Universidade do Minho
Escola de Economia e Gestão

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Behaviour-Based Price Discrimination with Retention Strategies

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Dissertação de Mestrado
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Trabalho realizado sob a orientação da
Professora Doutora Rosa Branca Esteves

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Behaviour-Based Price Discrimination with Retention Strategies

Abstract

In imperfect competitive markets firms have some market power, thus the practice of price discrimination is possible. In oligopolistic models, behaviour-based price discrimination is analyzed following two different approaches: the *switching costs approach* and the *brand preferences approach*. A recent Ofcom's report makes a reminder to the practice of firms implementing retention strategies, as a way to discourage customers to change the current supplier offering to all customers who show an intention to switch a special price discount.

The main objective of this Master Dissertation is to develop a theoretical model that analyzes the effects of retention strategies under the switching costs approach. After consumers have made their first-period consumption decisions and decide to change supplier in the second-period, they have to incur switching costs. It is a model that extends Chen (1997) by allowing firms to employ retention strategies. It is also a model based on Esteves and Rey (2010), that consider retention activity but under the brand preferences approach.

The results, when compared to those obtained without retention strategies, suggest *(i)* a lower deadweight loss due the less inefficient switching; *(ii)* a lower firms' profits; and *(iii)* a higher consumers' surplus.

Keywords: Behaviour-based price discrimination, retention strategies, switching costs

Discriminação de Preços com Estratégias de Retenção

Resumo

Em mercados de concorrência imperfeita, devido ao poder de mercado das empresas, a prática de *discriminação de preços* torna-se possível. Nos modelos de oligopólio, a discriminação de preços, com base no reconhecimento do perfil de compra do consumidor, é analisada segundo duas abordagens: a dos *custos de mudança associados à troca de empresa* ou a das *preferências (exógenas) do consumidor*. No relatório de 2010 da Ofcom, é feita uma chamada de atenção para a prática da implementação de *estratégias de retenção*, como forma de desencorajar os consumidores a trocarem de empresa, através da oferta de um desconto a todos aqueles que mostram intenção de trocar. O objetivo desta Dissertação de Mestrado consiste no desenvolvimento de um modelo teórico que analise os efeitos da implementação de estratégias de retenção, sob a abordagem que os consumidores têm custos associados à mudança de empresa, após terem feito a sua escolha inicial. É um modelo que resulta de uma extensão daquele que é apresentado no artigo de Chen (1997), considerando os mesmo pressupostos base e incluindo a capacidade das empresas em definirem estratégias de retenção. É igualmente um modelo assente no apresentado em Esteves e Rey (2010), que incorpora a capacidade das empresas em praticarem estratégias de retenção, mas seguindo a abordagem das preferências exógenas dos consumidores. Os resultados, quando comparados com os obtidos no caso da impossibilidade de implementação de estratégias de retenção, sugerem *(i)* uma menor perda de bem-estar, resultado da menor troca dos consumidores; *(ii)* um menor lucro para as empresas e *(iii)* um aumento no bem-estar dos consumidores.

Palavras-chave: Discriminação de preços, perfil de compra do consumidor, estretégias de retenção, custos de mudança

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Chapter 1

Introduction

"Price discrimination is a ubiquitous phenomenon. [...] Thus, the analysis of the forms that price discrimination can take and the effects of price discrimination on economic welfare are a very important aspect of the study of industrial organization."

Hal Varian (1989), Handbook of Industrial Organization, Vol. 1

In many imperfectly competitive markets firms can have the tools to engage in price discrimination practices. According to Stigler's (1987) definition, *price discrimination* is the ability of firms to charge different prices in the selling of the same or similar products where the ratios of their marginal costs are different.

Usually, we can see different forms of price discrimination in several day-to-day cases, such as students' discounts on cinema tickets or differential pricing on business and leisure airline travellers. According to Pigou (1920) there are three types of price discrimination. Under *first-degree price discrimination*, firms are able to charge a different price to different consumers according to their willingness to pay. In this case, price is equal to the consumer's reservation price and firms are able to extract the entire consumer surplus. *Second-degree price discrimination* occurs when firms charge nonlinear prices to consumers. The most common examples are the discounts to consumers who buy large amounts of the product (quantity discounts) or the imposition of a two-part tariff, i.e., firms require that consumers pay a fixed fee (regardless of the quantity bought) plus a variable component (that depends on the quantity bought). *Third-degree price discrimination* is probably the most common form of price discrimination and it arises when firms charge different prices according to the consumer's observable characteristics.

Third-degree price discrimination is the most common form of this business practice. Charging different prices to different consumer groups is the most commonly used. A new type of third-degree price discrimination has been implemented recently as a result of the developments in information technologies. Firms have been increasingly able to gather and record more information about consumers' preferences and use this information to charge different prices according to the consumers' purchase history. This type of price discrimination has been known in the economic literature as *Behaviour-Based Price Discrimination* (BBPD)[1]. Since firms are able to recognise their own customers and those of their rivals, firms can try to poach the customers of their rivals by offering them better deals. This may lead some customers to switch providers.

The analysis of BBPD has been done on the basis of two approaches. In the brand preferences approach purchase history discloses information about consumers exogenous brand preferences (e.g. Fudenberg and Tirole (2000)). In the switching costs approach purchase history discloses information about consumers' switching costs (Chen (1997)).

In a recent Ofcom's report [2] a new form of price discrimination is identified: save or retention strategies. This strategy is as a way to make it less attractive for a customer to search for and switch to a competing firm, i.e., save activity discourages customers to switch because the switching process is more expensive. In a Losing Provider Led (LPL) [3] process, for the consumers' switching process to be completed, customers have

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[1] Behaviour-based price discrimination is also known as *price discrimination based on purchase history* or *dynamic pricing.*

[2] Ofcom is an independent regulator and competition authority for the United Kingdom communications industries. See the report in "Strategic review of consumer switching", Ofcom, September 2010 (http://stakeholders.ofcom.org.uk/binaries/consultations/consumer-switching/summary/switching.pdf).

[3] An alternative to the LPL process is the Gaining Provider Led (GPL) process. Under the GPL process, customers only need to agree to a deal with their new provider who then contacts the customer's existing provider to complete the switching. In contrast with the LPL process, under the GPL process the switching process is easier but the risks of mis-selling are higher because customers have less information

to validate a code that has to be requested from the existing firm. In the United Kingdom, customers who want to switch their mobile telephone service must contact their existing provider and request a porting authorization code [4] which they then put through to their new provider in order to complete the switching process. So, this code request provides firms with the information that the consumers are willing to switch and allows firms to offer advantageous deals to those customers with the objective of retaining them. Since save activity can potentially make more difficult the switching processes, it is important to understand the economic and welfare effects of this business practice.

Motivated by the Ofcom report Esteves and Rey (2010) are the first to investigate the competitive and welfare effects of BBPD when firms can also engage in retention strategies. They do that in the context of the brand preference approach. They show moving from BBPD with no retention strategies to BBPD with retention strategies is bad for industry profits but good for consumers and overall welfare.

This Dissertation has two main goals. First, it aims to offer a review of the main results derived in the literature on BBPD taking into account the two approaches. Second, it aims to develop a theoretical model to investigate the economic and welfare effects of BBPD with retention strategies in the switching costs approach. Thus, the model developed in this thesis is based on Chen (1997) and Esteves and Rey (2010).

This Dissertation is structured as follows. Chapter 2 presents in detail the models who give rise to each approach as well as their main findings. We will also discuss the comparison between the results in both approches. First, it is presented the Chen's (1997) model of BBPD in the switching costs approach. Then, it is presented the same steps for the case of Fudenberg and Tirole's (2000) model of BBPD in the brand preferences approach. In the end of this Chapter it is presented some of the main extentions to the economic literature on BBPD.

Chapter 3 introduces retention strategies in the the two-approaches presented in

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about the implications of the switching process.

[4] The request of a porting authorisation code (PAC) occurs only when customers wish to change their mobile provider but want to maintain their existing phone number.

Chapter 2. Using the model of Esteves and Rey's (2010), Section 3.1 analyses the effects of BBPD with retention strategies under the brand preferences approach. The main contribution of this thesis can be found in Section 3.2 where it is developed a theoretical model with BBPD and retention strategies in the switching costs approach. The welfare analysis is presented in section 3.2.3. Finally, Chapter 4 presents the main conclusions of this work.

Chapter 2

Literature Review

In markets with repeated purchases, firms are able to recognise consumers' past purchase information and this feature allows firms to charge different prices to different consumers. We can find some examples in telecommunication markets or in banking sectors. In telecommunication markets it is usual that one firm offers a lower price to a customer who uses a rival's service. Or, in the banking sector it is usual that one banking company offers a lower interest rate to a customer who change company. We can notice that in these two examples there are two common features. First, the price that one firm charge depends on consumers' past purchases, namely whether or not the consumer bought from a competitor firm in the past. This implies a dynamic interaction in the marketplace. In these markets firms do not have to commit to their futures prices and can learn about consumers and segment the market in a better way. Second, these examples operate under imperfect competitive markets, namely in oligopolistic markets. This form of price discrimination is designed in the economic literature by behaviour-based price discrimination (BBPD). Thus, in broad terms, behaviour-based price discrimination is a form of price discrimination in which each firm charges a different price to different consumers according to their past purchases (consumers' purchase profile).

The literature for this study is mainly grounded on price discrimination in imperfectly competitive markets, specifically on the BBPD literature. Works from Chen (2005), Fudenberg and Villas-Boas (2006) and Esteves (2009b) gather the most important surveys on BBPD. On the other hand, Armstrong (2006) and Stole (2007) focus on price discrimination in imperfectly competitive markets.

There are two common approaches to modelling behaviour-based price discrimination: the *switching costs approach* and the *brand preferences approach*. The switching costs approach is based on the model presented in Chen (1997). In this approach firms and consumers interact for two periods. In the beginning of the game the firms' product are perfect substitutes, however after first period decisions have been made consumers are in some way locked-in to their previous supplier due to the existence of switching costs. Thus, purchase history reveals information about switching costs. The second approach is due to Fudenberg and Tirole (2000). They propose a two-period model where consumers have different brand preferences for the firms' products from the beginning. Here, purchase history reveals information about exogenous brand preferences.

The aim of this chapter is to present in detail the models who give rise to each approach as well as their main findings. We will also discuss the comparison between the results in both approches. First, it is presented the Chen's (1997) model with price discrimination and comparing it with the benchmark case without price discrimination. Then, it is presented the same steps for the case of Fudenberg and Tirole's (2000) model. Finally, we discuss some of the main extensions that have been proposed on the BBPD literature, as is the case of assuming that firms can engage in BBPD with retention strategies.

2.1 Switching Costs Approach

This section presents the main economic effects of behaviour-based price discrimination in the switching cost approach. We review the two period model developed in Chen (1997) in which firms offer in each period an homogeneous product but consumers have to incur a switching cost if they decide to change suppliers after the initial purchase [1]. Based on past purchase history firms can recognise a previous own customer and a rival's one and price discriminate accordingly. Chen called this practice *"paying customers to switch"*.

It is assumed that there are two firms, A and B, and that each firm produces an homogeneous product with constant and equal marginal cost c, $c \geq 0$.There is a unit mass of consumers and each consumer wants to buy one unit of the product either from firm A or B. Consumers' reservation price is given by v. In the first period, each firm chooses simultaneously its price, $p_1^i, i = A, B$. In the end of period 1, after consumers have made their decisions of consumption, each firm gets a proportion of the market, resulting in proportion α for firm A and $(1 - \alpha)$ for firm B, where $0 \leq \alpha \leq 1$. After consumers' first-period decisions have been made, in period 2 each firm can identify their own customers and the rival's customers. If a consumer switches to purchase from a different supplier he has to incur a switching cost, s uniformly distributed on $[0, \phi]$. In the second period, firms can price discriminate between their own customers and the rival's customers, choosing a set of prices $(p_{i2}^o,\, p_{i2}^r)$,with $i = A, B$. Firms and consumers discount second-period profits using the same discount factor, namely $\delta \in [0, 1]$.

2.1.1 Second-period Equilibrium

As usual the model is solved by backward induction. In the second period, each firm can recognise their own customers and the rival's customers and charge different prices

[1] According to Klemperer (1987) there are at least three types of switching costs: transaction costs, learning costs and artificial (or contractual) costs. *Transactions costs* are present when the switching occurs between identical services. A typical example is costs incurred when a consumer changes his bank account, which involves the closing of one set of accounts and the opening of another set in another bank. *Learning costs* are incurred when consumers only switch to a new provider (that is new for him). An example is the costs associated to get a new word processing system that has the same functions as others but that also has a manual written in a different style. The third type of switching costs is *artificial or contractual costs* and arises entirely at firms' discretion (and there are no social costs of brand switching). The two-first types of switching costs reflect real social costs of switching between brands.

at these different segments of consumers. In this way, firms charge a set of prices such that p^o_{i2} is the price for existing customers and p^r_{i2} is the price for rival's customers.

The indifferent consumer between choosing again firm i or switch to firm j in the second period is such that

$$v - p^o_{i2} = v - p^r_{j2} - s$$

with i, $j = A, B$ and $i \neq j$.

Thus,

$$s = p^o_{i2} - p^r_{j2}$$

Consumers with $s > p^o_{i2} - p^r_{j2}$ buy again from their currtent suppliers while those with $s < p^o_{i2} - p^r_{j2}$ do switch.

Let us q_{AA} represent all consumers that buy again from firm A (with high switching costs) and q_{BA} represent all consumers switch from firm A to firm B in the second period (with low switching costs). Given that $s \sim U[0, \phi]$, it follows that

$$q_{AA} = \alpha \int_{\overline{s}}^{\phi} f(s)ds = \frac{\alpha}{\phi}\left(\phi - p^o_{A2} + p^r_{B2}\right) \tag{i}$$

And, q_{BA} is given by

$$q_{BA} = \alpha \int_{0}^{\overline{s}} f(s)ds = \frac{\alpha}{\phi}\left(p^o_{A2} - p^o_{B2}\right) \tag{ii}$$

Doing the same for firm B it is straightforward to obtain:

$$q_{BB} = (1-\alpha) \int_{\overline{s}}^{\phi} f(s)ds = \frac{(1-\alpha)}{\phi}\left(1 - p^o_{B2} + p^r_{A2}\right) \tag{iii}$$

$$q_{AB} = (1-\alpha) \int_{0}^{\overline{s}} f(s)ds = \frac{(1-\alpha)}{\phi}\left(p^o_{B2} - p^r_{A2}\right) \tag{iv}$$

In this period, firm A and firm B's profits are, respectively,

$$\pi_{A2} = (p^o_{A2} - c)q_{AA} + (p^r_{A2} - c)q_{AB}$$

$$\pi_{B2} = (p^o_{B2} - c)q_{BB} + (p^r_{B2} - c)q_{BA}$$

Substituting equations *(i)*, *(ii)*, *(iii)* and *(iv)*, in the above profit functions we obtain:

$$\pi_{A2} = \frac{\alpha}{\phi}(p^o_{A2} - c)\left(1 - p^o_{A2} + p^r_{B2}\right) + \frac{(1-\alpha)}{\phi}(p^r_{A2} - c)(p^o_{B2} - p^r_{A2})$$

$$\pi_{B2} = \frac{(1-\alpha)}{\phi}(p^o_{B2} - c)\left(1 - p^o_{B2} + p^r_{A2}\right) + \frac{\alpha}{\phi}(p^r_{B2} - c)(p^o_{A2} - p^r_{B2})$$

Given that q_{ii} is the number of consumers that choose firm i in both periods (1 and 2), and q_{ji} the number of consumers that choose firm i in the first period and switch to firm j in the second period it follows that firm'i second-period profits are

$$\pi_i = (p^o_{i2} - c)q_{ii} + (p^r_{i2} - c)q_{ij},\ i = A, B$$

In this period each firm chooses simultaneously and non-cooperatively the pair of prices $(p^o_{i2},\ p^r_{i2})$ as a way to maximize π_i. From the profit maximization problem we obtain the following result.

