigma, w), \text{ with } L_i^m < 0, L_\varepsilon^m \le 0, L_\pi^m = L_r^m \ge 0, L_\gamma^m \ge 0,\ L_w^m \le 0. \ (74)$$

In case of no currency substitutability, all inequalities turn zero and the demand for domestic money simplifies to the closed economy form. From (31)-(36) and (25)-(30), an extended monetary aggregate comprehending money holdings denominated in domestic currency and in foreign currency (*x*=*m*+*f)*, will have the following properties:

$$x = L^x(c, i, \varepsilon, \pi, r, \gamma, \sigma, w), \text{ with } L_i^x < 0, L_\varepsilon^m > 0, L_\pi^x = L_r^x < 0, L_\gamma^x < 0,\ L_w^x > 0. \ (75)$$

Thus, when one moves from a money aggregate including assets denominated in domestic currency only to a broad monetary aggregate including real balances denominated in foreign currency, the signs of expected inflation and of real wealth change. In the second case, they are consistent with those postulated by the portfolio-balance approach.

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[19] With no surprise, this model has been used to test for the presence of currency substitution among currencies of countries and regions with developed financial markets (Joines, 1985, Bergstrand and Bundt, 1990, Mizen and Pentecost, 1994, and Lebre de Freitas, 2006).

A second interesting case occurs when prices and the exchange rate are perfectly correlated, as it tends to happen during hyperinflation episodes (Section 4.2). In that case, foreign money offers a perfect hedge against the inflation risk, but is dominated by the real asset in the store of value role. The implication is that, there will be no portfolio demand for moneys. Because the relevant opportunity cost of holding foreign money is the real interest rate, in case of currency substitutability, the demand for domestic money will become a positive function of the real interest rate (equations 40-42):

$$m = L^m(c,i,r), \text{ with } L_i^m < 0, L_r^m \geq 0. \qquad (76)$$

A question that arises is how this specification relates to the most popular one for hyperinflation episodes, proposed by Cagan (1956). In that specification, the nominal interest rate is replaced by an expected inflation term, using the Fisher principle. In our framework, if one used $di = dr + d\pi$ in the system (40)- (41), one would obtain a negative influence of expected inflation in the demand for domestic money ($dm/d\pi = -cv_{22}/\Delta < 0$), but the sign of the real interest rate would turn negative ($dm/dr = -c(v_{22} - v_{12})/\Delta < 0$). Of course, since during hyperinflations changes in the real rate of return tends to be negligible, specifying the inflation rate as the sole determinant of money velocity is not likely to involve a significant specification error. Note however that the omission of the real interest rate may render a coefficient on the expected inflation negative and significant, even when the nominal interest rate is included: using $di = dr + d\pi$ to eliminate the real interest rate in the system (40)-(41), one obtains $dm/di = -c(v_{22} - v_{12})/\Delta < 0$ and $dm/d\pi = -cv_{12}/\Delta < 0$. Thus, at least theoretically, omitting the real interest rate from the money demand specification could deliver a spurious relationship between expected inflation and the money demand, even after controlling for the domestic interest rate. In any case, this will only happen if foreign money competes with domestic money in the means of payment role. Did currency substitutability not exist and the demand for domestic money would simplify to the closed economy form, regardless the restrictions on foreign bond holdings.

A third case explored in this paper is when there is no exchange rate risk (Section 4.3), as it would be the case of a credibly fixed exchange rate regime. In this case, the two monies can also be interpreted as monetary assets denominated in domestic currency with different productivity in the production of liquidity services (like narrow money and quasi money). Since in this case both monies are dominated by a domestic bond, no money demand shall be influenced by the inflation rate. The demand for narrow money takes the form (48-51):

$$m = L^m(c, i, \varepsilon), \text{ with } L_i^m < 0, L_\varepsilon^m \leq 0, \qquad (77)$$

where ε denotes for the interest rate in the time deposit. The demand for broad money (m2=m+f), obeys to:

$$m2 = L^{m2}(c, i, \varepsilon), \text{ with } L_i^{m2} < 0, L_\varepsilon^{m2} > 0. \qquad (78)$$

We then analysed the case in which domestic bonds are not available. In this case, the optimal demand for domestic money obeys to a trade-off between risk and return. Because the relevant alternative to money holdings is the real asset, both money demands will be impacted negatively by increases in expected inflation. However, the role of inflation as opportunity cost of holding domestic money is mitigated by the mechanism referred above through which inflation can cause the demand for domestic money to increase. Formally, it is possible that the money demand depends positively on expected inflation (equations 53-53a). The key parameter influencing the sign of the partial derivative of domestic money in respect to the inflation rate is the co-variance between inflation and the exchange rate.

