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Mestrado em Matemática Económica e Financeira

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Ao meu marido, à minha mãe, ao meu irmão

e em particular ao meu pai,

um obrigado por tudo.

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Abstract

The main contribution of this thesis is to develop an original model of competitive behaviour-based price discrimination (BBPD) through targeted advertising in a product differentiated market. It also provides a review of the economic literature on informative advertising and on oligopoly price discrimination, particularly on BBPD. The competitive and welfare effects of both marketing strategies–i.e., BBPD and advertising–is already not well understood as there are few works in the field. One exception is Esteves (2009) who offers a first look at the dynamic effects of BBPD in homogeneous product markets, where firms need to invest in advertising to generate awareness. Broadly, this thesis extends her study to a product differentiated duopoly market. Thus, we consider a dynamic two-period model where two firms offer products that are differentiated à la Hotelling and it is assumed that through informative advertising consumers become imperfectly informed. In the first period price discrimination is not feasible. In the second period, based on consumers' purchase history firms can employ BBPD. This dissertation investigates the competitive and welfare effects of firms being able to price discriminate between their old and new costumers in a product differentiated market where through informative advertising consumers become imperfectly informed. We show that moving from no discrimination to BBPD decreases second period prices but increases first period prices. An important finding of this thesis is that BBPD boosts industry profits at the expense of consumer surplus and welfare. A common finding in the existing literature is that when the two firms can price discriminate BBPD leads firms to a prisoner's dilemma situation. This thesis shows that the existence of imperfect informed consumers through informative advertising can help firms to benefit from BBPD.

Keywords: Behaviour-based price discrimination, informative advertising, competitive and welfare effects

Resumo

O principal objectivo desta tese é desenvolver um modelo para aprofundar o conhecimento sobre as consequências a nível económico que derivam da capacidade das empresas praticarem preços diferentes para consumidores que revelam histórias de compra diferentes, fenómeno designado na literatura económica como "Behaviour-Based Price Discrimination" (BBPD). É apresentada uma revisão de literatura para sintetizar os diferentes estudos realizados em torno da publicidade informativa e da discriminação de preços em oligopólio, particularmente em BBPD. Do trabalho desenvolvido ao nível da publicidade, o estudo dos efeitos competitivos e o bem-estar, abrangendo estas duas estratégias de marketing (publicidade e BBPD) tem sido negligenciado. No entanto, os estudos iniciais sobre os efeitos dinâmicos da BBPD nascem com Esteves (2009) que apresenta um modelo em que as empresas oferecem o mesmo produto (produtos homogéneos) e os consumidores apenas tomam conhecimento da existência de um produto se receberem um anúncio da empresa. Portanto, o objectivo desta tese é estender o seu estudo a um duopólio com produtos diferenciados. Tendo em consideração esse propósito é apresentado um jogo sequencial de dois períodos onde as empresas oferecem produtos diferenciados "à la Hotelling". No primeiro período, as empresas não têm conhecimento das preferências dos consumidores e a discriminação de preços não é viável. No segundo período, após observarem o comportamento dos consumidores, as empresas distinguem, de forma imperfeita, os consumidores que no primeiro período compraram o seu produto ou o da empresa rival. Como no último período a discriminação de preços é permitida, as empresas oferecem preços diferentes a consumidores com histórias de compra distintas utilizando para o efeito publicidade informativa direccionada. Com esta análise mostra-se que enquanto a BBPD diminui os preços do segundo período relativamente ao caso em que não há discriminação, e pelo contrário, os preços do primeiro período são superiores. Uma descoberta importante desta tese é verificar que a BBPD impulsiona os lucros da indústria à custa do excedente do consumidor e do bem-estar. Uma descoberta semelhante à literatura existente é que, quando as duas empresas podem discriminar os preços, a BBPD leva as empresas a situação do dilema do prisioneiro. Esta tese mostra que a existência de consumidores imperfeitamente informados através de publicidade informativa podem ajudar as empresas a beneficiar da BBPD.

Palavras-chave: Discriminação de preços baseada no comportamento dos consumidores, publicidade informativa, efeitos competitivos e de bem estar.

Contents

Chapter 1

Introduction

1.1 Introduction

In many markets firms need to invest in advertising. Advertising reaches consumers thought television, radio, newspapers, magazines, billboards, passing buses, park benches, the mail, home telephones and the ubiquitous pop-up advertisements on our computers.

The actual impact that advertising has on market is a subject of some controversy. Some economists argue that the primary role of advertising is to provide information about products and their prices to potential consumers. The defenders of advertising argue that it provides consumers with useful information and encourages price competition. Without advertising, it would be impossible for new firms to become aware. Advertising, they say, promotes price competition, lowers prices and encourages a greater range of choice for consumers.

In the economic literature there are mainly two views of advertising. The first view is that advertising is *persuasive*, its main goal is to change consumers' tastes and create spurious product differentiation and brand loyalty. As a consequence, it has no "real" value for consumers, but rather induces artificial product differentiation. The second view is that advertising is *informative*. According to this approach, advertising has an important informative role to otherwise uniformed consumers. Many markets are caracterized by consumers imperfectly informed, since search costs may deter a consumer from learning each product's existence, price and/or quality. Advertising is the endogenous response to this problem: when a firm advertises, consumers receive information.

Regardless of the role of advertising, economists struggle with another important question: What marketing techniques are more efficient for advertisers? At this point, it is useful to remark on some recent trends in marketing: price discrimination and targeted advertising. Targeted advertising is meant to "target" specific advertisements (henceforth ads) towards specific consumer groups.

The rapid advance in information technology now makes it feasible for sellers to condition their price offers on consumers' prior purchase behavior. Innovations in web-based contextual advertising have helped firms to sharply focus their advertising. Now firms can selectively advertise their products to consumers based on web pages browsed and information searched by consumers. The use of modern information technologies has also given firms the ability to recognize customers with different past purchasing histories and send them targeted advertisements with different prices. When a firm charges different prices for the same good or service to different consumers, even though there is no difference in the cost to the firm of supplying these consumers, the firm is engaging in price discrimination. Consider the following examples. Mobile companies can offer a lower price to a customer who has been using a competitor's service; a credit card firm can offer a lower interest rate to a consumer who transfers balance from another credit card company; a cable company offers a lower monthly free to a customer who previously uses the satellite TV. The type of price discrimination in these examples has two common features. First, the prices depend on consumers'past purchases, and thus incorporate an explicit dynamic consideration. Furthermore, the information about a consumer's past purchase takes a particularly simple form, namely whether or not the consumer purchased from a rival in the past. Second, firms tend to operate under competition, often in oligopoly markets. Such price discrimination by purchase history has been named in the literature as Behaviour-Based Price Discrimination (BBPD) and has received much attention in the recent economic literature.

Several issues arise in models of behaviour-based price discrimination. The large body of previous literature on effects of price discrimination has been developed assuming that there is no role for advertising and that the market is fully covered. An exception is the work of Esteves (2009a) which offers a first look at dynamic effects of customer poaching in homogeneous product markets, where advertising is used by firms as a way to transmit relevant information to otherwise uninformed consumers. Extending this study to differentiated products would bring new interesting insights, improving the economic understanding of BBPD in competitive markets. Thus, the objective of this thesis is to broaden our understanding of price discrimination with imperfectly informed consumers in product differentiated markets. With this goal in mind this thesis extends the analysis of Esteves (2009a) to a product differentiated market.

This thesis is organised as follows. Chapter 2 offers a review of the more relevant literature. It begins with an historical overview of the development of the economic analysis of advertising wherein economists debate the purpose and the effects of advertising. It also discusses the key initial writings that are associated which each of two views of advertising. Then it offers a brief review of the literature on competitive price discrimination mainly on BBPD.

In order to evaluate the competitive effects of BBPD two benchmark cases are presented in chapter 3. We first present the benchmark case where consumers are imperfectly informed and price discrimination cannot occur either because firms have no information about consumer preferences or because it is illegal. Then we present the benchmark case where price discrimination is permitted but consumers are perfectly informed.

The main contribution of this thesis can be found in Chapter 4. It develops an *original* duopolistic model of product differentiation with repeated purchases where firms need to invest in informative advertising to become known and sell their products. We propose a two-period model in which firms choose prices and informative advertising intensities in the first period, and consumers make their purchasing decisions. In the second period, firms can distinguish between two types of consumers: those who bought from them and those who bought from the rival. Price discrimination through targeted advertising becomes possible. Here we investigate the competitive and welfare effects of price BBPD through advertising and show that new insights arise in comparison to Esteves (2009). Finally chapter 5 presents the thesis' main conclusions.

1.2 Related Literature

This thesis is mainly related to two strains of the economic literature. One is the literature on informative advertising; the other is the recent literature on competitive price discrimination, specifically the literature on behaviour-based price discrimination.