Proposition 1 *In the switching costs approach without retention strategies, second-period equilibrium prices are given by*

$$p^{o*}_{i2} = \frac{2}{3}\phi + c$$

$$p^{r*}_{i2} = \frac{1}{3}\phi + c$$

with $i = A, B$.

Proof. See the Appendix. ■

Thus, each firm charges a lower price to the rival's customers than to its own customers in the second period. Additionally, consumers with high switching costs ($s > \frac{1}{3}\phi$) will not change supplier in the second period while those with lower switching costs change supplier in period 2. An important remark is that in the second-period equilibrium, prices do not depend on firms' market shares. This means that in the first period, consumers will purchase from the firm with lower price and if prices are equal, firms split evenly the market.

Each firm has the following second-period equilibrium profits,

$$\pi^*_{A2} = \frac{\phi}{3}\left(\frac{1}{3} + \alpha\right) \tag{1}$$

$$\pi^*_{B2} = \frac{\phi}{3}\left(\frac{4}{3} - \alpha\right) \tag{2}$$

2.1.2 First-period Equilibrium

In the first period each firm chooses its first-period price taking into account how such price affects first-period profit as well as second-period profits. As firms' products are perfect substitutes in period 1 all consumers purchase from the firm which offers the lowest price. When first-period prices are equal it is assumed that a consumer buys from either firm with equal probability. The number of consumers who buy from firm A in period 1 is given by α and all the remaining consumers buy from firm B. This means that

$$q_{A1} = \alpha$$

and,

$$q_{B1} = 1 - \alpha$$

In broad terms, firm A overall profit can be written as

$$\pi_A = \pi_{A1} + \delta\pi^*_{A2}$$

$$\pi_A = (p_{A1} - c)\alpha + \frac{\delta\phi}{3}\left(\frac{1}{3} + \alpha\right)$$

Doing the same for firm B, it follows that its overall profit is

$$\pi_B = (p_{B1} - c)(1 - \alpha) + \frac{\delta\phi}{3}\left(\frac{4}{3} - \alpha\right)$$

Now, we have to analyse the different situations that can occur in the first period. From the Bertrand game, three situations are possible: *(i)* $p_{A1} = p_{B1}$; *(ii)* $p_{A1} < p_{B1}$; and *(iii)* $p_{A1} > p_{B1}$.

Consider first the case where $p_{A1} = p_{B1}$ and $\alpha = \frac{1}{2}$. In this case both firms have the same overall profit, given by

$$\pi_i = \frac{1}{2}(p_{i1} - c) + \frac{5}{18}\delta\phi$$

with $i = A, B$.

Consider now the case where $p_{A1} < p_{B1}$ and $\alpha = 1$. In this case, firm A's overall profit is

$$\pi_A = p_{A1} - c + \frac{4}{9}\delta\phi$$

and firm B's overall profit is given by

$$\pi_B = \frac{\delta\phi}{9}.$$

Finally, if $p_{A1} > p_{B1}$ it follows that $\alpha = 0$. In this case, firm A's overall profit is

$$\pi_A = \frac{\delta\phi}{9}$$

and firm B's overall profit is given by

$$\pi_B = p_{B1} - c + \frac{4}{9}\delta\phi.$$

Solving the equilibrium for the entire game we can show that the game has a unique subgame perfect equilibrium defined in the following proposition.

Proposition 2 *In the model with price discrimination in the switching costs approach, there is a unique subgame-perfect equilibrium, in which firm i's first-period equilibrium price is given by*

$$p_{i1}^{*} = c - \frac{\delta\phi}{3}$$

second period equilibrium prices are given by

$$p_{i2}^{o*} = \frac{2}{3}\phi + c$$

$$p_{i2}^{r*} = \frac{1}{3}\phi + c$$

and overall equilibrium profit is equal to

$$\pi_i^{*} = \frac{1}{9}\delta\phi.$$

Proof. See the Appendix. ■

Note that first-period equilibrium price is lower than marginal cost. The reason is that firms try to capture more customers in the first-period as a way to increase their base of locked-in customers.

In order to evaluate the economic effects of price discrimination in the switching costs approach we present next the benchmark case where price discrimination is for any reason not permitted.

2.1.3 No discrimination benchmark case in the switching costs approach

This analysis is also based in Chen (1997). Consider the same model as before except that now price discrimination cannot for any reason occur in period 2 (e.g. it is prohibited or firms cannot segment consumers). Thus, each firm charges a uniform price in period 2, namely p^u_{i2}, $i = A, B$. After observing the price offered by each firm, each consumer makes the decision of switching or not from his previous supplier. Look first into the second period.

Second-period equilibrium

The indifferent consumer between buying again from firm A at price p^u_{A2} or switching to firm B and pay p^u_{B2} is such that:

$$v - p^u_{A2} = v - p^u_{B2} - s$$

Thus,

$$\widetilde{s} = p^u_{A2} - p^u_{B2}.$$

The number of consumers who buy from firm A (q^u_A) are only those with high switching costs ($\widetilde{s} > p^u_{A2} - p^u_{B2}$). Firm B captures the remaining consumers. Therefore,

$$q^u_A = \alpha \int_{\widetilde{s}}^{\phi} \frac{1}{\phi} ds = \frac{\alpha}{\phi}(\phi - p^u_{A2} + p^u_{B2})$$

and

$$q^u_B = \alpha \int_0^{\widetilde{s}} \frac{1}{\phi} ds + (1-\alpha) \int_0^{\phi} \frac{1}{\phi} ds = \frac{\alpha}{\phi}(p^u_{A2} - p^u_{B2}) + (1-\alpha).$$

Each firm has the following profit function,

$$\pi^u_{A2} = (p^u_{A2} - c) q^u_A \frac{\alpha}{\phi}(p^u_{A2} - c)(\phi - p^u_{A2} + p^u_{B2})$$

$$\pi^u_{B2} = (p^u_{B2} - c) q^u_B = \frac{\alpha}{\phi}(p^u_{B2} - c)(p^u_{A2} - p^u_{B2}) + (1-\alpha)(p^u_{B2} - c)$$

Proposition 3 *In the switching cost approach with no price discrimination, second-period prices depend on first-period market share and are given by:*

$$p_{A2}^{u*} = \begin{cases} \frac{(1+\alpha)}{3\alpha}\phi + c, & \text{if } \alpha \geq \frac{1}{2} \\ \frac{(1+\alpha)}{3(1-\alpha)}\phi + c, & \text{if } \alpha < \frac{1}{2} \end{cases}$$

$$p_{B2}^{u*} = \begin{cases} \frac{(2-\alpha)}{3\alpha}\phi + c, & \text{if } \alpha \geq \frac{1}{2} \\ \frac{(2-\alpha)}{3(1-\alpha)}\phi + c, & \text{if } \alpha < \frac{1}{2} \end{cases}$$

Each firm has the following equilibrium profits

$$\pi_{A2}^{u*} = \begin{cases} \frac{(1+\alpha)^2}{9\alpha}\phi, & \text{if } \alpha \geq \frac{1}{2} \\ \frac{(1+\alpha)^2}{9(1-\alpha)}\phi, & \text{if } \alpha < \frac{1}{2} \end{cases} \tag{3}$$

$$\pi_{B2}^{u*} = \begin{cases} \frac{(2-\alpha)^2}{9\alpha}\phi, & \text{if } \alpha \geq \frac{1}{2} \\ \frac{(2-\alpha)^2}{9(1-\alpha)}\phi, & \text{if } \alpha < \frac{1}{2} \end{cases} \tag{4}$$

Proof. See the Appendix. ■

In contrast to the equilibrium under price discrimination, when firms cannot price discriminate second-period prices do depend on first-period market share: a firm's second-period price is an increasing function of its previous market share.

Corollary 1. *When $\alpha = \frac{1}{2}$ second-period uniform prices are equal to $p_{A2}^{u*} = p_{B2}^{u*} = \phi + c$. Thus, being permitted price discrimination decreases second-period prices. As a result of that second-period profits fall down with price discrimination.*

First-period equilibrium

Without price discrimination, a firm with higher market share will charge a higher price in the second period. Consumers are rational and take this into account when they make their consumption decisions in period 1. Considering for instance the case where $p^{u*}_{A2} \geq p^{u*}_{B2}$, where $\alpha \geq \frac{1}{2}$.

Given $\widetilde{s} = p^{u*}_{A2} - p^{u*}_{B2}$, in the first period and at given pair of prices $(p^{u*}_{A1},\ p^{u*}_{B1})$, the indifferent consumer between to buy from firm A or buy from firm B is

$$v - p^{u}_{A1} + \delta\left(v - \int_{\widetilde{s}}^{\phi} p^{u*}_{A2}\frac{1}{\phi}ds - \int_{0}^{\widetilde{s}} (p^{u*}_{B2} + s)\frac{1}{\phi}ds\right) = v - p^{u}_{B1} + \delta\left(v - p^{u*}_{B2}\right)$$

Simplifying, we can get that

$$p^{u}_{A1} - p^{u}_{B1} + \delta\left((p^{u*}_{A2} - p^{u*}_{B2}) - \frac{1}{2\phi}(p^{u*}_{A2} - p^{u*}_{B2})^2\right) = 0.$$

From second-period equilibrium prices we have that $(p^{u*}_{A2} - p^{u*}_{B2}) = \frac{(2\alpha-1)}{3\alpha}\phi$, for $\alpha \geq \frac{1}{2}$. Substituting in the equation above we find that

$$p^{u}_{A1} - p^{u}_{B1} + \frac{\delta\phi(2\alpha-1)(4\alpha+1)}{18\alpha^2} = 0$$

First-period profit for each firm is

$$\pi^{u}_{A1} = (p^{u}_{A1} - c)\alpha$$

and

$$\pi^{u}_{B1} = (p^{u}_{B1} - c)(1-\alpha)$$

Proposition 4 *Without price discrimination and under the switching costs approach, first-period equilibrium price is given by*

$$p^{u*}_{i1} = c + \frac{2}{3}\delta\phi$$

for $i = A, B$.

Proof. See the Appendix. ■

It is interesting to note that the ability of firms to recognise customers and price discriminate accordingly in period 2, reduces first and second-period prices. Second-period equilibrium prices are lower with price discrimination, as well the first-period equilibrium prices.

2.1.4 Welfare Analysis

This section aims to evaluate the welfare effects of BBPD in the switching cost approach. In this analysis we take into account the symmetric SPNE obtained with and without discrimination.

Look first at consumer surplus.

With BBPD without retention strategies, the overall consumer surplus is

$$CS^{nr} = v - p_{i1}^{*} + \delta \left(v - \int_{\overline{s}}^{\phi} p_{i2}^{o*} \frac{1}{\phi} ds + \int_{0}^{\overline{s}} (p_{i2}^{r*} + \overline{s}) \frac{1}{\phi} ds \right)$$

$$CS^{nr} = v - c + \frac{1}{3}\delta + \delta \left[v - \frac{2}{3}\left(\frac{2}{3}\phi + c\right) - \frac{1}{3}\left(\frac{1}{3}\phi + c + \frac{1}{6}\phi\right)\right]$$

$$CS^{nr} = (1+\delta)(v-c) - \frac{5}{18}\delta\phi$$

Without price discrimination and with $\alpha = \frac{1}{2}$, the overall consumer surplus is given by

$$CS^{nd} = v - p_{i1}^{u*} + \delta(v - p_{i2}^{u*})$$

$$CS^{nd} = v - c - \frac{2}{3}\delta\phi + \delta\left(v - \phi - c\right)$$

$$CS^{nd} = (1+\delta)(v-c) - \frac{5}{3}\delta\phi$$

We have already seen that when $\alpha = \frac{1}{2}$ all consumers pay a higher price in both periods under no price discrimination. Therefore, consumer surplus is higher under the case when firms "*pay consumers to switch*".

By the same reason, both firms' equilibrium profits are lower with price discrimination than without price discrimination. And, with BBPD without retention strategies and with $\alpha = \frac{1}{2}$, the overall profits of each firm, Π^{nr}, is

$$\Pi^{nr} = \pi_{i1}^* + \delta\pi_{i2}^*$$

$$\Pi^{nr} = \frac{1}{2}\left(c - \frac{\delta\phi}{3} - c\right) + \delta\phi\left(\frac{1}{9} + \frac{1}{6}\right)$$

$$\Pi^{nr} = \frac{\delta\phi}{9}$$

Without price discrimination and considering $\alpha = \frac{1}{2}$, the overall profits of each firm, Π^{nd}, is

$$\Pi^{nd} = \pi_{i1}^* + \delta\pi_{i2}^*$$

$$\Pi^{nd} = \frac{1}{2}\left(c + \frac{2}{3}\delta\phi - c\right) + \delta\left(\frac{1}{2}\phi\right)$$

$$\Pi^{nd} = \frac{5}{6}\delta\phi$$

With price discrimination we can note that firms' profits are lower than without price discrimination. Thus, price discrimination is bad for firms.

The overall welfare is given by the sum of consumer surplus and industry profits. With BBPD with no retention strategies, the overall welfare, W^{nr}, is given by

$$W^{nr} = CS^{nr} + \Pi_{ind}^{nr}$$

$$W^{nr} = \left((1+\delta)(v-c) - \frac{5}{18}\delta\phi\right) + \frac{2\delta\phi}{9}$$

$$W^{nr} = (\delta + 1)(v - c) - \frac{1}{18}\delta\phi$$

And, without price discrimination, the overall welfare, W^{nd}, is

$$W^{nd} = CS^{nd} + \Pi_{ind}^{nd}$$

$$W^{nd} = \left((1+\delta)(v-c) - \frac{5}{3}\delta\phi \right) + \frac{10}{6}\delta\phi$$

$$W^{nd} = (1+\delta)(v-c)$$

Note that,

$$W^{nr} - W^{nd} = -\frac{1}{18}\delta\phi < 0$$

Thus, we observe that price discrimination is bad for overall welfare due to the inefficient switching. As expected with price discrimination there is a deadweight loss in the second period, given by

$$DWL^{nr} = \delta \int_0^{\frac{\phi}{3}} s f(s) ds = \frac{\delta\phi}{18}$$

Without price discrimination and with $\alpha = \frac{1}{2}$, $p^{u*}_{A2} = p^{u*}_{B2} = \phi + c$ and $\widetilde{s} = 0$. Following this result, there is no deadweight loss without price discrimination.

The following table summarises the welfare effects of price discrimination.

	Consumer Surplus	Firm Profits	Welfare	Deadweight loss
Without	$(v-c)(1+\delta) - \frac{5}{3}\delta\phi$	$\frac{5}{6}\delta\phi$	$(1+\delta)(v-c)$	$-$
With	$(1+\delta)(v-c) - \frac{5}{18}\delta\phi$	$\frac{\delta\phi}{9}$	$(\delta+1)(v-c) - \frac{1}{18}\delta\phi$	$\frac{\phi}{18}$
	$\uparrow CS$	$\downarrow \pi$	$\downarrow W$	$\uparrow DWL$

With price discrimination, second-period and first-period prices are lower than the uniform price. Thus, the consumers surplus increases (more consumers pay a lower price) and firms profits descrease (for the same reason). Because some consumers change supplier and there is a cost of switching and a deadweight loss due the swictching.

