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On Resource Complementarity in Activity Networks

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Abstract:
The methodology of project management has been widespread in organizations of different functions and sizes. In this context, we address the issue of optimal resource allocation, and more specifically, the analysis of complementarity of resources (primary resource and supportive resource) in a project. We develop a conceptual system capable of determining the ideal timing, and the ideal mixture of resources allocated to the activities of a project, such that the project is completed on time, if not earlier, with minimal cost.

Keywords: *Sharing Resources, Allocation, Project Cost, Complementarity*

1 Introduction

This paper is concerned with the optimal resource allocation in activity networks under conditions of resource complementarity. It is easy to find in the literature papers on resource substitutability, in which one resource replaces another; for example, one may use semi-skilled labor instead of high skilled labor, or an old machine (m/c) instead of a new (and more efficient) one. A certain loss (or gain) is realized, perhaps in time or quality, which is offset by the gain in cost or availability. Alternatively, there are several studies dealing with the problem of multiple skills; see Arroub et al. (2009), Li and Womer (2009), Mulcahy (2005), Demeulemeester and Herroelen (1992) and Patterson (1984). The problem posed in this context is usually framed as seeking the most economical diversity that satisfies an uncertain demand with high enough probability. In such context there is cost incurred by the increased diversity of skills (Li & Womer, 2009) (e.g. a travel guide who speaks several languages, or a hand tool that can serve as a pair of scissors and a screwdriver) and there is gain secured by having a smaller number of service mechanisms.

The concept of complementarity which has been discussed based on economic view (Kremer, 1993) can be incorporated into the engineering domain as an enhancement of the efficacy of a "primary" resource (*P*-resource) by adding to it another "supportive" resource (*S*-resource). No replacement takes place. The gain achieved from such action is manifested in improved performance; e.g., shorter duration or improved quality, because of the enhanced performance of the *P*-resource. But such gain is usually achieved at an increased cost; namely the cost of the support resource(s). The issue then becomes: how much additional support should be allocated to project activities to achieve improved results most economically?

2 Problem description

Consider a project network in the activity-on-arc (AoA) representation: $G = (N, A)$, with the set of nodes $|N| = n$ (representing the "events") and the set of arcs $|A| = m$ (representing the "activities"). In general each activity requires the simultaneous use of several resources (Rudolph & Elmaghraby, 2009; Tereso, Araújo, Moutinho, & Elmaghraby, 2008, 2009a, 2009b; Vanhoucke, Demeulemeester, & Herroelen, 2002).

There is a set of "primary" resources, denoted by *P*, with $|P| = \rho$. Typically, a primary resource has a capacity of several units (say workers, m/c's, processors; etc.) (Mulcahy, 2005). Additionally, there is a pool of "support" resources, denoted by S, with $|S| = \sigma$ (such as less-skilled labor, or computers and electronic devices; etc.) that may be utilized in conjunction with the primary resources to enhance their performance.

The number of support resources varies with the resource, and the relevance of each to the *P*-resources may best be represented in matrix format as shown in Table 1 (ϕ indicates inapplicability).

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S-RESOURCE →	s_1	⋯	s_q	⋯	s_σ
↓ *P-RESOURCE*					
r_1	$v(1,1)$	⋯	$\emptyset$	⋯	$v(1,\sigma)$
⋮	⋮	⋮	⋮	⋮	⋮
r_p	$\emptyset$	⋯	$v(p,q)$	⋯	$v(p,\sigma)$
⋮	⋮	⋮	⋮	⋮	⋮
r_ρ	$v(\rho,1)$	⋯	$v(\rho,q)$	⋯	$\emptyset$

Table 1: Applicability and impact of support resources

In Table 1 an entry $v(r_p,s_q) \neq \phi$, measures the enhancement offered by *S*-resource s_q to *P*-resource r_p.

Although various models of the impact of the support resource may be constructed, we will discuss only two. The choice of the applicable model is decided empirically from data on the actual performance of the process.