Broadly, firms use advertising to improve the information available on the market and/or to entice consumers to buy their products. This information can be conveyed to consumers through the media such as internet, television, radio, newspapers, magazines and mail. Advertising is therefore an important feature of modern economic life. Usually it provides information on prices, quality and other attributes of the firms' products. Although till the end of the 19th century economists had little to say about advertising and its effects on economics, a vast literature emerged during the 20th century.

The initial studies on advertising are born with Marshall (1980, 1919) that distinguishes the constructive and the combative role of advertising. On the one hand, he believes that advertising can play a constructive role by alerting consumers to the existence and location of products carrying also information regarding their attributes and qualities. On the other hand, the combative role occurs when advertising involves repetitive messages, and whose apparent purpose is just removing consumers from one company to the rivals.

Later, Chamberlin (1933) sought to integrate formally advertising in economic theory. Chamberlim argued that a company can use advertising to differentiate its product from the rivals' ones. An important contribution of Chamberlim was to point to the contradictory effects of different purposes of advertising: to inform or to persuade. In this way, nowadays there are two important veins of research on advertising. One is the line of research on informative advertising, the other one is the line of research on persuasive advertising.

According to the persuasive advertising view the main role of advertising is to persuade consumers. By investing in advertising firms try to increase the consumers' willingness to pay for their products. According to the informative advertising view, advertising plays an important role by conveying relevant information such as product existence, availability and price to otherwise uninformed customers. As Nelson (1974) points out, advertising can serve as a tool for transmitting this information to consumers and therefore should not be considered as an unnecessary activity. Whilst in some models advertising is a sine qua non condition for demand existence, in others the main purpose of advertising is to transmit information about price.[1] The model proposed in this thesis is mainly related to the former type of advertising models. Hence, in what follows, we will focus on those models in which without advertising consumers are left out of the market. A common research concern in these models is the welfare effects of informative. Butters (1977) investigates whether there is too much or too little advertising. Remarkably, he finds that the market equilibrium level of advertising is socially optimal. This result was confirmed by Stahl (1994) who extended the latter model to oligopolistic markets and with more general demand curves and advertising technologies. Variations on Butters's (1977) model such as the introduction of product differentiation (Grossman and Shapiro (1984)), or heterogeneity among buyers (Stegeman (1991)) were shown to easily offset this result and helped establish the idea that increased competition stimulated additional advertising (the business stealing effect), while the incapability of the firm to appropriate the social surplus it generates acts as a deterrent to advertising (the nonappropriability of social surplus effect, Tirole (1988)). In addition, apart from Grossman and Shapiro (1984) who find that in a

[1] Within the stream of models where consumers are imperfectly informed only about price see, for instance, Bester and Petrakis (1995) and Moraga-González and Petrakis (1999). Basically, both works provide a duopoly model where though consumers know the existence of both firms, they are only informed about the price offered by the neighbourhood firm. The main role of advertising is, therefore, to transmit information about prices to consumers located at distant locations.

differentiated-product industry there is a single price equilibrium, those authors that have extended the Butters's model without departing from the homogeneous product assumption also obtain equilibrium outcomes displaying price dispersion (e.g. Stegeman (1991) and Stahl (1994)).

The developments in information technology have enabled firms to collect and use sophisticated databases of consumer information as a way to target specific messages to different types of consumers. Motivated by the information technology improvements, recent studies on informative advertising have been assuming that firms can target advertising messages to specific groups of consumers in a market. Hernández-Garcia (1997) and Esteban, Gil and Hernández-Garcia (2001) study targeted advertising in monopolistic markets. These studies have concentrated on the ability of the monopolist to target ads to those consumers with a higher valuation for the good. Shaffer and Zhang (1995), Bester and Petrakis (1996) and Moraga-González and Petrakis (1999) look at coupon targeting in duopoly settings. Iyer, et al. (2005) look also at the economic effects of targeted advertising in a duopoly market. In their work companies use targeted advertising to reach only those consumers who really want to buy their products. In these studies consumers are aware of the firm/product existence and so advertising is not needed to generate demand for a product.

The access to consumer information databases has also permitted firms to use targeted advertising messages as a tool to price discriminate. In many contexts, price discrimination emerges naturally as a dominant strategy. The simplest definition says that price discrimination means selling the same good at different prices. More generally price discrimination is present when two or more similar goods are sold at prices that are in different ratios to marginal costs (Varian, 1989, p 598).

Following the traditional definition of Pigou (1920) there are three types of price discrimination (PD). Under first-degree PD the firm is able to charge a different price per unit of product and per consumer, which under monopoly means that the firm is able to extract all consumer surplus. Second-degree PD is the practice of discriminating on the basis of unobserved consumer heterogeneity. The firm offers a menu of products and prices and consumers self-select into the appropriate niche of the market. Finally third-degree PD occurs when the firm can discriminate on the basis of observable and verifiable consumer characteristics (e.g. past purchasing decisions, age, gender, geographical location, etc.).

Recent typologies of price discrimination can now be encountered in the literature. Stole (2007) classification is based on whether the form of consumer heterogeneity is observable or not. Direct price discrimination is based on some observable demand related characteristic (e.g. third-degree, location based, behaviour-based, etc.). Indirect price discrimination arises if consumer heterogeneity is not directly observable and firms need to rely on self-selection mechanisms to indirectly separate consumers (e.g. nonlinear pricing). Within the classification of direct price discrimination, we can refer the practice of charging different prices to consumers with different past behavior. This form of price discrimination was introduced in the economic literature by Fudenberg and Tirole (2000).

While the literature of price discrimination is abundant in monopoly markets the same is not true in imperfect competitive contexts. An important finding in monopoly settings is that price discrimination always increases the firm's profits while the effect on welfare might be positive or negative. A relevant reference on competitive price discrimination is the paper by Thisse and Vives (1988). They examine competition between firms that are differentiated in geographical space. Two price strategies are presented: (i) uniform pricing and (ii) price discrimination. In the latter case they allow firms to set a different price to each consumer location. With price discrimination they show that firms face a prisoners' dilemma situation. Price discrimination is bad for profits but good for consumers.

As the model presented in chapter is mainly related to the literature on BBPD next we present the main works on this area of research. As mentioned BBPD is the practice of charging different prices to previous customers of a firm and to the rivals' previous customers (or new customers of a firm). Two approaches have been considered so far. In the brand preferences approach (e.g. Villas-Boas (1999), Fudenberg and Tirole (2000), Villas-Boas (2004), Esteves (2010)), purchase history discloses information about a consumer's exogenous brand preference for a firm. In the switching costs approach, consumers initially view the two firms as perfect substitutes; but in the second period they face a switching cost if they change supplier. In this setting, purchase history discloses information about exogenous switching costs (e.g. Chen (1997) and Taylor (2003)). Although the framework of competition differs in both approaches have some common prediction. Price discrimination leads firms to offer better deals to the rival's consumers than to its previous customers. Second, because both firms have symmetric information for price discrimination purposes and each firm regards its previous clientele as its strong market and the rival's clientele as its weak market—in the terminology of Corts (1998) there is best-response asymmetry—firms find themselves in the classic prisoner's dilemma. Third, there is socially excessive switching between firms. Nonetheless, important differences arise in both approaches when taking into account the effects of poaching on initial prices. While in the brand preferences approach when BBPD is permitted initial prices are high and then decrease (e.g. Fudenberg and Tirole (2000)), in the switching costs approach the reverse happens (e.g. Chen (1997)).

In the two previous approaches consumers are perfectly informed and there is no role for advertising. Esteves (2009a) offers a first look at dynamic effects of customer poaching in homogeneous product markets, where advertising is used by firms as a way to transmit relevant information to otherwise uninformed consumers. She shows that price discrimination might boost industry profits at the expense of consumers' surplus. Regarding the welfare effects, price discrimination is generally bad for welfare and consumers' surplus, though good for firms. The goal of this thesis is to extend Esteves (2009a) to a product differentiation market as a way to investigate whether or not new insights emerge about the economic effects of BBPD.

Chapter 2

Benchmark Models

2.1 Introduction

This chapter presents two benchmark models that will be useful to compare the results derived in the subsequent chapters where firms endogenously segment the market into imperfectly informed consumers in period 1 and are allowed to price discriminate in period 2. In the first benchmark firms choose advertising decisions and uniform prices in period 1 and in period 2 price discrimination cannot occur, either because firms have no information or because it is not permitted. In this model consumers are imperfectly informed about the firms' existence and prices and firms compete in uniform prices in both periods. In the second benchmark, consumers are perfectly informed and firms only compete in prices in both periods. However, in period 2 firms are allowed to price discriminate.

2.2 Competition with advertising and no price discrimination

2.2.1 The model

Consider first the case where firms need to invest in advertising to give information about the existence and price of their products. As usual in the literature of informative advertising we are assuming that without advertising consumers are uninformed and cannot buy the firms' products. Regarding price competition we start the analysis with the case where any form of price discrimination is not permitted, either because firms do not observe the first period decisions of individual consumers or because price discriminate is prohibited. This benchmark model will be useful to evaluate the competitive and welfare effects of price discrimination with advertising. The analysis here is similar to that of Tirole (1988) model and it is a simplification of Grossman and Shapiro (1984) model.