2.2 Brand Preferences Approach

Now it is assumed that consumers have brand preferences for the two products as in Fudenberg and Tirole (2000) which are present from the begining and fixed accross the two periods of the game. There are two firms, A and B, with constant marginal cost c, $c \geq 0$. Each consumer desires to buy one unit of product from either firm A or B, in each of the two periods. Each consumer willingness to pay v is sufficiently high such that no consumer stays out of the market. Consumers have exogenous preferences, θ, distributed on a Hotteling line of unit length, such that $\theta \in [0, 1]$. As made by Chen (2005) we present a simplified variation of the Fudenberg and Tirole (2000), by assuming that θ is uniformly distributed on $[0, 1]$. Firms are located at the endpoints of the Hotelling's line and consumers have to incur transportation costs t, per unit distance. This means that a consumer located at x incurs total cost $p_A + tx$ if decide to buy from firm A at price p_A; if he decides to buy from firm B at price p_B he has to incur total cost given by $p_B + t(1 - x)$. In the first period, firms cannot observe consumers' preferences so they quote a uniform price. After consumers' decisions have been made, each firm is able to recognise their own previous customers and the rival's previous customers and price discriminate accordingly.

2.2.1 Second-period Equilibrium

Suppose that from first period competition firm A serves all consumers to the left of θ_1 and firm B serves all consumers in the right of θ_1. So, when firms can recognise their own customers and rival's customers, they can charge different prices to those different customers' segment.

Let us first look at firm A's turf, where $\theta \in [0, \theta_1]$. In the second period, a consumer located at θ_A is indifferent between buying again from firm A or switching to firm B iff

$$p^o_{A2} + t\theta_A = p^r_{B2} + t(1 - \theta_A)$$

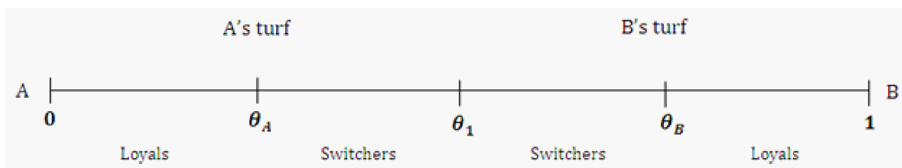
$$\theta_A = \frac{1}{2} + \frac{p^r_{B2} - p^o_{A2}}{2t}$$

Consumers with $\theta \in [0, \theta_A]$ have a strong preference for firm A and they decide to buy again from firm A. On the other hand, consumers with $\theta \in [\theta_A, \theta_1]$ decide to change from firm A to firm B in the second period.

Similarly, on firm B's turf we have that

$$\theta_B = \frac{1}{2} + \frac{p^o_{B2} - p^r_{A2}}{2t}$$

Graphically,



Customer poaching and brand switching (in Fudenberg and Tirole (2000))

Look again on firm A's turf. In this turf each firm (A and B) solves the following maximization problem:

$$\underset{p^o_{A2}}{Max} \left\{ (p^o_{A2} - c) \left(\frac{1}{2} + \frac{p^r_{B2} - p^o_{A2}}{2t} \right) \right\}$$

$$\underset{p^r_{B2}}{Max} \left\{ (p^r_{B2} - c) \left(\theta_1 - \frac{1}{2} - \frac{p^r_{B2} - p^o_{A2}}{2t} \right) \right\}$$

Solving the maximization problem we can find the following results

Proposition 5 *Under the brand preferences approach, second-period equilibrium prices are given by* *(i) if* $\frac{1}{4} \leq \theta_1 \leq \frac{3}{4}$:

$$p^{o*}_{A2} = \frac{1}{3}t(2\theta_1 + 1) + c; \quad p^{r*}_{A2} = \frac{1}{3}t(3 - 4\theta_1) + c$$

$$p^{o*}_{B2} = \frac{1}{3}t(3 - 2\theta_1) + c; \quad p^{r*}_{B2} = \frac{1}{3}t(4\theta_1 - 1) + c$$

(ii) if $\theta_1 \leq \frac{1}{4}$:

$$p^{o*}_{A2} = t(1 - 2\theta_1) + c; \quad p^{r*}_{A2} = \frac{1}{3}t(3 - 4\theta_1) + c$$

$$p^{o*}_{B2} = \frac{1}{3}t(3 - 2\theta_1) + c; \quad p^{r*}_{B2} = c$$

(iii) if $\theta_1 \geq \frac{3}{4}$:

$$p^{o*}_{A2} = \frac{1}{3}t(2\theta_1 + 1) + c; \quad p^{r*}_{A2} = c$$

$$p^{o*}_{B2} = t(2\theta_1 - 1) + c; \quad p^{r*}_{B2} = \frac{1}{3}t(4\theta_1 - 1) + c$$

Proof. See the Appendix. ■

In the interior solution, when $\frac{1}{4} \leq \theta_1 \leq \frac{3}{4}$, both firms have the same profit in the second period, given by

$$\pi^*_{i2} = \frac{5}{9}t(2\theta_1^2 - 2\theta_1 + 1) \tag{10}$$

with $i = A, B$.

Corollary 2. *With* $\theta_1 = \frac{1}{2}$*, each firm has the same profit in second period, given by* $\pi^*_{i2} = \frac{5}{18}t$.

2.2.2 First-period Equilibrium

Given $p_{i1}, i = A, B$, the indifferent consumer is located at θ_1 such that

$$p_{A1} + t\theta_1 + \delta\left[p^r_{B2} + t(1 - \theta_1)\right] = p_{B1} + t(1 - \theta_1) + \delta\left[p^r_{A2} + t\theta_1\right]$$

Simplifying and substituing p^{r*}_{B2} and p^{r*}_{A2}, first-period demand is

$$\theta_1 = \frac{1}{2} + \frac{3(p_{B1} - p_{A1})}{2t(3+\delta)}$$

In the first period, the equilibrium choices are about p_{A1} and p_{B1}. θ_1 is a function of first-period prices, such that $\theta_1 = f(p_{A1},\ p_{B1})$ and in the interior solution, when $\frac{1}{4} \leq \theta_1 \leq \frac{3}{4}$, the overall objective function for firm i's profits is

$$(p_{i1} - c)\theta_1(p_{i1}) + \delta\left[\frac{5}{9}t\left(2(\theta_1(p_{i1}))^2 - 2\theta_1(p_{i1}) + 1\right)\right]$$

with $i = A, B$.

Proposition 6 *In the model with price discrimination under brand preferences approach there is a unique equilibrium given by*

$$p^*_{i1} = \frac{t}{3}(3+\delta) + c$$

and second-period equilibrium prices, with $i = A, B$

$$p^{o*}_{i2} = \frac{2}{3}t + c \quad \text{and} \quad p^{r*}_{i2} = \frac{1}{3}t + c$$

and overall profits

$$\Pi_i = \frac{t}{18}(8\delta + 9)$$

Proof. See the Appendix. ■

2.2.3 No discrimination benchmark case in the brand preferences approach

If firms cannot recognise their own customers and rival's customers, they cannot price discriminate. Thus, in both periods firms behave like in the standart Hotelling model.

The indifferent consumer is located at θ such that:

$$p_A^u + t\theta = p_{B2}^u + t(1-\theta)$$

$$\theta = \frac{1}{2} + \frac{p_B^u - p_A^u}{2t}$$

In each period firm A and B solve the following problem:

$$\underset{p_A^u}{Max} \pi_A^u = (p-c)\theta$$

$$\underset{p_B^u}{Max} \pi_B^u = (p_B^u - c)(1-\theta)$$

It is straightforward to show the following result.

Proposition 7 *Without price discrimination, each firm price in period 1 and 2 is:*

$$p^{u*} = t + c.$$

Overall profit is

$$\pi^{u*} = \frac{1}{2} t \left(1+\delta\right).$$

Proof. See the Appendix. ■

2.2.4 Welfare Analysis

This section discusses the welfare effects of BBPD in the brand preferences approach. To achieve this goal we compute first overall welfare (W^{nd}), consumer surplus (CS^{nd}) and industry profits (Π^{nd}) with no discrimination.

Without price discrimination

As usual each period welfare can be conputed as $W = v - ETC$, where ETC is the expected transport cost. In each period welfare w is

$$w = v - \left(\int_0^{\frac{1}{2}} txdx + \int_{\frac{1}{2}}^{1} t(1-x)dx \right)$$

$$w = v - \frac{1}{4}t$$

Overall welfare with no discrimination is thus given by $W^{nd} = w_1 + \delta w_2$, so

$$W^{nd} = \left(v - \frac{1}{4}t \right)(1+\delta)$$

The profits of industry correspond to the sum of the two firms profits and is given by

$$\Pi^{nd} = t(1+\delta)$$

Consumer surplus is given by $CS^{nd} = W^{nd} - \Pi^{nd}$, thus

$$CS^{nd} = \left(v - \frac{5}{4}t \right)(1+\delta)$$

With price discrimination

We now compute overall welfare (W^{nr}), consumer surplus (CS^{nr}) and industry profits (Π^{nr}) with discrimination. When firms can price discriminate between old and new customers, second-period welfare is:

$$w_2^{nr} = v - \left[\int_0^{\theta_A} txdx + \int_{\theta_A}^{\theta_1} t(1-x)dx + \int_{\theta_1}^{\theta_B} txdx + \int_{\theta_B}^{1} t(1-x)dx \right]$$

In the equilibrium, $\theta_1 = \frac{1}{2}$, $\theta_A = \frac{1}{3}$ and $\theta_B = \frac{2}{3}$. So, second-period welfare is given by

$$w_2^{nr} = v - \left[\int_0^{\frac{1}{3}} txdx + \int_{\frac{1}{3}}^{\frac{1}{2}} t(1-x)dx + \int_{\frac{1}{2}}^{\frac{2}{3}} txdx + \int_{\frac{2}{3}}^{1} t(1-x)dx \right]$$

$$w_2^{nr} = v - \frac{11}{36}t$$

In the first period, the welfare is

$$w_1^{nr} = v - \left(\int_0^{\frac{1}{2}} txdx + \int_{\frac{1}{2}}^{1} t(1-x)dx \right)$$

$$w_1^{nr} = v - \frac{1}{4}t$$

Joining both expressions, overall welfare is given by

$$W^{nr} = w_1 + \delta w_2$$

$$W^{nr} = \left(v - \frac{1}{4}t \right) + \delta \left(v - \frac{11}{36}t \right)$$

$$W^{nr} = v(1+\delta) - \frac{1}{4}t - \frac{11}{36}\delta t$$

Overall industry is given by

$$\Pi^{nr} = \frac{2}{18}t(9 + 8\delta)$$

Given the overall welfare and industry profits, the overall consumer surplus is given by

$$CS^{nr} = W^{nr} - \Pi^{nr}$$

$$CS^{nr} = \left(v(1+\delta) - \frac{1}{4}t - \frac{11}{36}\delta t\right) - \left(\frac{2}{18}t(9+8\delta)\right)$$

$$CS^{nr} = v(1+\delta) - \frac{5}{4}t - \frac{43}{36}\delta t$$

The table below presents the welfare findings when we move from no discrimination to discrimination in the brand preference approach. It shows that industry profits and overall welfare decrease but consumer surplus increases.

	Welfare	Consumer Surplus	Industry Profits
Without	$\left(v - \frac{1}{4}t\right)(1+\delta)$	$\left(v - \frac{5}{4}t\right)(1+\delta)$	$t(1+\delta)$
With	$v(1+\delta) - \frac{1}{4}t - \frac{11}{36}\delta t$	$v(1+\delta) - \frac{5}{4}t - \frac{43}{36}\delta t$	$\frac{2}{18}t(9+8\delta)$
	$\downarrow W$	$\uparrow ECS$	$\downarrow \Pi$

Comparison between the two approaches

After the analysis of behaviour-based price discrimation under the switching costs and the brand preferences approach, we can make some remarks about the main results.

The results of two approaches have some common features. In both approaches price discrimination decreases second-period equilibrium prices. If firms can recognise their own customers and rival's customers through consumers' past purchase history, they can charge a lower second-period price to rival's customers in order to attract them. Moreover, price discrimination reduces equilibrium profits and the overall welfare due to inefficient consumer switching.

There are some diferences between the two approaches. In particular, under switching costs approach prices increases over time and under brand preferences approach prices decreases over time. In markets with repeated purchases consumers are "locked-in" because with switching costs it is more difficult to change supplier in second period. Thus, firms compete harder in order to acquire consumers in the first period and charge a lower initial price. And, since customers are locked-in, in the second period firms increase their prices. On the other hand, in the model present in Fudenberg and Tirole

(2000) it is assumed that consumers are forward looking and they preview a lower price in the second period (consumers become less elastic in period 1). Given this assumption, firms are able to rise their first-period price and prices decrease over time.

2.3 Some of the main extensions on BBPD

Considering the models presented in Chen (1997) and Fudenberg and Tirole (2000), some extentions have been made under each approach. The models were extended to multiple periods and multiple firms, to other distributions of consumers' preferences and to markets where advertising is needed to inform consumers.

Under the *switching costs approach*, Taylor (2003) generalizes the model presented in Chen (1997) considering n firms in the market, which means that the market becomes more competitive. In his study, Taylor (2003) investigates competition and consumer behaviour in subscription markets, where each firms knows its customers. In this market, a firm can attract new customers offering a low introductory price while simultaneously exploiting its own customers. In the case when there are only two firms operating in the market, competition is soft and each firm has positive profits. When there are three or more firms, each firm earns rent of its customer base but zero economic profits. That happens because each firm offers the switcher an introductory price below cost in order to attract more rival's customers, and later recoups this through nonswitchers. Compared with the duopolistic market, competitive markets can be less efficient because the lower introductory prices induce more consumers to switch.

Under the *brand preferences approach*, Villas-Boas (1999) considers a situation in a duopoly with infinitely lived firms and overlapping generations of consumers. Consumers have relative preferences for firms' products and, considering a two-period model, in each period new consumers arrive and old consumers leave the market. The main results are that *(i)* in equilibrium prices are lower because firms compete hard in order to attract rival's customers; *(ii)* greater consumer patience also lowers equilibrium prices because consumers become indifferent to which product to buy first, hence more sensitive to the current prices (it intensifies competition); and, *(iii)* greater firm patience softens the competitive interaction. Chen and Zhang (2009) consider the practice of dynamic targeted pricing based on consumer purchase history. Here, it is considered that firms and consumers behave strategically and that there are three segments of consumers in the market. The first two segments include those consumers who are loyal to firm A and firm B (and where consumers purchase always from this firms). The third segment of consumers consists of switchers, who always change provider in the second period. Firms only can recognise the segment of consumers after the first-period purchase decisions. With the presence of strategic consumers, there is a reduction of price competition, which implies that consumers are worse off, and firm's profits and social welfare increase. Considering the case where the distribution of consumers' preferences is discrete (binary distribution), Esteves (2010) analyses the competitive effects of price discrimination based on customer recognition in a duopolistic market. The welfare results and customer recognition do depend on what is learned about consumers' characteristics, which in turn depends on the distribution of preferences. The main conclusion of this work is that price discrimination based on customer recognition is bad for profits but good for consumers and social welfare. Esteves (2009a) incorporates the advertising as an important informative task, both for consumers and firms. It is considered the case where a firm recognises the consumers' purchase history and targets them with different advertising. Only the firm that advertises the highest price in the first period has information to engage in price discrimination. It is identified as "the race for discrimination effect", where each firm has an incentive to price high in the initial period. Because price discrimination softens competition rather than intensify it, all firms are better off even if only one firm can engage price discrimination. Consequently, consumers are worse off and social welfare decreases.