If $0 < v(r_p,s_q) \leq 1$, then it indicates the fraction by which the support resource s_q improves the performance of primary resource r_p. Typically, $v(r_p,s_q) \in [0.10, 0.50]$. In this case the performance of the allocation of P-resource r_p to activity a, which is denoted by $x(a,r_p)$, is augmented to,

$$x_{r_p}(a) = x(a,r_p) + v(r_p,s_p) \tag{1}$$

If $1 < v(r_p,s_q) < U < \infty$, then it indicates the multiplier of the *P*-resource allocation. Typically $1.10 \leq v(r_p,s_q) \leq 2.0$. In this case the performance of the allocation of *P*-resource r_p is augmented to

$$x_{r_p}(a) = x(a,r_p) \cdot v(r_p,s_q) \tag{2}$$

In the treatment below, we shall adopt mode (1). For the sake of simplicity, we make the following assumptions.

2.1 Assumption 1

The impact of the *S*-resources is additive: if a subset $\{s_q\}_{q=1}^{\sigma}$ of the *S*-resources is used in support of *P*-resource r_p in activity a then the performance of the former is enhanced to

$$x_{r_p}(a) = x(a,r_p) + \sum_{q=1}^{\sigma} v(r_p,s_q) \tag{3}$$

In the sequel we consider the possible addition of only a single *S*-resource; the discussion can be easily extended to multiple *S*-resources.

The primary resource $r_p \in P$ would accomplish activity a in time $y(a,r_p)$. If it is enhanced by the addition of *S*-resource s_q then its processing time decreases to $y_{r_p}(a)$, with $y_{r_p}(a) < y(a,r_p)$. The issue now is to express the functional relationship between the resource allocation (both primary and support) and the activity duration.

Let $w(a,r_p)$ denote the work content of activity a of *P*-resource r. Let $x(a,r_p)$ denote, as suggested above, the amount of primary resource r_p allocated to activity a.

2.2 Assumption 2

The duration of activity a when using resource r_p is given by (Tereso, Araújo, & Elmaghraby, 2004).

$$y(a,r_p) = \frac{w(a,r_p)}{x(a,r_p)} \tag{4}$$

If support resource s_q is added to the primary resource r_p then the duration becomes (considering model (1)),

$$y_{r_p}(a) = \frac{w(a,r_p)}{x_{r_p}(a)} \tag{5}$$

To illustrate, suppose an activity has work content $w(a,r_p) = 36$ man-days. Further, assume the *S*-resource s_q yields a rate $v(r_p,s_q) = 0.50$.

If $x(a,r_p) = 0.85$ then in the absence of the support resource the duration of the activity would be

$$y(a,r_p) = 36/0.85 = 42.35 \text{ days}.$$

But in the presence of the *S*-resource the duration would be only

$$y_{r_p}(a) = 36 \ / \ ((0.85 + 0.5) = 26.67 \text{ days},$$

a saving of approximately 37%.

If $x(a,r_p) = 1.5$ then in the absence of the *S*-resource the duration of the activity would be

$$y(a,r_p) = 36/1.5 = 24 \ days .$$

But in the presence of the *S*-resource the duration would be only

$$y_{r_p}(a) = {}^{36}\!/_{(1.5 + 0.5)} = 18 \text{ days},$$

a saving of 25%.

An activity normally requires the simultaneous utilization of more than one *P*-resource for its execution. The problem then becomes:

"At what level should each resource be utilized and which supportive resource(s) should be added to it (if any) in order to optimize a given objective?"

Recall that the processing time of an activity is given by the maximum of the durations that would result from a specific allocation to each resource (see a previous discussion on the evaluation of the duration considering multiple resources in Tereso et al. (2008, 2009a, 2009b).

$$y(a) = \underbrace{max}_{all\ r_p}\left\{y_{r_p}(a)\right\} \quad (6)$$

To better understand this representation, consider the miniscule project of Figure 1 and Figure 2 with three-activities. Assume that the project requires the utilization of four *P*-resources; not all resources are required by all the activities. The resource requirements of each activity are indicated in Table 2.

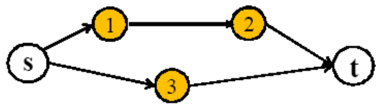

Figure 1: Project with 3 activities AoN.

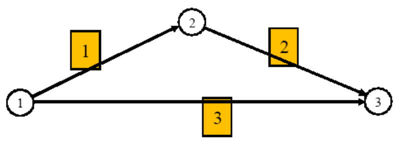

Figure 2: AoA representation.