Consider a market with two firms denoted by, $i = A, B$, producing differentiated products/brands, which we will refer to as brand A and brand B, respectively. Each firm is located at each end of the unit interval: firm A is located at $\theta = 0$ and firm B is located at $\theta = 1$. Consumers are uniformly distributed on the line segment $[0, 1]$ distributed with density 1 along this interval and they derive gross surplus v from consuming the good. Throughout the thesis we will assume that v is sufficiently large so that as long as informed consumers will always buy. Consumers incur a transportation cost t per unit of distance. Hence, a consumer located in address θ incurs transportation costs $t\theta$ if buying from firm A and $t(1-\theta)$ if buying the good of firm B. Assume also that each consumer buys at most one unit of either brand A or B, when informed.

There are two periods. In the first period, firms simultaneously choose brand advertising levels, $\phi_i \in [0, 1]$ as well as its price (denoted by p_i),$i = A, B$. In period 1, firms have no information about consumer preferences so that they cannot price discriminate and choose the uniform prices p_A and p_B.

After firms have sent their advertisements (henceforth ads) independently, a proportion ϕ_i and ϕ_j of customers are reached, respectively by firm i and j's advertising. Therefore, there are three types of consumers. There are consumers who purchase from the only known firm that we call captive consumers, namely $\phi_i\left(1-\phi_j\right)$. There are consumers who receive ads from both firms and we call them selective consumers, namely $\phi_i\phi_j$. There are also consumers who receive no ad from either firm, remain uninformed and out of the market. A captive consumer buys from the known firm as long as the price does not exceed v; if he receives ads from both firms he chooses the lowest full price if it does not exceed v.

Let the superscript nd identify the no-discrimination case. In period 1 firms A and B choose advertising intensities simultaneously and non-cooperatively and announce their prices under non-discrimination p_A^{nd} and p_B^{nd}.

Consider for instance the case of firm i. Its potential demand has size D_i. It can be decomposed in two parts. A fraction $\phi_i\left(1-\phi_j\right)$ does not receive an ad from firm j. It can thus be considered firm i's turf of captive consumers. A fraction $\phi_i\phi_j$ also receives at least an ad of firm j, and therefore constitutes a more elastic or competitive fragment of the demand. The consumers that receive no ad from either firm are uniformed and excluded from the market.

Firm A faces a demand of

$$D_A = \phi_A\phi_B \Pr\left(p_A^{nd} + t\theta < p_B^{nd} + t(1-\theta)\right) + \phi_A\left(1-\phi_B\right)$$

which simplifies to

$$D_A = \phi_A\phi_B\left(\frac{p_B^{nd} - p_A^{nd} + t}{2t}\right) + \phi_A\left(1-\phi_B\right). \tag{2.1}$$

Similarly, firm B's demand is given by

$$D_B = \phi_A\phi_B \Pr\left(p_A^{nd} + t\theta > p_B^{nd} + t(1-\theta)\right) + \phi_B\left(1-\phi_A\right),$$

which simplifies to

$$D_B = \phi_A \phi_B \left(\frac{p_A^{nd} - p_B^{nd} + t}{2t} \right) + \phi_B \left(1 - \phi_A\right). \tag{2.2}$$

Since in the second period customers are all anonymous to the firms and the prices announced through advertising in period 1 remain constant in period 2, each firm's demand is equal in both periods.

2.2.2 Advertising technology

Advertising is a costly activity for firms and conveys information on product existence and price. Let $\phi_i, (i = A, B)$ denote the fraction of consumers who receive an ad from firm i. The cost of reaching fraction ϕ_i of consumers is denoted $A(\phi_i)$. As usual in the literature (e.g. Butters (1977), Grossman and Shapiro (1984), Tirole (1988)) we assume that the cost of reaching consumers increases at an increasing rate, which formally can be written $\frac{\partial A}{\partial \phi} = A_\phi > 0$ and $\frac{\partial^2 A}{\partial \phi^2} = A_{\phi\phi} > 0$. The latter condition means that it is increasingly more expensive to inform an additional customer or likewise, to reach a higher proportion of costumers. It is also assumed that there are no fixed costs in advertising, that is, $A(0) = 0$. Following Tirole (1988), to simplify the computations, I will consider a quadratic advertising cost function given by $A_\phi = \frac{a}{2}\phi^2$ with a maximum advertising expenditure $\frac{a}{2}$. Butters (1977) and Grossman and Shapiro (1984) propose other technologies with the same mathematical properties but more complicated to manipulate.

2.2.3 Equilibrium analysis

As first period price decisions are valid for the two periods assuming that each firm discounts future profits using a common discount factor, $\delta \in (0, 1)$, firm i profit is equal to:

$$\pi_i = (1 + \delta) \left[p_i \phi_i \left(1 - \phi_j\right) + p_i \phi_i \phi_j \left(\frac{p_j - p_i + t}{2t} \right) \right] - A(\phi_i)$$

In period 1 firms simultaneously chooses prices and advertising levels. Each firm goal is to solve the following maximization problem:

$$\max_{p_i,\phi_i}\left\{(1+\delta)\left[p_i\phi_i\left(1-\phi_j\right)+p_i\phi_i\phi_j\left(\frac{p_j-p_i+t}{2t}\right)\right]-A\left(\phi_i\right)\right\}$$

Using the quadratic advertising costs $A\left(\phi_i\right)=\frac{a\phi_i^2}{2}$, and solving first the model with respect to p_i it follows that from the first order condition with respect to p_i we obtain firm i best response function given p_j:

$$p_i=\frac{1}{2\phi_j}\left(2t-t\phi_j+\phi_j p_j\right) \tag{2.3}$$

As we are looking for a symmetric equilibrium it must be true that $p_i=p_j=p^{nd}$ thus

$$\begin{aligned} p^{nd} &= \frac{1}{2\phi_j}\left(2t-t\phi_j+\phi_j p\right) \\ p^{nd} &= t\frac{\left(2-\phi_j\right)}{\phi_j} \end{aligned}$$

It is now straightforward to find the equilibrium level of advertising. From

$$\max_{\phi_i}\left\{(1+\delta)\left(2-\phi_j\right)t\left[\frac{\phi_i\left(1-\phi_j\right)}{\phi_j}+\frac{\phi_i}{2}\right]-A\left(\phi_i\right)\right\}$$

we obtain the FOC with respect to ϕ_i :

$$\phi_i=\frac{1}{a}p_i\left(1-\phi_j+\phi_j\left(\frac{p_j-p_i+t}{2t}\right)\right) \tag{2.4}$$

Note that from the profit maximization with respect to ϕ_i we obtain that in the best response function of firm i with respect to ϕ_j:

$$\underbrace{\frac{(1+\delta)\,t\left(2-\phi_j\right)^2}{2\phi_j}}_{MRA^{nd}}=\underbrace{A_\phi}_{MAC^{nd}}$$

The previous equation shows that in equilibrium the marginal revenue of sending an additional ad must be equal to the cost of sending that additional ad. With the quadratic advertising cost function the previous equation can be written as follows:

$$\frac{(1+\delta)\left(2-\phi_j\right)^2 t}{2\phi_j} = a\phi_i$$

As the model is symmetric we are looking for a symmetric equilibrium where $p_1^{nd} = p_2^{nd} = p^{nd}$ and $\phi_1^{nd} = \phi_2^{nd} = \phi^{nd}$. Solving the corresponding set of first-order conditions, we obtain the symmetric subgame perfect price-advertising nash equilibrium. So we can write the following proposition.

Proposition 1 *In the benchmark case without price discrimination, as long as $a \geq \frac{(1+\delta)t}{2}$, there is a symmetric subgame perfect nash equilibrium in which:*

(i) Firms choose an advertising reach, denoted $\phi^{nd} \in (0,1)$, equal to $\phi^{nd} = \frac{2}{1+\sqrt{\frac{2a}{t(1+\delta)}}}$.

(ii) Equilibrium prices in both periods are $p^{nd} = \sqrt{\frac{2at}{(\delta+1)}}$.

Proof. See the Appendix.

In the appendix we show $0 < \phi^{nd} \leq 1$ as long as $a \geq \frac{(1+\delta)t}{2}$. Before proceeding note also that as expected when $\delta = 0$ the equilibrium solution is equal to the static game presented in Tirole (1988).

Note that $\partial p^{nd}/\partial \phi^{nd} < 0$ whereas $\partial \phi^{nd}/\partial p^{nd} > 0$. Hence, greater levels of advertising stimulate price competition (i.e. lower prices) and higher prices stimulate advertising competition (i.e. higher levels of advertising). It is also easy to observe that price and advertising levels are increasing in product differentiation (t), while more costly advertising (a) induces less advertising and higher prices.