A recent extention was done by Esteves and Rey (2010), considering the case when firms can implement r*etention strategies*. Under the LPL process, the request of a code provides the firms information about the consumers that are willing to switch and allows them to suggest counter-offers with the intention of retaining them. According to the assumptions on Fudenberg and Tirole model, Esteves and Rey consider a two-period model where the second-period is separated in two stages: *(i)* in the first stage, each firm chooses a set of prices for their existing and new customers and all those who want to change providers must request an authorization code from their current provider in order to complete the switching process; and *(ii)* in the second stage, given that firms can recognise customers with a willingness to switch, they can employ save/retention activities in an attempt to make it less attractive for a customer to switch to a competing firm (firms offer a "secret" fixed discount to these customers). First-period equilibrium prices are lower in a situation where retention activities are possible than in a situation where firms practice a uniform price (save activity is forbidden). Furthermore, with retention strategies, consumer surplus and social welfare are increase and firms' profits decrease.

Chapter 3

Behaviour-Based Price Discrimination with Retention Strategies

In a recent report of the Ofcom it is made a reminder to the case when firms can engage in price discrimination and save activity is feasible. If behaviour-based price discrimination is possible and if firms are able to recognise their existing and rival customers through past customers' decisions of consumption, firms can make the switching more difficult for all consumers that show an intention to switch supplier. Moreover, firms can recognise their potential switcher customers because all customers who want to change supplier must contact their existing provider and request a code (PAC) in order to complete the switching process. In this way, all potential switchers give a signal of their willingness to switch and firms can implement retention strategies in order to discourage the switching. One possible way to retain customers is to offer them a secret discount.

Save activity is a form of price discrimination based not on the consumers' characteristics (such as the willingness to pay or the age) but on the consumers' behaviour. Thus, as explained in Esteves and Rey (2010) it is also a form of BBPD.

With the exception of Esteves and Rey (2010) [1] there is no other work looking on the competitive and welfare effects of BBPD when firms can also engage in retention strategies. Specifically, these authors investigate the issue of BBPD with retention strategies using the brand preference approach. They consider a two-period model where the second-period is separated in two stages: *(i)* in the first stage, each firm chooses a set of prices for their existing and new customers and those who want to change providers must request an authorization code from their current provider in order to complete the switching process; and *(ii)* in the second stage, given that firms can recognise customers with a willingness to switch, they can employ save/retention activities in an attempt to make less attractive for a customer to switch to a competing firm (firms offer a "secret" fixed discount to this customers). They show that first-period equilibrium prices are lower in a situation where retention activities are possible than in a situation where firms practice a uniform price (save activity is forbidden). Furthermore, with retention strategies, consumer surplus and social welfare are increased and firms' profits decrease.

The main proposal of this Master Dissertation is to do the same exercise made by Esteves and Rey (2010) but in the context of the switching costs approach, considering as a benchmark the model of Chen (1997). In order to investigate the effects of the implementation of BBPD with retention strategies it is developed a two-period model where in the second period firms offer a secret counter-offer (discount) for all customers who show an intention to change supplier. And, if consumers decide to change supplier they have to incur switching costs.

This Chapter is organised as follows. First, in Section 3.1., it is presented the model of Esteves and Rey (2010) with the main equilibrium results and the welfare effects of retention strategies under the brand preferences approach. Then, it is presented the

[1] Some authors have showing an interest about this subject. McGahan and Ghemawat (1994) develop a two period game-theoretic model, where duopolists attempt to retain old customers and attract new ones. Authors make an empirical analysis of their model on a sample of ordinary American insurance companies. Chen and Hitt (2002) develop and implement an approach to measure the magnitudes of switching costs and brand loyalty for online services providers based on the random utility modelling framework. In addition, Verhoef (2003) investigates the differential effects of customer relationship perceptions marketing instruments on customer retention and customer share development over time.

main contribution of this Master Dissertation - Section 3.2.. Following the assumptions present in Chen (1997) the model is extended considering the ability of firms to implement retention strategies. Initially, it is presented the model assumptions. Subsection 3.2.2. analyses the equilibrium of the game, presenting the results of second- and first-period equilibrium. Then, subsection 3.2.3. analyses the welfare effects of BBPD with retention strategies. In Section 3.3. it is presented a comparison between the two approaches with retention strategies.

3.1 Brand Preferences Approach with Retention Strategies

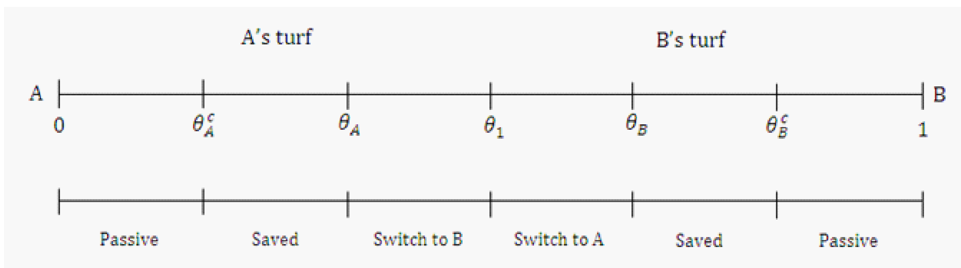
This section aims to present and discuss a model of BBPD with retention strategies in the context of brand preference approach. Motivated by the Ofcom report Esteves and Rey (2010) investigate the competitive and welfare effects of BBPD when firms can also engage in retention strategies. They do that in the context of the brand preference approach. As in Fudenberg and Tiorole (2000) there are two firms A and B selling a product/service to consumers whose brand preferences are represented by a parameter θ uniformly distributed on $[0, 1]$. Firm A is located at 0 and firm B at 1. Consumers want to buy one unit of product in each period either from A or B. In period 1 firms have no way to recognise customers, thus they charge a uniform price, namely p_{i1}. In the second period purchase history discloses information about consumers' preferences. In this period, there are two-stages. In the first-stage, firms choose two prices, one targeted to their old customers and one to the rival's customers, namely (p_i^o, p_i^r). In the second stage those customers with a real intention to switch need to contact their current supllier and ask a code which they must communicate to their new supplier in order to complete the switching process. With this contact, firms are able to recognise potential switchers and offer them a secret counter-offer. In the second stage each firm chooses a discount, $d_i, i = A, B$, to all customers that show an intention to leave.

3.1.1 Equilibrium Analysis

As usual, the model is solved by backward induction.

Second-period equilibrium

In the first period, the consumer located at θ_1 is indifferent between buying from firm A or firm B, which means that firm A's turf is given by $[0, \theta_1]$ and firm B's turf is $[\theta_1, 1]$.



Customer recognition with retention strategies (in Esteves and Rey (2010))

Look first on firm A's turf. In the group of A's consumers there is some of those who are willing to switch given the prices p^o_{A2} and p^r_{A2}. If all consumers who are willing to switch must contact their existing provider and request an authorization code to complete the switching process, firms are able to recognise the potential switchers and target them with special discounts in order to discourage the switching. So, in the group of potential switchers there is a proportion of them who will be saved.

So, in the second period in each firm's turf there are two different consumers: *(i) active consumers* which are those who are expressing an intention to switch; and *(ii) inactive consumers,* those who don't show any intention to switch. Only consumers who express an intention to leave the current supplier receive the discount.

In the second-stage of period 2, the indifferent consumer between buying again from firm A at price $p^o_{A2} - d_A$ or switching to firm B at price p^r_{B2} is located at θ_A:

$$p^o_{A2} - d_A + t\theta_A = p^r_{B2} + t(1 - \theta_A)$$

where,

$$\theta_A = \frac{1}{2} + \frac{p^r_{B2} - p^o_{A2} + d_A}{2t}$$

The indifferent consumer between acting as a passive or active is located at θ^c_A, where $d_A = 0$, and

$$\theta^c_A = \frac{1}{2} + \frac{p^r_{B2} - p^o_{A2}}{2t}$$

In firm B's turf, the results are similar to those obtained from firm A, such that

$$\theta_B = \frac{1}{2} + \frac{p^o_{B2} - p^r_{A2} - d_B}{2t}$$

$$\theta^c_B = \frac{1}{2} + \frac{p^o_{B2} - p^r_{A2}}{2t}$$

In the *second stage* each firm solves the following problem

$$\underset{d_A}{Max}(p^o_{A2} - d_A - c)(\theta_A - \theta^c_A)$$

$$\underset{d_B}{Max}(p^o_{B2} - d_B - c)(\theta^c_B - \theta_B)$$

Solving the problem, the secret discount offered by firm A is $d_A = \frac{1}{2}(p^o_{A2} - c)$ and by firm B is $d_B = \frac{1}{2}(p^o_{B2} - c)$.

In the *first stage* of period 2, each firm solves the following problem on firm A's turf:

$$\underset{p^o_{A2}}{Max} \quad \pi^o_{A2} = (p^o_{A2} - c)\theta^c_A + (p^o_{A2} - d_A - c)(\theta_A - \theta^c_A), \quad \text{where } d_A = \frac{1}{2}(p^o_{A2} - c)$$

$$\underset{p^r_{B2}}{Max} \quad \pi^r_{B2} = (p^r_{B2} - c)(\theta_1 - \theta_A), \quad \text{where } d_A = \frac{1}{2}(p^o_{B2} - c)$$

And, in firm B's turf each firm wants to

$$\underset{p^r_{A2}}{Max}\ \ \pi^r_{A2} = (p^r_{A2} - c)(\theta_B - \theta_1), \ \ \text{where } d_B = \frac{1}{2}(p^o_{B2} - c)$$

$$\underset{p^o_{B2}}{Max}\ \ \pi^o_{B2} = (p^o_{B2} - d_B - c)(\theta^c_B - \theta_B) + (p^o_{B2} - c)(1 - \theta^c_B), \ \ \text{where } d_B = \frac{1}{2}(p^o_{B2} - c)$$

with,

$$\theta_B = \frac{1}{2} + \frac{p^o_{B2} - p^r_{A2} - d_B}{2t}$$

$$\theta^c_B = \frac{1}{2} + \frac{p^o_{B2} - p^r_{A2}}{2t}$$

Proposition 8 *When firms can price discriminate between old/new customers and have the ability to implement retention strategies, second-period equilibrium is given by*

$$p^{o*}_{A2} = \frac{2}{5}t(2\theta_1 + 1) + c; \qquad p^{r*}_{A2} = \frac{2}{5}t(2 - 3\theta_1) + c$$

$$p^{o*}_{B2} = \frac{2}{5}t(3 - 2\theta_1) + c; \qquad p^{r*}_{B2} = \frac{2}{5}t(3\theta_1 - 1) + c$$

with,

$$d^*_A = \frac{1}{5}t(2\theta_1 + 1)$$

$$d^*_B = \frac{1}{5}t(3 - 2\theta_1)$$

Proof. See the Appendix. ■

From this result, it is straightforward that equilibrium profits for each firm is given by:

$$\pi^*_{A2} = \frac{1}{50}t(48\theta_1^2 - 36\theta_1 + 19)$$

$$\pi^*_{B2} = \frac{1}{50}t(48\theta_1^2 - 60\theta_1 + 31)$$

First-period equilibrium

Consumers and firms have the same discount factor δ, such that $\delta \in [0,1]$, and both are forward-looking. In the first-period, the indifferent consumer is located at θ_1, such that

$$p_{A1} + t\theta_1 + \delta\left[p^r_{B2} + t(1-\theta_1)\right] = p_{B1} + t(1-\theta_1) + \delta\left(p^r_{A2} + t\theta_1\right)$$

$$\theta_1 = \frac{1}{2} + \frac{p_{B1} - p_{A1} + \delta(p^r_{A2} - p^o_{B2})}{2t(1-\delta)}$$

Given the equilibrium prices of second-period (p^{r*}_{A2}, p^{o*}_{B2}), θ_1 is given by

$$\theta_1 = \frac{1}{2} + \frac{5(p_{B1} - p_{A1})}{2t(5+\delta)}$$

In the first period, the equilibrium choices are about p_{A1} and p_{B1}. θ_1 is a function of first-period prices, such that $\theta_1 = f(p_{A1},\ p_{B1})$, the overall objective function for firm A's profits is

$$(p_{A1} - c)\theta_1(p_{A1}, p_{B1}) + \delta\left[\frac{1}{50}t\left(48(\theta_1(p_{A1}, p_{B1}))^2 - 36(\theta_1(p_{A1}, p_{B1})) + 19\right)\right]$$

and firm B's objective function is

$$(p_{B1} - c)\theta_1(p_{A1}, p_{B1}) + \delta\left[\frac{1}{50}t\left(48(\theta_1(p_{A1}, p_{B1}))^2 - 60(\theta_1(p_{A1}, p_{B1})) + 31\right)\right]$$

Proposition 9 *When firms engage in BBPD with retention strategies, there is a symmetric subgame perfect Nash equilibrium where:*

(i) first-period equilibrium price is $p^*_{i1} = t(1 - \frac{\delta}{25}) + c$;

(ii) second-period equilibrium prices are $p^{o*}_{i2} = \frac{4}{5}t + c$ *and* $p^{r*}_{i2} = \frac{1}{5}t + c$, *with* $d^*_i = \frac{2}{5}t$;

(iii) overall profit is $\pi^*_i = \frac{1}{50}t(12\delta + 25)$.

Proof. See the Appendix. ■

Comparing the results with and without retention strategies, we can note that the second-period price for rival's customers are lower in the case when retention strategies are feasible; and, second-period prices for old customers are higher with retention strategies (however, note that a portion of old customers pay a lower price because they are saved and receive a discount). First-period equilibrium prices with retention strategies are lower than the case without retention strategies.

3.1.2 Welfare Analysis

In models where price discrimination is not feasible and in models with price discrimination (as in Fudenberg and Tirole (2000) model), first period equilibrium outcome is efficient because consumers choose the closer firm.

When firms can implement retention strategies, second-period welfare is

$$w_2^r = v - \int_0^{\theta_A} txdx - \int_{\theta_A}^{\theta_1} t(1-x)dx - \int_{\theta_1}^{\theta_B} txdx - \int_{\theta_B}^{1} t(1-x)dx$$

And, in the equilibrium $\theta_1 = \frac{1}{2}$, so $\theta_A = \frac{2}{5}$ and $\theta_B = \frac{3}{5}$. Thus, second-period welfare is

$$w_2^r = v - \int_0^{\frac{2}{5}} txdx - \int_{\frac{2}{5}}^{\frac{1}{2}} t(1-x)dx - \int_{\frac{1}{2}}^{\frac{3}{5}} txdx - \int_{\frac{3}{5}}^{1} t(1-x)dx$$

$$w_2^r = v - \frac{4}{50}t - \left(\frac{1}{10} - \frac{9}{200}\right)t - \frac{11}{200}t - \left(\frac{2}{5} - \frac{8}{25}\right)t$$

$$w_2^r = v - \frac{27}{100}t$$

In the first period, firms can not price discriminate and welfare is given by

$$w_1^r = v - \int_0^{\frac{1}{2}} txdx - \int_{\frac{1}{2}}^{1} t(1-x)dx$$

$$w_1^r = v - \frac{1}{4}t$$

The overall welfare is given by the welfare of two periods together and,

$$W^r = w_1^r + \delta w_2^r$$

$$W^r = v(1+\delta) - \frac{1}{4}t - \frac{27}{100}\delta t$$

In the second period, and in the equilibrium ($\theta_1 = \frac{1}{2}$), profit of each firm is $\pi_{i2}^* = \frac{13}{50}t$. So, second-period industry profits are

$$\pi_2^{ind} = \frac{13}{25}t$$

As the same way, in the first period, each firm's profit is given by $\pi_{i1}^* = \frac{t}{2}\left(1 - \frac{\delta}{25}\right)$ and the two firms together is

$$\pi_1^{ind} = t\left(1 - \frac{\delta}{25}\right)$$

Overall industry profits are given by

$$\Pi^{ind} = \pi_1^{ind} + \delta\pi_2^{ind}$$

$$\Pi^{ind} = \frac{1}{25}t(12\delta + 25)$$

Consumers' surplus is equal to the diference between the overall welfare and overall industry profits. In this way, the overall consumer surplus is given by

$$CS^r = W^r - \Pi^{ind}$$

$$CS^r = \left(v(1+\delta) - \frac{1}{4}t - \frac{27}{100}\delta t\right) - \left(\frac{1}{25}t(12\delta + 25)\right)$$

$$CS^r = v(1+\delta) - \frac{5}{4}t - \frac{3}{4}t\delta$$

The conclusion of Esteves and Rey's (2010) model is that with price discrimination based on customer recognition and with retention strategies, consumers are better off while firms are worse off. Overall welfare increases. It is straightforward to note that consumers surplus increase because more consumers pay a lower price (saved consumers receive a discount and for rival customers the price is lower). By the same reason, firms profits decrease and social welfare increase due to the consumer surplus. The brief of results are presented in the following table.