P-RESOURCE →	*1*	*2*	*3*	*4*
AVAILABILITY	2	1	3	2
Activity				
A1	16	0	12	12
A2	0	7	0	8
A3	20	22	0	0

Table 2: Work content (in man-days) of the activities of project 1.

Table 2 is to be read as follows. There are two units available of resources #1 & #4; one unit of resource #2 and 3 units of resource #3. Activity 1 requires 16 man-days of resource #1 and 12 man-days of each of resources #3 and #4. It does not require resource #2. Etc.

The relevance and impact of the support resources are represented in Table 3, which may be read as follows: *S*-resources 1 and 2 have availability of one unit each. *S*-resource 1 can support *P*-resources 1 & 3 and *S*-resource 2 can support *P*-resources 1 & 2; no support is available for *P*-resource 4.

	P-RES →	*1*	*2*	*3*	*4*
↓S-RES	*↓ AVAILABILITY*				
1	*1*	0.25	ϕ	0.25	ϕ
2	*1*	0.15	0.35	ϕ	ϕ

Table 3: The P-S matrix: Impact of S-resources on P-resources.

With little additional data processing, the problem can be enriched with the inclusion of the cost of the resource utilization at each level. Then in each cell in both the primary and secondary resource tables there shall be added the marginal cost for the resource per unit time. If the project gains a bonus for early completion and incurs a penalty for late completion then one can easily include such costs in the criterion function.

At time 0 we may initiate both activities A1 and A3 because their required *P*-resources are available (A1 requires *P*-resources 1, 3 & 4 and A3 requires *P*-resources 1 & 2.). Assume for the moment that no support resource is allocated to either activity. Further, suppose that each unit of the primary resource is devoted to its respective activity at level 1; i.e.,

$$x(1,r_1) = 1 = x(1,r_3) = x(1,r_4)$$
$$x(3,r_1) = 1 = x(3,r_2)$$

Observe that the *P*-resource availabilities have been respected: the two units of *P*-resource 1 have been equally divided between the two activities; *P*-resource 2 is not required by A1 and the unit available is allocated to A3, *P*-resources 3 & 4 are required only by A1. The *P*-resource allocation would look as shown in Table 4.

	P-RESOURCE			
ACTIVITY	1	2	3	4
A1	1	0	1	1
A3	1	1	0	0
TOTAL ALLOCATION	2	1	1	1

Table 4: The P-resources allocation at time 0.

The durations of the two activities shall be:

$$A1: y_1 = max\left\{\frac{16}{1}, \frac{12}{1}, \frac{12}{1}\right\} = 16 \text{ days}$$
$$A3: y_3 = max\left\{\frac{20}{1}, \frac{22}{1}\right\} = 22 \text{ days}$$

At time t = 16 activity A1 completes processing and A2 becomes sequence feasible. Unfortunately it cannot be initiated because *P*-resource 2, of which there is only one unit, is committed to A3 which is still on-going. Therefore activity 2 must wait for the completion of A3, which occurs at t = 22. When initiated at resource levels

$$x(2,r_2) = 1 = x(2,r_4)$$

it will consume $y_2 = \max\left\{\frac{7}{1}, \frac{8}{1}\right\} = 8$ days to complete.

The project duration would be

$$T^{(1)} = 22 + 8 = 30 \quad (7)$$

If the due date of the project were specified at $T_s = 24$, the project would be 6 days late.

2.2.1 Impact of the support resources

Suppose that at the start of the project both support resources were allocated to activity 3 as follows: $s_1 \rightarrow r_1$ and $s_2 \rightarrow r_2$; then

$$x_{r_1}(3) = 1 + 0.25$$
$$x_{r_2}(3) = 1 + 0.35$$

The duration of the A3 would change to

$$y_3 = max\left\{\frac{20}{1.25}, \frac{22}{1.35}\right\} \approx 16.30 \text{ days}$$

At t = 16.30 activity 2 can be initiated because primary resource 2 would be freed. If we continue with $x(2, r_2) = 1 = x(2, r_4)$ it will consume the same 8 days to complete and the project duration would be

$$T^{(2)} = 16.30 + 8 = 24.30 \quad (8)$$

The project is almost on time!

