YOUNG CHILDREN SOLVING PARTITIVE DIVISION TASKS

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This paper focuses on a study with 4- and 5-years-old children understanding of partitive division when discrete quantities are involved. The study analyses how young children understand the inverse divisor-quotient relationship when the dividend is the same. The participants were 30 kindergarten children from Braga, Portugal. Individual interviews were conducted when solving tasks involving the division of 12 and 24 discrete quantities by 2, 3 and 4 recipients. Results showed that 4- and 5-years-old children have some ideas of division, can estimate for the quotient when the divisor varies and the dividend is constant, and can justify their answers. Educational implications of these results are discussed for kindergarten activities.

Framework

Literature refers that kindergarten children possess an informal knowledge relevant for many mathematical concepts (see Nunes, 1992; Nunes & Bryant, 1997). This informal knowledge should provide the building of formal mathematical concepts. Concerning the division, several authors suggest that young children can divide discrete quantities successfully (see Frydman & Bryant, 1998; Pepper & Hunting, 1998; Kornilaki & Nunes, 2005; Squire & Bryant, 2002), arguing that these children possess some type of informal knowledge related to the division of quantities, understanding the inverse relation between the divisor and the quotient when the dividend is the same.

When considering the division of discrete quantities it becomes relevant to distinguish the partitive and the quotitive division. In partitive division problem a set of objects is given to be divided among recipients, and the share that each recipient has received is the unknown part. (e.g., there is a set of 10 candies to be shared among 5 children. How many candies does each child get?). In a partitive division problem, the divisor is the number of recipients and the quotient is the share they receive. In quotitive division, there is an initial quantity to be share into a known number of parts. The size of the parts is the unknown (e.g., Mary has 12 candies and wants to give 3 candies to each of her friends. How many friends are receiving the candies?). In quotitive division problems, the divisor is the share to be given to each recipient and the quotient is the number of recipients. Concerning these types of divisions Kornilaki and Nunes (2005) argued that children understand more easily the partitive division than the quotitive division.

Research presents several results of young children procedures when solving division tasks involving discrete quantities (see Piaget & Szieminska, 1971; Desforges & Desforges, 1980; Frydman & Bryant, 1998; Squire & Bryant, 2002). Particularly, Correa, Nunes and Bryant (1998) when investigating the development of the concept of division in young children, examined whether children who could share would be able to understand the inverse divisor-quotient relationship in partitive division tasks when asked to judge the relative size of 2 shared sets. The participants were 20 children of 5-year-olds, 20 of 6-years-old and 21 of 7-years-old from Oxford, England. The authors investigated the children's understanding of the three-term quantity relationship in division when the dividend was constant and the divisor varies. Their results showed that 9 of the 20 five-years-old performed significantly above chance and about 30% were able to verbalize this inverse relation in their justifications and 11 out of 20 of the 6-years-old scored above chance and verbalized the inverse relation between the divisor and the quotient in the partitive tasks. The authors also report age improvements between 5 and 7 years. Correa, Nunes and Bryant (1998) also analysed children's justifications according to children's age. Most of the 5-years-old were not able to give a mathematical justification for their choices and did not mention facts relevant to the solution of the task. The 6-years-old presented justifications that revealed a progress from some comprehension of sharing and numerical equivalence to the understanding of the inverse divisor-quotient relationship. The majority of the justifications presented by the 7-years-old showed a logicomathematical approach, referring the inverse divisor-quotient relationship.

More recently, Kornilaki and Nunes (2005) investigated whether the children could transfer their understanding of logical relations from discrete to continuous quantities. Among other things, the authors analysed 32 five-years-old, 32 six-years-old and 32 seven-years-old solving partitive division tasks involving discrete quantities. In this type of problems the number of recipients varied to produce two conditions: 1) in the same divisors condition, the size of the divisor was the same; 2) in the different

divisors condition, the number of recipients varied. The results showed that the different divisors condition was clearly more difficult than the same divisors condition. Thus, the authors argued that the inverse relation between the divisor and the quotient is understood later than the equivalence principle of division. The authors also pointed out that in partitive division tasks, one-third of the 5- and 6-years-old justified their responses as "the more recipients, the more they get", but this response decreased markedly with age as only slightly more 10% of the 7-years-old used this incorrect reasoning.

