Master's Thesis

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Numerical modelling of masonry vaults strengthened with transversal diaphragms

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DECLARATION

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ABSTRACT

Masonry vaults and arches are one of main structural elements present in most of historical constructions. Due to the impact of time, load and other construction features their strength capacity decreases making them more vulnerable to failure. Deflection and dislocation of units create internal condition of higher stress state, which in consequence can lead to premature collapse. That is why, to maintain the role of the vaults and arches and prevent them from failure, strengthening is needed. During the strengthening evaluation it cannot be forgotten that historical constructions are part of our cultural heritage and engineers are required to follow the conservation doctrine of minimum intervention. This condition involves detailed studies before proceeding with application of strengthening. Within, this framework, numerical modelling appears as a very useful method to define the accuracy of the intervention before it is applied.

The main objective of this thesis is numerical study of masonry arches and vaults strengthened by means of extrados stiffening diaphragms. Preparation and validation of numerical models was done according to an experimental arch tested by Paolo Girardello at University of Brescia, Italy. Based on the experimental parameters and geometry, two numerical models, built up on macro- and micro- approaches, were constructed in DIANA Finite Element Analysis program. The purpose of making two models was comparison of structural response of each one to monotonic, incremental load and conclude on usefulness of macro-modelling approach for masonry arch-type constructions.

Further, analysis of the efficiency of strengthening techniques throughout the non-linear analysis on both model types was performed. As a final step, parametric study of the strengthening technique was done for the macro-model of the arch. Data comparison between analyzed reinforced models was conducted and final conclusions on which parameter affects the structural response are stated.

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RESUMO

Os arcos e as abóbadas de alvenaria são dos principais elementos estruturais presentes na maioria das construções históricas. Devido às ações atuantes, efeito dapassagem do tempo e outrosaspetos da construção, a sua capacidade resistente diminui progressivamente tornando-os mais vulneráveis do ponto de vista estrutural. Movimentos parciais de partes dos arcos podem criar condições para um maior estado de tensão, que por consequência pode levar ao colapso prematuro. Neste sentido, o reforço surge como uma possibilidade para manter a função estrutural das abóbadas e arcos e impedi-los de ruir. Durante a avaliação da necessidade de reforço, não pode ser esquecido que as construções históricas são parte da nossa herança cultural e que aos técnicos se pede que observem a doutrina da intervenção mínima na conservação. Esta condição envolve estudos detalhados antes de se prosseguir com a aplicação de reforço. Dentro deste quadro, a modelação numérica aparece como um método muito útil para definir a precisão da intervenção antes desta se realizar.

O principal objetivo desta tese consiste no estudo numérico de arcos e abóbadas de alvenaria reforçadas por meio de diafragmas rígidos colocados no extradorso. A preparação e validação dos modelos numéricos foi feita de acordo com os resultados de um arco testado experimentalmente por Paolo Girardello da Universidade de Brescia, Itália. Com base nos parâmetros experimentais e geometria, foram construídosdois modelos numéricos no software DIANA, seguindo as estratégias de macro e micro-modelação. O propósito de fazer dois modelos foi permitir a comparação da resposta estrutural de cada modelo para carga monotónica incremental e concluir sobre a utilidade de uma abordagem baseada em macro-modelos para construções de alvenariado tipo arco.Posteriormente, foi analisada a eficácia da técnica de reforço através da realização de análises não lineares em ambos os tipos de modelo. Foi ainda feito o estudo paramétrico do reforço usando macro-modelo do arco. Finalmente, fez-se a comparação dosresultados entre os modelos reforçados analisados e discutem-se em detalheos resultados obtidos.

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ABSTRAKT

Sklepienia i łuki murowane są jednym z głównych elementów strukturalnych w wielu konstrukcjach historycznych. W związku z upływem czasu, działania obciążeń oraz różnych cech konstrukcji nośność obiektów historycznych maleje sprawiając, iż stają się bardziej podatne na uszkodzenie. Ugięcia i przemieszczenia fragmentów konstrukcji tworzą podwyższony stan naprężenia, który w konsekwencji może prowadzić do przedwczesnego zniszczenia. Aby zachować strukturalną funkcję łuków i sklepień oraz uchronić je przed zniszczeniem niezbędne jest zastosowanie wzmocnień konstrukcyjnych. Podczas oceny sposobu wzmacniania nie można zapomnieć, że obiekty historyczne należą do naszego dziedzictwa kulturowego i inżynierowie są zobowiązani do przestrzegania doktryn konserwatorskich, w których minimum interwencji jest jednym z głównych kryteriów. Aby dostosować się do wymogów doktryn przed zastosowaniem jakiegokolwiek wzmocnienia szczegółowa analiza konstrukcji jest niezbędna. Dlatego też, analiza numeryczna wydaje się być najlepszym sposobem na określenie skuteczności wzmocnienia przed zastosowaniem go.

Niniejsza praca magisterska pod tytułem: „Analiza numeryczna sklepień murowanych wzmocnionych membraną usztywniającą", ma na celu analizę zwiększenia nośności ceglanych łuków i sklepień poprzez zastosowanie usztywniających membran zamocowanych po stronie grzbietowej łuku.

Modele numeryczne stworzone na potrzeby niniejszego pracowania zostały przygotowane na podstawie łuków doświadczalnych testowanych przez Paolo Girardello na Universytecie w Brescia, Włochy. Bazując na geometrii i właściwościach materiałów otrzymanych z testów doświadczalnych dwa modele numeryczne, makro- i mikro-, zostały wykonane w programie DIANA Finite Element Analysis. Intencją utworzenia dwóch modeli było porównanie zachowania każdego z nich pod wpływem jednostajnie wzrastającego obciążenia oraz wyciągnięcie wniosków na temat użyteczności makro-modelu do analizy murowanych zakrzywionych konstrukcji.

Następnym krokiem była analiza efektywności proponowanej techniki wzmacniania przy użyciu nieliniowej analizy wykonanej na obu modelach. Końcową czynnością, było przeprowadzenie analizy parametrycznej wzmocnienia wykonanej na makro-modelu. Ostatnim etapem przeprowadzonego badania jest porównanie otrzymanych wyników z wszystkich wykorzystanych modeli oraz ostateczny wniosek, który z parametrów konstrukcji wpływa w największym stopniu na odpowiedź strukturalną konstrukcji.

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Chapter 1
INTRODUCTION

1.1. Background

Among various structural elements of historical constructions masonry arches and vaults deserve special attention. Thanks to them construction of some most beautiful and spectaculars buildings (like gothic cathedral) was possible. This was achievable due to remarkable properties of arches and vaults, which is compression state in which they work. During centuries, arches and vaults gave opportunity to create buildings on enormous scale with use of materials of low or almost null tensile strength, i.e. like masonry.

Historical constructions, over their lifetime, have undergone significant environmental influence by means of excessive load, water and wind erosion, settlements etc. Therefore, the current condition of many monuments and important buildings is recognized as endangered by collapse. That is why, the field of structural strengthening of existing constructions grows every year. Historical constructions are subjected to constantly acting ageing processes, thus it is of high importance to develop means of sustaining their existence and, if possible, restore their primary resistance of load capacity and stability.

Structural strengthening of historical construction is a delicate matter. It has to be preceded by careful studies of the nature of the building: its structural response to differential load

combinations, origins of the problems and, most of all, it's expected behaviour after strengthening intervention. That is why, every building has to investigated and treated as an individual case to address its needs best. It cannot be forgotten that a historical building is an object of great value form the cultural point of view, which leads to applying methods that will not alter its aesthetic side, as well as will allow to keep its function or give new one.

Over decades many preservation works have been done and a significant number of strengthening techniques was developed. Unfortunately, not all of them were successful as it happened that the intervention sometimes destroyed the cultural value of the building. To prevent repetition of poor restoration practice, the International Council on Monuments and Sites (ICOMOS) formulated some ground guidelines to help organizing work done on historical constructions so that monuments would be approached in a better way.

A body in ICOMOS, ISCARSAH (International Scientific Committee for Analysis and Restoration of Structures of Architectural Heritage) created a set of principles and recommendations aimed in ensuring rational methods of analysis and repair of historical construction appropriate to the cultural context. It recognizes that conventional calculation techniques and legal codes or standards oriented to the design of modern constructions may be difficult to apply, or even inapplicable, to historical structures. Additionally, recommendations state the importance of a scientific, multidisciplinary approach involving historical investigation, inspection, monitoring and structural modelling and analysis [1].

The principle of ISCARSAH state several important issues that have to be taken into consideration, always when approaching a historical construction. Authenticity of the structure with respect to the cultural context of the monument has to be preserved. The value and authenticity of architectural heritage cannot be assessed by fixed criteria because the respect due to each culture requires that its physical heritage is considered within the cultural context it belongs.

The value of each historic building is not only in the appearance of individual elements, but also in the integrity of all its components as a unique product of the specific building technology of its time and place. Thus, removal of the inner structures retaining only a façade does not satisfy conservation criteria.

The peculiarity of heritage structures, with their complex history, requires the organization of studies and analysis in steps. Condition survey, identification of the causes of damage and decay, choice of the remedial measures and control of the efficiency of the interventions has to be done. Nevertheless, no action should be undertaken without ascertaining the likely benefit and harm to the architectural heritage. Furthermore, where urgent safeguard measures are necessary to avoid imminent collapse they should avoid permanent alteration to the fabric.

A full understanding of the structural behaviour and material characteristics is essential for any conservation and restoration project. It is essential on the original and earlier states of the structure, on the techniques that were used and construction methods, on subsequent changes, on the phenomena that have occurred, and, finally, on its present state. Diagnosis of the state of the structure should be based on quantitative approaches (structural analysis, monitoring, in situ-laboratory tests, historical/ archaeological research) and qualitative ones, like direct observation of the damages and decays occurring in the building. A proper diagnosis has to be prepared aiming on determining the causes of existing problems.

After recognition of the origins of the damage and decay, a plan for remedial works and controls should be created. As mentioned, the plan should be addressed to fix the cause of the problems, rather than just symptoms. Only indispensable actions should be held to maintain architectural integrity of the constructions. Above all, rule of minimum intervention should be applied to show the respect towards historical and cultural significance, as well as to existing materials and structure.

Overall, while designing an intervention plan it has to be remembered that the above mentioned aims can be fulfilled by respecting some key orientations[1]:

- Safety evaluation and requirements; understanding of actions;
- Compatibility between original and newly added materials;
- Least intrusiveness;
- Durability;
- Reversibility;
- Controllability.

With the help of the ISCARSAH guidelines and with experience of the people working on restoration of a particular historical construction, the goal of maintaining world heritage, with its cultural, not altered value, should be achieved.

1.2. Motivation and objective of the thesis

The thesis is focused on analysis of strengthening techniques applicable to masonry arches and vaults present in historical construction. Elongation of life of structural elements in present construction is crucial, as many of historical buildings are defined as Cultural Heritage and cannot be alter in any way that will destroy its original meaning or function. Therefore, responsible strengthening of historical constructions is an important issue in modern restoration practice.

With the presence of new composite materials many traditional reinforcement methods were disregarded as not enough efficient in comparison with new technologies. The main aim of this

thesis is to carefully analyse one type of traditional strengthening technique, use of extrados stiffening elements made of masonry (sometimes called ribs).

Further, a comparison research is done on two types of numerical approaches: macro- and micro-modelling. In vast variety of researches done on masonry arches and vaults, the way to represent complex behaviour of an arch is done with use of micro-models. This is due to the fact, that representation of mortar joints as interfaces is of high importance to credibly reproduce arch behaviour. What is more, macro-modelling approach is considered to be able to realistically replicate only global behaviour of a structure, rather than some local phenomenon. Nevertheless, it has a big advantage that should be taken into account, which is higher and quicker feasibility of the model.

Regarding above mentioned issues, the thesis will try to simulate structural response of a masonry arch with the use of macro-modelling approach and to compare obtained results with the developed micro-model. This way a new numerical tool for analysis of masonry arches might be worth considering in the future analyses.

Detailed goals of the thesis are defined as follow:

- To compare and to evaluate the applicability and limitation of the finite element numerical methods applied to a masonry arch and to accurately demonstrate the behaviour of the structure;

- To define the reliability of the macro-modelling approach used for masonry arches in comparison with the micro-modelling method;

- To identify the effectiveness of the strengthening technique by means of numerical modelling and parametric study.

1.3. Thesis organization

The Master thesis is organized into seven chapters, of which the introduction chapter is considered as the first one.

Chapter two presents a brief introduction to the history of use of arches in construction. Description of the structural behaviour of arches and simple vaults are also presented. Main body of the chapter is devoted to variety of strengthening techniques applicable to historical constructions.

Chapter three contains some crucial issues that have to be defined prior to any analysis. Main types of analysis are listed and explained, with focus on applicability to masonry arches and vaults and advantages and drawbacks of each.

First part of chapter four describes the modelling of reference arch of the case study that is developed in the paper. Firstly, information about experimental tests is provided, with specifications of the geometry and materials of the masonry arch, as well as the type of strengthening applied. Further, two type of numerical models of experimental arch created in DIANA software are presented. Macro- and micro-model will be developed in order to define the differences in behaviour of each of model. The aim is to identify if a macro-model can represent complex behaviour of and arch as well as a micro-model. Characteristics of macro- and simplified micro-model are presented: material properties, geometry, mesh and constitutive laws used for each element.

Chapter five is focused on analysis of the strengthening technique for both numerical models of reference arch. Macro-model of strengthened arch will be considered in two manners. Firstly, the model will be built up as a continuous element (without distinction between arch and the reinforcing constituent). The whole model will be constructed of masonry treated as a composite material of uniform and homogenized properties. The result of analysis will allow to define the static behaviour and the influence of the reinforcement in terms of ultimate load capacity and stiffness. Secondly, an interface will be applied to the macro-model of strengthened arch to identify the difference in the structural behaviour between this and the first, fully continuous model. As the last step, the simplified micro-model of the experimental arch will be strengthened with extrados stiffening diaphragms built as an element of homogenized material properties. An interface between original arch and new element will be introduced to create a model closer to reality.

To fully exploit the utility of the macro-model, parametric study is presented in chapter six. Three modifications are applied to the reinforcement. Change in thickness and material properties allows to get differential results which help to conclude on the effectiveness of the strengthening technique.

Final remark, presented in chapter seven, are focused on comparison of behaviour and results between different types of numerical model of the same element. Furthermore, the conclusion on the efficiency of the strengthening technique is formulated based on observation of results from both numerical models. Results of the parametric study are presented with final statement on the influencing parameters. The chapter also includes a short review of opportunities for further research in the field of strengthening with extrados stiffening diaphragms.

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Chapter 2
MASONRY ARCHES AND VAULTS – OVERVIEW

Masonry structures are in general among constructions that are hardest to analyze. From point of view of the material parameters to the complexity of the construction it is never easy to define the state of a structure. Masonry, as a material, presents heterogeneous properties and its behaviour should not be considered as linear, especially under tensile stress state. Moreover, past builders and architects showed endless imagination in the manner they created new shapes and construction details which hinder the analysis of the structural performance of many buildings. Nevertheless, humanity should be thankful for their creativity because often, their innovative solution gave us the opportunity to admire their work.

2.1. Historical background on masonry arches and vaults

Vaulted constructions, suggested by nature itself, come after the use of shapes which are stable by only mobilizing compressive – arches and vaults. Used already by ancient civilizations in Mesopotamia, it was intensively exploited by Romans and has prevailed as the main roofing approach for large constructions up to the 19th-20th c. technological revolution [2].

True vaulting poses significant construction challenges, as the need for expensive and time-demanding shores and centering. Some cultures (Greek Mycenaean, Sumerian, Sasanian Persia etc.) skipped these difficulties by resorting to false vaults and arches built as sequence of self-

stable cantilevers. The technique affords only very limited spans to the cost of a millennium BC significant rise and high consumption of material.

True arch and vault construction has been achieved by the following means:

- Using true centering and shores (normally made of wood);

- Using earth or rubble fillings and mounds instead of centering;

- Using smart construction procedures avoiding or reducing the need for centering.

Younger civilizations (like Romans and the Byzantine Empire) noticed that use of arches and vaults allow to significantly enlarge the span between structural elements, and better transition of the forces towards the foundations. Therefore, its use allowed the construction of more spacious buildings of thinner structural elements, thus it reduced the cost of the erection of a structure.

From the Roman structures, to the Gothic ones, with each year the use of arches and vaults was exceeded up till modern times. The invention of combined steel-concrete material solved problems of tensile resistance and allowed construction of big span buildings without the need for arches.

2.2. Structural behaviour

Masonry arches and vaults owe their popularity in historical constructions to the remarkable property that they posses - state of compression in which they work. Nevertheless, their stability and safety highly depend on the mechanical properties of the masonry constituents as well as on the structures geometry. Due to the fact that masonry has negligible tensile strength, the safety condition of an arch is conditioned on the thrust line lying within the borders of each section of the arch itself. If the thrust line becomes tangent at any point of the arch section, formation of plastic hinge will occur. A hinge enhance further arch deformation, by material crushing in the compressed side of the section. This changes the structural behaviour of an arch and increases the probability of development of sufficient number of hinges (minimum four) to transform an arch into a mechanism and to cause its collapse [2], see Fig. 2.1.

a)

b)
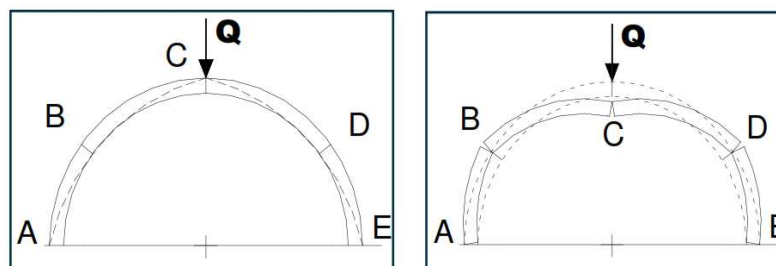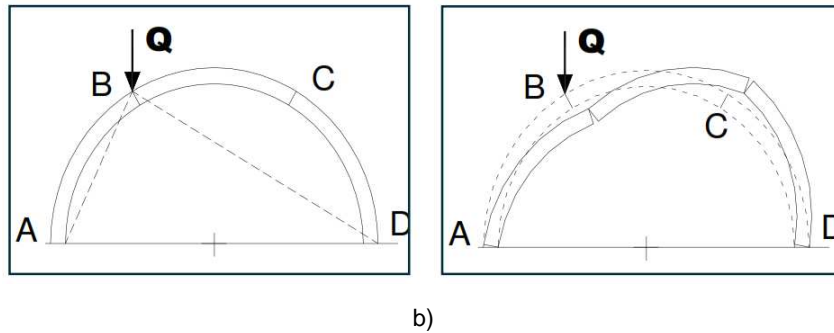
Fig. 2.1. Line of thrust and failure mechanism of an unstrengthened masonry arch subjected to
vertical load applied to: a) middle of the span; b) ¼ of the arch span [4].

Strengthening of a masonry arch can significantly change its structural behaviour as well as increase its ultimate load capacity (as proven in the literature mentioned along the following chapters). Nonetheless, the structural response of a strengthened arch varies depending on the type of the intervention applied. Some techniques prevent formation of plastic hinges (like FRP or SRG), other increase load-carrying capacity but do not alter the failure mechanism (i.e. lateral spandrel walls). Therefore, each strengthening solution has to be considered individually depending on the result that should be obtained.

The most significant impact on the collapse mechanism of a strengthened arch makes use of continuous reinforcing system, like in the case of FRP, SRP or SRG. As shown in the Fig. 2.2 presence of FRP strips or SRG layer, applied at the intrados or at the extrados of the vaults alters the mechanism of formation of the plastic hinges, because the reinforcement (fibers and steel cords) can bear the stresses occurring at the tensed edges. In those sections (which are in combined axial and bending stresses), as for concrete structures, the resistance depends on the masonry compression strength and on the fiber or steel tensile strength. In any case, the resistant mechanism is substantially enhanced.

In the case of extrados strengthening the line of thrust can fall outside the lower edge of the vault without any structural collapse (Fig. 2.2a). For the case of a vertical load applied to 1/4 of the span, the hinge formation in the extrados is prevented. As a consequence, the vault becomes an isostatic structure (it is a three hinges arch) consisting of two curved beams strengthened on their upper sides. Such a scheme allows one to obtain the stress parameters in every section of the structure by simple geometrical and equilibrium relationships.

In the case of a structure strengthened at the intrados, although the resultant static scheme is similar to the one adopted in the previous case, note that the distribution of the stress parameters is very different (Fig. 2.2b). First, the line of thrust falls outside the upper edge of the structure and the fibers prevent the hinge formation close to the point of application of the load. As a consequence, the external load is no longer in a nodal position, so the trend of the stress parameters along the vault changes. In particular, comparing the two cases, the flexural moment changes its sign and the shear stress at the springers is reduced [4].

a)

b)
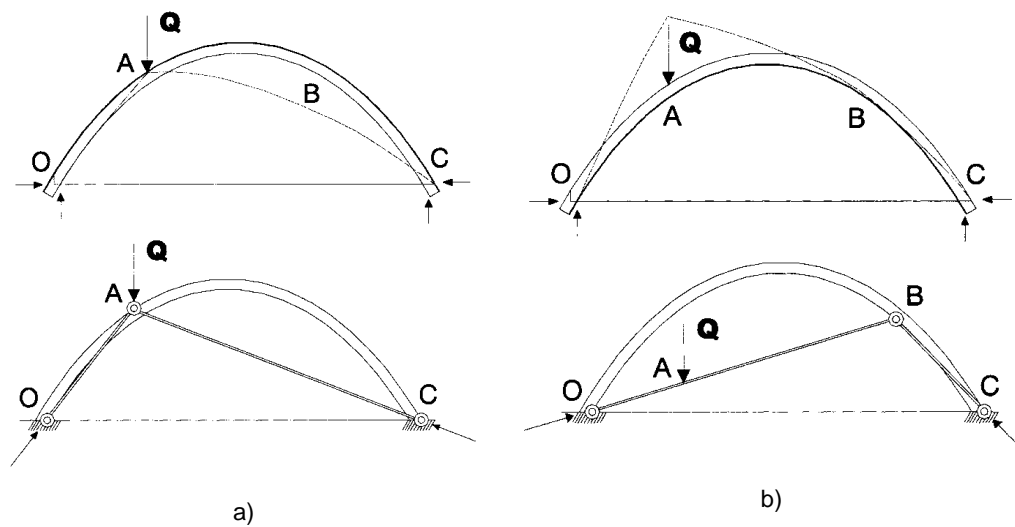
Fig. 2.2. Thrust line of an arch strengthened with use of FRP at:
a) extrados; b) intrados [4].