Given the equilibrium level of advertising ϕ^{nd}, it is straightforward to obtain each firm overall profit with no discrimination as a function of ϕ^{nd}:

$$\Pi_i^{nd} = \frac{(1+\delta)\left(2-\phi^{nd}\right)^2 t - a\left(\phi^{nd}\right)^2}{2}.$$

From Proposition 1 we get the following corollary.

Corollary 1 *For any $\delta \in]0,1]$, assuming that $a \geq \frac{(1+\delta)t}{2}$, each firm overall equilibrium profit equal to*

$$\Pi^{nd} = \frac{2a\sqrt{(\delta+1)}}{\left(1+\sqrt{\frac{2a}{t(1+\delta)}}\right)^2}. \tag{2.5}$$

As expected, profit increases in the degree of product differentiation, reflecting higher prices and a greater level of demand due to additional advertising. Somewhat unexpectedly, however, profit also increases with advertising costs. As firms engage in less advertising, the corresponding decrease in price competition overcompensates the direct tendency towards higher advertising costs. This is precisely the result found by Grossman and Shapiro (1984) and in Tirole (1988).

2.2.4 Welfare analysis

Social welfare is the sum of consumer surplus and industry profits, or equivalently the net utility for all consumers who buy the product in both periods minus overall advertising costs. Overall welfare is given by $W = w^1 + \delta w^2$.

First-period welfare is equal to:

$$\begin{aligned} w^{1,nd} &= \left(\phi^{nd}\right)^2\left(v - 2\int_0^{1/2} t\theta d\theta\right) + \phi^{nd}\left(1-\phi^{nd}\right)\left(v - 2\int_0^1 t\theta d\theta\right) - 2A\left(\phi^{nd}\right) \\ &= \left(\phi^{nd}\right)^2\left(v - 2t\left[\frac{\theta^2}{2}\right]_0^{1/2}\right) + \phi^{nd}\left(1-\phi^{nd}\right)\left(v - 4t\left[\frac{\theta^2}{2}\right]_0^1\right) - 2A\left(\phi^{nd}\right) \\ &= \left(\phi^{nd}\right)^2\left(v - \frac{t}{4}\right) + \phi^{nd}\left(1-\phi^{nd}\right)(v-t) - 2A\left(\phi^{nd}\right) \\ &= \phi^{nd}\left(v + \frac{1}{4}t\left(3\phi^{nd}-4\right)\right) - 2A\left(\phi^{nd}\right) \end{aligned}$$

while

$$\begin{aligned} w^{2,nd} &= \left(\phi^{nd}\right)^2\left(v - 2\int_0^{1/2} t\theta d\theta\right) + \phi^{nd}\left(1-\phi^{nd}\right)\left(v - 2\int_0^1 t\theta d\theta\right) \\ &= \phi^{nd}\left(v + \frac{t}{4}\left(3\phi^{nd}-4\right)\right) \end{aligned}$$

Therefore, it is straightforward to obtain that overall welfare with no discrimination is equal to:

$$\begin{aligned} W^{nd} &= (1+\delta)\,\phi^{nd}\left(v+\frac{t}{4}\left(3\phi^{nd}-4\right)\right)-2A\left(\phi^{nd}\right) \\ &= (1+\delta)\,\phi^{nd}\left(v+\frac{t}{4}\left(3\phi^{nd}-4\right)\right)-a\left(\phi^{nd}\right)^2 \end{aligned}$$

Using our previous computations industry profit with no discrimination can be written as a function of ϕ^{nd}. Thus, using the fact that $\Pi_{ind}^{nd}=2\Pi^{nd}$, then

$$\Pi_{ind}^{nd}=(1+\delta)\,t(2-\phi^{nd})^2-a\left(\phi^{nd}\right)^2$$

We can now compute expected consumer surplus given by $W^{nd}-\Pi_{ind}^{nd}$:

$$\begin{aligned} ECS^{nd} &= (1+\delta)\,\phi^{nd}\left(v+\frac{t}{4}\left(3\phi^{nd}-4\right)\right)-a\left(\phi^{nd}\right)^2 \\ &\quad -(1+\delta)\,t(2-\phi^{nd})^2+a\left(\phi^{nd}\right)^2 \\ &= (1+\delta)\,\phi^{nd}\left(v+\frac{t}{4}\left(3\phi^{nd}-4\right)\right)-(1+\delta)\,t(2-\phi^{nd})^2 \\ &= (1+\delta)\left(\phi^{nd}\left(v+\frac{t}{4}\left(3\phi^{nd}-4\right)\right)-t(2-\phi^{nd})^2\right) \end{aligned}$$

For the case where $\delta=1$ it follows that with no-discrimination:

$$\Pi_{ind}^{nd}=2t(2-\phi^{nd})^2-a\left(\phi^{nd}\right)^2$$

$$W^{nd}=2\phi^{nd}\left(v+\frac{t\left(3\phi^{nd}-4\right)}{4}\right)-a\left(\phi^{nd}\right)^2$$

$$ECS^{nd}=\phi^{nd}\left(2v+\frac{t\left(3\phi^{nd}-4\right)}{2}\right)-2t(2-\phi^{nd})^2$$

2.3 Competition with perfect informed consumers and price discrimination

2.3.1 The model

Consider next the benchmark case where consumers are perfectly informed about the firms' product existence and prices. Here we will assume that after consumers have made their buying decisions in period 1 firms can recognise their previous customers and those that bought from the rival and set prices accordingly. In other words, firms can price discriminate between old and new customers. The analysis here is similar to that of Fudenberg and Tirole (2000). Thus, suppose two firms, A and B, produce at zero marginal cost nondurable goods A and B. There are two periods, 1 and 2. On the demand side, there is population of consumers with mass normalized to 1, each of whom wishes to buy a single unit of either good A or B in each of the two periods. Consumer preferences are as specified in the Hotelling-style linear market of unit length with firms positioned at the endpoints. A consumer brand preference parameter θ is uniformly distributed on $[0, 1]$ and remains fixed for both periods of consumption. As usual a consumer located at θ incurs total cost $p_A + t\theta$ if he buys from firm A at price p_A, and he incurs total cost $p_B + t(1-\theta)$ if he buys the unit from B at price p_B.

Assume also suppose firms cannot commit to future prices. As in FT model consumers reveal information about their brand preference by their first-period choice. Suppose that standard competition à la Hotelling allows firm A to attract a fraction of θ_1. Thus, firm A's turf is the interval $[0, \theta_1]$, while firm B's turf is the remaining $[\theta_1, 1]$. In period 2 each firm is able to recognise its own previous customers and the rival's ones, and thus they can charge different prices to their own first-period customers p_i^O, and to the rival's previous customers p_i^R.

2.3.2 Equilibrium analysis

Second-period

In the second-period both firms can recognise a customer who belongs to its turf and to the rival's turf. Thus, both firms can charge different prices to their own first-period customers p_i^O, and to the rival's previous customers p_i^R.

Proposition 2. *When firms can price discriminate between old/new customers second period equilibrium prices are:*

$$p_A^O = \frac{1}{3}t\left(2\theta_1 + 1\right);\ p_A^R = \frac{1}{3}t\left(3 - 4\theta_1\right)$$

$$p_B^O = \frac{1}{3}t\left(3 - 2\theta_1\right);\ p_B^R = \frac{1}{3}t\left(4\theta_1 - 1\right).$$

It is straightforward to obtain that at the interior solution, i.e., if $\frac{1}{4} \leq \theta_1 \leq \frac{3}{4}$ both firms make the same profit in the second period, given by

$$\pi_i^2 = \frac{5}{9}t\left(2\theta_1^2 - 2\theta_1 + 1\right).$$

It is interesting to note that in this case each firm's second-period profit is minimized when firms share the first period market equally. The reason is that an equal initial market share generates the most informative outcome in the second period, and, in this setting with best response asymmetry, more information destroys profit. When initial market shares are very asymmetric, on the other hand, little is learned about most consumers' brand preferences, competition is less intense and profits increase.

Proof. The proof of this proposition can be found in Esteves and Rey (2010).

First-period

Turn now to first-period competition. Let p_i^1 represent firm i's first-period price, $i = A, B$. At an interior solution the indifferent consumer is located at θ_1 such that:

$$p_A^1 + t\theta_1 + \delta \left[p_B^R + t(1-\theta_1) \right] = p_B^1 + t(1-\theta_1) + \delta \left[p_A^R + t\theta_1 \right]$$

It is straightforward to obtain that

$$\theta_1 \left(p_A^1, p_B^1 \right) = \frac{1}{2} + \frac{3\left(p_B^1 - p_A^1 \right)}{2t(\delta+3)}.$$

Each firm overall objective function is

$$\Pi_A = p_A^1 \theta_1 \left(p_A^1, p_B^1 \right) + \delta \pi_A^2 \left[\theta_1 \left(p_A^1, p_B^1 \right) \right],$$

$$\Pi_B = p_B^1 \left[1 - \theta_1 \left(p_A^1, p_B^1 \right) \right] + \delta \pi_B^2 \left[\theta_1 \left(p_A^1, p_B^1 \right) \right].$$

Proposition 3 *There is a symmetric subgame perfect nash equilibrium in which:*

(i) First-period equilibrium prices are $p_A^1 = p_B^1 = t\left(1 + \frac{\delta}{3}\right)$ *and the first-period market is split symmetrically with* $\theta^1 \left(p_A^1, p_B^1 \right) = \frac{1}{2}$.