Whether or not such allocation of the support resources is advisable shall depend on the relative costs of the *S*-resources and tardiness. In fact, again depending on the relative costs, it may be advisable to have allocated *S*-resource 1 to activity 1 when it is initiated at time 0 and, when completed, continue as above with activity 3, since the gain in the project completion time may secure some bonus payment that would more than offset the cost of the added support. It is also possible to allocate more than one *S*-resource to complement the *P*-resources in some activities. All these, and other, possibilities should be resolved by a formal mathematical model.

2.3 Assumption 3

We assume that all costs are linear or piece-wise linear in their argument.

Let:

C^k: the *k*th uniformly directed cutset *(udc)* of the project network that is traversed by the project progression; $k = 1, \dots, K$.

$x(a, r_p)$: level of allocation of (primary) resource r_p to activity a (assuming integer values from 1 to $Q_P(p)$ if the activity needs this resource).

$x(a, (r_p, s_q))$: level of allocation of secondary resource s_q to primary resource r_p in activity a (assuming integer values from 0 to $Q_S(q)$).

$x_{r_p}(a)$: total allocation of resource r_p (including complementary resource) to activity a.

$v(r_p, s_q)$: degree of enhancement of *P*-resource r_p by *S*-resource s_q.

$w(a, r_p)$: work content of activity a when *P*-resource r_p is used.

$y_{r_p}(a)$: duration of activity a imposed by primary resource r_p (with or without enhancement from *S*-resource s_q).

$y(a)$: duration of activity a (considering all resources).

ρ : number of primary resources, $\rho = |P|$.

σ : number of secondary resources, $\sigma = |S|$.

$Q(p)(Q(q))$: capacity of *P*-resource r_p (*S*-resource s_p) available.

γ_p : marginal cost of *P*-resource r_p.

γ_q : marginal cost of *S*-resource s_q.

γ_E : marginal gain from early completion of the project.

γ_L : marginal loss (penalty) from late completion of the project.

t_i : time of realization of node i (AoA representation), where node 1 is the "start node" of the project and node n its "end node".

T_s : target completion time of the project.

The constraints are enumerated next. To avoid confusion with node designation we refer to an activity as *"a"* and to a node as i or j. The notation $a \equiv (i, j)$ means that activity a is represented by arc (i, j).

Respect precedence among the activities:

$$t_j \geq t_i + y(a), \quad \forall a \equiv (i, j) \in A \quad (9)$$

Define total allocation of resource r_p (including complementary resource) in activity a,

$$x_{r_p}(a) = x(a, r_p) + \sum_{s_q} \left(v(r_p, s_q) * x\left(a, (r_p, s_q)\right) \right) \quad (10)$$

Define the duration of each activity when using each *P*-resource; then define the activity's duration as the maximum of individual resource durations:

$$y_{r_p}(a) = \frac{w(a, r_p)}{x_{r_p}(a)} \quad (11)$$

$$y(a) = \max_{all\ r_p} \{y_{r_p}(a)\} \quad (12)$$

Respect the *P*-resource availability at each *udc* traversed by the project in its execution,

$$\sum_{a \in C^k} x(a, r_p) \leq Q_P(p), \qquad \forall p \in P, \forall C^k \quad (13)$$

in which *Q(p)* is the capacity (i.e., availability) of *P*-resource r_p (in the three activities example given above, the vector *Q (P)* = (2, 1, 3, 2)).

The difficulty in implementing this constraint stems from the fact that we do not know a priori the identity of the *udc*'s that shall be traversed during the execution of the project, since that depends on the resource allocations (both the *P*- and *S*-resources). A circularity of logic is present here: the allocation of the resources is bounded by their availabilities at each *udc*, but these latter cannot be known except after the allocations have been determined. Unfortunately, this vicious cycle cannot be broken by a blanket enumeration of all the *udc*'s of the project because that would over-constrain the problem. There are several ways to resolve this circularity, formal as well as heuristic. The formal ones are of the integer programming genre which, when combined with the nonlinear mathematical programming model presented above, present a formidable computing burden. The heuristic approaches are more amenable to computing; we propose such a heuristic approach below.