	Welfare	Consumer Surplus	Industry Profits
Without RS	$v(1+\delta) - \frac{1}{4}t - \frac{11}{36}\delta t$	$v(1+\delta) - \frac{5}{4}t - \frac{43}{36}\delta t$	$\frac{2}{18}t(9+8\delta)$
With RS	$v(1+\delta) - \frac{1}{4}t - \frac{27}{100}\delta t$	$v(1+\delta) - \frac{5}{4}t - \frac{3}{4}t\delta$	$\frac{1}{25}t(12\delta+25)$
	$\uparrow W$	$\uparrow ECS$	$\downarrow \Pi$

Corollary 3. *For any $\delta > 0$, comparing with BBPD without retention strategies under the brand preferences approach, BBPD with retention strategies boosts consumer surplus and overall welfare and decreases industry profits.*

3.2 Switching costs approach with retention strategies

As aforementioned this section presents the main contribution of this thesis: to investigate the economic and welfare effects of BBPD with retention strategies in markets with switching costs. Next we present the model assumptions.

3.2.1 Model

Following the assumptions presented in Chen (1997) we introduce the possibility of retention strategies. There are two firms, A and B, and each firm produces a non-durable homogenous good A and B, respectively, at a constant marginal cost, $c, c \geq 0$. There are two periods, 1 and 2. In the first period, firms have no information to engage in price discrimination, thus each firm simultaneously chooses a uniform price, p_{A1} and p_{B1}. Consumers have an identical reservation value, v. In each period, each consumer wants to buy one unit of the product, either from firm A or firm B. In the end of first period firm A's turf is given by α and firm B's turf is given by $1-\alpha$. Firms and consumers have an identical discount factor δ, such that $\delta \in [0,1]$. In a market with repeated purchases, purchase history allows firms to know their own customers and the rival's customers, thus behaviour-based price discriminations becomes possible. Following the same assumptions as in Esteves and Rey (2010) in this period it is considered a two-stage competition game. In the first stage, each firm is able to recognise their existing customers and those that bought from a rival firm. Firms choose a set of two different prices, one for old customers, p_{i2}^{o}, and one for the rival's firm customers, p_{A2}^{r}, $i = A, B$. If a customer decides to switch from his current provider he has to incur a switching cost s, uniformly distributed on $[0, \phi]$. All customers that wish to switch must contact their existing provider and request a code (under the LPL process) in order to complete the switching process. Due to this contact, each firm can recognise their potential switchers (all customers who give a signal of their intention to leave) and, at the second stage, firms can implement a retention strategy to discourage customers to switch. Each firm will offer a fixed discount to all consumers showing an intention to switch, d_i, $i = A, B$.

3.2.2 Equilibrium Analysis

After consumers have made their consumption's decisions in period 1, each firm can recognise their own customers and rivals' customers. So, in the second period each firm offers a set of price $(p_{i2}^{o},\ p_{i2}^{r})$, $i = A, B$. Additionaly, all those consumers who want to change supplier in the second period have to incur switching cost, s and need to contact their current supllier to obtain the code to complete the switching process. These consumers can then receive a discount. The game is solved working backwards from period 2.

Second-period

In the second period there are two different stages: *(i)* in the first stage, all consumers who want to change supplier have to contact their existing provider and request an authorization code in order to complete the switching process;and *(ii)* in the second stage, firms can implement retention strategies in order to discourage customers to switch and target them with a special discount price, $d_i, i = A, B$.

Second-stage

In the beginning of period 2, firm A has market share equal to α and firm B has a market share equal to $(1 - \alpha)$. Like in Esteves and Rey (2010) in each firm's turf there are two different consumers: *(i) active consumers* which are those who are expressing an intention to switch; and *(ii) inactive consumers,* those who don't show any intention to switch. Only consumers who express an intention to leave the current supplier receive the discount.

Look first into firm A's turf. In the second-stage of period 2, the indifferent consumer between buying again from firm A at price $p^o_{A2} - d_A$ or switching to firm B at price p^r_{B2} is located at s^*_A such that:

$$p^o_{A2} - d_A = p^r_{B2} + s^*_A$$

where,

$$s^*_A = p^o_{A2} - p^r_{B2} - d_A$$

The indifferent consumer between acting as a passive or active is located at s_A, where $d_A = 0$, and

$$s_A = p^o_{A2} - p^r_{B2}$$

Thus consumers with switching cost s_A are indifferent between acting as an active or passive consumer; and those with switching cost s^*_A are indifferent between accepting the discount and buy again from A or not accepting the discount and switch to B.

The number of consumers who bought from firm A in period 1 and switch for firm B in period 2 is given by q_{BA} which is equal to

$$q_{BA} = \alpha \int_0^{s_A^*} f(s)ds$$

On the other hand, the number of consumers who are saved and accept the discount offered by firm A, q_{AA}^s, is given by

$$q_{AA}^s = \alpha \int_{s_A^*}^{s_A} f(s)ds$$

And, the number of consumers bought from firm A and act as passive consumers in the second period, q_{AA}, is

$$q_{AA} = \alpha \int_{s_A}^{\phi} f(s)ds$$

Using $s_A^* = p_{A2}^o - p_{B2}^r - d_A$ and $s_A = p_{A2}^o - p_{B2}^r$ it is straightfoward to find that:

(i) switchers consumers in firm A's turf are

$$q_{BA} = \alpha \int_0^{s_A^*} \frac{1}{\phi} ds = \frac{\alpha}{\phi} \left(p_{A2}^o - p_{B2}^r - d_A\right)$$

(ii) saved consumers in firm A's turf are

$$q_{AA}^s = \alpha \int_{s_A^*}^{s_A} \frac{1}{\phi} ds = \frac{\alpha}{\phi} d_A$$

(iii) passive consumers in firm A's turf are

$$q_{AA} = \alpha \int_{s_A}^{\phi} \frac{1}{\phi} ds = \frac{\alpha}{\phi}(\phi - p_{A2}^o + p_{B2}^r)$$

A similar reasoning can be applied to firm B's turf. Thus,

(i) switchers consumers in firm B's turf are

$$q_{AB} = (1-\alpha)\int_0^{s_B^*}\frac{1}{\phi}ds = \frac{(1-\alpha)\left(p_{B2}^o - p_{A2}^r - d_B\right)}{\phi}$$

(ii) saved consumers in firm B's turf are

$$q_{BB}^s = (1-\alpha)\int_{s_B^*}^{s_B}\frac{1}{\phi}ds = \frac{(1-\alpha)}{\phi}d_B$$

(iii) passive consumers in firm B's turf are

$$q_{BB} = (1-\alpha)\int_{s_A}^{\phi}\frac{1}{\phi}ds = \frac{(1-\alpha)\left(\phi - p_{B2}^o + p_{A2}^r\right)}{\phi}$$

In the second stage, firm A wants to maximize the profit obtained with saved customers. So, the maximization problem is the following

$$\underset{d_A}{Max}(p_{A2}^o - d_A - c)q_{AA}^s$$

$$\underset{d_A}{Max}(p_{A2}^o - d_A - c)\left(\frac{\alpha}{\phi}d_A\right)$$

Firm B solves a similar same maximization problem as firm A. Thus,

$$\underset{d_B}{Max}(p_{B2}^o - d_B - c)q_{BB}^s$$

$$\underset{d_B}{Max}(p_{B2}^o - d_B - c)\left(\frac{(1-\alpha)}{\phi}d_B\right)$$

And, we get that

$$d_i^* = \frac{1}{2}(p_{i2}^o - c)$$

with $i = A, B$.

First-stage

Given the optimum discount price offered by each firm, such that $d_i = \frac{1}{2}(p^o_{i2} - c), i = A, B$, in the first-stage each firm wants to maximize their second-period profits. In the second period, each firm acts in both markets (strong and weak markets). In the firm A's turf, firm A solves the following maximization problem

$$\begin{aligned} \underset{p^o_{A2}}{Max}\pi^o_{A2} &= (p^o_{A2} - c)q_{AA} + (p^o_{A2} - d_A - c)q^s_{AA} \\ s.t. \;\; d_A &= \frac{1}{2}(p^o_{A2} - c) \end{aligned}$$

while firm B solves the problem:

$$\underset{p^r_{B2}}{Max}\pi^r_{B2} = (p^r_{B2} - c)q_{BA}$$

In firm B's turf, each firm has a similar maximization problem, such that

$$\begin{aligned} \underset{p^o_{B2}}{Max}\pi^o_{B2} &= (p^o_{B2} - c)q_{BB} + (p^o_{B2} - d_B - c)q^s_{BB} \\ s.t. \; d_B &= \frac{1}{2}(p^o_{B2} - c) \end{aligned}$$

and,

$$\underset{p^r_{A2}}{Max}\pi^r_{A2} = (p^r_{A2} - c)q_{AB}$$

Proposition 10 *When firms are able to implement BBPD with retention strategies in markets with switching costs, second-period equilibrium prices are given by:*

$$p^{o*}_{i2} = \frac{4}{5}\phi + c$$

$$p^{r*}_{i2} = \frac{1}{5}\phi + c$$

and the equilibrium discount is equal to:

$$d_i = \frac{2}{5}\phi$$

Second-period equilibrium profits is equal to:

$$\pi^*_{A2} = \frac{\phi}{25}(11\alpha + 1)$$

$$\pi^*_{B2} = \frac{\phi}{25}(12 - 11\alpha)$$

Proof. See the Appendix. ■

For rival's customers second-period equilibrium prices are lower when firms have the ability to implement retention strategies. For old customers, the price is higher with retention strategies. However, there is a proportion of old customers who are saved and pay a lower price because they receive a discount. As in Chen (1997) second-period prices do not depend on market share.

Corollary 4. *When firms split evenly the market in period 1,* $\alpha = \frac{1}{2}$ *and firms' profits are* $\pi^*_{A2} = \pi^*_{B2} = \frac{13}{50}\phi$.

First-period

As in Chen's (1997) model, in the first period no consumer is really attached to any firm in the market. Due to the perfect substitution assumption, consumers choose the firm which offers the lowest price in period 1. In the first period, firm A has α consumers in the market and firm B has $(1 - \alpha)$ consumers. It is easy to see that

$$\alpha = \begin{cases} 1 & if \quad p_{A1} < p_{B1} \\ \frac{1}{2} & if \quad p_{A1} = p_{B1} \\ 0 & if \quad p_{A1} > p_{B1} \end{cases}$$

In period 1, each firm chooses its first-period price simultaneously and non-cooperatively as a way to maximise overall profits. Firm A overall profit is:

$$\pi_A = \pi_{A1} + \delta\pi_{A2}^* = \left[(p_{A1} - c)\alpha + \frac{\delta\phi}{25}(11\alpha + 1)\right]_{\alpha=\frac{1}{2}}$$

$$\pi_A = \begin{cases} (p_{A1} - c) + \frac{12\delta\phi}{25} & if \quad p_{A1} < p_{B1} \\ \frac{1}{2}(p_{A1} - c) + \frac{13}{50}\delta\phi & if \quad p_{A1} = p_{B1} \\ \frac{\delta\phi}{25} & if \quad p_{A1} > p_{B1} \end{cases}$$

We prove the existence of an equilibrium by construction. Suppose as an hypothesis that in the first period both firms charge the same price, such that $p_{A1} = p_{B1}$ and $\alpha = \frac{1}{2}$. In this case we have that each firm profit is equal to

$$\pi_i = \frac{1}{2}(p_{i1} - c) + \frac{13}{50}\delta\phi, \text{ with } p_{i1} = p_{j1}$$

with $i, j = A, B$.

If firm A deviates and charges a lower price, i.e, $p_{A1}^d = p_{B1} - \varepsilon$, then it captures all the market and its profits from deviation is equal to $(p_{A1}^d - c) + \frac{12\delta\phi}{25}$. It is therefore easy to show that firm A has no incentive to deviate as long as $p_{A1} = c - \frac{11}{25}\delta\phi$. Considering now a deviation to a higher price it again follows that firm A has no incentive to deviate and charge a higher first-period price as long as $p_{A1} = c - \frac{11}{25}\delta\phi$. The same reasoning is applied to firm B. Thus, $p_{A1} = p_{B1}$ and $\alpha = \frac{1}{2}$ is an equilibrium as long as $p_{B1} = c - \frac{11}{25}\delta\phi$. This completes the proof.

We can therefore establish the following result.

Proposition 11 *In the model with BBPD with retention strategies in switching costs approach there is an unique symmetric subgame-perfect Nash equilibrium, in which:*

(i) first-period prices are given by $p_{i1}^* = c - \frac{11}{25}\delta\phi$;

(ii) second-period equilibrium is $p_{i2}^{o*} = \frac{4}{5}\phi + c$ *and* $p_{i2}^{r*} = \frac{1}{5}\phi + c$, *with* $d_i = \frac{2}{5}\phi$;

(iii) and, overall profits is $\Pi_i = \frac{\delta\phi}{25}$, *with* $i = A, B$.

3.2.3 Welfare Analysis

This section analyzes the welfare effects of behaviour-based price discrimination when retention strategies are possible in the switching costs approach. In the first period, the equilibrium is efficient because all consumers buy from the firm offering the lowest price and there are no switching costs.In the second period, because some consumers have to incur switching costs if they decide to change supplier, there is inefficient switching.

Let us look to the firms' profits. In the symmetric equilibrium firms split evenly the market and $\alpha = \frac{1}{2}$. Without retention strategies, the overall profits of each firm Π^{nr}, is

$$\Pi^{nr} = \frac{\delta\phi}{9}$$

With retention strategies, each firm's overall profit, Π^{r}, is given by

$$\Pi^{r} = \frac{\delta\phi}{25}$$

Thus it is straightforward to see that

$$\Pi^{r} - \Pi^{nr} = \frac{\delta\phi}{25} - \frac{\delta\phi}{9} = -\frac{16}{225}\delta\phi < 0.$$

Thus, firms are worse off with retention strategies. The reason is that firms compete more agressively with retention strategies.

Look next at overall consumer surplus. We have seen that with no retention strategies it is equal to:

$$CS^{nr} = (1+\delta)(v-c) - \frac{5}{18}\delta\phi$$

In the case when retention strategies are feasible, overall consumer surplus is

$$CS^{r} = (v - p_{i1}^{r*}) + \delta\left(v - \int_{0}^{s^*}(p_{i2}^{r*} + s)\frac{1}{\phi}ds - \int_{s^*}^{s}(p_{i2}^{o*} - d_i)\frac{1}{\phi}ds - \int_{s}^{\phi} p_{i2}^{o}\frac{1}{\phi}ds\right)$$

$$CS^r = v - c + \frac{11}{25}\delta\phi + \delta\left[v - \frac{2}{5}\left(\frac{4}{5}\phi + c\right) - \frac{2}{5}\left(\frac{4}{5}\phi + c - \frac{2}{5}\phi\right) - \frac{1}{5}\left(\frac{1}{5}\phi + c + \frac{1}{10}\phi\right)\right]$$

from which we obtain:

$$CS^r = (1+\delta)(v-c) - \frac{1}{10}\delta\phi$$

$$CS^r - CS^{nr} = (1+\delta)(v-c) - \frac{1}{10}\delta\phi - \left((1+\delta)(v-c) - \frac{5}{18}\delta\phi\right) = \frac{8}{45}\delta\phi > 0$$

Therefore, with retention strategies, overall consumer surplus is higher than that when firms cannot implement retention strategies. The reason is that a higher proportion of consumers pay lower prices (retained consumers and switchers). This proportion is equal to $\frac{3}{5}$.