2.3. Strengthening techniques

During decades of rehabilitation of historical constructions many strengthening techniques were developed. With the progress of technology, more sophisticated approaches were develop, therefore nowadays there is a wide range of interventions that can be used. Choosing the right method is a complicated process in which many factors have to be taken into consideration. The designer has to remember about factors such as:

- Cultural value of the structure, which will define where the intervention can be applied (i.e. many times the surface of the intrados in covered with painting, mosaics etc.);

- Toward what loading condition the strengthening is pointed out: seismic performance, upgrading load-carrying capacity or stabilisation of damaged construction;

- Reversibility of the technique and its interference into the original element, which is often limited due to the cultural value of the structure;

- Cost estimation is an important factor, as some techniques require high costs;

- Feasibility and efficiency of the method in the particular conditions.

In the following pages a short presentation of traditional and innovative strengthening techniques will be presented focusing on the properties of each one. The advantages and drawbacks of them will be pointed to show that all methods have to be carefully considered before applying to any historical construction.

2.3.1. Steel ties (external/internal)

Steel ties, located above the extrados or under the intrados are one of the most widely used strengthening method. This relatively easy-to-make technique helps to redistribute unfavourable stress condition that comes from thrust. Too high value of thrust creates outward movement of the abutment of the arches that, consequently, are subjected to crack formation that decreases structural capacity of the construction.

Use of ties influences the stiffness and stability of the construction. Ties have following effects:

a) Decrease the displacement of the abutment;
b) Decrease the formation of cracks in the arches and supporting walls;
c) Help in distribution of internal forces, even in the case of further damage.

Accurate performance of the tie, in a historical construction, is connected with the internal force introduced in and by the tie (see scheme at Fig. 2.3). A good solution, for obtaining the exact force needed, is primary pretensioning of the tie to the correct value corresponding with the compression capacity of the wall [6].

Unsuitable force introduced by the tie to the wall can lead to further cracking of the construction. Therefore, it is of high importance to determine the correct value before applying the solution. Furthermore, it is worth considering the way of anchoring the tie to the wall and its thermal deformations as it might affect the efficiency of the tie.

a)   b)
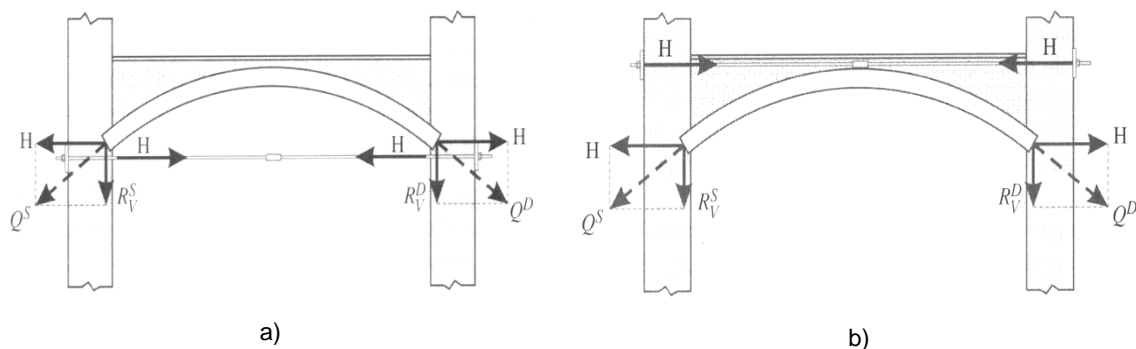
Fig. 2.3. Location of steel ties counteracting the thrust from the arch:
a) under the intrados; b) above the extrados [5].

An important issue while strengthening with the use of steel ties is the correct strategic positioning of the ties and accurate preparation of the wall. One of the rules used while tying historical constructions is transition of the forces at possibly big area as it is connected with low punctual compressive strength of the masonry wall.

Traditional steel tying has a big disadvantage, namely, they alter the visual aspect of the buildings through the anchors visible on the facades and/or through the ties themselves. Nowadays, this issue is often solved by the use of steel ties glued to the masonry wall [7]. This modern solution presents a set of advantages like: easier access to the wall from inside, shorter installation time and lower cost. Another modern solution regarding tying is use of ties made of composite materials, as explained in [8].

2.3.2. Reinforced concrete shells

Reinforced concrete shells is a good strengthening solution especially for vaults of big span. Its idea is to create a covering surface anchored to perimetric walls, that will take the structural role from deformed masonry vault [9].

Because of paintings or frescos frequently present in the intrados of arches and vaults the masonry element is suspended from the reinforced concrete shell. This method, quite controversial from the conservation point of view, is one of the few techniques which can help in strengthening vaults of large spans. However, it present set of important issues that have to be discussed. How to anchor properly the masonry vault if there is limited information about the state of the sections? Also, because the suspension is made in several points problems with shear stresses occurring in those places have to solved [6]. Sample application of this intervention solution can be seen in Fig. 2.4.
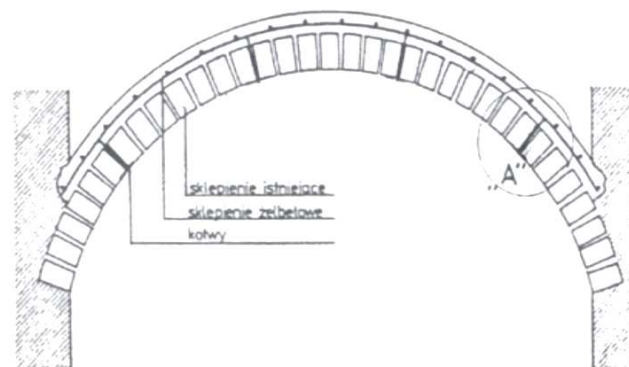
Fig. 2.4. Strengthening of an arch with use of reinforced concrete shell applied in the extrados [6].

The design of the reinforced concrete shells has to be done regarding the typical procedure. The quickest way of application in the extrados of the arch/vault is by shotcrete. Shotcrete undergoes placement and compaction at the same time due to the force with which it is projected from the nozzle. It has two type of consistency before application, therefore it can be distinguished as wet-mix and dry-mix versions .It can be impacted onto any type or shape of surface, including vertical or overhead areas [10]. The decision which concrete application method use (wet-mix or dry-mix) is of significant importance and has to be individually defined, depending on the possibilities and

needs of the construction site. Therefore, it cannot be generally classify which of the method is better.

Previous to use of reinforced concrete shells a deep examination of the applicability and effects on the construction has to performed. This technique might solve mechanical problems occurring in the vault, however it might cause other ones like non-reversibility of method and incompatibility of structural elements. Thanks to in-detail studies aforementioned difficulties might be avoided. Nevertheless, decision to use reinforced concrete shells has to reconsidered taking into account all the possible benefits of the strengthening, but even more, focusing on undesirable effects that the method can bring. If the benefits are insufficient in comparison to possible problems change of strengthening concept is an reasonable approach.

2.3.3. Suspensions of arches and vaults

Suspensions of arches and vaults is a technique based on creation of new load carrying construction that will take the structural role from the masonry element. In particular, this technique is used when other measures are inadequate and propping beneath the element cannot be a solution. The suspension structure can be made of timber elements of significant capacity. Wood is light and can easily adjust to the environmental conditions above the masonry vaults.

It consists in adding an active connection (sometime provided with dissipative elements) of the original structure with an upper structure (sometimes the roof structure) carrying part of the load, in order to stabilize and relive load from the original structure.

One of the most famous examples of strengthening with this method is strengthening of the vaults in the Basilica of Saint Francisco di Assisi [11]. Here the solution was complex and the engineering created a set of glulam ribs strengthened with AFRP to precisely imitate the shape of the vault as shown in the Fig. 2.5.

a)   b)

Fig. 2.5. Use of suspension solution in Basilica of Saint Francisco di Assisi [11].

The important factor in effectiveness of the solution is creating a precise curvature of the ribs. Specially, this is important when ribs are going to be made of glulam. High level of detail in the curvature can be avoided if the ribs are made of solid timber prepared in-situ. Nevertheless, this is connected with lower quality of the wood and, thus, smaller assurance of the safety of the construction.

As used in the Basilica of Saint Francisco di Assisi, a good improvement of the method is application of FRP to increase the load carrying capacity and limit the size of the ribs. It is also worth considering the use of lightweight, spatial steel trusses as the suspending construction. The main concern of this technique is the need for sufficient area of adherence as well as solving the problem of shear stresses occurring in the masonry-rib interface.

2.3.4. Grout injections; rebuilding

Apart from the range of techniques focused on strictly structural strengthening of constructions, restoring the original properties of the material pays a great role in reconstruction. Thanks to those actions an element might regain its mechanical properties and, consequently, there might be no need to apply structural interventions. Nevertheless, in general, those measures are considered more like a complementary works rather than as a way to strengthen a construction.

2.3.4.1.   Grout injections

The aim of injection is to restore the uniformity of the material as well as to seal all cavities in order to protect the inside of the masonry from harmful agent, like humidity. The act of injection is a delicate matter requiring many preliminary actions. Many factors have to be considered, among which the choice of the grout composition and its consistency are of the highest importance. Fig. 2.6 presents a sample of strengthening solution with the use of injection.
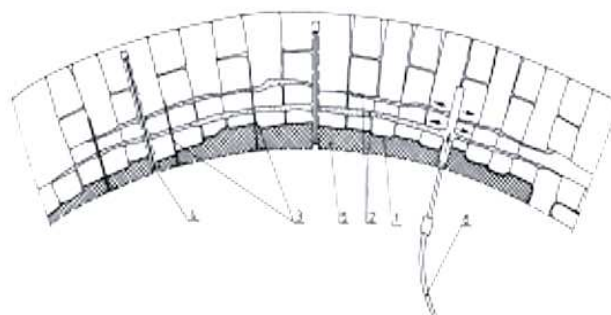
Fig. 2.6. Examples of application of injection into a masonry arch [12].

There are three main methods of injection regarding arches and vaults. They depend on the way how the injection is made. Namely, the following methods can be defined [13]:

1) Gravity injection – the grout is injected with the use of the gravity from a container placed above the vault;

2) Pressure injection- grout is injected with the use of a special unit;

3) Vacuum injection – grout is inserted inside the masonry thanks to underpressure created by a vacuum unit.

As mentioned before, the main aim of injection is to uniform the masonry element in its section so that it would perform like a full material. As well, to protect the material inside. However, there are exceptions that injection is used as a singular strengthening solutions. Though, it has to be remembered that injection does not solve the problems of the construction, therefore it can be used just if the destruction process is stabilized. Moreover, grout injection is an highly intrusive and non-reversible action, therefore it application has to be carefully reconsidered as if this kind of actions ascertain benefits higher than the cost of non-reversibility [12].

Main types of grouts are connected with the type of base matrix. They can be mineral (cementicious, gypsum, lime-gypsum) and based on polymers, i.e. epoxy [14]. Mineral grout are in general cheaper and compatible with existing masonry construction, nevertheless the properties of the old and new material have to be checked a priori to application. Epoxy grouts allow to create a substance of required properties, however possible incompatibility has to be taken into consideration.

2.3.4.2. Rebuilding

Rebuilding is a concept that can be applied to walls, but also to arches and vaults. The idea is to recover the original structural response by means of restoring continuity along a cracked area by remaking it [9]. In this technique, use of materials similar in composition, shape, dimensions, stiffness and strength, to those employed in the original wall is preferable, to pursue compatibility and homogeneity in the global behaviour. This method is more a repair than strengthening of the element, therefore an important matter is to remove the origins of the problems.

Fig. 2.7 presents an example of the rebuilding works done on a vault in St. Fermo Church in Verona, Italy [14]. Here the rebuilding was just a first step of intervention aiming at repair of a deformed vault. Further works were connected with strengthening the vault with use of FRP strips.

Fig. 2.7. Reconstruction of a part of a soldier masonry vault
of the St. Fermo Church in Verona, Italy [14].

2.3.5. Fiber Reinforced Polymers (FRP)

Among the materials used to repair or to upgrade existing historical and modern structures, there has been a significant interest in the use of fiber-reinforced polymer (FRP) composites, either in the form of externally bonded or near-surface mounted reinforcements. This interest in FRP is connected with some features that it possesses, like: low weight, corrosion immunity, high tensile strength, adaptability to curved surfaces, and ease of application, which makes it attractive to be used in repair or strengthening works. Several studies were performed to define the properties of arches strengthened with this technique: [16], [17], [18], [19]. In all of them FRP present a set of advantages, but it has some drawbacks, among which are: high cost and low fire resistance, durability uncertainty and possible mechanical and physical incompatibilities. As well, brittle failure of FRP is a feature that has to be carefully considered when applying FRP to a masonry element.

Continuous fiber-reinforced materials with polymeric matrix (FRP) can be considered as composite, heterogeneous, and anisotropic materials with a prevalent linear elastic behaviour up to failure. Composites for structural strengthening are available in several geometries from laminates used for strengthening of members with regular surface to bi-directional fabrics easily adaptable to the shape of the member to be strengthened. Those materials are also suitable for applications where the aesthetic of the original structures needs to be preserved (buildings of historic or artistic interest) or where strengthening with traditional techniques cannot be effectively employed.

There are three main types of FRP used in civil engineering, which depend on the type of fiber used. Those are: carbon, glass and aramid fiber, which differ in the mechanical properties (Fig. 2.8). Recently an increased interest in use of basalt fibers was noticeable, due to its similar properties to glass fibers in terms of ductility and load capacity (according to [20]) and its much lower cost in comparison with carbon or aramid fibers, thus seem accurate for low-cost interventions. Furthermore, basalt fibers are in-toxic, no environmental restrictions apply to it. Additionally, because of its thermal isolating properties, is suited for use as fire-protection application.

FRP can vary also in the type of matrix used: epoxy of mortar based. FRPs come with several type of fabrics: bars, sheets and laminates. With such big assortment of fabrics it is essential to define the type most suitable for the need.
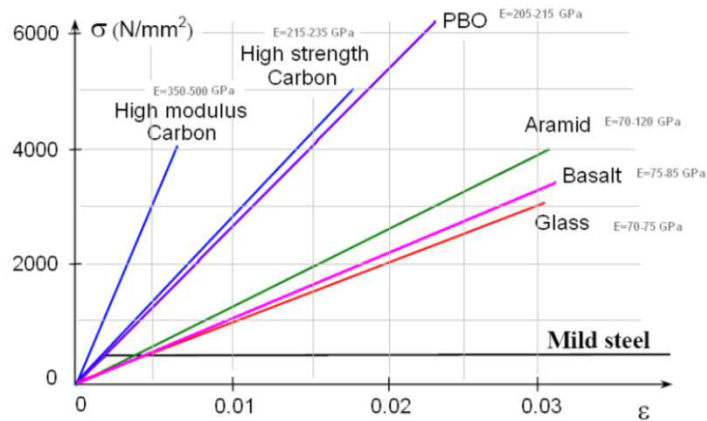
Fig. 2.8. Comparison between different types of FRP
regarding used fiber and mechanical properties of each [20].

As explained in Chapter 2.2 the application of the strengthening material modifies the static behaviour of the arch by inhibiting the formation of the fourth plastic hinge. Therefore, the collapse of the structure is due to other mechanisms, which are dependent on the limits of strength of the constituent materials (original vault and reinforcement) and on the structural interactions of them at the local level.

Because a design criterion taking into account the premature failure of the masonry against the reinforcement has been adopted, the following possible mechanisms of collapse are considered:

1. Crushing of the masonry

A first formulation of the model for the evaluation of the ultimate strength of strengthened sections under combined compressive and bending stresses, similar to the one assumed for RC structures, was developed by Triantafillou in [21]. As for the constitutive laws of the materials, a linear elastic behaviour was adopted in the model for the reinforcement whereas a rectangular stress-block law was considered for the masonry (Fig. 2.9). The state of stress in a section of the vaults can easily be obtained by relations of equilibrium.
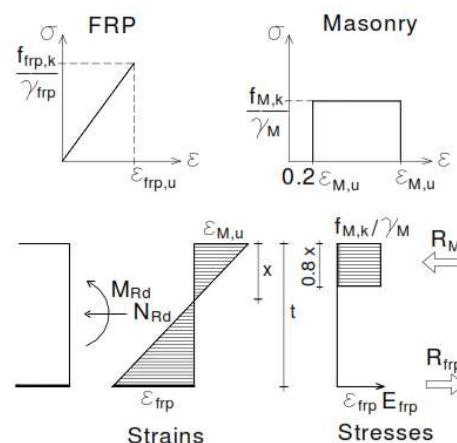
Fig. 2.9. Constitutive laws and stress-strain distribution in section [4].

2. Detachment of the adhesion system

Reinforcement's detachment from the support is due to the presence of a component perpendicular to the plane of the tensed fibers. This effect provokes a mechanism that could be defined as a ''tear''. Fig. 2.10 shows a portion AB of the laminate, having length d_s and radius of curvature R, whose internal tensile force per unit width T is assumed constant; d_N is the component normal to the segment AB that is responsible for the phenomenon. By using simple geometric relationships, both the arch equation and the tensile stress of the fiber are known and it is possible to evaluate in any section the normal component d_N.

$$\frac{\delta N}{2} = T \cdot \sin\left(\frac{\delta\phi}{2}\right)$$

$$\delta N = T \cdot \delta\phi$$

$$\frac{ds}{\delta\phi} = R$$
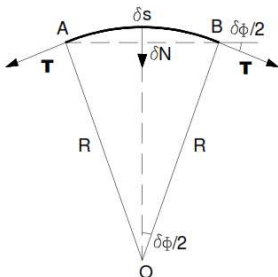
$$\frac{dN}{ds} = \frac{T}{R}$$

Fig. 2.10. Formulation of the "tear" mechanism of the detachment of the FRP from the masonry arch [4].

The simplest way to deal with this phenomenon was to equal the perpendicular tensile strength of masonry (measured through pull-off tests) to the calculated normal component d_N/d_s. Therefore, it is possible to find backwards which is the maximum acceptable tensile force T that can be assigned to the reinforcement and, consequently, the maximum external loads (e.g. see[15], [22]). Another possible solution to this problem is use of anchors to increase the resistance against the detachment of the FRP system [24].

3. Mortar-brick sliding due to shear stresses

Such a mechanism is caused by the presence of the shear stress component in each section. The mathematical model describing the resistant mechanism is still under calibration by experimental tests. It is supposed that the sliding resistance is caused by two main components, R_m due to the masonry and R_{frp} to the FRP laminate. As for the masonry contribution, a friction model following the Coulomb law is assumed and a dowel action effect is considered for the reinforcement. Fig. 2.11 shows a scheme of a section of the vault in the failure conditions. The experimental tests have allowed analysis of the above mentioned mechanisms and proposal of simplified mathematical models.
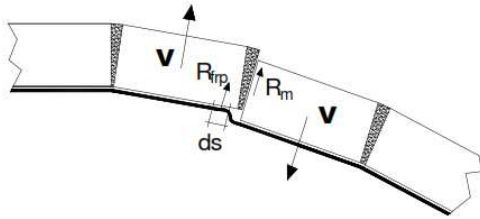
Total shear strength:

$$R_{tot} = R_m + R_{frp}$$

Coulomb-type law (masonry):

$$R_m = \mu N$$

Dowel-type action affect (fibers):

$$R_{frp} = \sigma_{a,max} \cdot A_{frp}$$

Fig. 2.11. Sliding mechanism between mortar and brick and
formulation of the shear resistance of the section [4].

The typical and most effective application of FRP is on the whole length of the arch. This was confirmed by experimental tests with use of partial length reinforcement [23]. In these case, the structure cannot be handled as a reinforced beam, since the failure mechanism is someway similar to that of the unreinforced structure. This means that the hinges' position is changed by the presence of the FRP and they tend to develop at the edges of the reinforcement.

2.3.6. Innovative composite materials (SRP, SRG and TRM)

Steel Reinforced Polymer, Steel Reinforced Grout and Textile Reinforced Mortar are three modern solutions in the topic of strengthening of masonry arches and vaults. Their development can be associated with the new conservation trend to use traditional materials. But more important are the properties that those methods combine thanks to the constituent materials that are used: steel provides ductility for the structure, avoiding brittle failure of the structural element; use of mortar prevents incompatibility problems between the matrix and the substract. Also a new trend is the use of natural fibers, due to their low cost, low elastic modulus and moderate strength is observable. Furthermore, low fire resistance, relatively high cost and other disadvantages of FRP forced search of new technology.

2.3.6.1.   Steel Reinforced Polymer and Grout

Due to the above mentioned difficulties with the use of FRP a new family of composite materials, based on high strength twisted steel wires (shown in Fig. 2.12) embedded within a cementitious grout (referred as SRG) or in epoxy matrix (referred as SRP) was developed. Particularly, SRG combines the traditional advantages of composite materials with higher fire resistance and lesser cost. However, SRG and SRP are still relatively new techniques and few studies have been done to define its properties ([4], [24], [25]). Although, an increase of interest regarding use of SRP and, specially, SRG has to be noted, especially in the use for masonry arches and vaults [26].
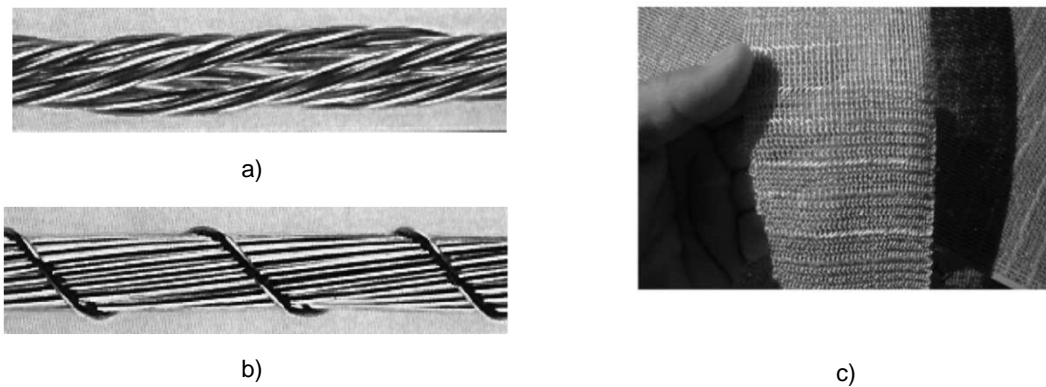
Fig. 2.12. Reinforcing component of the SRP/SRG technique: a) steel cord made by twisting two-wire strands around three wire strand; b) steel cord with 12 wires wrapped by one wire;
c) cords held together by two knit yarns made from polyester [25].