(ii) Second-period equilibrium prices are $p_A^O = p_B^O = \frac{2}{3}t$ *and* $p_A^R = p_B^R = \frac{1}{3}t$.

Proof. The proof of this proposition can be found in Esteves and Rey (2010).

2.3.3 Welfare analysis

As usual total welfare is given by the sum of consumer surplus and industry profits. When consumers are fully informed and firms can price discriminate between old/new customers second period welfare w^2 is given by

$$\begin{aligned} w^2 &= v - \int_0^{\frac{1}{3}\theta_1 + \frac{1}{6}} t\theta d\theta - \int_{\frac{1}{3}\theta_1 + \frac{1}{6}}^{\theta_1} t(1-\theta) d\theta - \int_{\theta_1}^{\frac{5}{6} - \frac{1}{3}\theta_1} t\theta d\theta - \int_{\frac{5}{6} - \frac{1}{3}\theta_1}^{1} t(1-\theta) d\theta \\ &= v - \frac{11}{36} t \end{aligned}$$

In period 1 consumers buy from the closer firm, thus first-period welfare is equal to

$$w^1 = v - 2\int_0^{\frac{1}{2}} t\theta d\theta = v - \frac{t}{4}$$

Thus, under BBPD and perfect informed consumers overall welfare is given by $W = w^1 + \delta w^2$, or

$$W = v\left(1+\delta\right) - \frac{1}{4}t - \frac{11\delta}{36}t \tag{2.6}$$

In equilibrium industry profit under BBPD with perfect informed consumers is

$$\Pi_{ind} = \frac{1}{9}t\left(8\delta + 9\right) \tag{2.7}$$

It is now straightforward to obtain consumer surplus, denoted $ECS = W - \Pi_{ind}$. At the interior solution equilibrium solution we have:

$$ECS = v - \frac{5}{4}t - \frac{43}{36}t\delta + v\delta \tag{2.8}$$

Chapter 3

Competition with Advertising and Price Discrimination

3.1 Introduction

Advertising plays an important informative role mainly in new product markets by conveying information to otherwise uninformed consumers. When firms and consumers interact more than once, by collecting information about the "reach" of their advertising firms may learn the identity/address of consumers that receive one of their ads. If firms realise that some of these informed consumers do not buy from the firm currently but rather from the rival, they can try to induce them to switch by offering them selective price discounts. Thus, advertising is a tool for price discrimination practices.

This chapter offers the main contribution of this thesis by proposing an original model of targeted advertising with behaviour-based price discrimination. As said in the literature review, with the exception of Esteves (2009a) the literature on BBPD as been developed assuming that consumers are perfectly informed and there is no role for informative advertising. As Esteves (2009a) we depart from this assumption by assuming that firms need to invest in advertising to inform consumers about the existence and price of their very new products.

We consider a two period duopoly model with anonymous consumers who may buy from a firm only if they receive an ad from it. In the first-period firms need to invest in advertising to become known and also give information about prices. As prices can change faster than consumers' awareness, in the second period, the level of awareness is constant and firms can only change prices. As in Esteves (2009a) advertising plays two different tasks. First, as all consumers are initially uninformed, advertising decisions endogenously create consumer heterogeneity in awareness of the firms' existence and prices. Second, by collecting information about the "reach" of their advertising, firms learn the identity of informed consumers who bought from them in period 1 and send later advertising messages with different prices to their own and to the rival's previous customers. The possibility of firms to reconnect and communicate with "lost" customers and entice them back is nowadays possible and has been known as retargeting[1]. As is explained in Esteves 2009, "once a potential customer is aware of a firm's website (e.g. through normal advertising channels) and visits it, a cookie is passed to the consumer's browser that records his behavior on the site and identifies him as either a nonpurchaser or a customer that bought from the firm. Then, at a determined time, loyal customers and potential consumers are retargeted with messages specific to them.[2]"

The main contribution of this thesis is to extend Esteves (2009a) to a product differentiation model. While Esteves (2009a) deals with a homogeneous product market we propose a model with horizontal product differentiation. Our aim is to investigate how the ability to price discriminate affects: (i) the firms' pricing and advertising strategies and (ii) the level of profits and social welfare.

This chapter is organized as follows. Next we present the model. Section 3.2, analyses the second stage of the game where firms compete with price discrimination.

[1]This marketing practice is also referred to as behavioral retargeting, remarketing or remessaging.

[2]Boomerang, DoubleClick's one-to-one targeting group, gives the following retargeting example. "A consumer goes to an online shoe retailer and leaves the site without making a purchase. Then by utilizing a retargeting technology, the shoe retailer can catch the consumer the next time (when he's visiting a news site, perhaps). By visiting a site, a consumer has let that site know he is interested in the product and Retargeting helps the advertiser entice the consumer to return and buy its product (e.g. receive 10 percent off if you buy today)." See http://www.websmartamerica.com/behavioral-retargeting.php.

Section 3.3 presents the first stage of the model in which advertising decisions are taken in a non-cooperative way. Section 3.4 and 3.5 discusses, respectively, the competitive effects of price discrimination and advertising. Section 3.6 analyses the welfare effects of price discrimination through advertising.

3.2 The model

The model presented in this chapter is similar to the model presented in section 2.2 except that now in period 2 we allow firms to use the information about consumer's past behaviour to employ BBPD. Therefore, suppose two firms, A and B, are launching a new differentiated good. In period 1 firms need to invest in advertising to provide information about their product existence and prices. In this period firms choose simultaneously an advertising intensity and price. In the first period, firms cannot price-discriminate because they have no information about consumers' type. In period 2, after having observed the consumers' previous decisions firms are able to distinguish their previous consumers from those that bought from the rival before and set prices accordingly.

As in section 2.2 firms need to invest in advertising to generate demand. This means that at the begining of the game consumers are uniformed about the product existence. After advertising decisions have been made there are four segments of consumers: firm A's captive (monopoly) segment $(\phi_A (1-\phi_B))$, firm B's captive segment $(\phi_B (1-\phi_A))$, the selective segment $(\phi_A \phi_B)$ and non-informed consumers $((1-\phi_A)(1-\phi_B))$. The group of captive consumers purchase from the only known firm as long as the price offered is below or equal to v. A selective consumer receives ads from both firms therefore he chooses the lowest full price if it does not exceed v. Consumers are uniformly distributed on the unit interval $[0,1]$. Firm A (product A) is located at point $\theta = 0$, while firm B (product B) is located at point $\theta = 1$. Hence for a consumer located at $\theta \in [0,1]$, $t\theta$ is the transport cost of choosing product A and $t(1-\theta)$ is the transport cost of choosing product B.

Look next on each firm's demand. Consider for instance the case of firm A. A captive consumer to firm A buys product A as long as $p_A + t\theta \leq v$.[3] A selective consumer who is indifferent between the two firms is located at θ, thus

$$p_A^1 + t\theta = p_B^1 + t(1-\theta).$$

Hence, a consumer located at θ is indifferent between buying product A or B if

$$\theta = \frac{p_B^1 - p_A^1 + t}{2t}.$$

We can now compute firm A and B demand, respectively given by:

$$D_A = \phi_A \phi_B \left(\frac{p_B^1 - p_A^1 + t}{2t} \right) + \phi_A \left(1 - \phi_B\right). \tag{3.1}$$

and

$$D_B = \phi_A \phi_B \left(\frac{p_A^1 - p_B^1 + t}{2t} \right) + \phi_B \left(1 - \phi_A\right). \tag{3.2}$$

In period 2 firms are constrained to reach the same consumers. However, in this period by observing the consumers' purchase history each firm will be able to recognise its previous customers and those that receive one its ads in period 1 but decided to buy the product from the rival. In this period, each firm will choose a different price to its own customers (p_i^O) and to the rival's customers (p_i^R). Note however that firms can only try to poach a selective consumer who bought from the rival before.

