Numerical approaches in a problem of management of hydroelectric resources

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Abstract

In this paper we consider a simplified model for a system of hydro-electric power stations with reversible turbines. The objective of our work is to obtain the optimal profit of power production satisfying restrictions on the water level in the reservoirs. Two different numerical approaches are applied and compared. These approaches centers on a global optimization technique (Chen-Burer algorithm) and on a Projection Estimation Refinement method (Bushenkov' PER method) used to reduce the dimension of the problem.

1 Introduction

In [5] it is considered a simplified model for a cascade of hydro-electric power stations where some of the stations have reversible turbines. The objective is to optimize the profit of power production. The problem is treated in the framework of optimal control theory and sufficient conditions of optimality in a certain local sense are deduced.

In the actual paper we consider one of such cascades with two hydro-electric power stations. The fluxes of water to turbine or pump on each power station are defined as control variables and the maximization of the profit of energy sale is the objective function. The state variables represent the water level in reservoirs which are subject to some constraints. The presence of these constraints and the nonconvexity of the cost function contribute to an increase of complexity of the problem. Solving this problem amounts to maximizing a nondefinite quadratic form subject to linear constraints in a Hilbert space.

The nonconvexity of the cost function enables the existence of several local minimum and the application of a global optimization method would be convenient. The Chen-Burer algorithm is a method for global optimization of a general quadratic function subject to linear and bound constraints in $\mathbb{R}^n$ (for details, see [4]).

After constructing a discretization of our problem, we numerically obtain its global solution using Chen-Burer algorithm. Two different approaches are discussed and compared. In the first one, we directly apply the Chen-Burer algorithm. In the second approach we use a specific structure of the cost function that allows us to reduce the dimension of the problem constructing a projection of the set of feasible solutions onto a subspace of the cost function arguments. Then the Chen-Burer algorithm is applied to the "projected" lower-dimensional problem. After that the solution to the original discrete problem is obtained solving a simple convex programming problem.

The solution to the original discrete problem can now be used as an initial guess when applying a local optimization software.

2 Problem statement

For a cascade of 2 hydro-electric power stations, we consider the optimal control problem with dynamics of water volumes at time t , $V_k(t)$ $(K=1,2)$, described by the following control system

$$\begin{array}{rcl} \dot{V}_1(t) & = & A - u_1(t) \\ \dot{V}_2(t) & = & u_1(t) - u_2(t) \end{array}$$

where the controls $u(t) = (u_1(t), u_2(t))$ are the turbined/pumped volumes of water for each reservoir, at time t, and A is the incoming flow.

The control variables and the water volumes satisfy the following technical constraints :

$$\begin{array}{l} V_k(0) = V_k(T), \\ V_k(t) \in [V_k^m, V_k^M], \\ u_k(t) \in [u_k^m, u_k^M]. \end{array}$$

Here V_k^m and V_k^M, $k=1,2$, stand for the imposed minimum and maximum water volumes, respectively; u_k^m and u_k^M, $k=1,2$, are the the imposed minimum and maximum turbined/pumped water volumes. The objective is to find optimal controls $\hat{u}_k(\cdot)$ and respective volumes $\hat{V}_k(\cdot)$, that lead to an optimal profile for the management of water in the system:

$$J(u(\cdot), V(\cdot)) =$$

$$\int_0^T c(t) \left[u_1(t) \left(\frac{V_1(t)}{S_1} + H_1 - \frac{V_{2)}(t)}{S_2} - H_2 \right) + u_2(t) \left(\frac{V_{2)}(t)}{S_2} - H_2 \right) \right] dt \longrightarrow \max,$$

where $c(\cdot)$ is the price of the energy, H_k, $k=1,2$, are the liquid surface elevations and S_k, $k=1,2$, are the areas of the reservoirs.

In this work we assume the price $c(\cdot)$ to take constant values c_1 and c_2 respectively on the intervals $[0, \frac{T}{2}[$ and $[\frac{T}{2}, T]$. With this $c(\cdot)$ and replacing the control variables by equivalent expressions obtained from dynamic equations, we can write after some manipulation the cost function as

$$J(u(\cdot), V(\cdot)) =$$

$$-\frac{Ac_1}{s_1}\int_0^{T/2} V_1(t)dt - \frac{Ac_2}{s_1}\int_{T/2}^{T} V_1(t)dt + H_1(c_2-c_1)V_1(0) + \frac{c_2-c_1}{2s_1}V_1^2(0) - H_1(c_2-c_1)V_1\left(\frac{T}{2}\right) - \frac{c_2-c_1}{2s_1}V_1^2\left(\frac{T}{2}\right)$$
$$+H_2(c_2-c_1)V_2(0) + \frac{c_2-c_1}{2s_2}V_2^2(0) - H_2(c_2-c_1)V_2\left(\frac{T}{2}\right) - \frac{c_2-c_1}{2s_2}V_2^2\left(\frac{T}{2}\right)$$

A discret version of this model is now constructed. Taking an even number N of discretization steps and defining new variables x and y as

$$x = \left[V_1(0), V_1\left(\frac{N}{2}\right), V_2(0), V_2\left(\frac{N}{2}\right) \right],$$

$$y = \left[V_1(1), \cdots, V_1\left(\frac{N}{2}-1\right), V_1\left(\frac{N}{2}+1\right), \cdots, V_1(N-1), V_2(1), \cdots, V_2\left(\frac{N}{2}-1\right), V_2\left(\frac{N}{2}+1\right), \cdots, V_2(N-1) \right],$$

the cost function comes as

$$I(x,y) = \langle a, x\rangle + \langle b, y\rangle + \langle x, Qx\rangle \rightarrow \min \tag{1}$$

where a and b are appropriate vectors gathering the linear part of the cost relative to x and y and Q is an appropriate matrix difining the quadratic part of cost function.