According to Borri et al.[4], SRP and SRG presents some significant advantages when comparing with FRP:

- Steel cords have shear strength which might simplify problems regarding connection and anchorage of the intervention;

- Use of steel changes the type of failure behaviour of the arch: from brittle to more ductile;

- Use of mortar matrix provides higher fire resistance than use of epoxy matrix as well as prevents occurrence of incompatibility between original element and the applied intervention.

As it was mentioned in section 2.2, the collapse mechanism of unreinforced arches is connected with formation of at least four plastic hinges which change the arch into a mechanism. With the use of SRP and SRG the structural response changes. The failure mechanism looks like the one of an arch strengthened with FRP in the extrados (explained in previous section). The SRP/SRG layer prevents development of plastic hinges, thus the failure mechanism has its origins from the limits of strength of the constituent materials (masonry and reinforcement). Apart from the above mentioned, failure can originate from the structural interaction of the materials resulting i.e. in debonding or localized shear. Depending on the properties of the materials, position and amount of the reinforcement there can be distinguished several collapse models: masonry crushing, reinforcement rupture or debonding and shear sliding along mortar joints.

An experimental study by Borri et al. [4] compared FRP, SRG and SRG reinforcement in different configurations and provided some interesting result that will be summarize here. Arches strengthened in the intrados were more efficient in terms of ultimate load capacity, inverted trend was observed in terms of ultimate deflection. Steel cords allowed greater strength increase than carbon fibers, both in intrados and extrados arrangement. Failure mechanisms were caused by shear sliding (extrados arrangement) or laminate debonding (intrados) for all types of strengthening techniques (materials).

2.3.6.2.   Textile Reinforced Mortar

The drawbacks of FRP, like physical and chemical incompatibility with masonry, enforced research of new techniques. Textile Reinforced Mortar came with the idea of substituting organic matrix present in the FRP technique with inorganic one – mortar. This replacement shows important advantages of the TRM, such as: water vapour; permeability and compatibility with masonry; it is appropriate to use it in humid substrate (which in the case of SRP and SRG is not advised due to corrosion possibility); fire resistance; ease of application - even over an irregular shape. One of the important issues is need for compatible strengthening core in textile form.

One of the possible textile fibers that might be used in TRM are basalt fibers shown in Fig. 2.13, an idea presented in [27]. Basalt fibers have similar properties like glass fibers, which means better ductility and ultimate load capacity. Furthermore, shows set of other advantages listed in section 2.3.5.

a)   b)

Fig. 2.13. Basalt-Textile applied to a) a specimen; b) an experimental arch [27].

Experimental work done on masonry arches, presented by Garmendia et al. in [27] show that Basalt-Textile Reinforced Mortar (BTRM) highly increases the structure's ultimate load capacity and deformability. The cement-based mortar, provides compatibility (tested in case of stone masonry). Moreover, the intervention technique presented good structural behaviour, working until the structure's collapse. The failure mechanism was not connected with debonding like in many polymer-based reinforcements. This is an important feature in case that the strengthening would have to be removed from real building (no ripping of the substrate).

2.3.7. Extrados stiffening elements

Use of extrados stiffening elements (sometimes also called ribs) as a strengthening solution is an old and new idea at the same time. It can be said that nowadays it is being rediscovered by engineers thanks to its effectiveness, simplicity in concept and change in the conservation approach towards traditional materials.

In order to increase arch/vault resistance and stiffness, traditionally spandrel masonry walls are proposed, simple or reinforced with composite materials [28]. Nevertheless, it has to be remember that application of this technique is connected with additional deadload which might come with some issue to be solved. Therefore, some innovative approaches are developed in which the strengthening ribs are made from composite materials [29] or glued timber [30], [31]. Depending on the concept, ribs can be understood as a continuous elements located throughout the extrados or spandrel walls placed on both sides of the arch at the extrados.

Strengthening solution with the use of continuous ribs is presented in two papers, however there are some important differences in the approaches. Bednarz [30] proposes the use of a class GL25 glulam wooden beam attached to the brick using steel anchors, glued at the extrados oh the arch, shown in Fig. 2.14. In this experimental work many strengthening techniques were used (i.e. use of CFRP strips or TRM) in order to make a comparison work between methods when subjected to static, unsymmetrical load condition. The results obtained show that use of the glulam rib increased, with the higher rate, ultimate load capacity of the arch. The failure occurred in the connection anchor-arch and not in the wooden rib, therefore it can be concluded that the strengthening solution works until the collapse of the arch.

a)   b)
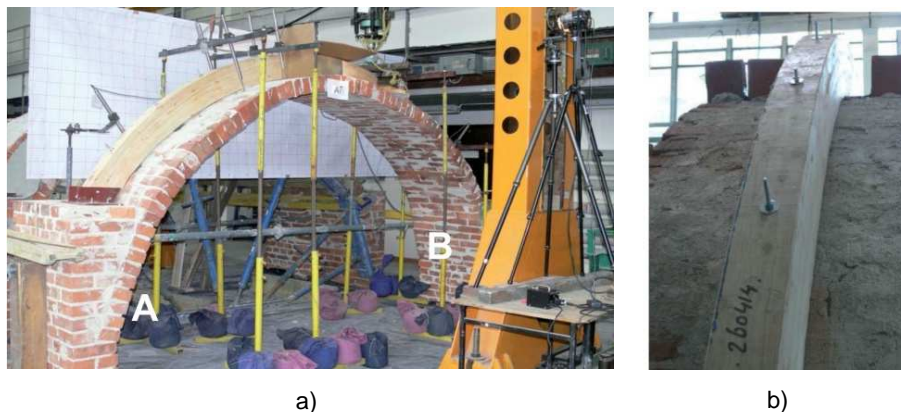
Fig. 2.14. a) View of the experimental arch strengthened with glulam beam;
b) Detail of anchor of the glulam beam-arch [30].

In [29] the idea behind the work was to create a strengthening solution to upgrade the seismic performance of single leaf vault (Fig. 2.15b). Lightweight ribs proposed are simply overlaying the vault extrados profile and design to provide a passive confinement to the vault. No shear transfer other that friction in allowed along the vault-to-rib interface. In this way the vault decompression is prevented or limited and in static conditions the dead load is supported by the existing masonry vault, which maintain the original compression state. In seismic condition the rib constrain the vault, thus providing passive confinement. The rib tubular cross section is made of lime mortar reinforced with glass fiber plaster mesh; the inner core is made of polystyrene element (Fig. 2.15a).
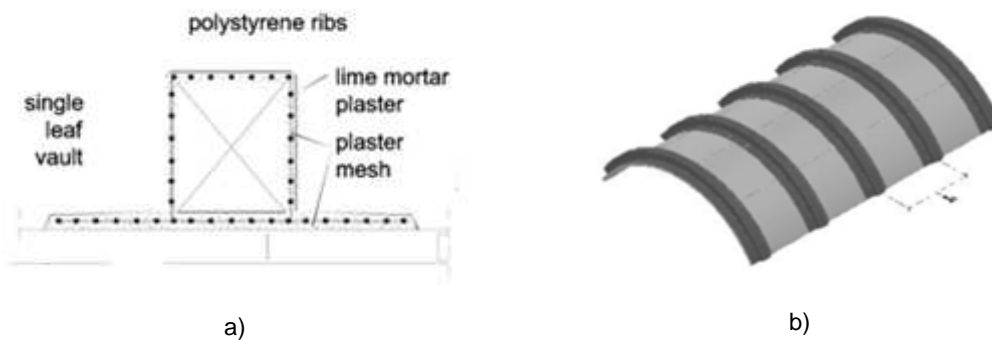
Fig. 2.15. a) Lightweight rib cross section; b) Continuous ribs placed over a vault [29].

Ribs treated as spandrel walls were also proposed as a strengthening solution in [28], [31]. Girardello, [28] performed an experimental work on masonry arches, using different strengthening techniques (SRG, SRP, CFRP and BTRM) under monotonic or cyclic loading. Two models were strengthened with use of "frenelli" – extrados stiffening diaphragms reinforced with SRP and SRG (Fig. 2.16a). Experimental results showed that this technique presents an overall better performance in terms of displacement and failure mechanisms than single SRP or CFRP reinforcement.

The maximum loads were the lowest of the experimental campaign but the failure mode was interesting. The two samples exhibited a ductile collapse with the damage zone concentrated on the stiffening diaphragms (Fig. 2.16b) and notable global displacement. The collapse appears after the formations of sufficient number of plastic hinges on the vaults.
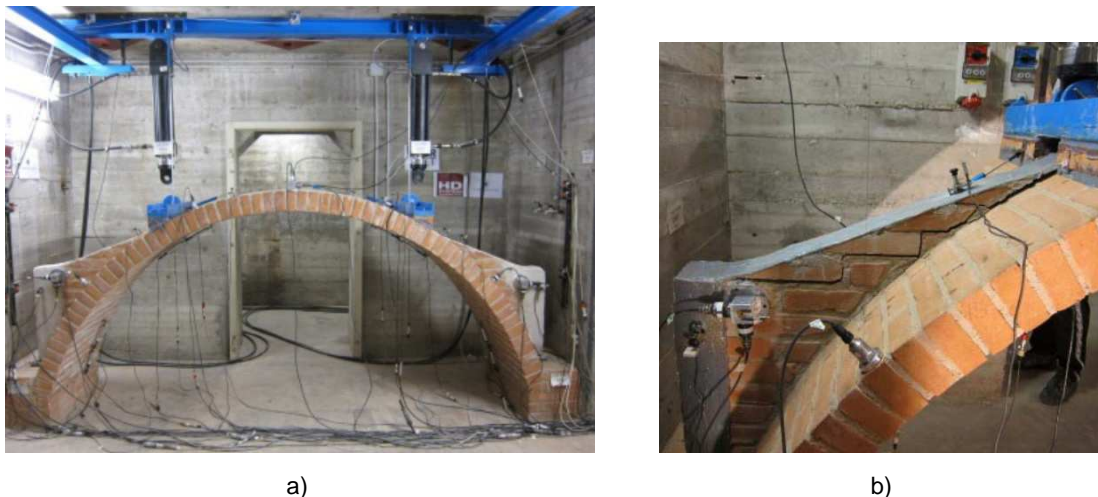
Fig. 2.16. a) Vault with extrados stiffening diaphragms; b) damage of diaphragm reinforced with SRP [28].

In the PhD thesis by Ferrario, [31] the objective of the work was to increase seismic performance of single barrel vaults but without changing its original structural response. Throughout experimental testing it was proven that the use of plywood spandrel element, a priori defined by means of an analytical procedure, significantly enhances the structure's seismic capacity.

Furthermore, a big advantage of the method is lightness and reversibility, because the ribs are made of glulam and are connected with the vault and abutments with the use of steel, punctual anchors. This connection is considered as to be isostatic, thus allows some movements so that the vault and lateral spandrel work better.

Chapter 3
NUMERICAL MODELLING OF ARCHES AND VAULTS

3.1. Masonry modelling challenges

Masonry vaults are one of the most common structural shapes present in the architectural heritage of the world. These elements are defined as structures in which the load bearing is clearly associated with the distribution of material in space. A growing interest in the preservation of historic structures has created a need of methods for the analysis of load-bearing unreinforced masonry structures, such as arches, vaults, and buttresses.

The limit analysis methods, first applied to medieval structures in detail by Heyman [2], provide a useful and intuitive approach to the understanding of the behaviour of masonry arches and vaults. However it usefulness in performing structural assessments of multipart structures has limitations. The difficulty of analysis mostly come from the level of sophistication of historical construction and impossibility of defining the real thrust line before the collapse.

Thanks to the development of technologies, nowadays modelling became an essential part of any structural analysis because it provides variety of possibilities of arranging the properties of constructions. Specially, in the field of historical construction, where simple analytical formulations are often insufficient, modelling provides a wide range of activities that can be performed on a structure. Nevertheless, the analyst should always control the simulation and calculation done by software, by means of coarse but reasonable approaches derived from past formulations.

Numerical models are today preferred because of:

1) Economical reasons –preparing and testing analogical or experimental models is costly and time consuming. They normally permit a single use only.

2) Versatility – computer permits many variations, improvements, changes. Many different loading hypothesis can be analyzed with limited effort.

3) Capability – nowadays there are many alternative and powerful numerical formulations for the analysis of complex structures (including the masonry ones). Continuous research keeps on providing improved models and computer applications.

In any case, the model used in the structural analysis is usually a compromise between realism and cost. The need for difficult data, which not always can be determined by means of reasonably inexpensive laboratory or in situ tests, must also be considered in selecting the modelling approach.

The structural model must take into consideration and simulate all the aspects influencing the structural response, including: geometry and morphology, material properties, actions, existing alterations and damage, interaction of the structure with the soil.

Structural modelling and analysis must be done in combination with activities of empirical nature (historical research, inspection, experiments, monitoring). History, inspection and experiments are needed to furnish input data to construct the structural model. These activities provide empirical information for validation. The process of calibrating/validating is in fact a direct application of the scientific method. Hypotheses are implicitly formulated by building up the model, which is then validated by comparing its results with the empirical evidence provided by history, inspection (including experiments) and monitoring.

Possible formulations of the modelling strategies usually vary in the way of treatment of the material. They can be basic (like linear elastic analysis, limit analysis or generalized matrix formulation) or more sophisticated like macro-modelling, micro-modelling or distinct element method.

3.2. Types of analysis of masonry structures

3.2.1. Limit analysis method

Limit analysis have its origins in 17th century idea developed by Hooke, who stated that the arch equilibrium can be associated with inverted centenary shape. Further development of Hooke's formulation brought new definition of arch equilibrium, graphic statics and finally the idea of a thrust line: the arch is stable if it is possible to fit any thrust line within its boundaries. Graphic statics supplied a practical method consistently based on the catenary principle useful especially

to arch-type constructions. As well, the thrust line concept is of higher difficulty that it primary seems, as in a stable arch exist infinite number of thrust lines that goes through its body, between its borders.

In 1966 Heyman [32] formalised the study of block assemblies of any configuration through the application of plastic limit analysis theorems and generalised them for masonry structures. Using rigid-perfectly plastic block elements to represent an assembly of elements, it is possible to evaluate the load capacity and failure mechanisms of structures. This assumption means that the deformations in the elastic field are considered to be negligible with respect to those produced in the plastic field. Consequently, for the limit analysis there are some simplificationsthat must be considered while analysis, namely:

1. *Masonry has no tensile strength*. Masonry pieces have indeed tensile strength, but the joints between them may be dry or made with weak mortar;

2. *Masonry has an infinite compressive strength*. In ancient masonry structures the compressive stresses are usually small compared with the corresponding strength, so, crushing is often not applicable;

3. *Sliding between masonry blocks cannot occur*. This is a simplification accepted by Heyman (1966) for his hand calculations, although he recognized that sliding failures occur sometimes;

Limit analysis method can be regarded as a practical computational tool, since it requires a reduced number of material parameters and is relatively simple in idea and computation. It can also provide a good insight into the structural behaviour, failure pattern and limit load. This method also allows for quick evaluation of the effect of interventions on the structural behaviour. It provides first approach which may assist in taking decisions that a more sophisticated analysis is to be carried out. In case of limited time, or limited resources, or scarce data, limit analysis provides an acceptable solution [35].

3.2.2. Linear elastic analysis

Linear elastic analysis is commonly used in the calculation of steel and reinforced concrete structures. However, its application to masonry structures is, in principle, inadequate because it does not take into account the non-tension response and other essential features of masonry behaviour. Masonry shows a complex non-linear response even at low or moderate stress levels, due to its very limited capacity in tension. Thus, simple linear elastic analysis cannot be used to simulate masonry strength responses, typically observed in arches and vaults, characterized by the development of partial subsystems working in compression. Attempts to use linear elastic analysis to arches may result in very conservative or inaccurate approaches. Linear elastic analysis is not useful, in particular, to estimate the ultimate response of masonry structures and should not be used to conclude on their strength and structural safety [35]. Some actions can be

simulated by associating the appearance of tensile stresses to possible cracking, however it only can provide limited qualitative information (Fig. 3.1).

During the last years, non-linear analysis is becoming more popular thanks to larger software availability and increasing computer capacity. However, linear analysis is always performed, prior to the application of more sophisticated approaches, to allow a quick and first assessment of the adequacy of the structural models. Moreover, due to its availability and reduced computer cost, it has been used as an auxiliary tool assisting in diagnosis even in more recent times, [36].
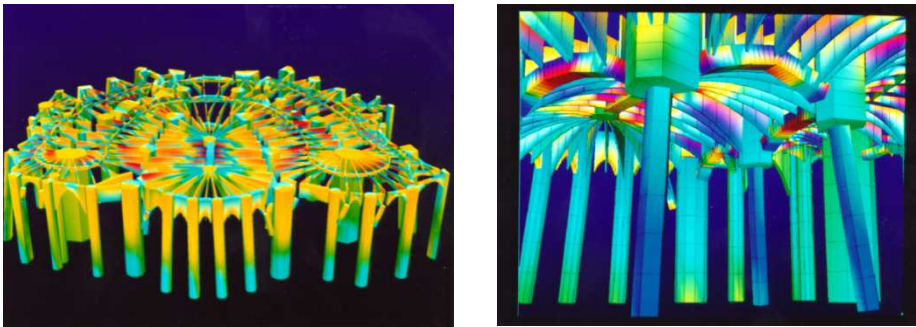
Fig. 3.1. Linear elastic analysis of the Crypt of the Güell Colony in Barcelona, [37].

3.2.3. Non-linear analysis

In mechanics a problem is considered non-linear when the action-response relationship is not proportional (it does not follow Hooke's law). In a structural element it can be distinguished four types of non-linearities coming from different origins:

1. Geometric non-linearities, when the difference between the deformed and undeformed configurations cannot be ignored;

2. Material non-linearities, when departure of the material constitutive behaviour from Hooke's law cannot be ignored;

3. Non-linear displacement boundary conditions;

4. Non-linear applied forces or tractions.

Particularly for masonry, the non-linearity is connected with the material discontinuities present in masonry material, thus it is called non-linear material analysis. This analysis includes constitutive equations describing material features causing non-linear behaviour: cracking, yielding and crushing in compression, frictional sliding, etc. Models are supported by alternative theories like plasticity or damage theory.

Because of, previously mentioned, non-linearities occurring in the masonry material most of the analysis are now performed in the non-linear range to simulate, in a more accurate way, real behaviour of masonry element. Specially this progress was and is enhanced by the development of computer technologies which provide powerful devices that can cope with the high needs of non-linear analysis.

3.2.4. Finite element representation

Masonry is a material fully heterogeneous, therefore its properties vary depending on the direction of applied load. Specially, the mortar joints are points of vulnerability and can be treated like planes of weakness. That is why, the strategies toward masonry modelling can be divided in: modelling of individual components (units and joints) and interfaces between them called micro-modelling. Another approach (macro-modelling) treats masonry like one composite material of uniform and homogenized properties [36]. Graphical representation of each modelling approach is presented in Fig. 3.2.
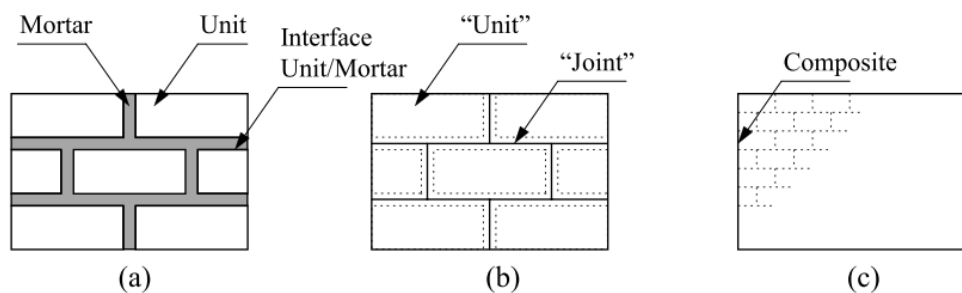
Fig. 3.2.Modelling strategies for masonry structures: a) detailed micro-modelling;
b) simplified micro-modelling; c) macro-modelling [36].

The decision on which strategy to use for analysis of masonry structures has to considered depending on what level of accuracy is needed. The following aspects also should be carefully considered in process of model type selection:

1) Scale of the study (local, medium, large);
2) Detail (or refinement -minimum dimension resolved in terms of stresses, deformation, damage…);
3) Comprehensiveness of the model (individual member, macro-element, global).

The formulation selected should generally be the simplest and less costly one, although still able to simulate the features or phenomena intended.

3.2.4.1. Micro-modelling

The different components (units, mortar, interfaces), are modelled separately. Specific constitutive equations are utilized for each type of component. In particular, interfaces are described so that they can represent frictional sliding/separation between units. A possible simplification consists of

using blocks to model of combined response of units and mortar. In that case, the model, only consisting of blocks and interfaces, allows significant reduction of computer effort. Due to their computational needs, micro-models can only be used to analyze simple members (solid or hollow walls).

As mentioned above, several micro-modelling strategies can be defined depending how in detail the structure should be analysed, i.e.:

1. Detailed micro-modelling (Fig. 3.2a) – units and mortars are represented by continuum elements whereas the behaviour of the unit-mortar interface is represented by discontinuous elements.

2. Simplified micro-modelling (Fig. 3.2b) – expanded units are represented by continuum elements whereas the behaviour of the mortar joints and unit-mortar interface is lumped in discontinuous elements.

3. Distinct element method - a method allowing finite displacements and rotations of bodies. Contact is modelled by means of point-contact approach. It recognizes new contacts automatically.

3.2.4.2. *Macro-modelling*

In macro-modelling – units, mortar and unit-mortar interface are smeared out in the continuum (Fig. 3.2c). Masonry is described as an homogeneous material characterized by a set of average (or homogenized) properties. A criteria or technique is needed to derive the homogenized properties from those of the individual components (units –stone blocks, bricks-, mortar and the unit-mortar interface). Macro-models describe the response of masonry with acceptable accuracy. However, they might fail to simulate failure modes involving separation or sliding between different parts.

Macro-modelling is considered a good approach of structural analysis of objects of high complexity and size, as it requires less data input and the computational cost is much lower than in the case of micro-modelling. Its main advantages are relatively simple pre- and post-processing, simple theoretical framework and efficient computations which provide results in global scale of the construction. The main difficulty comes with the definition of the continuum parameters of the assigned materials. Nevertheless, results obtained from well calibrated models are credible, thus macro-model is one of the most popular structural modelling type in present times. However, sometimes using FEM macro-modelling to analyze entire large structures is still challenging for the capacity of modern computers.