The overall welfare with retention strategies, W^r, is given by

$$W^r = CS^r + \Pi^r_{ind}$$

$$W^r = \left((1+\delta)(v-c) - \frac{1}{10}\delta\phi\right) + \frac{2\delta\phi}{25}$$

$$W^r = (1+\delta)(v-c) - \frac{1}{50}\delta\phi$$

And, as we saw in the benchmark case, without retention strategies the overall welfare, W^{nr}, is

$$W^{nr} = (1+\delta)(v-c) - \frac{1}{18}\delta\phi$$

Thus,

$$W^r - W^{nr} = (1+\delta)(v-c) - \frac{1}{50}\delta\phi - \left((1+\delta)(v-c) - \frac{1}{18}\delta\phi\right) = \frac{8}{225}\delta\phi > 0$$

We can therefore conclude that welfare increases when firms use retention strategies. This happens because less consumers switch in equilibrium. In the model with switching costs without retention strategies (Chen (1997)), all consumers with switching cost lower than $\frac{\phi}{3}$, will change supplier in the second period. With retention strategies, only consumers with switching cost lower than $\frac{\phi}{5}$ will change provider in the second period. While the deadweight loss with no retention strategies is equal to $\frac{\phi\delta}{18}$, with retention strategies the deadweight loss is equal to $\frac{\delta\phi}{50}$.

The next table summarises the effects on welfare in the cases with and without retention strategies.

	Consumer Surplus	Profits	Welfare	Deadweight Loss
Without RS	$(1+\delta)(v-c)-\frac{5}{18}\delta\phi$	$\frac{\delta\phi}{9}$	$(1+\delta)(v-c)-\frac{1}{18}\delta\phi$	$\frac{\phi}{18}$
With RS	$(v-c)(1+\delta)-\frac{1}{10}\delta\phi$	$\frac{\delta\phi}{25}$	$(1+\delta)(v-c)-\frac{1}{50}\delta\phi$	$\frac{\phi}{50}$
	$\uparrow ECS$	$\downarrow \Pi$	$\uparrow W$	$\downarrow DWL$

Corollary 5. *The analysis of BBPD with retention strategies under the switching costs approach, suggests that retention strategies are good for consumers and overall welfare but bad for firms, for any $\delta > 0$.*

3.3 Comparison between the two approaches

In this section it is done a comparison between the equilibrium and welfare effects of BBPD with retention strategies under the switching costs approach and the brand preferences approach. The table below shows the main results of each approach.

		Switching Costs Approach	Brand Preferences Approach
2nd Period	p_{i2}^{o*}	$p_{i2}^{o*} = \frac{4}{5}\phi + c$	$p_{i2}^{o*} = \frac{4}{5}t + c$
	p_{i2}^{r*}	$p_{i2}^{r*} = \frac{1}{5}\phi + c$	$p_{i2}^{r*} = \frac{1}{5}t + c$
	d_i^*	$d_i^* = \frac{2}{5}\phi$	$d_i^* = \frac{2}{5}t$
1st Period	p_{i1}^*	$p_{i1}^* = c - \frac{11}{25}\delta\phi < c$	$p_{i1}^* = t(1 - \frac{\delta}{25}) + c > c$
Welfare	W	$\uparrow W (\downarrow DWL)$	$\uparrow W$
	CS	$\uparrow ECS$	$\uparrow ECS$
	Π	$\downarrow \Pi$	$\downarrow \Pi$

Relatively to second-period equilibrium prices, in both approaches second-period equilibrium prices are below the uniform second-period price. If we compare second-period prices with BBPD and no retention strategies in both approaches we have that passive consumers pay a higher prices but all the others (retained consumers and switchers) pay a lower, which suggest that consumers are better off with BBPD and retention strategies.

Regarding to first-period prices under the switching costs approach first-period equilibrium price decreases when we move from BBPD with no retention strategies to BBPD with retention strategies. The same happens in the brand preferences approach, i.e, first period price with retention strategies is below its no retention counterpart . However, while in the brand preferences approach with no retention strategies first period price is above the uniform price, with retention strategies first-period price is below the uniform price. And marginal cost while under brand preferences approach first-period price is higher than marginal costs.

The welfare effects of BBPD with retention strategies are the same in both approaches, such that: (i) consumer surplus increase; (ii) firms profits decrease; and, (iii) overall welfare increases. Thus,the two models suggest that retention strategies are good for consumers but bad for firms.

Chapter 4

Conclusion

The literature on BBPD is quite new and the analysis of the effects of BBPD in competitive markets considers the ability of firms to charge different prices to old and new customers according their past purchases. The analysis of BBPD is done under two different approaches - the switching costs approach and the brand preferences approach. Both approaches claim that price discrimination is bad for profits and can decrease social welfare due to inefficient switching.

The main goal of this Dissertation was to provide a review of the economic literature on BBPD and develop a theoretical model to investigate the ability of firms to implement BBPD with retention strategies as a way to reduce consumer switching. While Esteves and Rey (2010) investigate the economic and welfare effects of BBPD with retention strategies under the brand preferences approach, the model developed in this dissertation has looked at the same issue in the switching costs approach.

The obtained results suggest that with retention strategies less consumers change supplier in the second period ($\frac{1}{5}$ of consumers against $\frac{1}{3}$ of consumers without retention) because some of potential switchers receive a discount and decide not to change supplier.

In comparison with the equilibrium without retention strategies, second-period prices for old customers, p_{i2}^{o*}, are higher than without retention. In turn, second-period equilibrium prices for rival's firm customers, p_{i2}^{r*}, are lower with retention than without retention strategies (competitive effects in order to increase the number of customers poached from the rival, even those who can accepted the discount). First-period price is, as in the benchmark case without retention (Chen (1997)), lower than marginal costs. Moreover, first-period price is lower with retention than without retention strategies because firms compete harder in order to increase their base of locked-in customers.

Comparing the results of BBPD with retention strategies under the swithing costs approach and the brand preferences approach, we can note that the analysis considering retention supports the results without retention strategies. As in models with BBPD (without retention), under both approaches second-period prices are lower than uniform prices. And, the different feature is related to the trend of prices over the time. While under switching costs approach prices increases over time, under brand preferences approaches prices decreases over the time.

The welfare effects are as follows: (i) deadweight loss is lower under retention activity because less consumers change supplier in the second period (less inefficient switching); (ii) firms' profits are lower with retention strategies because $\frac{3}{5}$ of consumers pay a lower price - customers who really change supplier and customers who are saved; and, (iii) consumer surplus is higher with retention strategies because more customers pay a lower price in the second period. Thus, retention strategies are good for consumers and bad for profits. In general, there is an increase in social welfare due to less inefficient switching.

Chapter 5

Appendix

In this section is presented the proofs of the propositions.

Proof of Proposition 1. Each firm second period profit equals:

$$\pi_{A2} = \frac{\alpha}{\phi}(p^o_{A2} - c)\,(1 - p^o_{A2} + p^r_{B2}) + \frac{(1-\alpha)}{\phi}(p^r_{A2} - c)(p^o_{B2} - p^r_{A2})$$

$$\pi_{B2} = \frac{(1-\alpha)}{\phi}(p^o_{B2} - c)\,(1 - p^o_{B2} + p^r_{A2}) + \frac{\alpha}{\phi}(p^r_{B2} - c)(p^o_{A2} - p^r_{B2})$$

Each firm wants to maximize its profits with respect to p^o_{i2} and with respect to p^r_{i2}, $i = A, B$. The first-order conditions are

$$\frac{\partial \pi_{A2}}{\partial p^o_{A2}} = \frac{\alpha}{\phi}\,(\phi - p^o_{A2} + p^r_{B2}) - \frac{\alpha}{\phi}(p^o_{A2} - c) = 0$$

$$p^o_{A2} = \frac{\phi + p^r_{B2} + c}{2}$$

$$\frac{\partial \pi_{A2}}{\partial p^r_{A2}} = \frac{(1-\alpha)}{\phi}(p^o_{B2} - p^r_{A2}) - \frac{(1-\alpha)}{\phi}(p^r_{A2} - c) = 0$$

$$p^r_{A2} = \frac{p^o_{B2} + c}{2}$$

$$\frac{\partial \pi_{B2}}{\partial p^o_{B2}} = \frac{(1-\alpha)}{\phi}\,(\phi - p^o_{B2} + p^r_{A2}) - \frac{(1-\alpha)}{\phi}(p^o_{B2} - c) = 0$$

$$p^o_{B2} = \frac{\phi + p^r_{A2} + c}{2}$$

$$\frac{\partial \pi_{B2}}{\partial p^r_{B2}} = \frac{\alpha}{\phi}(p^o_{A2} - p^r_{B2}) - \frac{\alpha}{\phi}(p^o_{A2} - c) = 0$$

$$p^r_{B2} = \frac{p^r_{A2} + c}{2}$$

Second-order conditions are also satisfied. Solving the systems of equation above, p^o_{i2} and $p^r_{i2}, i = A, B$, are given by

$$p^{o*}_{A2} = p^{o*}_{B2} = \frac{2}{3}\phi + c \tag{A}$$

$$p^{r*}_{A2} = p^{r*}_{B2} = \frac{1}{3}\phi + c \tag{B}$$

■

Proof of Proposition 2. We have seen that:

(i) If $p_{A1} = p_{B1}$ and $\alpha = \frac{1}{2}$,

$$\pi_i = \frac{1}{2}(p_{i1} - c) + \frac{5}{18}\delta\phi$$

(ii) If $p_{A1} < p_{B1}$ and $\alpha = 1$, firm A's overall profit is

$$\pi_A = p_{A1} - c + \frac{4}{9}\delta\phi$$

and firm B's overall profit is given by

$$\pi_B = \frac{\delta\phi}{9}.$$

(iii) If $p_{A1} > p_{B1}$, firms interchange positions thus $\alpha = 0$. Thus,

$$\pi_A = \frac{\delta\phi}{9}$$

and

$$\pi_B = p_{B1} - c + \frac{4}{9}\delta\phi.$$

To find the first-period equilibrium price suppose as an hipothesys that $p_{A1} = p_{B1}$ and $\alpha = \frac{1}{2}$. If for instance firm A decides to deviate to $p^d_{A1} > p_{B1}$ it would loose its first period demand and profit. Its profit from deviation would be equal to $\frac{\delta\phi}{9}$. Firm A has no incentives to deviate from $p_{A1} = p_{B1}$ as long as

$$\frac{1}{2}(p_{A1} - c) + \frac{5}{18}\delta\phi = \frac{\delta\phi}{9}$$

from which we obatin $p_{A1} = c - \frac{1}{3}\delta\phi$. Consider now the case where firm A decides to deviate to $p^d_{A1} < p_{B1}$. In this case it would capture all the market in period 1. Its profit from deviation would be equal to $p_{A1} - c + \frac{4}{9}\delta\phi$. Firm A has no incentives to deviate from $p_{A1} = p_{B1}$ as long as

$$p_{A1} - c + \frac{4}{9}\delta\phi = \frac{1}{2}(p_{A1} - c) + \frac{5}{18}\delta\phi$$

from which we obatin $p_{A1} = c - \frac{1}{3}\delta\phi$. Doing the same for firm it is straightforward to prove that firm B has no incentive to deviate from $p_{A1} = p_{B1}$ as long as $p_{B1} = c - \frac{1}{3}\delta\phi$. Therefore, first period equilibrium price is equal to $p^*_{i1} = c - \frac{\delta\phi}{3}$ with $i = A, B$. ■

Proof of Proposition 3. Each firm has the following profit function,

$$\pi^u_{A2} = \frac{\alpha}{\phi}(p^u_{A2} - c)(\phi - p^u_{A2} + p^u_{B2})$$

$$\pi^u_{B2} = \frac{\alpha}{\phi}(p^u_{B2} - c)(p^u_{A2} - p^u_{B2}) + (1 - \alpha)(p^u_{B2} - c)$$

Solving the maximization problem of each firm, the first order conditions are

$$\frac{\partial \pi^u_{A2}}{\partial p^u_{A2}} = (p^u_{A2} - c)\left(-\frac{\alpha}{\phi}\right) + \frac{\alpha}{\phi}(\phi - p^u_{A2} + p^u_{B2}) = 0$$

$$p^u_{A2} = \frac{\phi + c + p^u_{B2}}{2}$$

$$\frac{\partial \pi^u_{B2}}{\partial p^u_{B2}} = \frac{\alpha}{\phi}(p^u_{A2} - p^u_{B2}) + (p^u_{B2} - c)\left(-\frac{\alpha}{\phi}\right) + (1-\alpha) = 0$$

$$p^u_{B2} = \frac{1}{2}(p^u_{A2} + c) + \frac{(1-\alpha)}{2\alpha}\phi$$

Using the equations above, uniform prices are given by

$$p^{u*}_{A2} = \frac{(1+\alpha)}{3\alpha}\phi + c$$

$$p^{u*}_{B2} = \frac{(2-\alpha)}{3\alpha}\phi + c$$

According to the results, $p^{u*}_{A2} \geq p^{u*}_{B2}$, if and only if $\alpha \geq \frac{1}{2}$.

Solving the model for the case when $p^u_{A2} \leq p^u_{B2}$, $\widetilde{s} = p^u_{B2} - p^u_{A2}$ and the amount of consumers that choose each firm is given by

$$q^u_B = (1-\alpha)\int_{\widetilde{s}}^{\phi} \frac{1}{\phi} d(s)$$

$$q^u_B = \frac{(1-\alpha)}{\phi}(\phi - p^u_{B2} + p^u_{A2})$$

$$q^u_A = (1-\alpha)\int_0^{\widetilde{s}} \frac{1}{\phi} d(s) + \alpha \int_0^{\phi} \frac{1}{\phi} d(s)$$

$$q^u_A = \frac{(1-\alpha)}{\phi}(p^u_{B2} - p^u_{A2}) + \alpha$$

The profits of each firm are

$$\pi^u_{A2} = (p^u_{A2} - c)q^u_A$$

$$\pi^u_{A2} = \frac{(1-\alpha)}{\phi}(p^u_{A2} - c)(p^u_{B2} - p^u_{A2}) + \alpha(p^u_{A2} - c)$$

$$\pi^u_{B2} = (p^u_{B2} - c)q^u_B$$

$$\pi^u_{B2} = \frac{(1-\alpha)}{\phi}(p^u_{B2} - c)(\phi - p^u_{B2} + p^u_{A2})$$

Solving the FOC, we obtain similar results, such that

$$p_{A2}^{u*} = \frac{(1+\alpha)}{3(1-\alpha)}\phi + c$$

$$p_{B2}^{u*} = \frac{(2-\alpha)}{3(1-\alpha)}\phi + c$$

and, $p_{A2}^{u*} \leq p_{B2}^{u*}$, if and only if $\alpha \leq \frac{1}{2}$.

Therefore, without price discrimination, the two firm's equilibrium strategies are

$$p_{A2}^{u*} = \begin{cases} \frac{(1+\alpha)}{3\alpha}\phi + c, & \text{if } \alpha \geq \frac{1}{2} \\ \frac{(1+\alpha)}{3(1-\alpha)}\phi + c, & \text{if } \alpha < \frac{1}{2} \end{cases}$$

$$p_{B2}^{u*} = \begin{cases} \frac{(2-\alpha)}{3\alpha}\phi + c, & \text{if } \alpha \geq \frac{1}{2} \\ \frac{(2-\alpha)}{3(1-\alpha)}\phi + c, & \text{if } \alpha < \frac{1}{2} \end{cases}$$

■

Proof of Proposition 4. Looking into the first case where $\alpha \geq \frac{1}{2}$ and $p_{A2}^{u*} \geq p_{B2}^{u*}$.