The studies of Correa, Nunes and Bryant (1998) and Kornilaki and Nunes (2005) give evidence that, at age of 6 and 7, children have an insight into relations between the division terms, long before they are introduced to this operation at school. If previous research reports some success with 5-years-old children, how would children of 4-years-old would perform? Besides, it becomes relevant to get a better insight on young Portuguese children's informal knowledge of division.

This paper focuses on young Portuguese children understanding of division of discrete quantities, when solving partitive division problems. For that we tried to address three questions: 1) How do children estimate the quotient in a partitive division in which the divisor varies and the dividend is kept constant? 2) How do children perform the partitive division tasks involving discrete quantities? 3) What procedures do they use in this process?

Methods

A study focused on young children's ideas of partitive division was conducted to address these questions. The participants were 15 four-years-old (11 boys and 4 girls, mean age 4 years and 6 months) and 15 five-years-old (7 boys and 8 girls, mean age 5 years and 6 months) from Braga, Portugal.

The participants were interviewed individually by one of the researchers when solving the problems. Each problem was presented to each child using a story and manipulatives representing the items involved in each story were available.

Each child was presented to 6 problems: 3 involving the division of 12 units (carrots) by 2, 3 and 4 recipients (rabbits), respectively; and 3 problems involving the division of 24 units (cabbage) by 2, 3 and 4 recipients (rabbits).

In the interview, first children were invited to estimate the effects on the quotient of increasing the divisor keeping the dividend constant. Then they were asked why they thought so. The idea was to have an insight on children's understanding of the inverse divisor-quotient relationship when the dividend is constant. Then children were asked to carry out the division. In this process, their ability to perform the division was assessed as well as the procedures used by them.

The story presented to the children involved a context in which a white little rabbit had 12 carrots. Then he had to share them fairly with his friend, the brown rabbit. At this moment the child was asked: "Do you think that the white rabbit would be with more or less carrots? Why?". Them the child was invited to accomplish the division between the two rabbits. Them the child was asked: "Do you think that both rabbits are happy with this division of the carrots? Why?", "How many carrots did each received?". Then a little grey rabbit came around and they had to put all the carrots together again and share them among the three rabbits. "Do you think that each rabbit is going to have more or fewer carrots now?"; "Can you help the rabbits to share the carrots?"; "Do you think that all the carrots are happy with this division? Why?". The story continues to include the black rabbit. The same questions were asked. When the 24 units were involved, an analogous story was presented to them but now involving the 2, 3 and 4 rabbits and 24 cabbages.

Each child took approximately 20 minutes to solve all the problems, in spite of having no limit of time for it.

Results

In order to understand children's ability to estimate the quotient in a partitive division in which the divisor varies and the dividend is kept constant, their correct responses and justifications were analysed. Table 1 resumes the percentage of correct estimates and the respective valid justifications for the division of 12 and 24 units, according to the age. A valid justification is an argument in which a child expresses some ideas of the inverse divisor-quotient relationship, such as "because there are more rabbits and each one get fewer carrots." or "they will have fewer carrots because now there is the X rabbit".

Table 1: Percentage of correct responses and valid arguments when estimating for the quotient with the dividends of 12 and 24 units, respectively.

	4-years-old		5-years-old	
	Correct response	Valid argument	Correct response	Valid argument
12 units	67%	43%	72%	67%
24 units	71%	52%	78%	83%

It is interesting to note that children's performance in the estimating tasks improved from the first part of the problems (involving 12 units) to the second one (involving 24 units), in spite of the sizes of the initial sets. Perhaps this is due to the fact that when the problems involving the 24 units were presented to the children, they were not a novelty anymore. Among those of the 5 years of age, there were 78% of correct responses when 24 units were involved, and in 83% of them the children presented a valid argument.