Chapter 4
MODELLING OF REFERENCE ARCH

This chapter deals with numerical modelling of an unstrengthened masonry arch resorting to two different modelling strategies, namely macro- and micro-modelling. The arch was tested in the laboratory and available results were used to calibrate two numerical models here developed.

4.1. Experimental model and data collection

The geometry, material properties and the results needed for the numerical model are taken from a PhD thesis under the title 'Rinforzo di volte in muratura con materiali compositi innovativi' (Strengthening of masonry vaults with innovative composite materials, [31]), written by Paolo Girardello in 2013, at Universita' degli Studi di Brescia, under the supervision of professor Francesca da Porto. The thesis is an overview of experimental and numerical analysis procedure of masonry vaults structures. Strengthening techniques of arches and vaults are listed and explained in detail. Experimental program consist of tests on constituent materials used in the build up of vaults and theirs reinforcement.

For the experimental part of arch testing the constructions were made of solid clay brick (250×120×55mm) type Rosso Vivo A6R55W produced by San Marco-Terreal Italia (Noale-VE, Italy) and joints of lime mortar T30 V produced by Tassullo (Tassullo-TN, Italy), [29]. Flexural tests were performed on 12 clay bricks, and pieces obtained after each failure were subjected to compressive and splitting tests. Eight T30 V mortar specimens 40×40×160 mm were tested in

flexure and in compression; elastic modulus was measured on other eight T30 V prisms of mortars. Four masonry panels were tested in compression and initial shear strength was measured on twelve three-bricks specimens. The results obtained in the material test are listed in the Table 4.1.

Table 4.1. Mechanical properties of materials (CoV in brackets), [29].

Material	Compressive strength N/mm^2	Flexural strength N/mm^2	Splitting strength N/mm^2	Elastic modulus N/mm^2	Tensile strength N/mm^2	Shear strength N/mm^2
Brick	17.68 (6.2%)	4.43 (10.2%)	2.99 (11.2%)	7200	-	-
Mortar T30V	1.75 (15.6%)	0.74 (14.4%)	-	3570 (21.2%)	-	-
Masonry	5.97	-	-	1193	-	0.173 α = 28.48°

Set of 8 vaults was constructed, of which 7 were with reinforcement, each of different kind (SRG, SRP, CFRP, BTRM and extrados stiffening diaphragms with SPR and SRG strips). The unreinforced arch and one reinforced with SRG was loaded in monotonic way, while all other strengthened arches were subjected to cyclic loads. The loads were applied in ¼ and ¾ of the arch span. The arrangement of arch and load applicator is illustrated in Fig. 4.1.
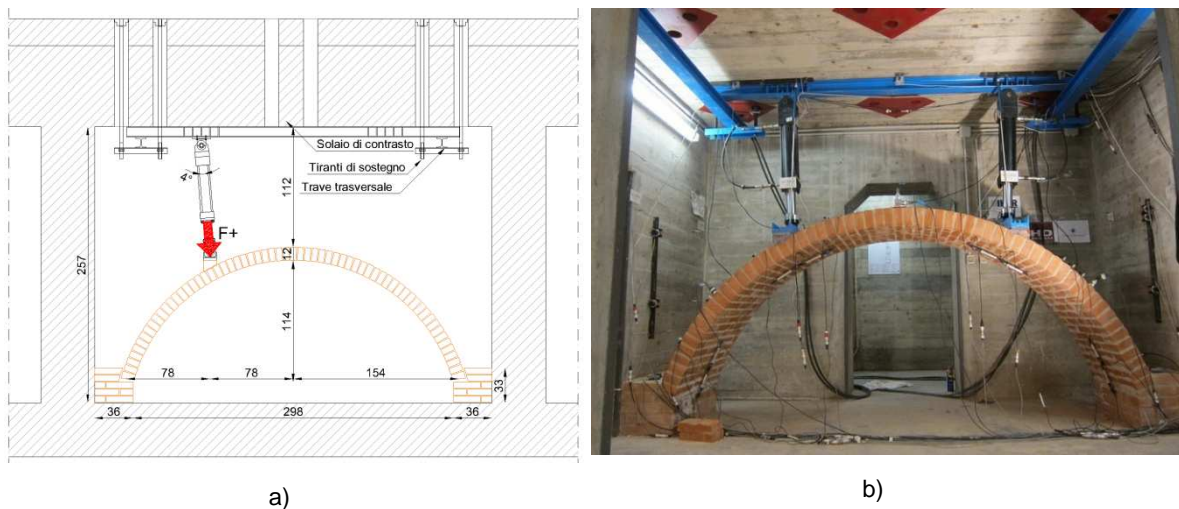
Fig. 4.1. Experimental arch: a) scheme of the arch with position of load application;
b) real view of the setup of the construction, [31].

The experiment provided data about displacement and failure mode of the unreinforced arch. A set of LVDT sensors was applied in strategic positions in the structure. During the test the ultimate load of the arch was measured of value 1,38kN, with the corresponding displacement of the keystone equal to 0,39mm. The collapse happened due to formation of four plastic hinges, creating a mechanism. Location of hinges is presented in Fig. 4.2.

a)   b)
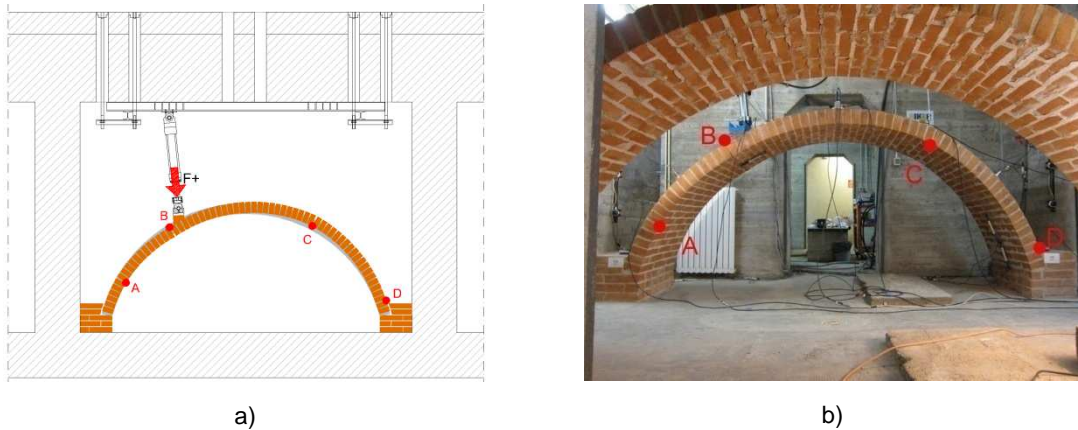
Fig. 4.2. Unreinforced arch: a) scheme of the arch with position plastic hinges;
b) real view of the location of plastic hinges, [31].

For the purpose of calibration of the numerical modelling of this master thesis the results of the plane vault will be used. They are in Table 4.2.

Table 4.2. Results of the experimental test, [29].

Arch	F_{ymax} kN	d_{max} mm	Failure mode
Unreinforced subjected to monotonic load	1,38	0,39	Four-hinge mechanism

4.2. Macro-model of the reference arch

4.2.1. Finite element model adopted

To define the load bearing capacity and structural behaviour of masonry arch, a simulation of experimental arch was carried out. First approach toward modelling of the arch is with use of macro-model for which the material nonlinearities of masonry are the governing parameters.

The finite element two-dimensional model was created in DIANA 9.4. software. The masonry arch had a 2980mm span, 1140mm rise, 120mm voussoir thickness (full geometry in detail in Fig. 4.3) and total width 770mm. During the analysis, subsequently to the application of the arch self-weight, a monotonic incremental load was applied at the quarter span.
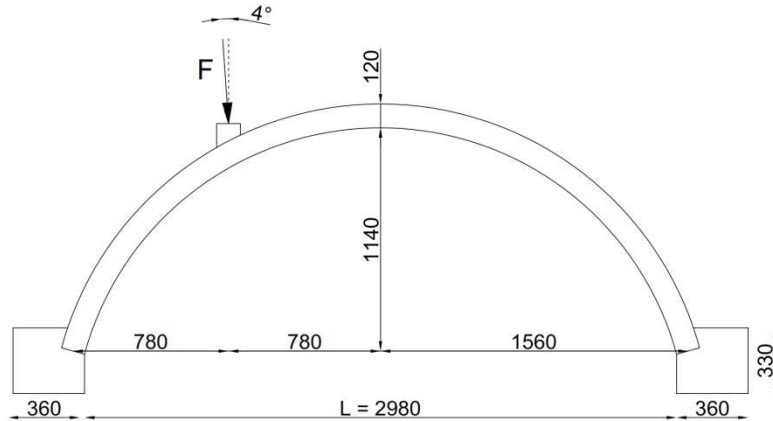
Fig. 4.3. Geometry of the arch model.

The mesh adopted in the analysis was of quadrilateral eight-nodded plane stress elements (Fig. 4.4).
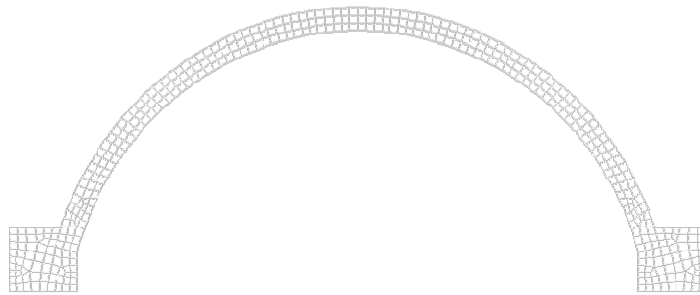
Fig. 4.4. Eight-nodded quadrilateral element mesh applied to the model.

The constitutive model employed intended to adequately simulate the complex behaviour of masonry, oriented to simulate the real behaviour by means of nonlinear properties of masonry. Therefore the most important part of the modelling was connected with defining the material parameters that were not obtained in the experimental tests of constituent materials.

Preliminary parametric numerical study was carried out to define the correct parameters of the masonry. The study helped to define material properties to create representative numerical model, in terms of behaviour and ultimate load, for the experimental arch. Those defined mechanical properties allowed to perform a second parametric study based on reliable data (presented in Chapter 6).

Of the properties responsible for nonlinear behaviour in masonry, tensile strength plays the biggest role. For that reason, and the fact that tensile strength was not defined in tests, analysis of the numerical models with different values of tensile strength was performed.

As known, masonry has almost null tensile strength, nevertheless for need of structural design some guidelines are defined. In terms of practical design rules for tensile strength of masonry a

value equal to 10% of masonry compressive strength is applied. However, in the analysis of existing structures this indication seem too high and often the value is decreased to 5% of compressive strength.

This procedure of defining masonry tensile strength was used in this thesis. Starting from 5% of compressive strength (f_c = 5,97MPa) the value of tensile resistance defined as 0,3MPa was established. For this value of masonry parameters first nonlinear analyses were performed. Obtained results were far from experimental ones, that is why further reduction of value of tensile strength was carried out up to the value of 0,04MPa (equivalent of 0,7% of compressive strength), for which the nonlinear behaviour of the numerical model was sufficiently corresponding with the experimental behaviour.

As constitutive law used to simulate the behaviour of the masonry smeared crack model was used for quasi-brittle materials. The model describes the behaviour in tension and compression of the material through a stress-strain relationship. The adopted behaviour in tension is described by the laws of softening of the exponential type and parabolic diagram in compression, shown in Fig. 4.5.
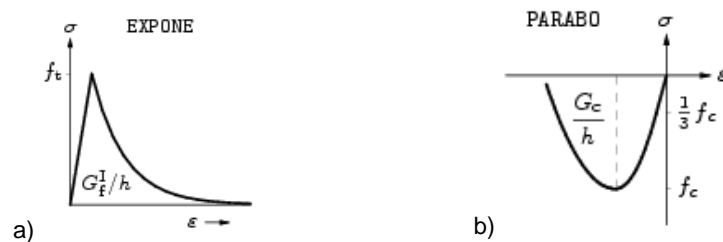
Fig. 4.5. Damaged plasticity model, behaviour in a) tension and b) compression, [39].

As described in part 0, most of the properties used to simulate the masonry were characterized in [31] by means of experimental tests. All the elastic and inelastic properties adopted for modelling are included in Table 4.3.

Table 4.3. Elastic and inelastic properties of the masonry

	Elastic modulus	Poisson ratio	Tension		Compression	
	E	ν	f_t	G_f^I	f_c	G_{fc}
	N/mm^2	-	N/mm^2	N/mm	N/mm^2	N/mm
Masonry	1193	0,15	0,04	0,02	5,97	9,55

4.2.2. Numerical results

To identify structural behaviour, ultimate load capacity and the failure mechanism, an analysis of plain, unstrengthened arch was performed. This analysis gave an overall view of the behaviour of the structure under increasing load. As well, calibrated model was further used while performing analysis of strengthened arch. Diagrams in Fig. 4.6 present comparison of displacement in two

particular points of the construction, keystone and loading point. The evaluation is made on the resultant displacement of those two points, because those are obtained experimental results.

As presented in Fig. 4.6 the results of numerical analysis show good agreement with experimental results in terms initial stiffness of the arch and peak load. After the ultimate load, the observable drop in the load carrying capacity is connected with high damages present in the structure. Failure was, in general, characterized by brittle behaviour, typical for masonry structures.

Up to value equal to 35% of ultimate load (0,50kN) the stiffness of numerical model is almost perfectly overlying the experimental one. Above 0,50kN some differences are present as the nonlinear behaviour of arch tends to prevail. Additionally, experimental results present irregularities probably coming from noise during the test.
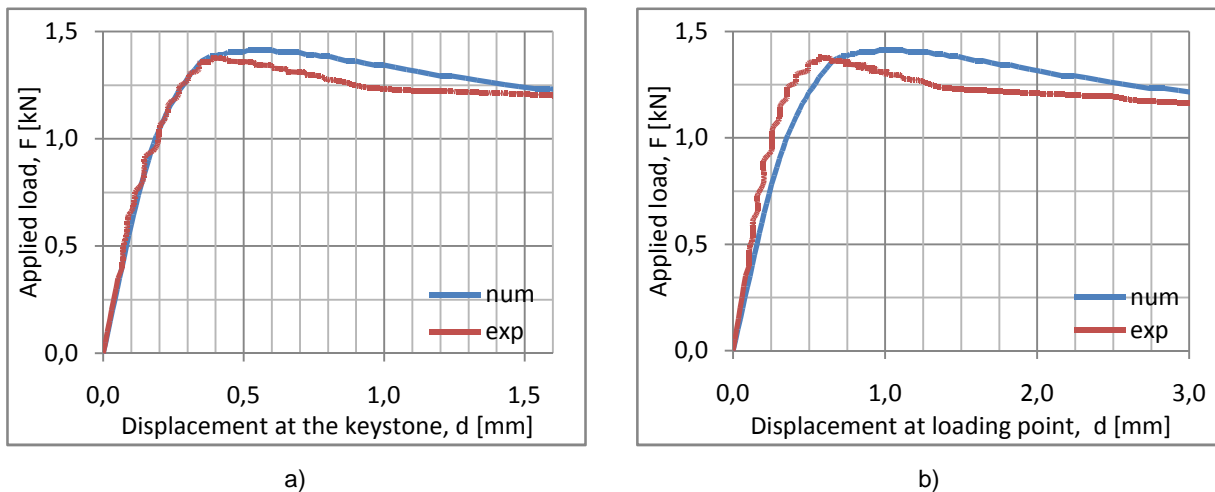
Fig. 4.6. Comparison between experimental and numerical arch results
presented on load - displacement curve for a) the keystone, b) loading point.

The peak load of numerical model is close in value with the experimental. Also the model shows nearby results in terms of resultant displacement measured for the experimental arch. The initial stiffness was calculated for values up to 0,5kN after which nonlinear behaviour starts. Comparison of results is presented in Table 4.4.

Table 4.4. Comparison of results of experimental and numerical tests.

Type of arch	Ultimate load capacity kN	Displacement at the keystone mm	Initial stiffness kN/mm
exp	1,38	0,39	6,46
num	1,41	0,52	6,03

Numerical model allows to accurately identify the formation of hinges along the loading procedure. Therefore the appearance and sequence of four hinges development before the collapse of the

mechanism are presented in Fig. 4.7. The figures present the deformation of arch shape as well as the distribution of average principal tensile strains, which might be associate with crack pattern. On each frame of Fig. 4.7 an approximate location of hinges is represented by a dot. Every subsequent frame has one dot more symbolizing appearance of new hinge up to formation of all four hinges.

The frame a) represents early stage of loading, 0,55kN, that has been defined as the beginning of nonlinear behaviour of the arch. After this step the hinge formation is accelerated and structure undergoes deformation up to the point of collapse. All four hinges appear before the maximum load, nevertheless the arch is self-stable up to the point of peak value, after which the collapse occur due to high displacements of elements. Hinge 1 and 2 appear close to the position in which they appear in the experimental tests. Hinge 3 is difficult o locate accurately as the tensile strains are distributed along part of the extrados. As for the hinge 3 its location varies slightly from the experimental one (which was further from the abutment). This might be connected with the geometrical division of the model. All the arch is treated like one continuous structural component, that is why the strain concentration happens at the abutment, where the geometry experiences some geometrical changes.

d)
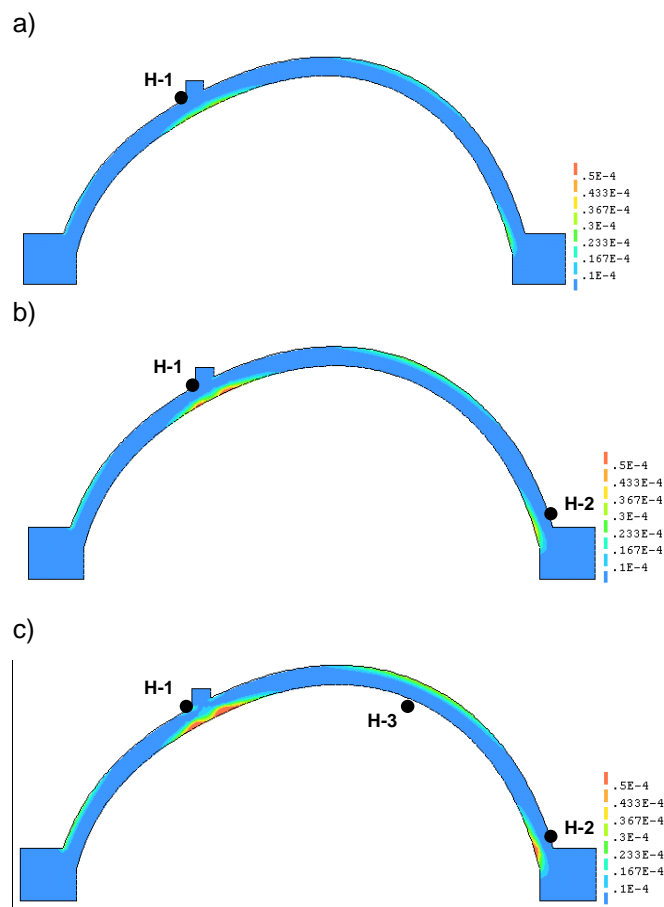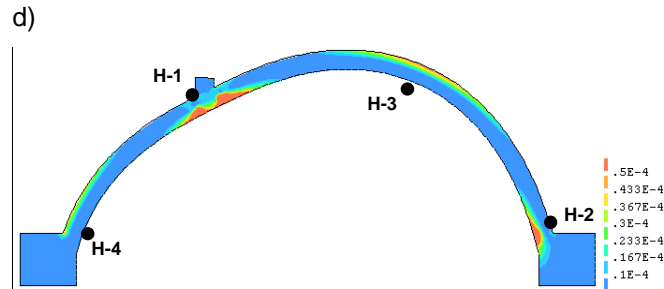
Fig. 4.7. Numerical sequence of hinges formation for the unstrengthened masonry
arch defined as principal strains imitating crack appearance.

For more in detail representation of hinge development before the collapse a load-displacement
curve is presented in Fig. 4.8. In the curve, each hinge, marked with a blue dot, result in slight
stiffness changes along the structural response path. All four hinges formed before the peak load
and were detected approximately at the beginning of their development.
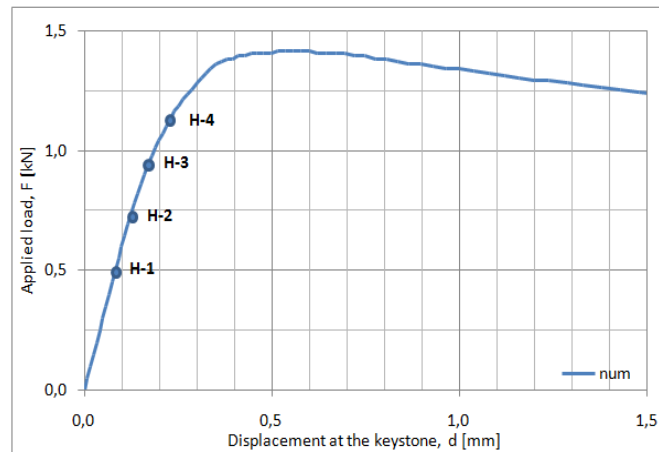
Fig. 4.8. Identification of the hinges' formation on the load-displacement curve.

4.3. Micro-model

4.3.1. Adopted numerical model

In the simplified micro-model approach the model consists of units, which represents brick, and of
interfaces, which imitate the behaviour of mortar joints. In the modelling process, the unit is
treated like a continuous elastic material, while all the nonlinearity of the masonry is concentrated
in the properties, and thus behaviour, of the interface. Geometry of the micro-model of the
reference arch is presented in Fig. 4.3.

To define the complex masonry behaviour in more credible way for the analysis procedure of the
masonry interface a crack-shear-crush multi-surface model was selected (Fig. 4.9). The model
sets a nonlinear relation between tractions (i.e. stresses) and relative displacements across the

interface. The tractions are normal (t_n) and shear tractions (t_t), [40]. Zero thickness interface was assumed between the units.
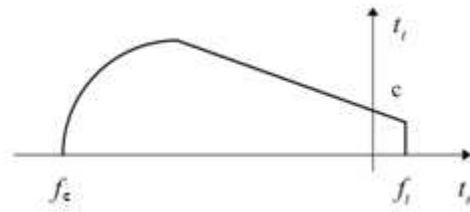
Fig. 4.9. Composite interface model, [40].