Given $\widetilde{s} = p_{A2}^{u*} - p_{B2}^{u*}$, in the first period and at given pair of prices $(p_{A1}^{u*},\ p_{B1}^{u*})$, the indifferent consumer between to buy from firm A or buy from firm B is

$$v - p_{A1}^{u} + \delta\left(v - \int_{\widetilde{s}}^{\phi} p_{A2}^{u*}\frac{1}{\phi}ds - \int_{0}^{\widetilde{s}}(p_{B2}^{u*} + s)\frac{1}{\phi}ds\right) = v - p_{B1}^{u} + \delta\left(v - p_{B2}^{u*}\right)$$

Simplifying, we can get that

$$p_{A1}^{u} - p_{B1}^{u} + \delta\left((p_{A2}^{u*} - p_{B2}^{u*}) - \frac{1}{2\phi}(p_{A2}^{u*} - p_{B2}^{u*})^2\right) = 0$$

From second-period equilibrium prices we have that $(p_{A2}^{u*} - p_{B2}^{u*}) = \frac{(2\alpha-1)}{3\alpha}\phi$, for $\alpha \geq \frac{1}{2}$. Substituting in the equation above we find that

$$p_{A1}^{u} - p_{B1}^{u} + \frac{\delta\phi(2\alpha-1)(4\alpha+1)}{18\alpha^2} = 0$$

First-period profit for each firm is

$$\pi^u_{A1} = (p^u_{A1} - c)\alpha$$

and

$$\pi^u_{B1} = (p^u_{B1} - c)(1 - \alpha)$$

Notice that α is a function of first-period prices, so $\alpha(p^u_{A1}, p^u_{B1})$ and the first-order condition, for firm A is

$$\frac{\partial \pi^u_{A1}}{\partial p^u_{A1}} = \alpha + (p^u_{A1} - c)\frac{\partial \alpha(p^u_{A1}, p^u_{B1})}{\partial p^u_{A1}} = 0$$

and using the implicit differentiation rule, we have

$$\frac{\partial \alpha(p^u_{A1}, p^u_{B1})}{\partial p^u_{A1}} = -\frac{\partial \alpha(p^u_{A1}, p^u_{B1})}{\partial p^u_{B1}} = -\frac{9\alpha^3}{\delta\phi(\alpha + 1)}$$

Thus, the first-order condition for firm A becomes

$$\frac{\partial \pi^u_{A1}}{\partial p^u_{A1}} = \alpha - (p^u_{A1} - c)\frac{9\alpha^3}{\delta\phi(\alpha + 1)} = 0$$

where,

$$p^u_{A1} = \frac{\phi\delta(\alpha + 1)}{9\alpha^2} + c$$

For firm B, we have that the first-order condition is

$$\frac{\partial \pi^u_{B1}}{\partial p^u_{B1}} = (1 - \alpha) + (p^u_{B1} - c)\frac{9\alpha^3}{\delta\phi(\alpha + 1)} = 0$$

and,

$$p^u_{B1} = (1 - \alpha)\frac{\phi\delta(\alpha + 1)}{9\alpha^3} + c$$

Substituing into equation ix, we find that

$$\left(\frac{\phi\delta(\alpha+1)}{9\alpha^2}+c\right)-\left((1-\alpha)\frac{\phi\delta(\alpha+1)}{9\alpha^3}+c\right)+\frac{\delta\phi(2\alpha-1)(4\alpha+1)}{18\alpha^2}=0$$

$$\alpha=\frac{1}{2}$$

Thus, with $\alpha=\frac{1}{2}$ we have that p^u_{A1} and p^u_{B1} are given by

$$p^u_{A1}=c+\frac{2}{3}\phi\delta$$

$$p^u_{B1}=c+\frac{2}{3}\phi\delta$$

When $\alpha<\frac{1}{2}$, $p^{u*}_{A2}<p^{u*}_{B2}$ and the indifferent consumer between choose firm A or firm B in the first period is such that

$$v-p^u_{A1}+\delta\left(v-p^{u*}_{A2}\right)=v-p^u_{B1}+\delta\left(v-\int_{p^{u*}_{B2}-p^{u*}_{A2}}^{\phi}p^{u*}_{B2}\frac{1}{\phi}ds-\int_0^{p^{u*}_{B2}-p^{u*}_{A2}}(p^{u*}_{A2}+s)\frac{1}{\phi}ds\right)$$

And, by analogy the result obtain is given by

$$p^u_{B1}-p^u_{A1}+\frac{\delta\phi(1-2\alpha)(5-4\alpha)}{18(1-\alpha)^2}=0$$

And, the unique equilibrium occurs when $\alpha=\frac{1}{2}$ where first-period prices are given by

$$p^{u*}_{A1}=c+\frac{2}{3}\phi\delta$$

$$p^{u*}_{B1}=c+\frac{2}{3}\phi\delta$$

■

Proof of Proposition 5. In firm A's turf, given the maximization problem of each problem, such that

$$\underset{p^o_{A2}}{Max}\left\{(p^o_{A2}-c)\left(\frac{1}{2}+\frac{p^r_{B2}-p^o_{A2}}{2t}\right)\right\}$$

$$\underset{p^r_{B2}}{Max}\left\{(p^r_{B2}-c)\left(\theta_1-\frac{1}{2}-\frac{p^r_{B2}-p^o_{A2}}{2t}\right)\right\}$$

Solving the problem we find that firm A' best response and firm B' best response are

$$p^o_{A2}=\frac{t}{2}+\frac{p^r_{B2}+c}{2}$$

$$p^r_{B2}=t\theta_1-\frac{t}{2}+\frac{p^o_{A2}+c}{2}$$

It thus follows that

$$p^{o*}_{A2}=\frac{1}{3}t(2\theta_1+1)+c$$

$$p^{r*}_{B2}=\frac{1}{3}t(4\theta_1-1)+c$$

In the firm B's turf, each firm wants

$$\underset{p^r_{A2}}{Max}\left\{(p^r_{A2}-c)\left(\frac{1}{2}+\frac{p^o_{B2}-p^r_{A2}}{2t}-\theta_1\right)\right\}$$

$$\underset{p^o_{B2}}{Max}\left\{(p^o_{B2}-c)\left(1-\frac{1}{2}-\frac{p^o_{B2}-p^r_{A2}}{2t}\right)\right\}$$

Firm A's best response is

$$p^r_{A2}=\frac{t}{2}-t\theta_1+\frac{p^o_{B2}+c}{2}$$

and firm B's best response is

$$p^o_{B2}=\frac{t}{2}+\frac{p^r_{A2}+c}{2}$$

The result obtain is given by

$$p_{A2}^{r*} = \frac{1}{3}t(3 - 4\theta_1) + c$$

$$p_{B2}^{o*} = \frac{1}{3}t(3 - 2\theta_1) + c$$

If in the first-period firms split equally the market, this means that $\theta_1 = \frac{1}{2}$, and $p_{A2}^{o} = p_{B2}^{o} = c + \frac{2}{3}t$ and $p_{A2}^{r} = p_{B2}^{r} = c + \frac{1}{3}t$.

In the endpoints, each firm does not have an incentive to charge a price higher that zero (because consumers are loyal and do not switch), such that $p_{i2}^{r} = 0$, $i = A, B$. When $p_{A2}^{r} = 0$, $\theta_1 = \frac{3}{4} + \frac{3}{4t}c \geq \frac{3}{4}$, and when $p_{B2}^{r} = 0$, $\theta_1 = \frac{1}{4} - \frac{3}{4t}c \leq \frac{1}{4}$.

So, the set of equilibrium prices $(p_{i2}^{o*}, p_{i2}^{r*})$, $i = A, B$,are:

(i) if $\frac{1}{4} \leq \theta_1 \leq \frac{3}{4}$:

$$p_{A2}^{o*} = \frac{1}{3}t(2\theta_1 + 1) + c; \quad p_{A2}^{r*} = \frac{1}{3}t(3 - 4\theta_1) + c$$

$$p_{B2}^{o*} = \frac{1}{3}t(3 - 2\theta_1) + c; \quad p_{B2}^{r*} = \frac{1}{3}t(4\theta_1 - 1) + c$$

(ii) if $\theta_1 \leq \frac{1}{4}$:

$$p_{A2}^{o*} = t(1 - 2\theta_1) + c; \quad p_{A2}^{r*} = \frac{1}{3}t(3 - 4\theta_1) + c$$

$$p_{B2}^{o*} = \frac{1}{3}t(3 - 2\theta_1) + c; \quad p_{B2}^{r*} = c$$

(iii) if $\theta_1 \geq \frac{3}{4}$:

$$p_{A2}^{o*} = \frac{1}{3}t(2\theta_1 + 1) + c; \quad p_{A2}^{r*} = c$$

$$p_{B2}^{o*} = t(2\theta_1 - 1) + c; \quad p_{B2}^{r*} = \frac{1}{3}t(4\theta_1 - 1) + c$$

■

Proof of Proposition 6. Given the indifferent consumer such that θ_1 is

$$\theta_1 = \frac{1}{2} + \frac{3(p_{B1} - p_{A1})}{2t(3+\delta)}$$

For each firm, first-period overall profits are given by

$$\pi_{A1} = (p_{A1} - c)\theta_1 + \delta\pi^*_{A2}$$

$$\pi_{B1} = (p_{B1} - c)(1-\theta_1) + \delta\pi^*_{B2}$$

Notice that θ_1 is a function of first-period prices, such that

$$\pi_{A1} = (p_{A1} - c)\theta_1(p_{A1}, p_{B1}) + \delta\left(\frac{5}{9}t(2(\theta_1(p_{A1}, p_{B1}))^2 - 2\theta_1(p_{A1}, p_{B1}) + 1)\right)$$

and

$$\pi_{B1} = (p_{B1} - c)(1 - \theta_1(p_{A1}, p_{B1})) + \delta\left(\frac{5}{9}t(2(\theta_1(p_{A1}, p_{B1}))^2 - 2\theta_1(p_{A1}, p_{B1}) + 1)\right)$$

Solving the maximization problem, we have that

$$\frac{\partial \pi_{A1}}{\partial p_{A1}} = 0$$

$$\frac{5t\delta}{9}\left(\frac{3}{t\,(\delta+3)} + \frac{6}{t\,(\delta+3)}\left(\frac{3(p_{B1}-p_{A1})}{2t\,(\delta+3)} - \frac{1}{2}\right)\right) - \frac{(p_{A1}-c)}{2t\,(\delta+3)} - \frac{3(p_{B1}-p_{A1})}{2t\,(\delta+3)} + \frac{1}{2} = 0$$

$$p_{A1} = -\frac{1}{4\delta - 18}\left(9p_{B1} + 9c + 9t - 7\delta p_{B1} + 3c\delta + 6t\delta + t\delta^2\right)$$

As the game is symmetric, $p_{A1} = p_{B1}$, we get that

$$p^*_{A1} = p^*_{B1} = \frac{t}{3}(3+\delta) + c$$

At this prices, firms split evenly the market and $\theta_1 = \frac{1}{2}$.

First-period profits are

$$\pi^*_{A1} = \pi^*_{B1} = \frac{t}{6}(3+\delta)$$

Thus, the overall profits for each firm is

$$\Pi_i = \pi^*_{i1} + \delta\pi^*_{i2}$$

$$\Pi_i = \frac{t}{6}(3+\delta) + \frac{5}{18}\delta t$$

$$\Pi_i = \frac{t}{18}\left(8\delta + 9\right)$$

■

Proof of Proposition 7. If firms cannot recognise their own customers and rival's customers, each firm charges the same price to all consumers in the market. In the second period, the indifferent consumer is located at θ_1 and he is indifferent to buying again from firm A or changing supplier and buying from firm B if

$$p^u_{A2} + t\theta_1 = p^u_{B2} + t(1-\theta_1)$$

$$\theta_1 = \frac{1}{2} + \frac{p^u_{B2} - p^u_{A2}}{2t}$$

Each firm wants to maximize their profit and solves the following problem

$$\underset{p^u_{A2}}{Max}\pi^u_{A2} = (p^u_{A2} - c)\theta_1$$

$$\underset{p^u_{B2}}{Max}\pi^u_{B2} = (p^u_{B2} - c)(1-\theta_1)$$

The first-order condition of firm A is given by

$$\frac{\partial \pi^u_{A2}}{\partial p^u_{A2}} = \frac{1}{2} + \frac{p^u_{B2} - p^u_{A2}}{2t} + (p^u_{A2} - c)\left(-\frac{1}{2t}\right) = 0$$

$$p^u_{A2} = \frac{t}{2} + \frac{1}{2}\left(p^u_{B2} + c\right)$$

For firm B we have the similar results, such that

$$\frac{\partial \pi^u_{B2}}{\partial p^u_{B2}} = \frac{1}{2} - \frac{p^u_{B2} - p^u_{A2}}{2t} + \left(p^u_{B2} - c\right)\left(-\frac{1}{2t}\right) = 0$$

$$p^u_{B2} = \frac{t}{2} + \frac{1}{2}\left(p^u_{A2} + c\right)$$

Solving the problem, we find that without price discrimination, second-period equilibrium prices are

$$p^{u*}_{A2} = p^{u*}_{B2} = t + c$$

In the first period, firms' problem is similar and the first-period equilibrium prices are the same as in the second-period. So, first-period equilibrium prices are given by

$$p^{u*}_{A1} = p^{u*}_{B1} = t + c$$

Thus, the equilibrium prices for this static game are

$$p^{u*}_{A1} = p^{u*}_{A2} = p^{u*}_{B1} = p^{u*}_{B2} = t + c$$

Firm's profits in each period are the same and given by

$$\pi^{u*}_{A} = \pi^{u*}_{B} = \frac{1}{2}t$$

■

Proof of Proposition 8. In the second-stage of period 2, the indifferent consumer between to buying again from firm A at price $p^o_{A2} - d_A$ or switching from firm B at price p^r_{B2} is located at θ_A:

$$p^o_{A2} - d_A + t\theta_A = p^r_{B2} + t(1 - \theta_A)$$

where,

$$\theta_A = \frac{1}{2} + \frac{p^r_{B2} - p^o_{A2} + d_A}{2t}$$

The indifferent consumer to acting as a passive or active is located at θ^c_A, where $d_A = 0$. Thus,

$$\theta^c_A = \frac{1}{2} + \frac{p^r_{B2} - p^o_{A2}}{2t}$$

The game is symmetric and in firm B's turf, the results are similar to those obtained from firm A, and

$$\theta_B = \frac{1}{2} + \frac{p^o_{B2} - p^r_{A2} - d_B}{2t}$$

$$\theta^c_B = \frac{1}{2} + \frac{p^o_{B2} - p^r_{A2}}{2t}$$

(i) second stage

In the *second stage* each firm wants to maximize the return of saved customers and solves the following problem

$$\underset{d_A}{Max}(p^o_{A2} - d_A - c)(\theta_A - \theta^c_A)$$

$$\underset{d_B}{Max}(p^o_{B2} - d_B - c)(\theta^c_B - \theta_B)$$

Given $(\theta_A - \theta^c_A) = \frac{d_A}{2t}$, taking the first derivative, we get

$$\frac{\partial}{\partial d_A} = -\frac{d_A}{2t} + (p^o_{A2} - d_A - c)\frac{1}{2t} = 0$$

$$d_A = \frac{1}{2}(p^o_{A2} - c)$$

By analogy,

$$d_B = \frac{1}{2}(p^o_{B2} - c)$$

(ii) first stage

In the *first stage* each firm solves the following problem on firm A's turf:

$$\begin{aligned} \underset{p^o_{A2}}{Max}\ \ \pi^o_{A2} &= (p^o_{A2} - c)\theta^c_A + (p^o_{A2} - d_A - c)(\theta_A - \theta^c_A), \\ \text{with } d_A &= \frac{1}{2}(p^o_{A2} - c) \end{aligned}$$

$$\begin{aligned} \underset{p^r_{B2}}{Max}\ \ \pi^r_{B2} &= (p^r_{B2} - c)(\theta_1 - \theta_A), \\ \text{with } d_A &= \frac{1}{2}(p^o_{B2} - c) \end{aligned}$$