The constraints of the problem are translated into

$$V_i(k) \in [V_i^m, V_i^M], \text{ for } k = 0, \cdots, N-1 \text{ and } i = 1,2$$

$$(u_1(k) =)\ V_1(k) + A - V_1(k+1) \in [u_1^m, u_1^M], \text{ for } k = 0, \cdots, N-2$$
$$(u_2(k) =)\ V_2(k) + V_1(k) + A - V_1(k+1) - V_2(k+1) \in [u_2^m, u_2^M], \text{ for } k = 0, \cdots, N-2$$
$$V_1(N-1) + A - V_1(0) \in [u_1^m, u_1^M]$$
$$V_2(N-1) + V_1(N-1) + A - V_1(0) - V_2(0) \in [u_2^m, u_2^M]$$

3 Numerical methods

To the discretized problem we apply directly the Chen-Burer algorithm. This algorithm combines a finite branching based on the first order Karush-Kuhn-Tucker conditions and polyhedral-semidefinite relaxations of completely positive programs (see [4]. The time taken by this algorithm is very long and we consider another approach. Before applying Chen-Burer algorithm we reduce the dimension of the problem using the projection estimation refinement method (PER) of Bushenkov.

With this method the orthogonal projection P of a polytope X onto a subspace is approximated by a sequence of polytopes $P^0, P^1, ..., P^k, ...$ that tend to P, and $P^k \subset P$ for $\forall k$. The number of vertices of polytopes increase by one on each iteration. Every next polytope is constructed on the basis of the previous one using procedures of computing the support functions for the projection P and Fourier-Motzkin convolution method ([2]). In [3] a robust algorithm for solving this task was proposed.

For approximating polyhedra, two descriptions are constructed simultaneously - one as a set of their vertices and the other as a system of linear inequalities. Knowing inequalities of internal approximating sets and the values of the corresponding support functions, it is easy to find external approximating sets $\bar{P}^0, \bar{P}^1, ..., \bar{P}^k$, which will contain the sought-for projection P, i.e. $P^k \subset P \subset \bar{P}^k$ for $\forall k$.

More computational details and a discussion about these techniques of polyhedral approximation can be found in [6].

Returning to our discretized problem, define a new variable $z = \langle b, y \rangle$ and exclude the variable $V_1(1)$ which can be written as

$$V_1(1) = -\left(\frac{s_1}{Ac_1} z + V_1(2) + \ldots + V_1\left(\frac{N}{2} - 1\right) + \frac{c_2}{c_1}\left(V_1\left(\frac{N}{2} + 1\right) + \ldots + V_1(N-1)\right)\right).$$

The cost function (1) can be expressed in terms of x and z

$$\langle \bar{a}, \bar{x} \rangle + \langle \bar{x}, \bar{Q}\bar{x} \rangle \to \min, \tag{2}$$

where $\bar{x} = (x, z)$, $\bar{a} = (a, 1)$, and $\bar{Q} = \begin{pmatrix} Q & 0 \\ 0 & 0 \end{pmatrix}$.

The projection of the set of feasible solutions onto the subspace of variables $(V_1(0), V_1(N/2), V_2(0), V_2(N/2), z)$ can be constructed using Buchenkov'method. With this projection and cost function (2) we get a lower-dimensional optimization problem. The Chen-Burer algorithm is applied to this problem and a solution is obtained. A simple convex programming problem is then used to get the solution of original discrete problem. Finally, this solution can now be used as an initial guess when applying a local optimization software.

Next we present numerical results obtained when the following data for parameters of problem are considered

$$V_1^m = 86.7, \quad V_1^M = 147, \quad V_2^m = 48.3, \quad V_2^M = 66, \quad u_1^m = -0.3456, \quad u_1^M = 0.4392, \quad u_2^m = 0, \quad u_2^M = 0.8316,$$

$$s_1 = 81.7, \quad s_2 = 44.5, \quad H_1 = 3, \quad H_2 = 1, \quad c_1 = 2, \quad c_2 = 20, \quad A = 0.1589, \quad N = T = 24.$$

3.0.1 Results from Chen-Burer Algorithm

The results of Chen-Burer algorithm applied directly to the discretized problem are shown in Figure 3.0.1.

This global solution has cost -308.918 euros. The execution time was high (24 hours).