3.2.1 Advertising technology

Advertising is a costly activity for firms and conveys information on product existence and price. Let $\phi_i, (i = A, B)$ denote the fraction of consumers who receive an ad from firm i. The cost of reaching fraction ϕ_i of consumers is denoted $A(\phi_i)$. As usual in the literature (e.g. Butters (1977), Grossman and Shapiro (1984), Tirole (1988)) we assume

[3] In the subsequent analyisis we will assume that the condition $p_A + t \leq v$ is always satisfied to guarantee that nobody stays out of the market. This is a standard assumption in the Hotelling framework.

that the cost of reaching consumers increases at an increasing rate, which formally can be written $\frac{\partial A}{\partial \phi} = A_{\phi} > 0$ and $\frac{\partial^2 A}{\partial \phi^2} = A_{\phi\phi} > 0$. The latter condition means that it is increasingly more expensive to inform an additional customer or likewise, to reach a higher proportion of costumers. It is also assumed that there are no fixed costs in advertising, that is, $A(0) = 0$. Following Tirole (1988), to simplify the computations, I will consider a quadratic advertising cost function given by $A_{\phi} = \frac{a}{2}\phi^2$ with a maximum advertising expenditure $\frac{a}{2}$. Butters (1977) and Grossman and Shapiro (1984) propose other technologies with the same mathematical properties but more complicated to manipulate.

3.3 Equilibrium analysis

3.3.1 Second-period

After period 1, when customers can be recognized, firms may price discriminate between their old and new costumers. When a firm achieves that type of knowledge, it may have incentives to send targeted ads with better deals to the selective consumers, in an effort to poach them from the rival firm. Each firm has the ability identify the selective consumers who bought from the rival firm in period 1 but they cannot distinguish from their own consumers who are captive or selective consumers. This means that in period 2, firms can only recognize the old and new costumers. In other words, there are two segments each of size $\phi_A(1-\phi_B)$ and $\phi_B(1-\phi_A)$ which consists of consumers who are in one firm's database but not in the other, that is, each firm group of captive consumers. There is no competition for these consumers in period 2.

In second period, when price discrimination is allowed, firms select a pair of second-prices, $\{p_i^O, p_i^R\}$, where p_i^R is the price offered by firm i to the customers that bought from firm j in period 1, and p_i^O is firm i's price for customers who purchase from firm i in the first stage. Note that firm i' captive consumers buy from i in period 1 iff $p_1^i + t\theta < v$, where p_1^i is firm i's first period price.

Given the firms' first period prices there is a cutoff $\theta^* \in [0,1]$. The consumer located at θ^* is indifferent between buying good A and B. The marginal consumer, $\hat{\theta}_2^A$, who bought from A in the first period will be indifferent between buying again from A at price p_A^O and switching to firm B paying p_B^R iff

$$p_A^O + t\hat{\theta}_2^A = p_B^R + t\left(1 - \hat{\theta}_2^A\right).$$

It follows that,

$$\hat{\theta}_2^A = \frac{p_B^R - p_A^O + t}{2t}.$$

Similarly, the marginal consumer, $\hat{\theta}_2^B$, who bought from firm B in the first period will be indifferent between continuing to so at price p_B^O and switching to firm A paying the price p_A^R iff

$$p_A^R + t\hat{\theta}_2^B = p_B^O + t\left(1 - \hat{\theta}_2^B\right),$$

from which we obtain

$$\hat{\theta}_2^B = \frac{p_B^O - p_A^R + t}{2t}.$$

Look first at firm A's second period profits. Its second period profit comes from the customers that buy from firm A again and from consumers that switched from B to A in second period. Therefore, total second-period profit for firm A equals:

$$\pi_A^2 = \pi_A^O + \pi_A^R.$$

Firm A's second period profit from old consumers is equal to

$$\pi_A^O = p_A^O \phi_A (1 - \phi_B) + p_A^O \phi_A \phi_B \left(\frac{p_B^R - p_A^O + t}{2t}\right)$$

while its profit from the poached selective group is

$$\pi_A^R = p_A^R \phi_A \phi_B \left(\hat{\theta}_2^B - \theta^*\right) = p_A^R \phi_A \phi_B \left(\frac{p_B^O - p_A^R + t}{2t} - \theta^*\right).$$

We can now compute firm A second period profit which is equal to

$$\begin{aligned}\pi_A^2 &= p_A^O\phi_A(1-\phi_B)+p_A^O\phi_A\phi_B\left(\frac{p_B^R-p_A^O+t}{2t}\right)\\ &\quad +p_A^R\phi_A\phi_B\left(\frac{p_B^O-p_A^R+t}{2t}-\theta^*\right)\end{aligned}$$

Firm A's goal is to choose p_A^O and p_A^R as a way to maximise π_A^2.[4] From the maximisation problem, and from the first-order condition with respect to p_A^O it follows that $\frac{\partial \pi_A^O}{\partial p_A^O}=0$ from which we obtain firm A best response function given p_B^R :

$$p_A^O=\frac{t(2-\phi_B)+\phi_B p_B^R}{2\phi_B}$$

Likewise, the first-order condition with respect to p_A^R is given by $\frac{\partial \pi_A^R}{\partial p_A^R}=0$, thus firm A best response function given p_B^O :

$$p_A^R=\frac{p_B^O+t(1-2\theta^*)}{2}.$$

Analogous expressions hold for firm B. Therefore, firm B's second period profit is

$$\pi_B^2=\pi_B^O+\pi_B^R$$

where

$$\pi_B^O=p_B^O\phi_B(1-\phi_A)+p_B^O\phi_A\phi_B\left(\frac{p_A^R-p_B^O+t}{2t}\right)$$

and

$$\pi_B^R=p_B^R\phi_A\phi_B\left(\theta^*-\frac{p_B^R-p_A^O+t}{2t}\right)$$

[4] Given the strict concavity of the profit function we don't need to care about second order conditions.

We can now compute firm B second period profit which is equal to

$$\begin{aligned}\pi_B^2 &= p_B^O \phi_B (1-\phi_A) + p_B^O \phi_A \phi_B \left(\frac{p_A^R - p_B^O + t}{2t}\right) \\ &+ p_B^R \phi_A \phi_B \left(\theta^* - \frac{p_B^R - p_A^O + t}{2t}\right).\end{aligned}$$

From the first-order condition with respect to p_B^O i.e, $\frac{\partial \pi_B^O}{\partial p_B^O} = 0$, we obtain firm B's best response function given p_A^R :

$$p_B^O = \frac{t(2-\phi_A) + \phi_A p_A^R}{2\phi_A}$$

and from the first-order condition with respect to p_B^R, $\frac{\partial \pi_B^R}{\partial p_B^R} = 0$, we have that firm B's best response function given p_A^O is

$$p_B^R = \frac{p_A^O + t(2\theta^* - 1)}{2}.$$

From the four best response functions it is straightforward to obtain the following second period equilibrium prices:

$$p_A^O = \frac{t[4 + \phi_B(2\theta^* - 3)]}{3\phi_B}$$

$$p_B^O = \frac{t[4 + \phi_A(-2\theta^* - 1)]}{3\phi_A}$$

$$p_A^R = \frac{t[2 + \phi_A(1 - 4\theta^*)]}{3\phi_A}$$

$$p_B^R = \frac{t[2 + \phi_B(4\theta^* - 3)]}{3\phi_B}$$

As expected when consumers are fully informed about firms' existence, that is when $\phi = 1$ we obtain the Fudenberg and Tirole (2000) second-period equilibrium prices given the uniform distribution of consumers preferences. Specifically, when $\theta^* = \frac{1}{2}$, $p_A^O = p_B^O = \frac{2}{3}t$ and $p_A^R = p_B^R = \frac{1}{3}t$.

We can now specify both firms' second period profit function as

$$\pi_A^2 = \frac{t}{18\phi_A\phi_B}\left[\phi_A^2\left(2\phi_B\theta^* - 3\phi_B + 4\right)^2 + \phi_B^2\left(\phi_A - 4\phi_A\theta^* + 2\right)^2\right],$$

and

$$\pi_B^2 = \frac{t}{18\phi_A\phi_B}\left[\phi_B^2\left(\phi_A + 2\phi_A\theta^* - 4\right)^2 + \phi_A^2\left(4\phi_B\theta^* - 3\phi_B + 2\right)^2\right].$$

3.3.2 First-period

With the above results in hand, we can now analyze the initial period where firms make their advertising and pricing decisions rationally anticipating how such decisions will affect their profits in the subsequent period.