Most of the parameters of the materials used in the modelling procedure were identify experimentally as explained in the beginning of the chapter. Values of properties that were not obtained from experiments were defined from other experimental and numerical works present in literature, [40] and [41]. Some properties, like tensile strength and mode I fracture energy, were estimated by means of numerical analysis trials up to the moment of calibration of the micro-model (like in the case of macro-model). All material properties are defined in

Table   4.5   and Table 4.6.

Table 4.5. Elastic properties of the masonry and interface.

Element	Elastic modulus E N/mm^2	Poisson ratio ν -	Normal stiffness k_n N/mm^3	Shear stiffness k_s N/mm^3
Brick	7200	0,15	-	-
Interface	-	-	21	8,4

Table 4.6. Inelastic properties of the interface.

	Tension		Shear				Compression		
	f_t N/mm^2	G_f^I N/mm	c N/mm^2	$tan\varphi$ -	$tan\phi$ -	G_f^{II} N/mm	f_c N/mm^2	G_{fc} N/mm	κ_p -
Interface	0,072	0,025	0,173	0,43	0	0,05	5,97	9,55	10

The mesh consists of two types of elements that accurately describe the mechanisms happening in the particular parts of the arch. Eight-nodded quadrilateral elements were applied to all units and abutments of the arch. A six-nodded interface elements was employed during the analysis. Monotonic incremental load was applied like in the case of macro-model, in quarter-span of the arch. To perform the nonlinear analysis, the arc-length method and the crack mouth opening displacement CMOD technique were employed to surpass instabilities caused by nonlinearities. The adopted mesh of the model is presented in Fig. 4.10.

a)   b)
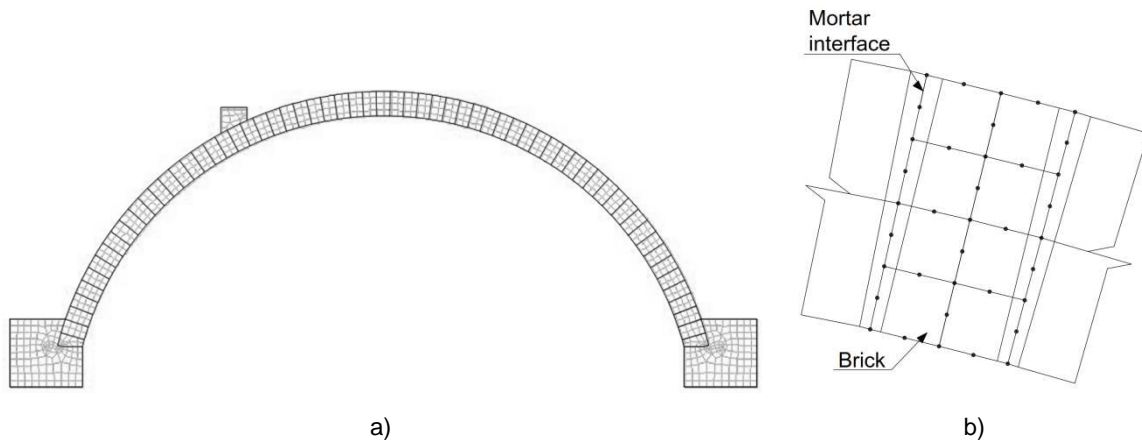
Fig. 4.10. Meshing type use in the model: a) general view, b) detail of the unit.

4.3.2. Modelling results

An analysis of micro-model was done to identify, in detail, the structural response, ultimate load and the collapse of mechanism of the reference arch. Calibration of the model was a crucial issue for subsequent modelling of strengthening technique, as for comparison of two numerical modelling approaches.

Micro-model does not replicate the behaviour of experimental arch as expected. Structural response to the incremental load shows that the model is less stiff than the experimental arch but tends to have brittle behaviour. It has long range of behaviour similar to elastic. Close to the ultimate capacity the behaviour tends to become nonlinear. After reaching the peak load the model fails to replicate the softening branch (Fig. 4.11).

Detected, in the numerical program, nonlinear behaviour starts approximately at force value 0,65kN. This means that the inelastic actions start relatively "late" (45% of maximum load) as typically for masonry arch constructions (assumed usually at 30% of ultimate load).

In terms of ultimate capacity the model exceeds the results from the experiment, while at the same time the maximum displacement under peak load is smaller than in the real arch (keystone section). The initial stiffness of the arch is reasonably close to the experimental value. All the comparable results are listed in Table 4.7.
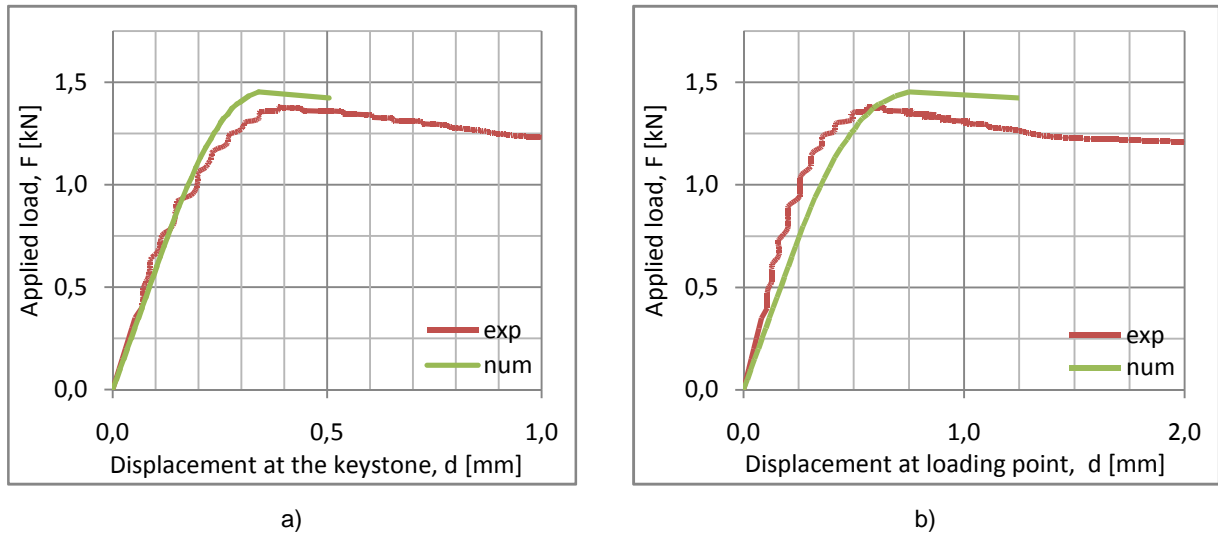
Fig. 4.11.Comparison of results between experimental and numerical arch
presented on load - displacement curve for a) the keystone, b) loading point.

Table 4.7. Comparison of results of experimental and numerical tests.

Type of arch	Ultimate load capacity kN	Displacement at the keystone mm	Initial stiffness kN/mm
exp	1,38	0,39	6,46
num	1,45	0,34	5,85

The sequence of hinge formation during the load increment was detected in the numerical program. It is presented in Fig. 4.12 on deformed mesh with distributed average principal tensile strains. Each hinge is marked as a black dot and each frame represents one new hinge location. The position of each hinge was identified with higher accuracy that in the case of macro-model. It was possible thanks to the deformations noticeable in the interface mesh. Furthermore, the location of hinges is closer to the original position from the experimental arch.

The first frame (Fig. 4.12a) presents the step of first nonlinear behaviour of the arch, at value 0,65kN. Each subsequent frame presents new locations of increased principal tensile strains which can be associated with cracks, and thus the hinges can be detected. All hinges, as expected, were developed before the ultimate load. Nevertheless, the arch stayed stable (most probably due to self-weight) developing nonlinear behaviour up to the peak load after which brittle failure happened. Fig. 4.13 presents openings of joints due to hinge development in the load step that corresponds with formation of last, fourth hinge.

a)

a)

b)

b)

c)

c)

d)

d)
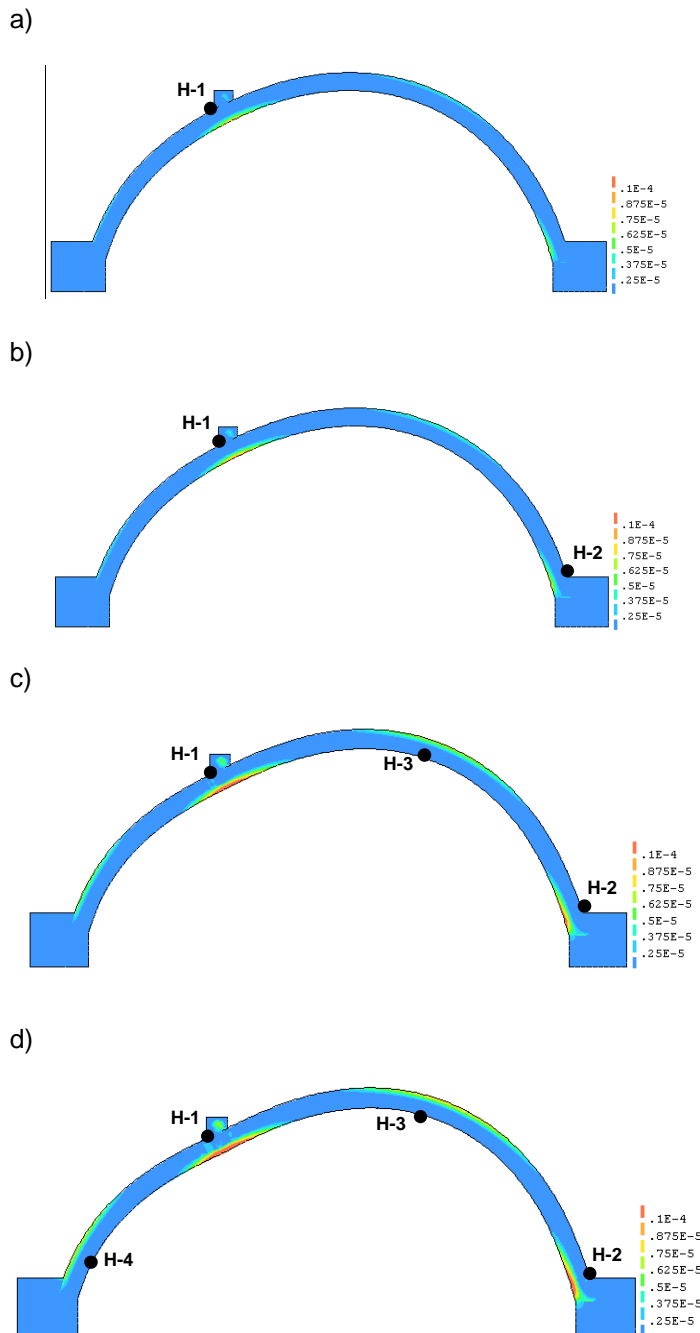
Fig. 4.12. Sequence of hinge formation on a deformed model with distributed principal strains.
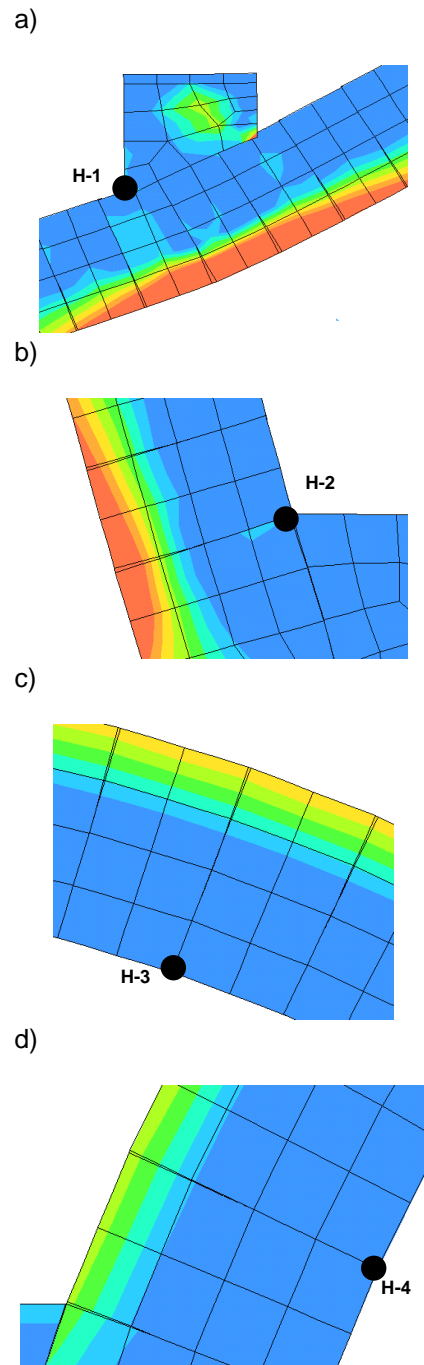
Fig. 4.13. Detail of joints opening caused by each hinge for the load step corresponding with formation of the last hinge.

A load-displacement curve presented on Fig. 4.14 shows the moment of hinge formation. Each hinge comes with small change in the stiffness of the arch, not detectable on the graph, but identified in the output file of the numerical program. All were approximately discovered at the beginning of formation.
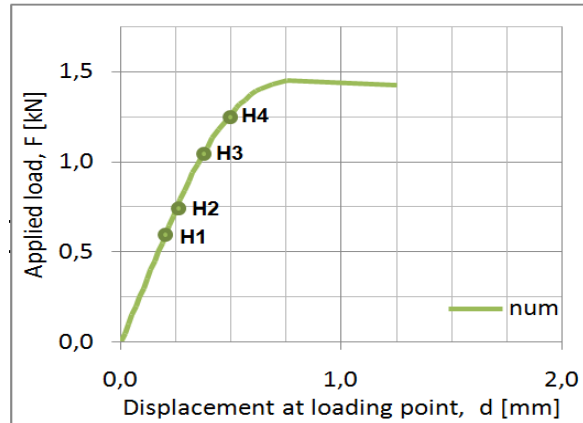
Fig. 4.14. Identification of the plastic hinges formation on the load-displacement curve.

4.4. Comparison of the numerical models

The idea of creating two different models, with different degree of accuracy, imitating a real arch was performed in this thesis. The current chapter provides all relevant information to analyse the reliability of each model. Results are listed in Table 4.8.

The macro-model represented the reference arch very realistically. Its initial stiffness and load carrying capacity are of nearby values with the experimental ones. The location of hinges and sequence of formation was reasonably reproduced.

The micro-model gave more accurate location of hinges along the loading process. However, the results in terms of initial stiffness and peak load were further from the experimental ones. The displacement under the peak load was smaller than of the real arch, which signifies that the arch in general is stiffer and behaves in more brittle manner.

Table 4.8. Comparison of results of experimental and numerical tests.

Type of arch	Ultimate load capacity kN	Displacement at the keystone mm	Initial stiffness kN/mm
Experimental	1,38	0,39	6,46
Macro-model	1,41	0,52	6,03
Micro-model	1,45	0,34	5,85

Numerical modelling is always just an attempt to realistically replicate an experiment. Therefore it is not possible to define which of the models is better. Macro-model has a big advantage of simpler pre-processing in terms of model creation, as well it requires less properties. What is more, the analysis procedure involves less computational cost and, as seen in the graph, can replicate more of the post peak behaviour. On the other hand, micro-model shows behaviour of the units which is more credible.

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Chapter 5

MODELLING OF AN ARCH STRENGTHENED WITH STIFFENING DIAPHRAGMS

In Chapter 5, numerical analysis of a strengthened masonry arch is presented. The reinforcement technique that will be modelled is the use of extrados stiffening masonry diaphragms. Unfortunately, there are no experimental results of the strengthened arch as in [31], because the diaphragms studied by Girardello were additionally reinforced with SRP and SRG strips.

After the validation of the reference arch model (explained in previous chapter), a constitutive macro-model of strengthened arch was employed with intention to simulate the complex behaviour of masonry arch and its interaction with the stiffening diaphragms, by means of interfaces. As a final step, a micro-modelling strategy was adopted to simulate more precisely the interaction between the masonry arch and strengthening solution.

The idea of strengthening an arch or a vault with a use of stiffening diaphragms (also called ribs) is a new and old idea at the same time. It can come in variety of geometry and material configuration (as presented in detail in part 2.3.7).

5.1. Macro-model of strengthened arch

5.1.1. Numerical model adopted

The thesis is focused on a particular case of this technique. A partial stiffening element will be located at the extrados of an arch in a symmetric manner. The idea came from a Doctoral thesis

written by Paolo Girardello [31]. In the thesis geometry of the diaphragm was, somehow, forced by the arrangement of the load application jack available in the laboratory. Therefore the geometry will be constant throughout all the analysis program and will be as shown in Fig. 5.1. The width of the arch is as previously, 770mm. The stiffening diaphragm is of thickness 120mm and located in the middle of the width of the arch.
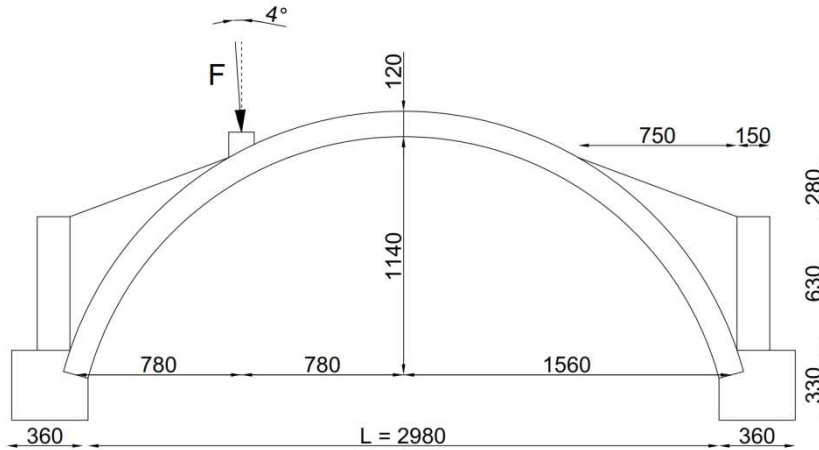
Fig. 5.1. Geometry of the strengthened arch model.

To fully define the influence of the stiffening part on the arch two approaches are used: model without an interface between arch and strengthening element (perfect bond) and a model with a structural interface.

5.1.1.1.   Macro-model without interface

In the model without interface between the arch and stiffening diaphragm, the structure is expected to work as one uniform element. The modelling is limited to adding the strengthening part of the same material properties like the arch, listed in Table 5.1.

Table 5.1. Elastic and inelastic properties of the masonry diaphragm

	Elastic modulus	Poisson ratio	Tension		Compression	
	E N/mm^2	ν -	f_t N/mm^2	G_f^I N/mm	f_c N/mm^2	G_{fc} N/mm
Masonry	1193	0,15	0,04	0,02	5,97	9,55

The mesh adopted in the analysis includes eight-nodded quadrilateral plane stress elements to represent the segments of masonry arch and six-nodded triangular elements for the diaphragms, as presented in Fig. 5.2.
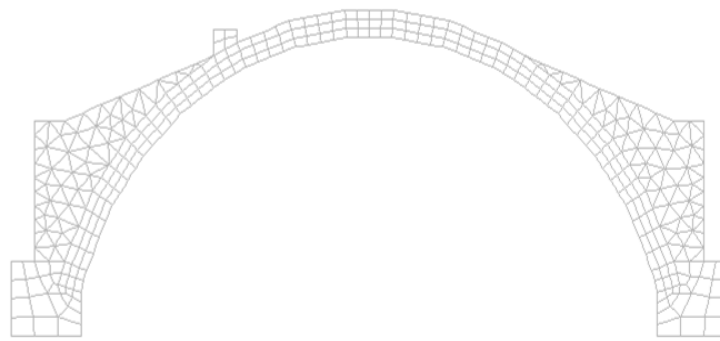
Fig. 5.2. General view on the adopted mesh.

5.1.1.2. Macro-model with interface

The macro-model with interface between the arch and the stiffening part is considered to be a more realistic representation of the reality that the model without interface. This is explained by the fact that two elements, one added to another should not be treated as one uniform continuous element. This is a reasonable approach especially in case of historical constructions, where strengthening is done much later than the construction was erected. Material properties and, even, the idea of attaching a new part implies that there has to undergo some interaction between new and old element of the structure.

Another important issue to consider interface elements in modelling is masonry material itself. Masonry is an anisotropic material where the orientation of the joints plays a crucial role in the determination of the elastic properties and strength. The description of the tensile behaviour of masonry should include tension normal and parallel to the joints. Taking this into account, in the case of macro-model approach for an arch with strengthening, properties of the interface arch-diaphragm had to be defined. Values of normal and shear stiffness, as well as tensile strength will have a significant impact on the performance of the structure under increasing load.

For the need of this thesis a different estimation procedure was used to derive the values of normal and tangential stiffness of the interface arch-diaphragm. The formula was established on hypothesis that a masonry-masonry element should have the same relative displacement under load application as an element masonry-brick with a zero thickness interface in the middle. From this assumption a range of probable values of the normal stiffness was obtained (11-114 N/mm). Furthermore, because the value used in the interface unit-unit in the micro-model of plain arch fits within this range, it was decided to use this later stiffness value for the interface arch-diaphragm.

For macro-model with interface a behavioural model like in the micro-model interface between two units was adopted. The type selected was crack-shear-crush model with constant mode II fracture energy (for details please go to section 4.2.2). The behaviour of masonry was kept like in case of

plain arch (section 4.2.1) and was applied to both, arch and stiffening element. The material properties used in the numerical model as are presented in Table 5.2 and Table 5.3.

Table 5.2. Elastic properties of the masonry and interface.