Another remarkable point is the success observed among the 4-years-old when asked to estimate and justify their judgement. Almost half of the children presented a valid justification for their correct answer when dividing the 12 units; when they were asked to divide the 24 units, their valid justifications increased slightly above 50%. These results suggest that children of 4-years-old may have some ideas about the inverse divisor-quotient relationship presented in these conditions.

Children performance was analysed solving division tasks involving 12 and 24 units by 2, 3 and 4 recipients, respectively. Table 2 resume the percentage of children's correct responses by age level, in these problems.

Table 2: Percentage of correct responses by age level when solving the division of 12 and 24 units by 2, 3 and 4 recipients.

	4-years-old (n=15)		5-years-old (n=15)	
	12 units	24 units	12 units	24 units
Division by 2	87%	60%	87%	80%
Division by 3	67%	86%	80%	74%
Division by 4	67%	67%	80%	80%

The results suggest that for young children it becomes more difficult to accomplish the division of 24 units than the division of the 12 units set, possibly due to the magnitude of the set.

As the children's performance was not normally distributed a Mann-Whitney U Test was conducted in order to analyse children's performance dividing 12 and 24 units according to the age level. The results show no significant differences on children's performance when dividing 12 units according to the age levels (age 4, Mdn=3, age 5, Mdn=2, U=149, n.s.) and when dividing 24 units according to the age levels (age 4, Mdn=3, age 5, Mdn=3, U=128, n.s.). Thus, results give evidence that there is no difference of 4- and 5-years-old children's performance in this division tasks.

Trying to explain these results, children's procedures were analysed when dividing 12 and 24 units by 2, 3 and 4 recipients, respectively. The same procedures were observed when children were dividing 12 and 24 units. The procedures observed were: sharing relying on the one-to-one correspondence by the recipients; counting procedures that were adjusted in the final to produce equal shares; sharing based on perceptual influence ignoring the size of the shares; and sharing combined with counting to produce equal shares.

Figure 1 shows a child using one-to-one correspondence when dividing the 12 carrots among the 4 rabbits. Figure 2A-2B shows children using counting procedures when sharing the carrots between 3 and 2 rabbits, respectively. Figure 3A-3B gives examples of children using procedures based on perception only, ignoring the size of the shares obtained. Figure 4A-4B gives examples of children using sharing activity combined with counting to produce equal shares.

Figure 1: A child using one-to-one correspondence.

Figure 2A-2B: Two children using counting procedures.

Figure 3A-3B: Two children using procedures based only on perception ignoring the size of the shares.

Figure 4A-4B: Two children using sharing combined with counting to produce equal shares.

Table 3 resumes the observed procedures used by the children of both age groups when solving the division problems of 12 and 24 units, respectively. The procedures used by children did not change much according to the magnitude of the set to divide.

Table 3: Children's procedures solving the division of 12 and 24 units by 2, 3 and 4 recipients, according to the age level.

Type of procedure	4-years-old (n=15)						5-years-old (n=15)					
	12 units			24 units			12 units			24 units		
	By 2	By 3	By 4	By 2	By 3	By 4	By 2	By 3	By 4	By 2	By 3	By 4
Sharing & corresp.	10	9	9	8	8	8	7	9	9	9	6	6
Counting	0	0	1	2	2	2	0	0	1	2	2	3
Sharing ignore shares	3	5	3	1	3	4	6	5	4	4	4	4
Sharing & counting	2	1	2	4	2	1	2	1	1	0	3	2

As each child was presented to 6 problems, there were a total of 90 resolutions by age level. The procedure mostly used in these resolutions by both age groups of children was correspondence one-to-one, used by 57% of the resolutions of the 4-years-old and by 51% of the resolutions of the 5-years-old. This procedure conducted children to correct resolutions. The procedures using sharing activity based on perceptual influence ignoring the size of the shares were also popular among children of both age groups, conducting them to wrong answers. It was used by 21% of the resolutions of the 4-years-old and by 30% of the resolutions of the 5-years-old. The relevance of producing equal shares when accomplishing a sharing procedure seems to be an issue for young children, but it is essential to understand fundamental relations in a partitive division situation.