To further investigate the case when both moneys are dominated, we then considered two extreme cases regarding the size of the co-variance between prices and the exchange rate.

The first case is when purchasing power parity holds instantaneously, so that foreign money becomes dominated by the real asset in the store of value role. In this case, the demand for domestic money gets the following properties (equations 56-59):

$$m = L^m(c, \pi, r, \sigma, w), \text{ with } L_\pi^m < 0\ ,\ L_r^m < 0\ L_w^m > 0. \qquad (79)$$

In this setup, moving from a money aggregate that includes assets denominated in domestic currency only (m) to a money aggregate that includes assets denominated in both currencies ($x=m+f$) does not change qualitatively the properties of the money demand (equations 56-63).

As a final exercise, we considered the case in which there is no exchange rate risk and the agent is constrained on her holdings of domestic and foreign bonds. In this case, the two monies deliver the same risk, though they differ in terms of provision of liquidity services. The properties of the demand for domestic/narrow money are as follows (equations 64-67):

$$m = L^m(c, \varepsilon, \pi, \sigma, w), \text{ with } L_\pi^m < 0, L_\varepsilon^m < 0, \ L_w^m > 0. \quad (80)$$

A broad money aggregate comprehending both types of money (m2=m+f) will have the following properties:

$$m2 = L^{m2}(c, \varepsilon, \pi, \sigma, w), \text{ with } L_\pi^{m2} < 0, L_\varepsilon^{m2} > 0, \ L_w^{m2} > 0. \quad (81)$$

Where in this case, ε denotes for the interest rate in the time deposit.

7. Conclusions

In this paper, we investigated the circumstances under which the demand for domestic money shall depend on the inflation rate, using an optimizing model where money holdings help reduce transaction costs. In particular, we explored the case in which the range of assets available to the representative agent includes a real asset offering a perfect hedge against the inflation risk.

As demonstrated in the earlier literature, under complete bond markets, the money demand shall not be influenced by portfolio considerations. If however agents face a binding constraint in their bond holdings, the demand for like-denominated money will become driven by portfolio considerations. Exploring this direction, we first analysed the case in which agents are constrained in their holdings of foreign bonds, but not on their holdings of domestic bonds. In this case, the eventual influence of expected inflation on the demand for domestic money is, at most, positive, never negative.

We then introduced a binding constraint in the holdings of domestic bonds. In this case, the influence of expected inflation on the money demand obeys to a balance between two effects: on one hand, an increase in expected inflation may cause the individual to decrease its holdings of foreign money, which may imply an increase in the demand for domestic money through the currency substitution channel. On the other hand, the increase in expected inflation rises the cost of holding domestic money, inducing a lower demand. On balance, the second effect is more likely to dominate, though formally this is not a general case.

We also explored a version of the model where there is no exchange rate risk, so that foreign money becomes equivalent to a second category of domestic money, like time deposits, which pay a positive nominal return but are less productive in terms of liquidity services. We showed that such modification does not change the main proposition that, as long as money is dominated by an interest-bearing asset, its demand should not depend negatively on the inflation rate. In order for the money demand to depend negatively on the inflation rate, one has to suppress domestic bonds from the range of alternative assets.

All in all, in no case we found an institutional setup where both the inflation rate and the nominal interest rate influence negatively the money demand, as usually postulated by the portfolio balance approach.

The main message of the paper is that the money demand properties are context specific. The optimal demand for money may be driven by portfolio considerations or not, depending on the range of assets that are at disposal of the optimising agent. Because economies are composed by heterogeneous agents, no particular setup shall be seen as applying for an economy as a whole. Thus, rather than relying on ad hoc specifications for the money demand (as suggested by the portfolio-balance approach), the researcher may try instead to take opportunity of modern econometric techniques, to disentangle in each particular context the sign and significance of the different variables in the long run money demand relationship, and then infer about the dominant constraints regarding asset availability.

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