Assuming a common discount factor, $\delta \in]0,1]$, the present value of firm A's overall profit can be written as follows:

$$\Pi_A = \pi_A^1 + \delta\pi_A^2$$

where

$$\pi_A^1 = p_A^1\phi_A\left[\left(1-\phi_B\right) + \phi_B\left(\frac{p_B^1 - p_A^1 + t}{2t}\right)\right] - A\left(\phi_A\right) \tag{3.3}$$

and

$$\pi_A^2 = \frac{t}{18\phi_A\phi_B}\left[\phi_A^2\left(2\phi_B\theta^* - 3\phi_B + 4\right)^2 + \phi_B^2\left(\phi_A - 4\phi_A\theta^* + 2\right)^2\right] \tag{3.4}$$

Given the cutoff $\theta^* \in [0,1]$, it follows that type θ^* is indifferent between buying good A in period 1 at p_A^1 and then switch to B in period 2 and pay p_B^R, or buying B in period 1 at price p_B^1 and then switch to A and pay p_A^R. In other words, type θ^* is defined by

$$v - p_A^1 - t\theta^* + \delta\left(v - p_B^R - t\left(1-\theta^*\right)\right) = v - p_B^1 - t\left(1-\theta^*\right) + \delta\left(v - p_A^R - t\theta^*\right). \tag{3.5}$$

Solving the previous equation in order to θ^* yields

$$\theta^* = \frac{t\delta\left(2\phi_B - 2\phi_A + \phi_A\phi_B\right) + 3\phi_A\phi_B\left(t - p_A^1 + p_B^1\right)}{2t\phi_A\phi_B\left(\delta+3\right)}. \tag{3.6}$$

Firm A and B objective is to choose the price and advertising intensity as a way to maximise overall profit. Look first on firm A decisions. From $\frac{\partial \Pi_A}{\partial p_A^1} = 0$ it follows that: $\frac{d\pi_A^1}{dp_A^1} + \frac{d\pi_A^2}{dp_A^1} = 0$. However, we have that $\frac{d\pi_A^2}{dp_A^1} = \frac{d\pi_A^2}{d\theta^*}\frac{d\theta^*}{dp_A^1}$. Thus, the derivative of overall profit with respect to p_A^1 is

$$\frac{d\pi_A^1}{dp_A^1} + \frac{d\pi_A^2}{d\theta^*}\frac{d\theta^*}{dp_A^1} = 0.$$

This yields:

$$p_A^1 = \frac{10t\phi_A\left(\delta+1\right) + 2t\phi_B\left(\delta+\delta^2+4\right) - t\phi_A\phi_B\left(\delta+3\right)^2 + \phi_A\phi_B p_B^1\left(3\delta-1\right)}{2\phi_A\phi_B\left(4+3\delta\right)}.$$

Similar derivation of Π_B of with respect to p_B^1 gives:

$$p_B^1 = \frac{10t\phi_B\left(\delta+1\right) + 2t\phi_A\left(\delta+\delta^2+4\right) - t\phi_A\phi_B\left(\delta+3\right)^2 + \phi_A\phi_B p_A^1\left(3\delta-1\right)}{2\phi_A\phi_B\left(4+3\delta\right)}.$$

Therefore, firm i best response function given firm j's price is:

$$p_i = \frac{10t\phi_i\left(\delta+1\right) + 2t\phi_j\left(\delta+\delta^2+4\right) - t\phi_i\phi_j\left(\delta+3\right)^2 + \phi_i\phi_j p_j\left(3\delta-1\right)}{2\phi_i\phi_j\left(4+3\delta\right)}. \tag{3.7}$$

Now consider the equilibrium choice of advertising intensity. Plugging θ^* into the overall expected profit of firm i, it is straightforward to find that the first-order condition respect to ϕ_i. This gives firm i's best-response function with respect to ϕ_j :

$$\begin{aligned} a\phi_i &= -\frac{p_i}{2t\left(\delta+3\right)}\left(t\left(3\phi_j + \delta\phi_j - 6\right) + 3\phi_j\left(p_i - p_j\right)\right) \\ &\quad + \frac{2\delta}{3\phi_i^2\left(\delta+3\right)^2}\left(2t\left(\phi_i - \phi_j\right)\left(\delta-2\right) + 5\phi_i\phi_j\left(p_i - p_j\right)\right) \end{aligned}$$

This equation states the equality between the marginal cost of advertising $a\phi_i$ and the marginal benefit of informing an additional consumer.

As the game is symmetric we are looking for a symmetric subgame perfect nash equilibrium such that $p_A^1 = p_B^1 = p^*$ and $\phi_A = \phi_B = \phi^*$.

Proposition 4 *There is a symmetric subgame perfect nash equilibrium in which:*

(i) In period 1 the equilibrium level of advertising ϕ^* is equal to $\phi^* = \frac{t(\delta+6)-\sqrt{t^2\delta^2+72at}}{t(\delta+3)-6a}$ and satisfies the condition $0 < \phi^* \leq 1$ as long as $a \in R\backslash\left\{\frac{t(\delta+3)}{6}\right\}$. The equilibrium price is equal to $p^* = t(1+\frac{\delta}{3})\left(\frac{2-\phi^*}{\phi^*}\right)$, and both firms share equally the market in period 1, that is $\theta^*\left(p_A^1, p_B^1\right) = \frac{1}{2}$.

(ii) In period 2, equilibrium prices are $p^O = \frac{2t}{3}\left(\frac{2-\phi^*}{\phi^*}\right)$ and $p^R = \frac{t}{3}\left(\frac{2-\phi^*}{\phi^*}\right)$.

As usual in the literature of BBPD firms offer lower prices to the rival's previous customers than to old customers. It is interesting to note that firms offering lower second period prices to relatively price-sensitive segments than first period price. Loyal customer pay two times more than new customers. A key of this model is that a consumer's purchase of a rival's product in the first period implies a weaker demand of the consumer towards the firm's product in the second period. This motivates each firm to offer lower prices to its rival's customers in the second period because each firm wants to attract the competitor's previous customers.

Given the equilibrium level of advertising ϕ^*, each firm second-period equilibrium profit is equal to

$$\pi_2^* = \frac{5}{18}t\left(2-\phi^*\right)^2$$

while each firm' s first period equilibrium profit equals

$$\pi_1^* = \frac{t}{2}\left(\phi^*-2\right)^2\left(\frac{1}{3}\delta+1\right) - A\left(\phi^*\right)$$

Corollary 2 *For any $\delta \in \left]0,1\right]$, overall equilibrium profit under BBPD with advertising is equal to*

$$\Pi^* = \frac{t}{18}\left(\phi^*-2\right)^2\left(8\delta+9\right) - A(\phi^*),$$

while overall equilibrium profit without BBPD is given by

$$\Pi^{nd} = \frac{t}{2}\left(2 - \phi^{nd}\right)^2 (1 + \delta) - A\left(\phi^{nd}\right).$$

3.4 Competitive effects of price discrimination

With the above results in hand, we are now in a position to investigate how price discrimination affects the equilibrium outcomes - i.e., prices, advertising and profits. To implement this exercise we will compare the equilibrium outcomes with BBPD and advertising with those presented in the non-discrimination case (section 2.2).

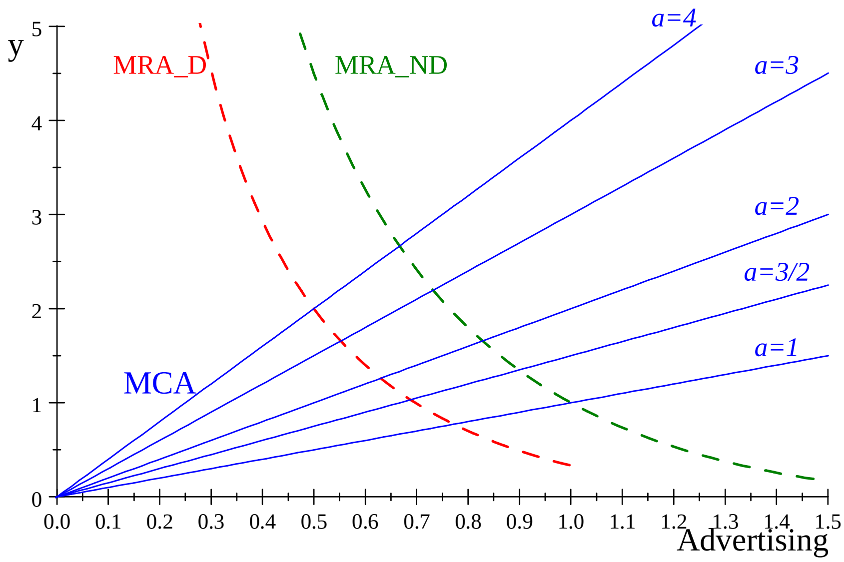
Advertising

Regarding advertising decisions if we compare the firms's advertising choices with and without advertising we observe that firms choose more advertising under no-discrimination than under discrimination. Remember that the equilibrium level of advertising with BBPD and advertising must satisfy the condition $a \in R \backslash \left\{\frac{t(\delta+3)}{6}\right\}$. The equilibrium level of advertising with no discrimination and advertising must satisfy the condition $a \geq t$. From the comparison of ϕ^* and ϕ^{nd} we can establish the following result.