Respect for the *S*-resources availability for each *udc* traversed by the project in its execution is handled as follows. Note that the requirement that an *S*-resource is applied only to its relevant *P*-resources is taken care of in the *P*-*S* matrix (see Table 3); what this constraint accomplishes is to limit its use to each resource's total availability. The requisite constraints are similar to constraints (13),

$$\sum_{a \in C^k} x\left(a, \left(r_p, s_q\right)\right) \leq Q_S(q) \quad \forall q \in S, \forall C^k \quad (14)$$

in which $Q_S(q)$ is the capacity of *S*-resource s_q (in the three-activities example given above, the vector $Q_S(q) = (1,1)$).

Define earliness and tardiness by,

$$e \geq T_s - t_n \quad (15)$$

$$d \geq t_n - T_s \quad (16)$$

$$e, d \geq 0 \quad (17)$$

The criterion function is composed of two parts: the cost of use of the *P*- and *S*-resources, and the gain or loss due to earliness or tardiness, respectively, of the project completion time (t_n) relative to its due date.

For simplicity, we make the following two assumptions:

(i) the cost of resource utilization is quadratic in the allocation (Rudolph & Elmaghraby, 2009; Tereso, et al., 2004), for the duration of the activity, which renders the cost linear in the resource allocation (recall that the work content is assumed a known constant),

$$c_R(a, r_p) = \left(\gamma_p * x(a, r_p) + \gamma_q * \sum_{\text{all } s_q} x\left(a, \left(r_p, s_q\right)\right)\right) * w(a, r_p) \quad (18)$$

$$c_R(a) = \sum_{\text{all } r_p} c_R(a, r_p) \quad (19)$$

And

(ii) the earliness-tardiness costs are linear in their respective marginal values, as shown in Figure 3;

$$c_{ET} = \gamma_E \cdot e + \gamma_L \cdot d \quad (20)$$

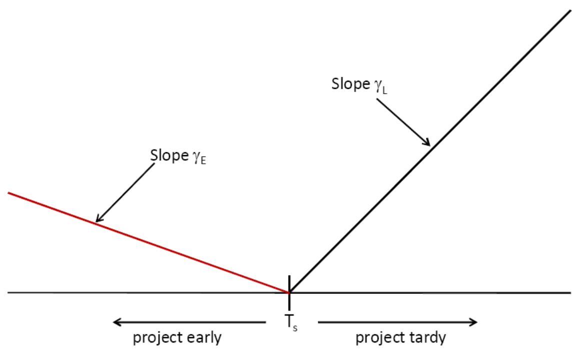

Figure 3: Linear cost of earlier and tardier end of Project.

The desired objective function may be written simply as

$$\min z = \sum_{a \in A} c_R(a) + c_{ET} \quad (21)$$

Now, addressing the problem raised above; namely, how can one constrain the aggregate use of the *P*- and the *S*-resources when the identity of the *udc* to which the constraining relation should be applied is known only after the allocations have been made? As a heuristic, it is propose the following iterative procedure.

At the start node 1 the *udc* is known, hence constraints (13) and (14) can be imposed. Assume abundant availability of the resources in all subsequent *udc*'s – hence these constraints need not be considered. The solution obtained shall identify the next node to be realized the earliest. Repeat the same optimization step at the new node, taking into account the committed resource(s) to the on-going activities from the previous step, assuming abundant availability of the resources in all subsequent *udc*'s. Continue until the project is completed. Observe that the solution obtained is feasible, hence its value constitutes an upper bound on the optimum cost.

It remains to either demonstrate empirically that the solution obtained using this heuristic does not deviate significantly from the optimum, or prove mathematically that it does produce the optimum.

3 Conclusion

The goal of this paper is to provide a formal model to some unresolved issues in the management of projects, especially as related to the utilization of supportive resources. The relevance of the problem is the opportunity to shape a system that allows not only that we improve the allocation of often scarce resource(s), but also result in reduced uncertainties within the projects, combined with increased performance and lower project costs. There still remains the implementation of the model in an easy-to-use computer code that renders it practically usable[*].

This research also unveils several research avenues to be explored. These can be gleaned from the assumptions made. Relaxation of one or more of these assumptions would go a long way towards the resolution of more real life problems.

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