3.0.2 Results with Bushenkov' method

From Bushenkov'method we get a feasible set for the projected problem. The Chen-Burer algorithm is applied to this problem and the solution obtained is

$$\hat{\bar{x}} = [140.66, \ 147, \ 48.30, \ 49.16, \ -68.18]$$

The solution of the discretized problem is recovered by solving a simple convex quadratic programming problem. The function *QuadProg* from the Matlab.

$$\begin{aligned} \text{minimize} \quad & \|\Pi(y) - \hat{\bar{x}}\|^2 \\ \text{s.t.} \quad & Ay \le b \\ & A_{eq} y = b_{eq} \\ & LB \le y \le UB \end{aligned}$$

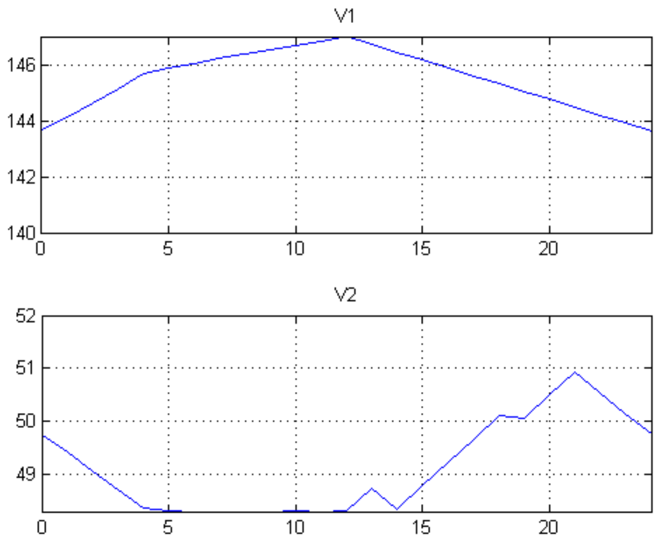

Figure 1: Results for the Discretized problem

where $y = (V_1(0),\ V_1(1),\ \cdots,\ V_1(N-1),\ V_2(0),\ V_2(1),\ \cdots,\ V_2(N-1))$, $\Pi(y) = (V_1(0),\ V_1(N/2),\ V_2(0),\ V_2(N/2))$ and $\hat{\bar{x}} = (\hat{V}_1(0),\ \hat{V}_1(N/2),\ \hat{V}_2(0),\ \hat{V}_2(N/2))$

This solution together with $\hat{\bar{x}}$ is then used as an initial guess for the optimization package from Bushenkov and Smirnov (see [12]).

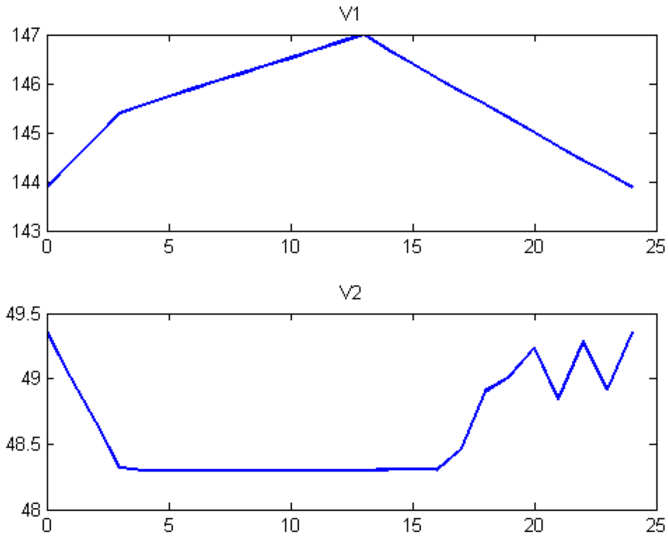

Figure 2: Results with new approach

	Discretized Problem	New approach
	Chen - Burer Algorithm (directly)	Projection Chen - Burer Algorithm QuadProg Determine initial guess OC
Total time execution	24 hours	1.48 min

Table 1: Comparison of methods

The cost associated of this solution is -308.8.

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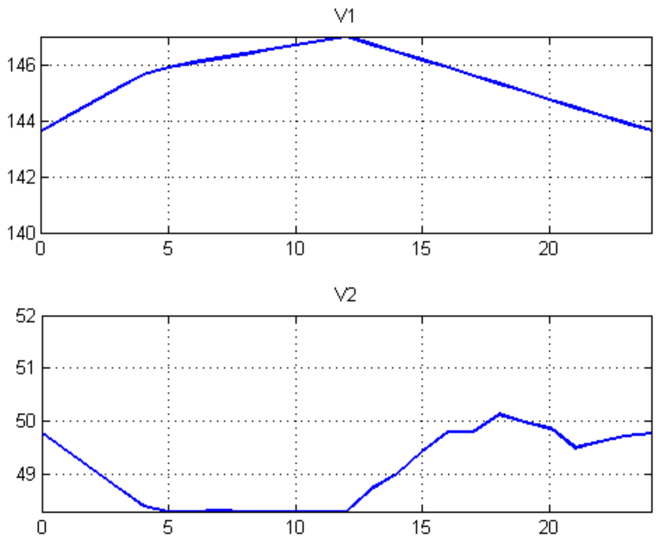

Figure 3: Final results with new approach