Discussion and conclusions

The findings of the study reported here suggest that young children possess some ideas related to the division of quantities, understanding the inverse relation between the divisor and the quotient when the dividend is the same. The analysis conducted here give evidence that children of 4-years-old reveal some understanding of the effect of increasing the number of recipients when the amount to share is constant. These children were able to estimate the result of division. In agreement with Frydman and Bryant (1998), Correa, Nunes and Bryant (1998) and Kornilaki and Nunes (2005), who previously studied these issues, the results of this small study also suggest that children have some ideas of the inverse divisor-quotient relationship in partitive division tasks, when asked to judge the relative size of shared sets.

The study reported here has some similarities with some presented previously in the literature (see Correa, Nunes & Bryant, 1998; Kornilaki & Nunes, 2005), but also offers some original contributions. Correa, Nunes and Bryant (1998) investigated 5- to 7-years-old children's understanding of inverse divisor-quotient relationship, when partitive division was involved. Their findings give evidence that 5-years-old children can succeed in these tasks. Also Kornilaki and Nunes (2005) give evidence of 5-years-old children success when solving this type of tasks. In our study we analysed how children of 4- and 5-years-old behave when dealing with this type of problems. Some positive signs arise from this investigation. Four-years-old children are able to understand some ideas of divisor-quotient relations in particular conditions.

The procedures used by the children of this study suggest that one-to-one correspondence can play an important role on children's sharing activity and on their accomplishment of division. Some authors argue that sharing activities can be relevant in the understating of the inverse relation between the divisor and the quotient (see Correa, Nunes & Bryant, 1998) and that understanding the sharing activity helps children to understand the relation between the dividend, the divisor and the quotient (see Kornilaki & Nunes, 2005). In agreement with these ideas, one-to-one correspondence sustaining the sharing activity seems to allow young children to understand the logical relations involved in the division of quantities.

These findings suggest that kindergarten activities can include sharing activities in particular conditions, in order to stimulate children's early ideas of division. These ideas are crucial to understand some complex mathematical concepts such as fractions, later on in the formal traditional school.

References

Correa, J., Nunes, T. & Bryant, P.: 1998, Young Children's Understanding of Division: The Relationship between Division Terms in a Noncomputational Task, *Journal of Educational Psychology*, 90, 321-329.

Cowan, R., & Biddle, S.: 1989, Children's understanding of one – to – one correspondence in the context of sharing, *Education Psychology, Vol.* 9, Nº2, 133-140.

Desforges, A., & Desforges, C.: 1980, Number-based strategies of sharing in young children, *Educational Studies*, 6, 97-109.

Frydman, O., & Bryant, P.: 1988, Sharing and understanding of number equivalence by young children, *Cognitive Development*, 3, 323-339.

Kornilaki, E., & Nunes, T.: 2005, Generalizing Principles in Spite of Procedural Differences: Children's understanding of division, *Cognitive Development*, 20, 388-406.

Nunes, T.: 1992, Ethnomathematics and everyday cognition, in: *Handbook of research on mathematics teaching and learning*, D.A. Grouws Editor, pp.557-574.

Nunes, T., & Bryant, P.:1997, *Crianças Fazendo Matemática*. Porto Alegre: Editora Artes Médicas.

Pepper, K., & Hunting, R.: 1998, Preschoolers' Counting and Sharing, *Journal for Research in Mathematics Education*, Vol. 29, nº 2, 164–183.

Piaget, J. & Szeminska, A.: 1971, *A génese do número***.** Translated by Cristiano M. Oiticica. 3ª Edição. Rio de Janeiro, Zahar.

Squire, S., & Bryant, P.: 2002, The influence of sharing on children's initial concept of division, *Journal of Experimental Child Psychology*, 81, 1–43.