Element	Elastic modulus E N/mm^2	Poisson ratio ν -	Normal stiffness k_n N/mm^3	Shear stiffness k_s N/mm^3
Masonry MS1	1193	0,15	-	-
Interface INT	-	-	21	8,4

Table 5.3. Inelastic properties of the masonry and interface

	Tension		Shear				Compression		
	f_t N/mm^2	G_f^I N/mm	c N/mm^2	$tan\varphi$ -	$tan\phi$ -	G_f^{II} N/mm	f_c N/mm^2	G_{fc} N/mm	κ_p -
Masonry MS1	0,04	0,02	-	-	-	-	5,97	9,55	-
Interface	0,072	0,025	0,173	0,43	0	0,05	5,97	9,55	10

The mesh of the model consists of two types of elements, eight-nodded quadrilateral on the arch and six-nodded triangular on the strengthening part, like in the case of model without interface. For the interface six-nodded interface elements were adopted to simulate the masonry joint interface (as presented in Fig. 5.3). Zero thickness interfaces was assumed for the arch-element joints.
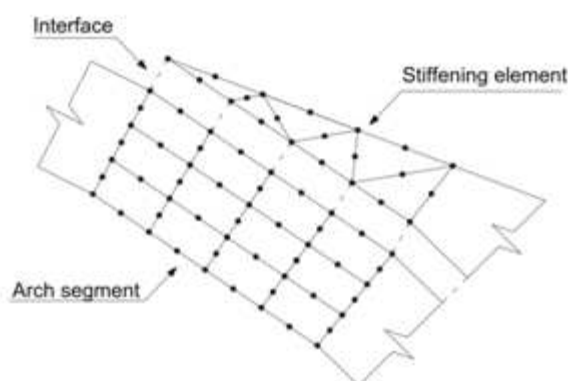
Fig. 5.3. Detailed view on the mesh elements applied for particular model elements.

5.1.2. Numerical results of the macro-modelling analysis

The numerical results obtained from the strengthened arch allowed to define the rate in which the strengthening technique is efficient in increasing structural capacity of the original arch. Comparison graphs for unstrengthened and strengthened numerical model are presented in

following pages. They are focused on two points of the structure, keystone and the point of load application.

5.1.2.1. Macro-model without interface

In this case the strengthening technique show significant increase of the load carrying capacity of the arch. The structural response is illustrated in Fig. 5.4. The same figure presents comparison of two cases of the arch, strengthened and non-strengthened. The nonlinear behaviour starts approximately for force value of 25% of the ultimate load (0,5kN) which might be explained with additional self-weight of the construction creating higher stress state in the arch. The behaviour of both models is of similar type, nevertheless arch with strengthening shows increased peak load, higher initial stiffness and vertical displacement under the maximum applied force. Comparison of results of vertical displacement between both models is listed in Table 5.4.

a) b)
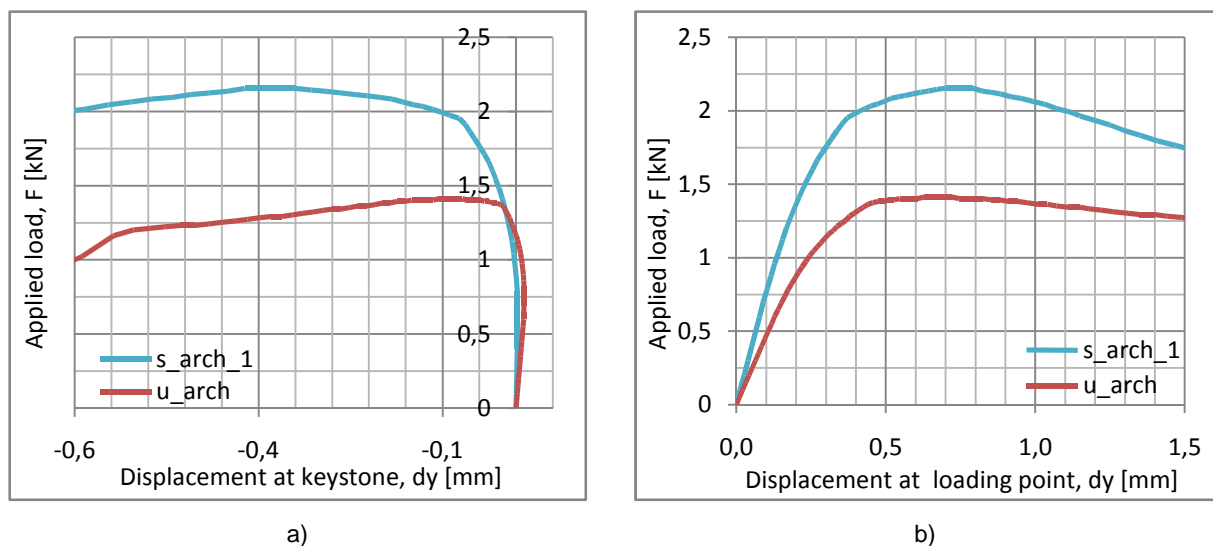
Fig. 5.4.Comparison of results between unstrengthened and strengthened arch presented on load - displacement curve for a) the keystone, b) loading point.

Table 5.4. Comparison of results for unstrengthened and strengthened (perfect bond) models.

Type of arch	Ultimate load capacity kN	Increase of peak load %	Initial stiffness kN/mm
u_arch	1,41	-	6,03
s_arch_1	2,15	34	12,60

The strengthened arch exhibits similar structural behaviour and collapse mechanism, when compared with the plain arch, see Fig. 5.5 where the principal tensile strains are illustrated on the deformed mesh. The sequence of hinge formation is like of the unstrengthened arch. This was expected as the strengthening technique does not provide any kind of continuous confinement

that could alter the mechanism of failure, i.e. by preventing the formation of hinges on any of the sides. However, the hinges were shifted because of the presence of stiffening element. In the springer on the opposite side of the load application point the location of hinge can be estimated as on the edge of the diaphragm. This is different from the plain arch hinge position (compare with Fig. 4.7).
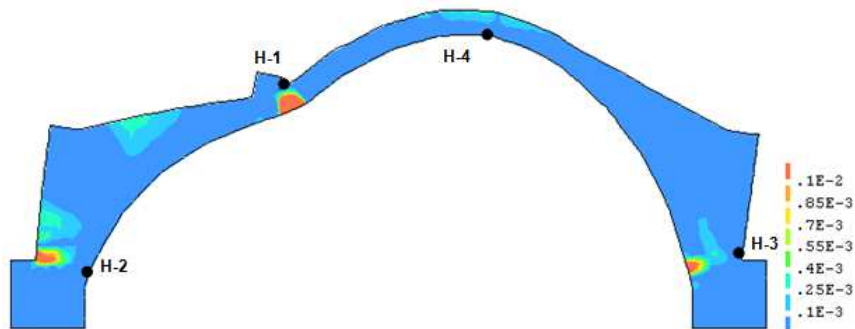
Fig. 5.5. Deformation and hinge location of the numerical arch (s_arch_1) under the peak load (2,15kN).

5.1.2.2.   Macro-model with interface

In the model with interface between arch and stiffening diaphragm, the structural behaviour is different than in the case of continuous strengthened model. The failure is located mostly in the arch itself, but also in the left stiffening element. The hinges occurred approximately in the same positions like in the unstrengthened arch, but the cracks appear also in the diaphragms. After the ultimate load is reached, a drop in load carrying capacity is noticeable. Failure was, in general, characterized by brittle behaviour and was recognized as mechanism.

As it can be seen in Fig. 5.6 (which presents vertical displacement at the keystone and under the load applicator) the peak load is increased with regards to the plain arch. The maximum value of reinforced arch is just 1,72kN which means raise from the original arch of 22%. In terms of initial stiffness, the strengthening increases it double (presented in

Table 5.5 Table 5.5). Nonlinear behaviour starts relatively earlier around 0,6kN (around 35% of ultimate load).

Fig. 5.6.Comparison of results between unstrengthened and strengthened arch presented on load - displacement curve for a) the keystone, b) loading point.

Table 5.5. Comparison of results for unstrengthened and strengthened (with interface) models.

Type of arch	Ultimate load capacity kN	Increase of peak load %	Initial stiffness kN/mm
u_arch	1,41	-	6,03
s_arch	1,72	22	11,06

As in the case of the unstrengthened arch the appearance of hinges along load application was identify. The sequence of formation is presented in Fig. 5.7. Each frame of the Fig. 5.7 presents the deformation of the arch with the average principal tensile strains depicted on it. Tensile strains are associated with crack appearance, therefore location of hinges was possible to identify (marked with black dot). It can be observed that the development of all four hinges happens before the ultimate load occurred.

a)

b)

c)

d)
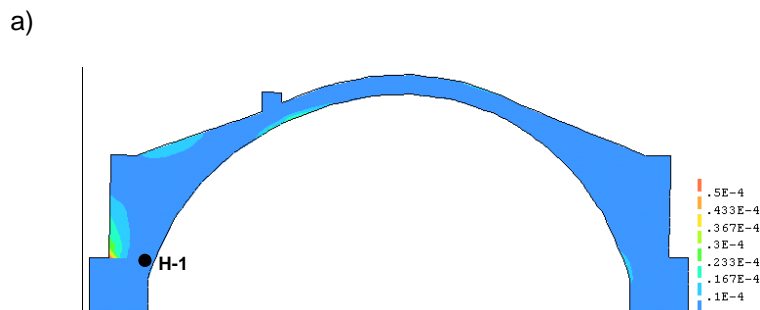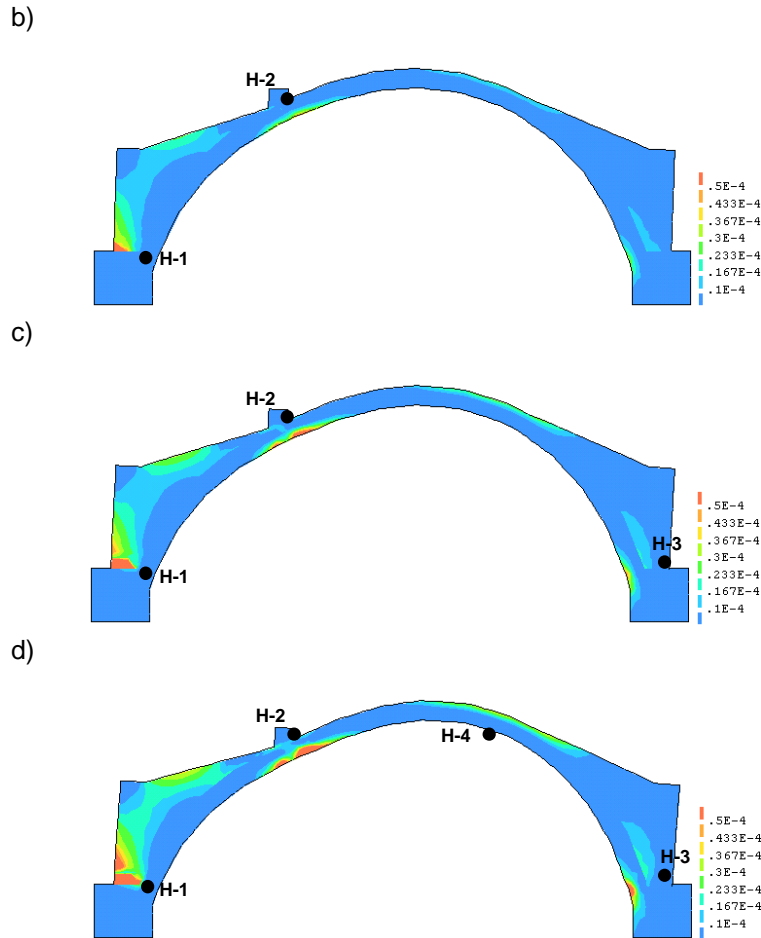
Fig. 5.7. Numerical sequence of hinges formation for the strengthened masonry
arch defined as principal strains imitating crack appearance.

The sequence of formation of each hinge varies from the one that happened in plain arch. For the arch with stiffening diaphragms first hinge to appear is the one located in the intrados, close to the springer of the arch, located on the same side as the load application point (Fig. 5.7a) and second hinge (Fig. 5.7b) is the one located on the extrados, next to the load. The third hinge (Fig. 5.7c) develops in the extrados, near the other springer of the arch. The last hinge appears in the intrados of the arch ring, at the position of the biggest displacement (Fig. 5.7d). In the position of hinge creation, a tendency can be notice, in which the hinges form alternately, once on intrados, once on extrados. Nevertheless, the sequence is different from the one of the plane arch, which must be connected with the presence of the stiffening element, which forces different structural behaviour of the arch. It seems that self-weight of the diaphragms, with the increasing load application create a stress concentration, sufficiently high to alter the typical behaviour of an arch. However, the failure comes from the same origins like in the plain arch, i.e. the development of a four-hinge mechanism in the arch.

5.1.3. Results summary

The presence of interface alters the performance of strengthened arch in comparison with the one without interface. Without interface, the structure works like two continuous elements, without zones of potential discontinuities represented by the interface. Thanks to that, the model without interface showed higher load carrying capacity and higher displacement under peak load. The difference in behaviour of each model type can be observed in Fig. 5.8 where load-displacement curves for each arch are illustrated. Table 5.6 consists of all numerical results, of the unstrengthened arch (u_arch) and both types of strengthened model (s_arch_1 representing model without interface and s_ach with interface). From the results it can be noticeable (as from Fig. 5.8) that model without interface provides higher strengthening effect. This might be explained with the presence of discontinuities represented by interface which are regions of lower mechanical parameters, therefore possible failure zones.
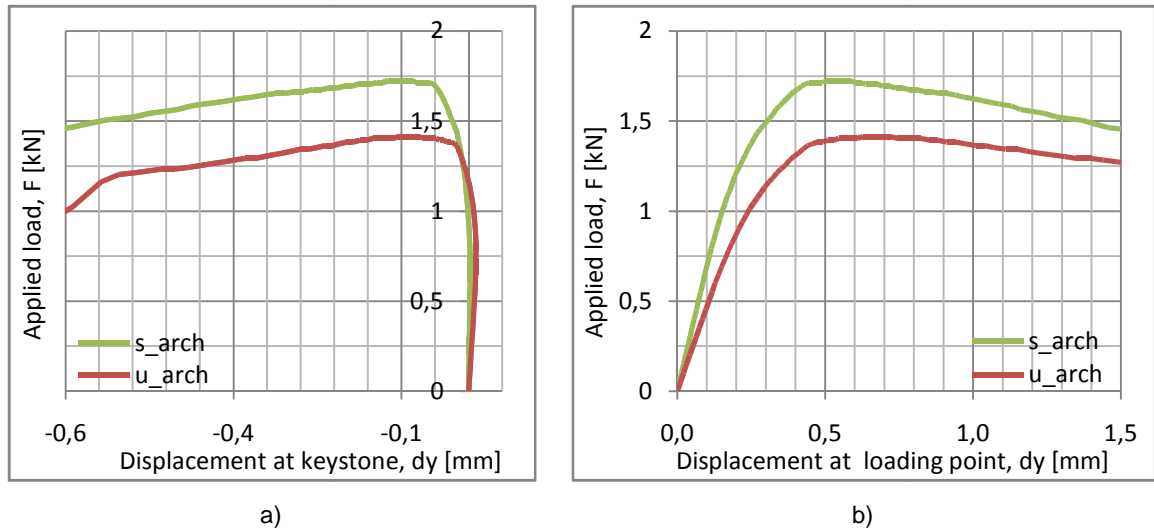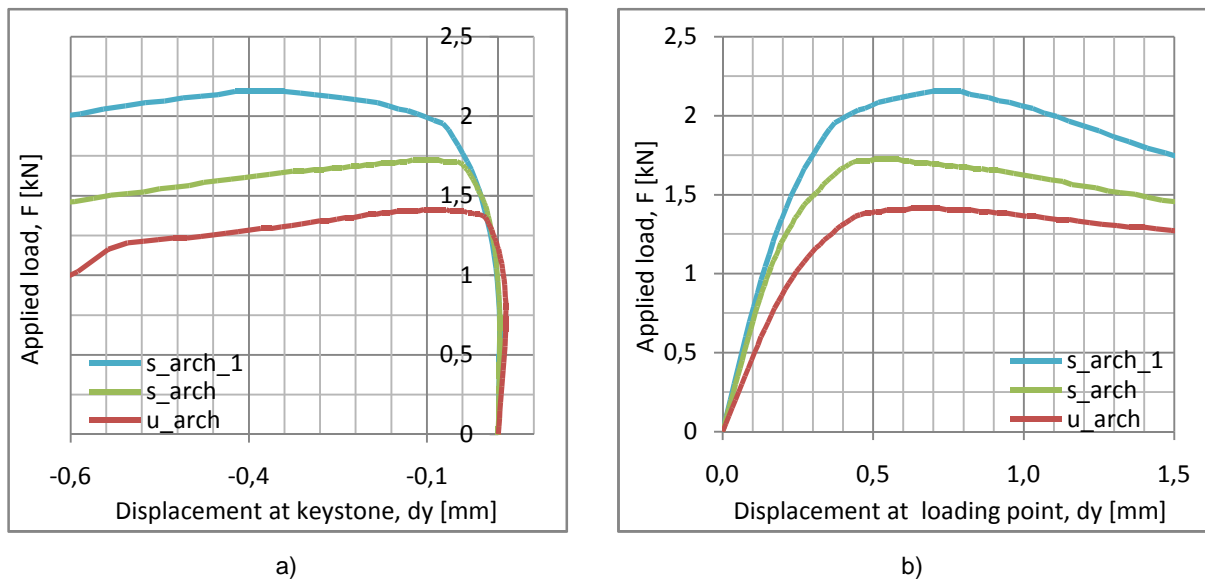

Fig. 5.8.Comparison of results between unstrengthened and strengthened arch
presented on load - displacement curve for a) the keystone, b) loading point.

Table 5.6. Comparison of results for unstrengthened and strengthened models.

Type of arch	Ultimate load capacity kN	Initial stiffness kN/mm
u_arch	1,41	6,03
s_arch	1,72	11,06
s_arch_1	2,15	12,06

An interesting matter is the fact of different hinge formation in each strengthened model. As presented previously for the reinforced arch without interface, the hinge development happened in

the same sequence like in the plain arch, while in the model with interface the progression was different. Possibly this can be explained with the presence of interface. The first strengthened arch, treated like continuous element of composite, uniform material behave like one arch element just of variable thickness along the span. Stress was distributed uniformly, without concentration in any particular parts, consequently the arch developed the first hinge at the same location like in plain arch.

This argumentation seems reasonable when we notice that the first hinge in the model with interface occurs in the springer on the side of the load application (Fig. 5.7a). This is connected with stress concentration transmitted into weak plane of the interface between the abutment and stiffening element on which self-load of the diaphragm and incremental live load are applied.

Since the stress concentration starts for smaller load, the model with interface can withstand lower force values and, thus reaches its failure quicker. This is illustrated in Fig. 5.9, where for the same load value (1,72kN) different principal strain distribution occurs. In the arch without interface the range of strains in much smaller than in the model with interface. As a result, the model s_arch_1 shows better performance and capacity.

a)

b)
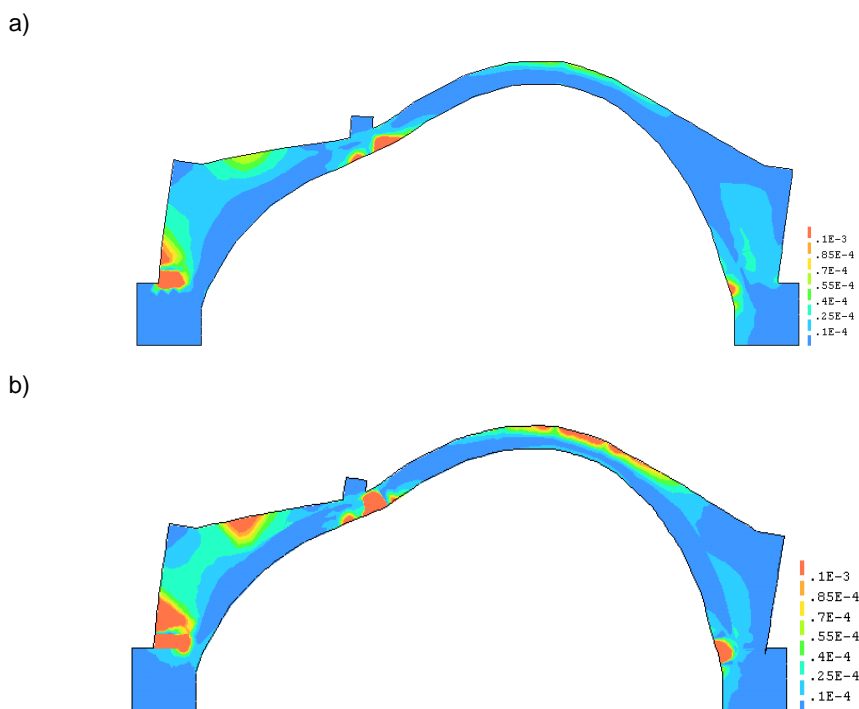
Fig. 5.9. Deformation of the arch and distribution of principal tensile strains under the force 1,72kN for model: a) without interface (s_arch_1); b) with interface (s_arch).

Nevertheless, it has to be reminded that, as explained previously, the model without interface is a poorer representation of reality. Consequently, it was taken here as a theoretical curiosity that helps to define the significance and importance of interface in modelling, rather than basis for defining the efficiency of the strengthening technique.

5.2. Micro-model of strengthened arch

5.2.1. Characteristics of the micro-model

Micro-modelling is a more complex type of representation of reality. As explained in section 4.2.2 the model consist of units, imitating the bricks in the arch, and interfaces reproducing the mortar joints. In case of the strengthened arch the situation becomes even more complicated. The model is created based on the reference arch with additional elements, representing the extrados stiffening diaphragm. The geometry of the strengthened micro-model is like in the case of macro-model presented in Fig. 5.1. Both parts are connected with use of interfaces. In order to replicate the reinforced arch, different materials were used for each component of the model (Fig. 5.10). In this way the model is describing the real materials in more convincing manner.
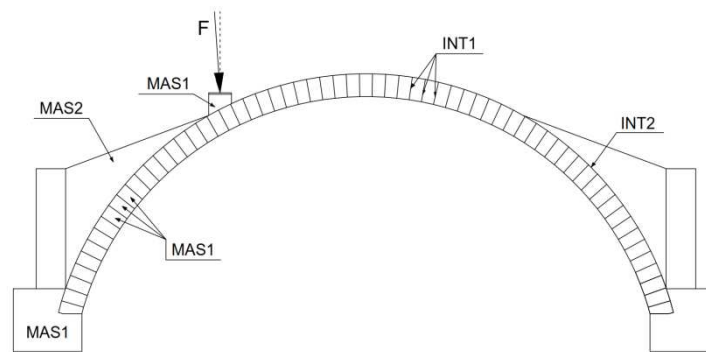
Fig. 5.10. Geometry of the strengthened arch with used material types for each element.