Solving the maximization problem for each firm in firm A's turf:

$$\frac{\partial \pi^o_{A2}}{\partial p^o_{A2}} = 0$$

$$\left(\frac{1}{2} + \frac{p^r_{B2} - p^o_{A2}}{2t}\right) + (p^o_{A2} - c)\left(-\frac{1}{2t}\right) + \frac{1}{2}\left(\frac{1}{4t}(p^o_{A2} - c)\right) + \frac{1}{2}(p^o_{A2} - c)\left(\frac{1}{4t}\right) = 0$$

$$p^o_{A2} = \frac{2}{3}t + \frac{2}{3}p^r_{B2} + \frac{1}{3}c$$

and,

$$\frac{\partial \pi^r_{B2}}{\partial p^r_{B2}} = \left(\theta_1 - \frac{1}{2} - \frac{p^r_{B2}}{2t} + \frac{1}{4t}(p^o_{A2} + c)\right) + (p^r_{B2} - c)\left(-\frac{1}{2t}\right) = 0$$

$$p^r_{B2} = t\theta_1 - \frac{1}{2}t + \frac{1}{4}p^o_{A2} + \frac{3}{4}c$$

Given the previous results, we obtain that p^o_{A2}, d_A and p^r_{B2} are

$$p^o_{A2} = \frac{2}{5}t(2\theta_1 + 1) + c$$

$$d_A = \frac{1}{5}t(2\theta_1 + 1)$$

$$p^r_{B2} = \frac{2}{5}t(3\theta_1 - 1) + c$$

Looking now to firm B's turf, we know that

$$\theta_B = \frac{1}{2} + \frac{p^o_{B2} - p^r_{A2} - d_B}{2t}$$

$$\theta^c_B = \frac{1}{2} + \frac{p^o_{B2} - p^r_{A2}}{2t}$$

Each firm wants to

$$\begin{aligned} \underset{p^r_{A2}}{Max} \;\; \pi^r_{A2} &= (p^r_{A2} - c)(\theta_B - \theta_1), \\ \text{where } d_B &= \frac{1}{2}(p^o_{B2} - c) \end{aligned}$$

$$\begin{aligned} \underset{p^o_{B2}}{Max} \;\; \pi^o_{B2} &= (p^o_{B2} - d_B - c)(\theta^c_B - \theta_B) + (p^o_{B2} - c)(1 - \theta^c_B), \\ \text{where } d_B &= \frac{1}{2}(p^o_{B2} - c) \end{aligned}$$

Solving the firms' problem, we obtain that

$$p^o_{B2} = \frac{2}{5}t(3 - 2\theta_1) + c$$

$$d_B = \frac{1}{5}t(3 - 2\theta_1)$$

$$p^r_{A2} = \frac{2}{5}t(2 - 3\theta_1) + c$$

Gathering the above results, when firms can implement retention strategies, second-period equilibrium prices are given

$$p^{o*}_{A2} = \frac{2}{5}t(2\theta_1 + 1) + c; \qquad p^{r*}_{A2} = \frac{2}{5}t(2 - 3\theta_1) + c$$

$$p_{B2}^{o*} = \frac{2}{5}t(3-2\theta_1) + c; \qquad p_{B2}^{r*} = \frac{2}{5}t(3\theta_1 - 1) + c$$

with,

$$d_A^* = \frac{1}{5}t(2\theta_1 + 1)$$

$$d_B^* = \frac{1}{5}t(3 - 2\theta_1)$$

■

Proof of Proposition 9. With retentiton strategies and in the first period, the indifferent consumer is located at θ_1, such that

$$p_{A1} + t\theta_1 + \delta\left[p_{B2}^r + t(1-\theta_1)\right] = p_{B1} + t(1-\theta_1) + \delta\left(p_{A2}^r + t\theta_1\right)$$

$$\theta_1 = \frac{1}{2} + \frac{p_{B1} - p_{A1} + \delta(p_{A2}^r - p_{B2}^o)}{2t(1-\delta)}$$

Given the equilibrium prices of second-period $(p_{A2}^{r*} - p_{B2}^{o*}) = \frac{2}{5}t(3-6\theta_1)$. Thus, θ_1 is given by

$$\theta_1 = \frac{1}{2} + \frac{5(p_{B1} - p_{A1})}{2t(5+\delta)}$$

It is straightforward that θ_1 is a function of first-period prices, such that $\theta_1 = f(p_{A1},\ p_{B1})$. The overall objective function for firm A's profits is

$$\Pi_A = (p_{A1} - c)\theta_1(p_{A1}, p_{B1}) + \delta\left[\frac{1}{50}t\left(48(\theta_1(p_{A1}, p_{B1}))^2 - 36(\theta_1(p_{A1}, p_{B1})) + 19\right)\right]$$

and firm B's objective function is

$$\Pi_B = (p_{B1} - c)\theta_1(p_{A1}, p_{B1}) + \delta\left[\frac{1}{50}t\left(48(\theta_1(p_{A1}, p_{B1}))^2 - 60(\theta_1(p_{A1}, p_{B1})) + 31\right)\right]$$

Substituing the expression of θ_1 into the above overall profit function and taking the first derivative for each firm, we get that

$$p_{A1} = \frac{\left(125c - 250p_{B1} + 125t + 70\delta p_{B1} + 25c\delta + 20t\delta - t\delta^2\right)}{95\delta - 125}$$

and,

$$p_{B1} = -\frac{\left(125p_{A1} + 125c + 125t - 95\delta p_{A1} + 25c\delta + 20t\delta - t\delta^2\right)}{70\delta - 250}$$

Joining both equations above, we get that first-period equilibrium price for each firm is

$$p^*_{A1} = p^*_{B1} = t(1 - \frac{\delta}{25}) + c$$

Second-order condition is given by

$$\frac{\partial^2 \Pi_A}{\partial A^2} = \frac{(7\delta - 25)}{t\,(\delta + 5)^2}$$

which is negative for all $\delta \in [0, 1]$. ■

Proof of Proposition 10. Solving the model by backward induction.

(i) second stage

Given

$$q_{BA} = \frac{\alpha}{\phi}\,(p^o_{A2} - p^r_{B2} - d_A)$$

$$q^s_{AA} = \frac{\alpha}{\phi} d_A$$

$$q_{AA} = \frac{\alpha}{\phi}(\phi - p^o_{A2} + p^r_{B2})$$

in the second stage, firm A wants to maximize its return obtained with saved customer. Tthe maximization problem is the following

$$\underset{d_A}{Max}(p^o_{A2} - d_A - c)q^s_{AA}$$

$$\underset{d_A}{Max}(p^o_{A2} - d_A - c)\left(\frac{\alpha}{\phi} d_A\right)$$

The first order condition can be written as

$$\left(-\frac{\alpha}{\phi}\right) d_A + (p^o_{A2} - d_A - c)\left(-\frac{\alpha}{\phi}\right) = 0$$

$$d^*_A = \frac{1}{2}(p^o_{A2} - c)$$

Looking now into the firm B's turf, which has $(1-\alpha)$ market share in the beginning of period 2. The indifferent consumer to buying again from firm B at price p^r_{B2} to firm A and pay p^r_{A2} is

$$v - p^o_{B2} = v - p^r_{A2} - s$$

$$s = p^o_{B2} - p^r_{A2}$$

Within the active consumers, I created two groups: switchers and saved consumers. At s^*_B the number of switchers is given by

$$q_{AB} = (1-\alpha)\int_0^{s^*_B} \frac{1}{\phi} ds$$

$$q_{AB} = \frac{(1-\alpha)}{\phi}(p^o_{B2} - p^r_{A2} - d_B)$$

Saved consumers are

$$q^s_{BB} = (1-\alpha)\int_{s^*_B}^{s_B} \frac{1}{\phi} ds$$

$$q^s_{BB} = \frac{(1-\alpha)}{\phi} d_B$$

And, the number of inactive consumers is

$$q_{BB} = (1-\alpha)\int_{s_B}^{\phi} \frac{1}{\phi} ds$$

$$q_{BB} = \frac{(1-\alpha)}{\phi}(\phi - p^o_{B2} + p^r_{A2})$$

Firm B solves the same maximization problem as firm A. Thus,

$$\underset{d_B}{Max}(p^o_{B2} - d_B - c)q^s_{BB}$$

$$\underset{d_B}{Max}(p^o_{B2} - d_B - c)\left(\frac{(1-\alpha)}{\phi}d_B\right)$$

And, the first-order condition is written as

$$-\left(\frac{(1-\alpha)}{\phi}d_B\right) + (p^o_{B2} - d_B - c)\left(\frac{(1-\alpha)}{\phi}d_B\right) = 0$$

$$d^*_B = \frac{1}{2}(p^o_{B2} - c)$$

(ii) first stage

In the firm A's turf, each firm solves the following maximization problem

$$\begin{aligned} \underset{p^o_{A2}}{Max}\pi^o_{A2} &= (p^o_{A2} - c)q_{AA} + (p^o_{A2} - d_A - c)q^s_{AA} \\ s.t.\ \ d_A &= \frac{1}{2}(p^o_{A2} - c) \end{aligned}$$

and,

$$\begin{aligned} \underset{p^r_{B2}}{Max}\pi^r_{B2} &= (p^r_{B2} - c)q_{BA} \\ s.t.\ \ d_A &= \frac{1}{2}(p^o_{A2} - c) \end{aligned}$$

Finding the first-order condition to firm A, it is obtained that

$$\frac{\partial \pi^o_{A2}}{\partial p^o_{A2}} = 0$$

$$\frac{\alpha}{\phi}(\phi - p^o_{A2} + p^r_{B2}) + (p^o_{A2} - c)\left(-\frac{\alpha}{\phi}\right) + \frac{1}{2}\left(\frac{\alpha}{2\phi}(p^o_{A2} - c)\right) + \frac{1}{2}(p^o_{A2} - c)\left(\frac{\alpha}{2\phi}\right) = 0$$

$$p^o_{A2} = \frac{2}{3}\phi + \frac{2}{3}p^r_{B2} + \frac{1}{3}c$$

And firm B's first-order condition is

$$\frac{\partial \pi^r_{B2}}{\partial p^r_{B2}} = \frac{\alpha}{\phi}(\frac{1}{2}p^o_{A2} - p^r_{B2} + \frac{1}{2}c) + (p^r_{B2} - c)\left(-\frac{\alpha}{\phi}\right) = 0$$

$$p^r_{B2} = \frac{1}{4}p^o_{A2} + \frac{3}{4}c$$

Joining the equations above I obtain that

$$p^o_{A2} = \frac{2}{3}\phi + \frac{2}{3}\left(\frac{1}{4}p^o_{A2} + \frac{3}{4}c\right) + \frac{1}{3}c$$

$$p^{o*}_{A2} = \frac{4}{5}\phi + c$$

So, the equilibrium is given by

$$p^{o*}_{A2} = \frac{4}{5}\phi + c$$

$$p^{r*}_{B2} = \frac{1}{5}\phi + c$$

$$d^*_A = \frac{2}{5}\phi$$

Next, in firm B's turf each firm has a similar maximization problem, such that

$$\begin{aligned} \underset{p^o_{B2}}{Max}\pi^o_{B2} &= (p^o_{B2} - c)q_{BB} + (p^o_{B2} - d_B - c)q^s_{BB} \\ s.t.\ \ d_B &= \frac{1}{2}(p^o_{B2} - c) \end{aligned}$$

and,

$$\begin{aligned} \underset{p^r_{A2}}{Max} \pi^r_{A2} &= (p^r_{A2} - c) q_{AB} \\ s.t. \;\; d_B &= \frac{1}{2}(p^o_{B2} - c) \end{aligned}$$

Where first-order conditions are

$$\begin{aligned} \frac{\partial \pi^o_{B2}}{\partial p^o_{B2}} &= 0 \\ 0 &= \frac{(1-\alpha)}{\phi}(\phi - p^o_{B2} + p^r_{A2}) + (p^o_{B2} - c)\left(-\frac{(1-\alpha)}{\phi}\right) + \frac{1}{2}\left(\frac{(1-\alpha)}{2\phi}(p^o_{B2} - c)\right) + \\ &+ \left(\frac{1}{2}(p^o_{B2} - c)\right)\frac{(1-\alpha)}{2\phi} \end{aligned}$$

$$p^o_{B2} = \frac{2}{3}\phi + \frac{2}{3}p^r_{A2} + \frac{1}{3}c$$

$$\frac{\partial \pi^r_{A2}}{\partial p^r_{A2}} = \frac{(1-\alpha)}{\phi}\left(\frac{1}{2}p^o_{B2} - p^r_{A2} + \frac{1}{2}c\right) + (p^r_{A2} - c)\left(-\frac{(1-\alpha)}{\phi}\right) = 0$$

$$p^r_{A2} = \frac{1}{4}p^r_{B2} + \frac{3}{4}c$$

With the two equations above, I get that

$$p^o_{B2} = \frac{2}{3}\phi + \frac{2}{3}\left(\frac{1}{4}p^o_{B2} + \frac{3}{4}c\right) + \frac{1}{3}c$$

$$p^{o*}_{B2} = \frac{4}{5}\phi + c$$

and,

$$p^{r*}_{A2} = \frac{1}{5}\phi + c$$

$$d_B = \frac{2}{5}\phi$$

So, second-period equilibrium is given by

$$p_{A2}^{o*} = \frac{4}{5}\phi + c; \quad d_A^* = \frac{2}{5}\phi; \quad p_{A2}^{r*} = \frac{1}{5}\phi + c$$

$$p_{B2}^{o*} = \frac{4}{5}\phi + c; \quad d_B = \frac{2}{5}\phi; \quad p_{B2}^{r*} = \frac{1}{5}\phi + c$$

Equilibrium profits for each firm is given by

$$\begin{aligned}\pi_{A2}^* &= (p_{A2}^o - c)\left(\frac{\alpha}{\phi}(\phi - p_{A2}^o + p_{B2}^r)\right) + (p_{A2}^o - d_A - c)\left(\frac{\alpha}{\phi}d_A\right) + \\ &+ (p_{A2}^r - c)\left(\frac{(1-\alpha)}{\phi}(p_{B2}^o - p_{A2}^r - d_B)\right)\end{aligned}$$

$$\begin{aligned}\pi_{B2}^* &= (p_{B2}^o - c)\left(\frac{(1-\alpha)}{\phi}(\phi - p_{B2}^o + p_{A2}^r)\right) + (p_{B2}^o - d_B - c)\left(\frac{(1-\alpha)}{\phi}d_B\right) + \\ &+ (p_{B2}^r - c)\left(\frac{\alpha}{\phi}(p_{A2}^o - p_{B2}^r - d_A)\right)\end{aligned}$$

where,

$$\pi_{A2}^* = \frac{\phi}{25}(11\alpha + 1)$$

$$\pi_{B2}^* = \frac{\phi}{25}(12 - 11\alpha)$$

For firm A, second-order condition is given by $-\frac{3\alpha}{2\phi}$ and for firm B is $\frac{3(\alpha-1)}{2\phi}$, which are negative for any $\alpha \in [0, 1]$.

■

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