The mesh adopted consists of eight-nodded quadrilateral elements for the arch, six-nodded triangular for the extrados stiffening diaphragms and six-nodded structural interfaces. For both interfaces a three-noded integration scheme was chosen. Like in previous cases the model was analysis in terms of structural response to a monotonic, incremental load. As in the micro-model of reference arch, here also the arc-length method with CMOD control technique was employed. The mesh applied to the model is shown in Fig. 5.11.

a)   b)
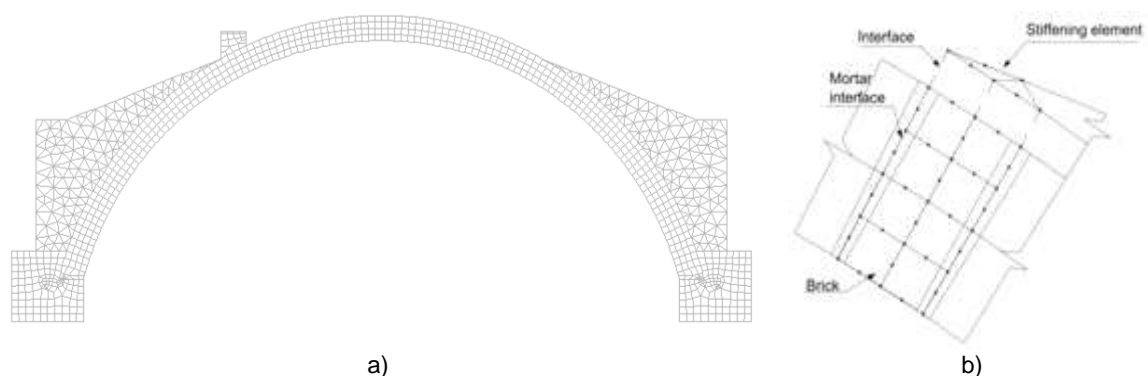
Fig. 5.11. Meshing type use in the model: a) general view, b) detail of the unit-diaphragm connection.

Material properties and behavioural models for the arch were explained in section 4.3.14.2.2 and for the extrados element in section 5.1.1. By keeping the same model of the applied strengthening, comparison between both numerical models will be possible. All material properties used for this model are listed in Table 5.7 and Table 5.8.

Table 5.7. Elastic properties of the masonry and interface.

Element	Elastic modulus E N/mm^2	Poisson ratio ν -	Normal stiffness k_n N/mm^3	Shear stiffness k_s N/mm^3
Masonry MS1	7200	0,15	-	-
Masonry MS2	1193	0,15	-	-
Interface INT1 and INT2	-	-	21	8,4

Table 5.8. Inelastic properties of the masonry and interface

	Tension		Shear				Compression		
	f_t N/mm^2	G_f^I N/mm	c N/mm^2	$tan\varphi$ -	$tan\phi$ -	G_f^{II} N/mm	f_c N/mm^2	G_{fc} N/mm	κ_p -
Masonry MS2	0,04	0,02	-	-	-	-	5,97	9,55	-
Interface INT1	0,072	0,025	0,173	0,43	0	0,05	5,97	9,55	10
Interface INT2	0,04	0,02	0,173	0,43	0	0,05	5,97	9,55	10

5.2.2. Numerical results of the micro-model

The results of the analysis of the micro-model with strengthening show some surprising features. The load-displacement curve is showed in Fig. 5.12. It can be seen that the displacement in the vertical direction of the keystone is very low, around 0,01mm. Also, this might be linked with the fact that unstrengthened model did not replicate the post-peak behaviour credibly and failed before any further displacement. Therefore, the graph looks rather strange in comparison with all previous ones, but is just a graphical manner due to the lower values on the horizontal axis representing vertical displacement.

The plain arch in the keystone goes downwards constantly until the peak load is reached, after which brittle collapse happens and displacement of the keystone inverts the direction of movement (part of connection between units is lost and keystone goes up). The strengthened arch shows a more ductile behaviour. The loading point moves as expected, downwards with increasing load. The keystone, though, firstly behaves like in the plain arch, but with increasing load the deformation of the arch is grater and affects the displacement of the keystone which starts to go upwards (Fig. 5.12a). The displacement of the loading point was as expect, with increasing load goes downward until the maximum load is achieve (Fig. 5.12b).

a)   b)
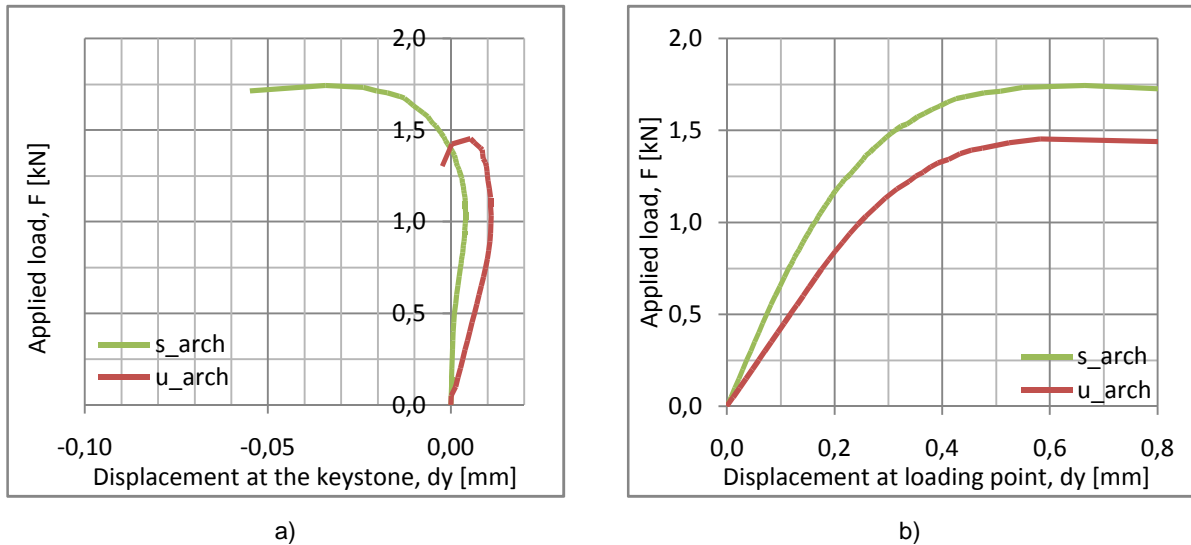
Fig. 5.12.Comparison of results between unstrengthened and strengthened arch
presented on load - displacement curve for a) the keystone, b) loading point

The presence of masonry diaphragm alters the structural response of the arch in terms of ultimate capacity and initial stiffness. Initial stiffness in the vertical direction is increased highly, almost doubled its value. All the comparable parameters are listed in Table 5.9.

Table 5.9. Comparison of results for unstrengthened and strengthened models.

Type of arch	Ultimate load capacity kN	Increase of peak load %	Initial stiffness kN/mm
u_arch	1,45	-	5,85
s_arch	1,74	20	11,54

Like in the case of macro-model, also for the micro-model the sequence of hinge formation could be tracked and is represented in Fig. 5.13. However, in the case of micro-model the appearance of hinges from the distribution of average principal tensile strains is not as obvious as in the case of macro-model. This aspect is understandable due to the fact that in micro-model for each structural element a different material was assigned. And with, in general, average stress state in the structure, the strains were increasing more in lower quality material. Therefore, to make it easier to define developing hinges, corresponding principal tensile stresses (for the same each load step like the tensile strains) are also plotted in Fig. 5.14.

a)

a)

b)

b)

c)

c)

d)

d)
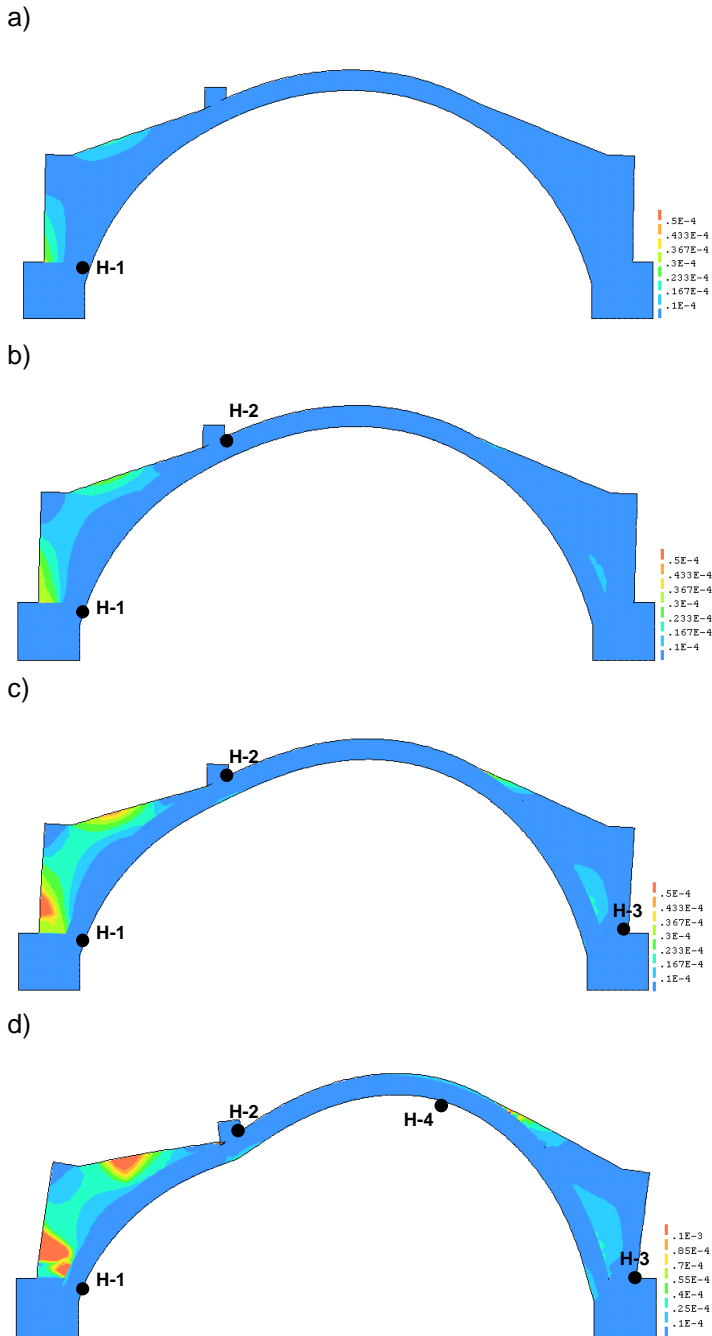
Fig. 5.13. Deformed arch with distribution of principal tensile strains for steps of a-c) hinge formation, d) ultimate load.
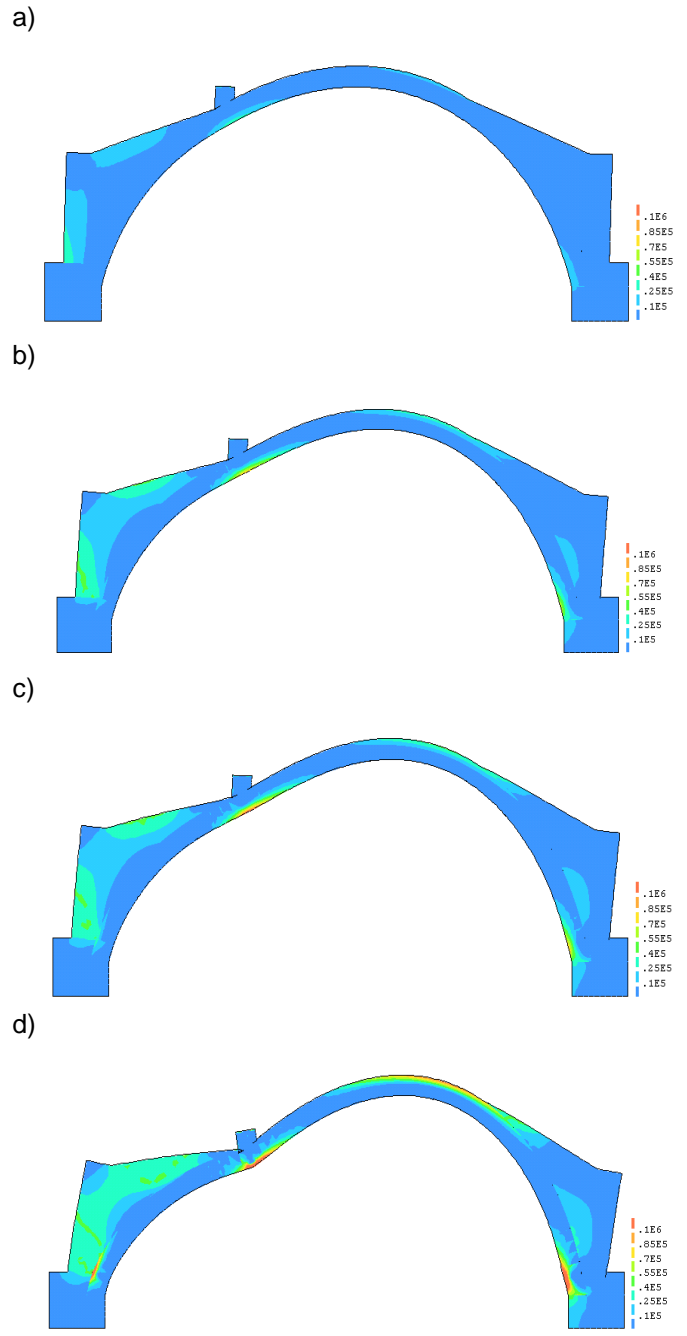
Fig. 5.14. Deformed arch with distribution of principal tensile stresses for steps of a-c) hinge formation, d) ultimate load.

Thanks to corresponding view on principal strains and stresses it was possible to detect the approximated location of hinges. The first hinge creates strain concentration in the stiffening element located on the same side like the load application point (Fig. 5.13a). The second hinge is placed under the loading point. Although, increase of strains in this point is not visible in Fig. 5.13b, stress concentration at this location is noticeable at Fig. 5.14b and considered to be the effect of deformation caused by hinge number two. The third hinge affects two regions in the structures creating both strain concentration zones on the other extrados element (Fig. 5.13c) and

stress state in the adjacent springer (Fig. 5.14c). In the last frame of Fig. 5.14 a new tensile stress zone is observable on the extrados of the arch. This location is regarded as position of the last hinge, although, again in the strain image is barely detectable. Late formation of the last hinge might explain so brittle behaviour of the structure. As a final remark, it can be clearly state that the biggest deformations happens firstly and mostly in the stiffening part and not in the arch. Probably this causes the higher ultimate load of the strengthened arch.

5.3.   Comparison of models

The different modelling strategies followed aimed at understanding the effect of the stiffening diaphragms on the arch behaviour and to clarify the influence of the degree of detail introduced in the used models. The application of the reinforcement with use of extrados stiffening diaphragms modified the arch static behaviour, however the collapse mechanism was connected with development of four plastic hinges (as in the case of plain arch).

The macro-model was able to replicate the post-peak behaviour of the strengthened arch further than the micro-model, thus, its behaviour can be considered more ductile. It was easier in pre-processing and the analysis required less time and input parameters. Both cases of macro-model were described to see the difference in performance and to state the influence of the interface element in the behaviour of the strengthened arch.

Micro-model required higher computational cost, more element and parameter input, was also more time consuming in creation. Also it required more input data. Its structural response to the incremental load was more brittle, which might be seen as a closer response to the real arch.

The results in terms of ultimate capacity of both reinforced models were very similar (1,72kN and 1,74kN, for macro- and micro-, respectively). The same was observed in the values of initial stiffness, both models were working in range around 11kN/mm. The macro-model showed higher capacity in the vertical displacement which signifies more ductile behaviour of it. This might be connected with the uniform, continuous material used all over the arch model. Less discontinuities defined in the macro-model allowed bigger deformation.

As mentioned in the section 4.4 numerical modelling is just a way to represent reality with use of mathematical and physical phenomenon defined as sets of equations and hypothesis. Because both models have partially different assumptions it is difficult to state which of them is closer to reality, and thus better. The most significant base for such a conclusion would be evaluation of the hinge appearance in a real construction and comparison with results of both models. Unfortunately, this is impossible as there was no experimental research done on this type of strengthening.

Nevertheless, because the results in terms of ultimate load capacity and initial stiffness are similar for macro- and micro-model it can be concluded that the strengthening technique is efficient and

always worth considering while thinking about future reinforcement applied to masonry arches. However, it should be used for construction which do not need high increase of load baring capacity or in combination with other strengthening techniques.

Chapter 6
PARAMETRIC STUDY
OF THE DIAPHRAGM'S ROLE

6.1. Model selection

To fully use the potential of numerical modelling a parametric study was established to understand the response of the strengthened arch to variations of material parameters and geometry of the extrados stiffening diaphragm. Three changes of the constituent strengthened model were made. None of it implies changes in the mesh of the numerical model (in the plane stress directions), due to time constrains.

The decision to use the macro-model as a base for the parametric study was connected with higher simplicity of the model and smaller computational cost of the analysis. Additionally, the macro-model was performing better in terms of post-peak behaviour which allows to interpret better the influence of each strengthening change.

Calibration of the numerical model of plain arch with the use of experimental data give a wide possibility to perform numerous tests on the arch. After exploiting the simplest strengthened model (described in chapter 5) which geometry was based on the experimental program done by Girardello in [28] other alterations were introduced into the numerical model to define which parameters have the highest influence on the performance of the strengthened arch.

Three types of parametric analysis were issued:

1) Change of the thickness of the extrados stiffening diaphragm from 12cm to 24cm;

2) Increase of the properties of the masonry treated like a composite material (doubling of Young modulus and compressive and tensile strength);

3) Increase of the properties of the interface representing the mortar joints between arch and the diaphragm (doubling of tensile strength, 10x higher value of normal and shear stiffness, increase of cohesion).

6.2. Arch strengthened with thicker extrados stiffening diaphragm (s_arch_2)

Enlargement of stiffening element is the first type of the alteration to the original strengthening technique. Since the arch is of 77cm wide application of another diaphragm of 12cm thickness seems legit as it does not exceed 50% of the extrados area of the arch.

Fig. 6.1 present a comparison of test results between unstrengthened arch (u_arch) and two cases of strengthened arch: one with diaphragm of 12cm of thickness (s_arch) and other with 24cm of thickness (s_arch_2).

Fig. 6.1.Comparison of results between unstrengthened and strengthened arch presented on load - displacement curve for: a) the keystone, b) loading point.

As it can be understood from Fig. 6.1, the double thickness of the extrados stiffening diaphragm influences the structural behaviour of the arch. It increases the ultimate load capacity of the arch of another 14% in regards with first strengthening. The initial stiffness is significantly increased in comparison with unstrengthened arch. Also the stiffness between the two strengthened models is different, however the rise is smaller than between u_arch and s_arch. Table 6.1 presents summary of results.

Table 6.1. Comparison of results for models of different properties.

Type of arch	Thickness of the diaphragm cm	Ultimate load capacity kN	Increase of peak load %	Initial stiffness kN/mm
u_arch	-	1,41	-	6,03
s_arch	12	1,72	22	11,06
s_arch_2	24	1,95	38	14,06

The addition of the second diaphragm does not alter the hinge location or sequence of formation (Fig. 6.2). The order is as in the original strengthening type (s_arch). As a result, it can be concluded that failure mechanism does not depend on the thickness of the extrados stiffening element.

a)

b)
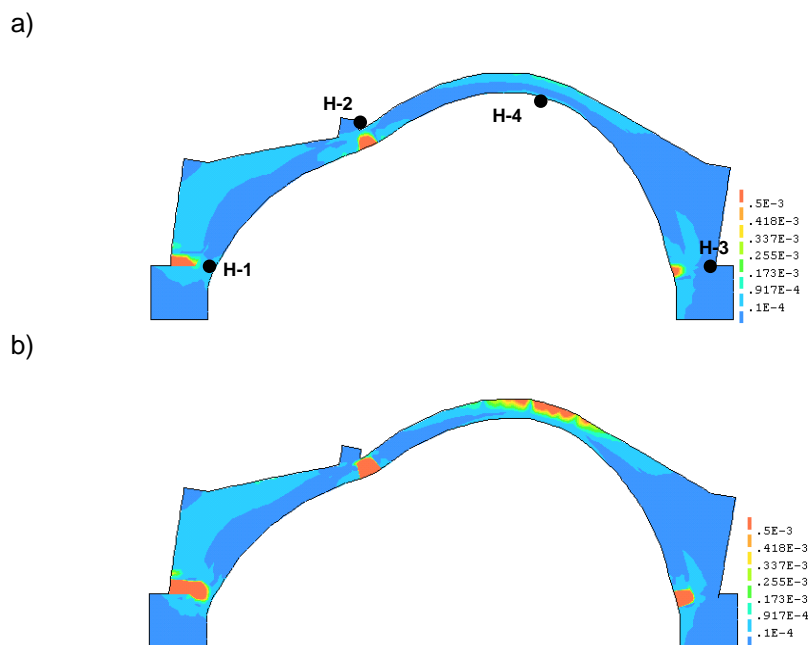
Fig. 6.2. Deformation and distribution of principal tensile strains for model s_arch_2:
a) under peak load; b) in moment of final displacement.

6.3. Strengthened arch with extrados stiffening diaphragm of better masonry quality (s_arch_3)

Another part of the parametric analysis was connected with the change of properties of masonry used for extrados stiffening diaphragm. Geometry of the stiffening element is kept without any change. The masonry properties used are as in Table 6.2. Masonry MS1 refers to the arch while masonry MS2 (the only being improved) refers to the diaphragms.

Table 6.2. Material properties of masonry of extrados stiffening diaphragm.

	Elastic modulus	Tension		Compression	
	E N/mm^2	f_t N/mm^2	G_f^I N/mm	f_c N/mm^2	G_{fc} N/mm
Masonry MS1	1193	0,04	0,02	5,97	9,55
Masonry MS2	2400	0,08	0,02	12	9,55

Fig. 6.3 shows a comparison of results between unstrengthened arch (u_arch), originally strengthened one (s_arch) and arch strengthened with the use of extrados stiffening diaphragm made of higher quality masonry (s_arch_3). As it can be seen, the upgrade of the material of the transversal diaphragm does not make a significant impact into to the performance of the already strengthened structure. In terms of load capacity the peak value increases from 1,72kN to 1, 79kN. The value of displacement corresponding with the peak load decreases slightly, but is of the same value as for the arch with primary strengthening.
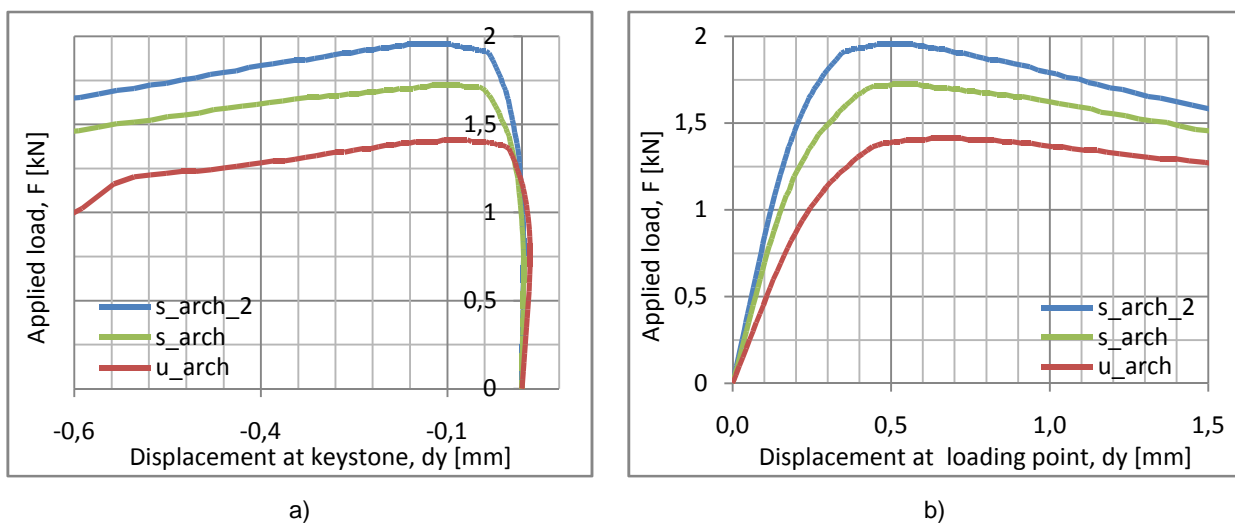

Fig. 6.3.Comparison of results between unstrengthened and strengthened arch
presented on load - displacement curve for: a) the keystone, b) loading point.

From above graphs some interesting facts might be stated. The initial stiffness of the arch is significantly increased in comparison of the stiffness of plain arch, although the difference between two strengthening techniques is minor. Furthermore, after certain displacement occurs in the arch a more significant drop in load happens, which did not occur in the case of unstrengthened or originally strengthened arch. All the results of the analysis are presented in Table 6.3.

Table 6.3. Comparison of results for models of different properties.

Type of arch	Ultimate load capacity kN	Increase of peak load %	Initial stiffness kN/mm
u_arch	1,41	-	6,03
s_arch	1,72	22	11,06
s_arch_3	1,79	27	13,02

The change of material properties of the reinforcing element did not alter the performance of the arch in terms of location and sequence of hinge development (Fig. 6.4).

a)

b)

Fig. 6.4. Deformation and distribution of principal tensile strains for model s_arch_3:
a) under peak load; b) in moment of final displacement.

6.4. Strengthened arch with higher mortar properties (s_arch_4)

The last test in this parametric study section was connected with changes in the properties of the mortar used for connecting the extrados stiffening diaphragm to the arch. Therefore the properties of the interface were enhanced and are presented in Table 6.4.

Table 6.4. Properties of the interface element (s_arch_4).

Element	Normal stiffness k_n N/mm^3	Shear stiffness k_s N/mm^3	Tensile strength f_t N/mm^2	Cohesion c N/mm^2
Interface INT	210	84	0,14	0,25

Fig. 6.5 illustrates that an increase of tensile strength, cohesion and normal and shear stiffness of the arcg-diaphragm interface give a small raise of load capacity of the arch. The peak load is of value 1,79kN which means a 4% of amplification with regards to originally strengthened arch. Also, the difference between strengthening types, s_arch and s_arch_4, is very minor, the new method gives just extra 4% of load capacity. In terms of initial stiffness the increase of normal and shear stiffness of interface made a difference in the global stiffness of the arch. Comparison of results between all types of structures is presented in Table 6.5.

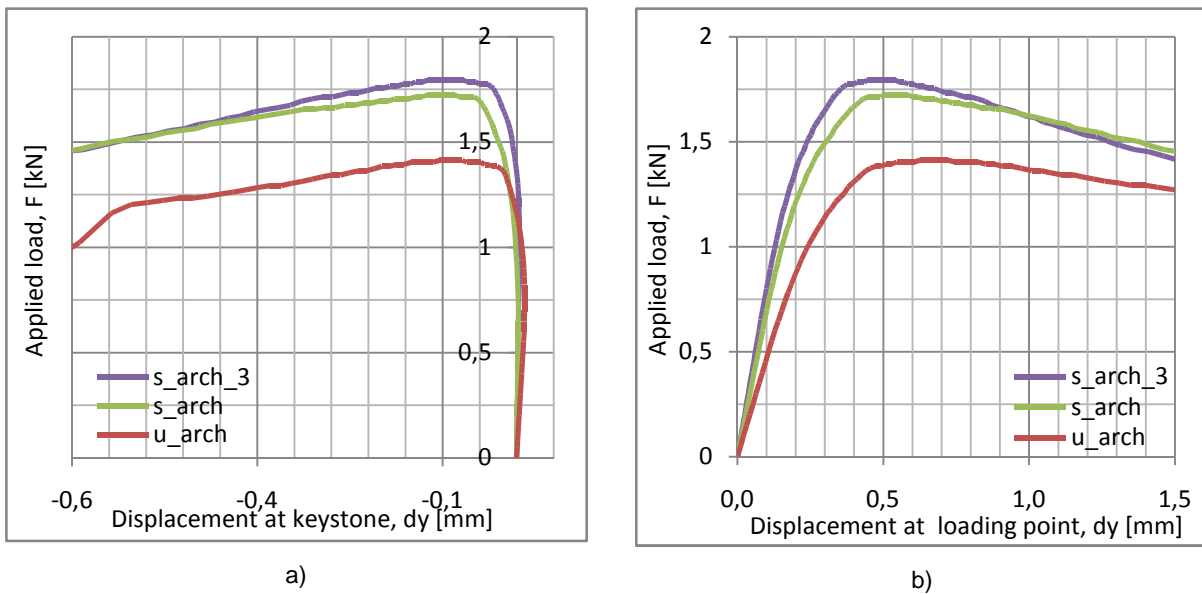a)   b)
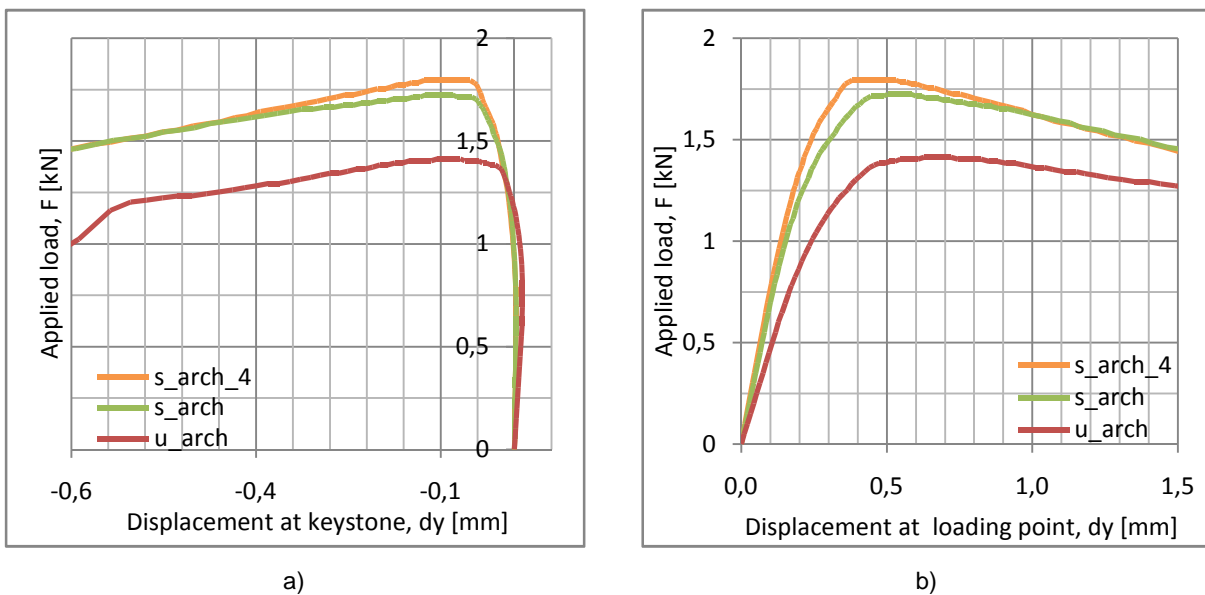
Fig. 6.5.Comparison of results between unstrengthened and strengthened arch presented on load - displacement curve for a) the keystone, b) loading point.

Table 6.5. Comparison of results for models of different properties.

Type of arch	Ultimate load capacity kN	Increase of peak load %	Initial stiffness kN/mm
u_arch	1,41	-	6,03
s_arch	1,72	22	11,06
s_arch_4	1,79	27	12,41

The hinge formation happens like in previous cases, no alterations are noticeable. The graphical representation of the average principal tensile strains distribution under peak load and ultimate displacement is presented in Fig. 6.6.
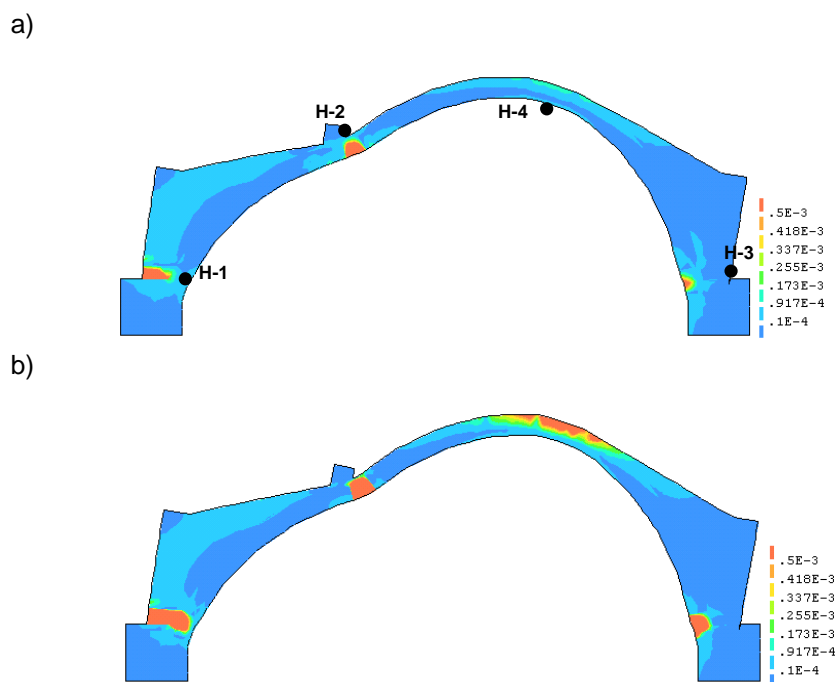
a)

b)
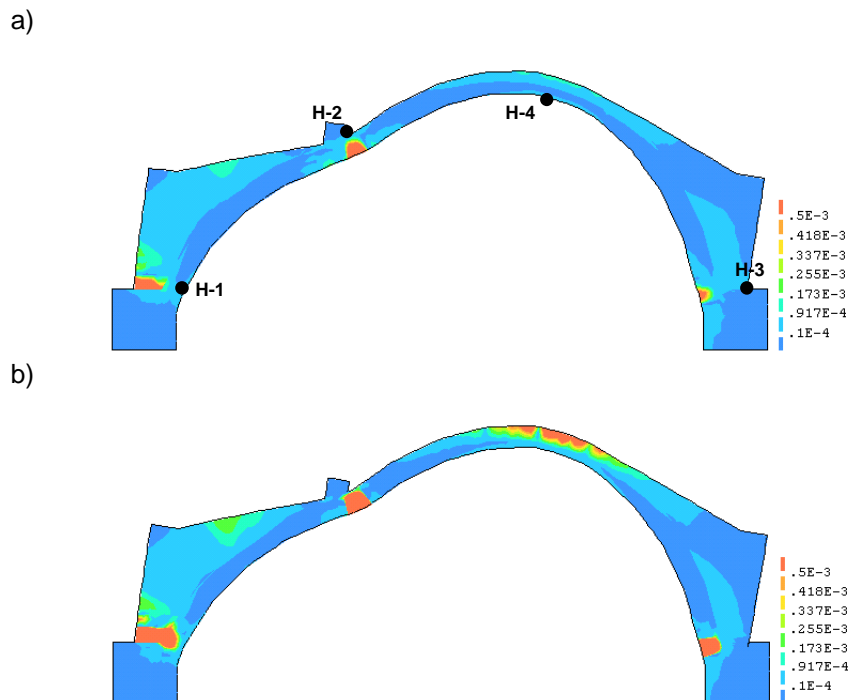
Fig. 6.6. Deformation and distribution of principal tensile strains for model s_arch_4:
a) under peak load; b) in moment of final displacement.

6.5.   Comparison of results

As expected, each alteration in the extrados stiffening diaphragm provides a variation in terms of the load carrying capacity that the arch can sustain. Comparison of results of all parametric variations is illustrated in Fig. 6.7 and listed in Table 6.6.

The biggest influence is due to the use of another diaphragm of the same thickness like the original one (s_arch_2). Its impact on the arch was the biggest and allows to reach a peak load of value 1,95kN. Also arch with this reinforcement showed the highest initial stiffness, as was expected from the use of another extrados element.

Use of better quality masonry (s_arch_3) for the diaphragm element also did not bring as big changes in the load bearing capacity as expected. Increase of only 4% from the value of the originally strengthened arch (s_arch) does not seem worth further consideration. A better idea is to take masonry fully compatible (mechanically, physically and chemically) to the existing one and disregard this small increase of capacity.

The last alteration was made on the properties of the mortar, connecting the diaphragm to the arch, that in terms of modelling meant changes in the interface. Increase of the strength and

stiffness of the interface improved the structural response. The ultimate load capacity was obtained as in case of better masonry. This allow to conclude that the changes in the material of the diaphragm is worth considering when comparing with the increase of mortar.
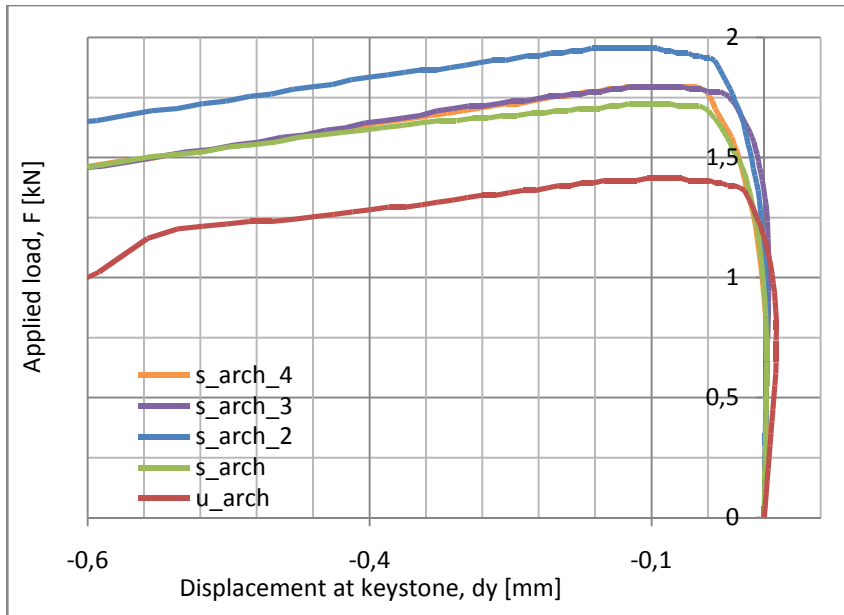
a)
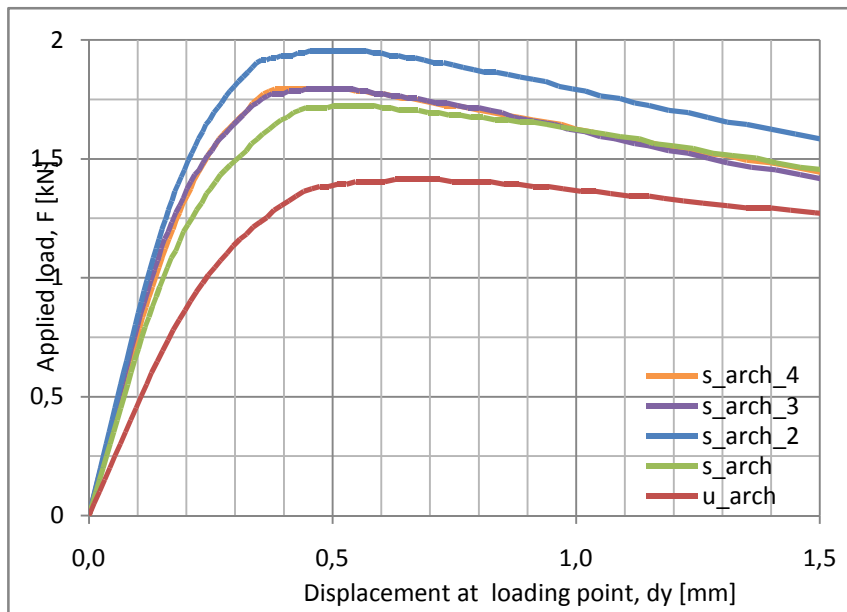
b)

Fig. 6.7.Comparison of results between all types of analyzed arches presented on load - displacement curve for: a) the keystone, b) loading point.

Table 6.6. Comparison of results for all models of parametric studies.

Type of arch	Ultimate load capacity kN	Increase of peak load %	Initial stiffness kN/mm
u_arch	1,41	-	6,03
s_arch	1,72	22	11,06
s_arch_2	1,95	38	14,06
s_arch_3	1,79	27	13,02
s_arch_4	1,79	27	12,41

As can be concluded from the parametric study, additional alteration to the original strengthening technique does not alter significantly the maximum load carrying capacity nor the initial stiffness of the arch. However, among three proposed changes the one worth considering is the use of a thicker stiffening element.

It is not clear if simultaneous application of all proposed changes could significantly increase the carrying capacity and initial stiffness of the original arch. Another alteration that might change the behaviour of the arch is the use of different geometry for the transversal stiffening diaphragm, i.e. continuous on the length of extrados with constant or variable height. Finally, the strengthening of the arch-diaphragm sub-structure resorting to the use of externally bonded fibers (either FRP, SRP or SRG) applied at the extrados of the arch-diaphragm sub-structure would definitely increase the load capacity and change the failure mode, but this solution has not been pursuit within this dissertation.

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Chapter 7
FINAL REMARKS

7.1.   Main conclusions

This Master thesis was focused on the numerical modelling of strengthened masonry arches and vaults. This topic was developed by many researches for several years, but most of the time it was concentrated on the use of innovative materials like FRP, and more recent on SRG, SRP or textile reinforced mortar (TRM). This approach towards strengthening of arches is reasonable as many constructions require significant increase of load bearing capacity and the mentioned methods provide good experimental results in terms of increasing peak load. This thesis tried to look on the strengthening methods from another point of view. Therefore, it was proposed to use only masonry-made extrados stiffening diaphragms as reinforcement technique. The method has some advantages as it is fully compatible with the arch material.

In terms of masonry arches modelling, the most rigorous approach is micro-modelling. Therefore, thesis was also focused on creation of two complementary models of an arch, micro- and macro-modelling, to be able to state conclusions on the utility of macro-modelling in case of masonry arch constructions.

The two ways of approaching the numerical modelling of masonry arches gave some interesting outcome. Both models of the reference arch were calibrated with sufficiently high approximation of results. In terms of initial stiffness and ultimate load, both models reached values very close to the experimental one. Although, the micro-model showed some difficulties in reproducing the post-peak behaviour. Also, models of the strengthened arch showed similar response. This allows to

conclude that the numerical representation of the reinforcement technique was done in a convincing way.

The proposed strengthening technique showed an influence on the structural response of the arch. In case of the macro- and micro-model the rate in which the stiffening element increased ultimate load capacity and the initial stiffness was comparable. This means that both models worked in a similar way and it can be concluded that the results provided by strengthening can be consider reasonable.

The parametric study performed on the macro-model showed a variety of results. Change of the masonry of the stiffening element or in the properties of mortar did not alter the response of the structure much. On the other hand, the thickness of the diaphragm reveal an important impact on the capacity of the arch.

As a final statement the most significant contributions of this thesis are listed below:

- This thesis was devoted to the numerical study of an unusual but fully compatible strengthening solution for arches and vaults, based on the use of transversal stiffening masonry diaphragms;

- The micro- and macro-modelling strategies were applied to a case study. Both techniques were able to reproduce the observed experimental behaviour of the unstrengthened arch, although the macro-modelling approach has performed better;

- Extrados stiffening diaphragms are a valid technique for improving the structural response of masonry arches and vaults, particularly in terms of better initial stiffness. However, if reinforcement is mainly focused on higher maximum load, the technique should be combined with another or, simply, disregarded;

- For the type of structures analysed the macro-model can be a good substitution for micro-model as it gives comparable good results at a lower cost;

- Numerical modelling is a powerful tool for reality representation; however its results have to be interpreted with common sense and combine with technical knowledge.

7.2. Further research

The present study could not cover all possible solutions regarding the strengthening of arch with extrados stiffening diaphragm. Therefore, as part of the present study, a number of areas for further research have been identified.

To validate the strengthened micro- and macro-model, developed for the thesis, and to confirm the results obtained from the analysis a set of experimental tests on arches strengthened with extrados stiffening diaphragm would be a rational step. This way, the efficiency of the strengthening technique would be confirmed.

In the present thesis the shape of the geometry of the strengthening element was defined by the experimental work done by Paolo Girardello. However, for development of this strengthening technique it is worth to consider other geometries of the stiffening diaphragm. For example, a continuous element going through the entire length of the extrados might modify the structural behaviour of the arch in a higher manner. Variable or constant height of the diaphragm, along the length of the extrados, is another way to verify the impact of the stiffening element on the arch behaviour.

Finally, another field of research can be connected with additional strengthening technique used simultaneously with the stiffening diaphragms. Use of continuous FRP strips or other innovative techniques (like SRG etc.) applied on the top of the diaphragms and extrados of the arch might result in significant increase of the arch's structural performance.

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