Proposition 5 *From the comparison between ϕ^* and ϕ^{nd} it is true that $\phi^{nd} > \phi^*$, thus firms advertise more with no discrimination than with BBPD.*

Proof. See Appendix

Consider next the effect of price discrimination on firms' advertising decisions. To plot the functions we will assume that $\delta = 1$ and $t = 1$. From $a \in R \backslash \left\{\frac{2t}{3}\right\}$ and $a \geq t$, we will only consider that $a \geq t$, thus $a \geq 1$. The figure illustrates the downward sloping curves which represent the marginal revenue of advertising with discrimination (MRA_D). The upward sloping curves are the marginal advertising cost using the quadratic advertising cost function.The optimal level of advertising is given by the intersection between a MRA curve and the corresponding MCA. Thus, the intersection between an MRA D and MCA provides the equilibrium level of advertising with price discrimination. Similarly, the intersection between an MRA ND and MCA provides the equilibrium level of advertising with no price discrimination. From the above picture it is evident that as long as $a \geq t$ it always the case that $\phi^{nd} > \phi^*$. Note that when $t = 1$ and $a = 1$, $\phi^{nd} = 1$. For other values such that $a > 1$, $\phi^* < \phi^{nd} < 1$.



Equilibrium Advertising Level

Prices

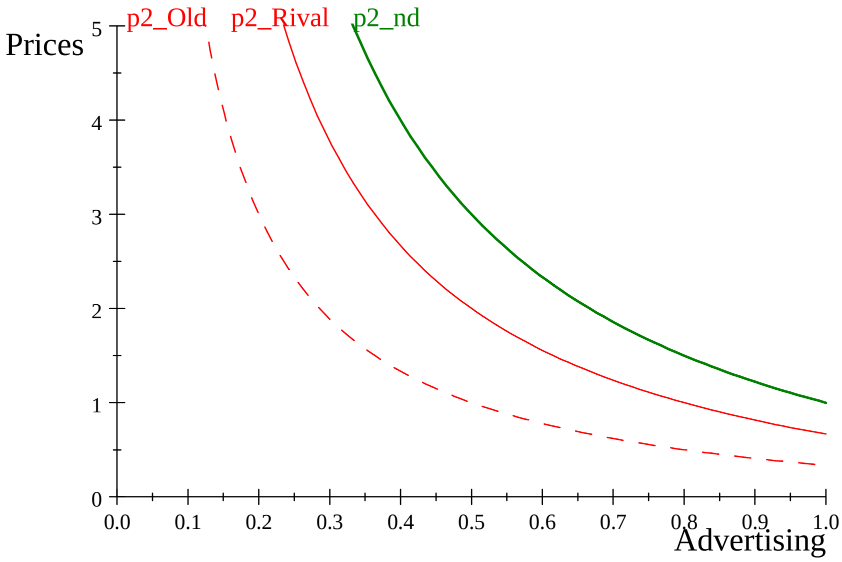
Look first at the impact of BBPD on second period prices.

Second-period prices Here we compare second period prices of the dynamic price discrimination game with second period prices under advertising and uniform pricing. We can therefore establish the following result.

Proposition 6 *When we move from the no-discrimination and advertising case to the BBPD with advertising case, second period prices fall down, thus* $p^R < p^O < p^{nd}$.

Proof. See the Appendix.

Next figure plots the equilibrium price to old and new customers (p2_Old and p2_Rival) when firms are able to price discriminate and the second-period equilibrium price under non-discrimination (p2_nd). As firms advertise more under no-discrimination, the figure shows that consumers pay always *lower* prices when firms are allowed to price discriminate. In other words, we see that second-period prices are *below* the non-discrimination counterparts. A ban on price discrimination would make consumers pay higher second-period equilibrium prices.



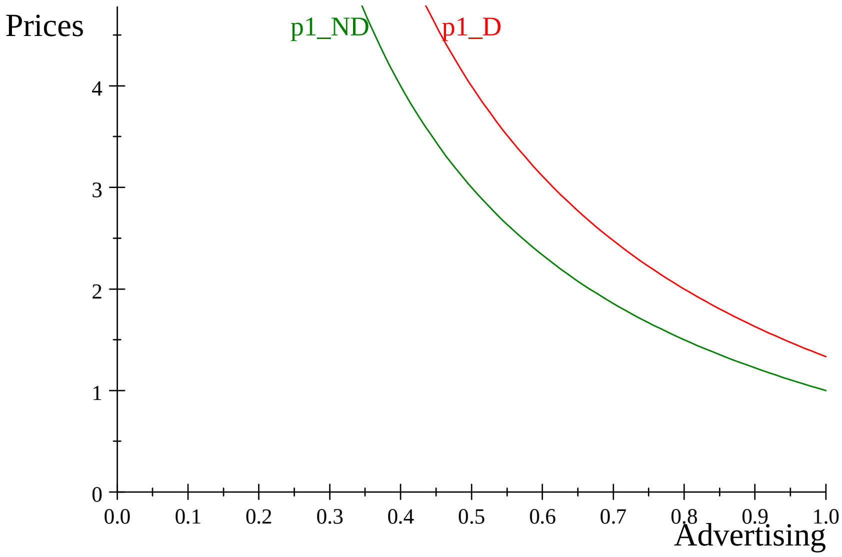
Second-period equilibrium prices

First period prices Now we compare first period equilibrium prices with BBPD and advertising with the first period equilibrium price derived in the benchmark case with advertising and no-discrimination (section 2.2). We can establish the following result.

Proposition 7 *When we move from the no-discrimination and advertising case to BBPD with advertising case, first-period prices increase, that is* $p^1 > p^{nd}$.

Proof. See the Appendix.

Next we plot the behaviour of first period prices with and with no discrimination as a function of ϕ. As firms advertise more under no-discrimination from a comparison between p_1 and p_1^{nd}, we observe that $p_1 > p_1^{nd}$ for any $\phi \in]0,1[$. Thus, first-period equilibrium prices are lower under uniform pricing than under discriminating. As in Fudenberg and Tirole (2000) price discrimination raises first period prices but reduces second-period prices. The reason is that as consumers foresee lower second-period prices due to discrimination they become less price sensitive. Consequently, firm raise the first-period price.



First-period equilibrium prices

Profits

Next we compare equilibrium profits with and without price discrimination both in period 2 and 1.

Proposition 8 *When we move from the no-discrimination and advertising case to BBPD with advertising, second period equilibrium profit decreases while first-period equilibrium profit increases.*

Proof. See the Appendix.

Proposition 9 *Firms are better off under price discrimination than under non-discrimination because overall profits increase when we move from non-discrimination to discrimination.*

Proof. See the Appendix.

This is a very relevant result of this model because it shows that in contrast to the extant literature n BBPD, price discrimination in the context of imperfect informed consumers *do not lead* to the usual prisoner's dilemma situation. A standard result in the literature on BBPD with perfect informed consumers is that overall profits decrease when firms can price discriminate. Here as in Esteves (2009a) price discrimination can benefit firms. However, the intuition for our result is different from that in Esteves (2009a). In Esteves (2009a) only one of the firms (the high-price firm in period 1) has information to discriminate in period 2. This gives rise to the race for discrimination effect according to both firms have incentives to price above non-discrimination levels in period 1. In contrast to Esteves (2009a) in the present model both firms can price discriminate in period 2. Overall profits with discrimination are above the non-discrimination levels because (i) consumers become less price sensitive in period 1 and so first period prices increase and (ii) because firms choose less advertising with discrimination which softens price competition in both periods.

The next table summarizes all the results obtained in this section.

Table 1: Main Results

Variable	Comparison
Advertising levels	$\phi^* < \phi^{nd}, a > t$
First-period prices	$p^1 > p^{nd}$
Second-period prices to old customers	$p^O < p^{nd}$
Second-period prices to rival customers	$p^R < p^{nd}$
First-period profits	$\pi_1^* > \pi_1^{nd}$
Second-period profits	$\pi_2^* < \pi_2^{nd}$
Overall profits	$\Pi^* > \Pi^{nd}$

3.5 Competitive effects of imperfect information

Basically the aim of this section is to investigate how first and second period prices behave as we move from BBPD with perfect to BBPD with imperfect information. To do so, we compare the equilibrium price solutions of BBPD with perfect information ($\phi = 1$) with the equilibrium solutions with BBPD under imperfect information. Remember that the case of BBPD with perfect information is the case of Fudenberg and Tirole (2000) which was presented in section 2.3.

The impact of information on prices is obtained by comparing the equilibrium prices under full and imperfect information.

Second-period prices We have seen that when information is perfect ($\phi = 1$) $p_A^O = p_B^O = \frac{2}{3}t$ and $p_A^R = p_B^R = \frac{1}{3}t$. In contrast under imperfect information we have found that $p_A^O = p_B^O = \frac{2t}{3}\left(\frac{2-\phi}{\phi}\right)$ and $p_A^R = p_B^R = \frac{t}{3}\left(\frac{2-\phi}{\phi}\right)$. It is easy to see the perfect information prices are below the imperfect information counterparts as long as $\phi < 1$. Thus, the discriminatory prices under imperfect information are above those under perfect information. Note that as firms advertise more, consumers become better informed and the group of selective consumers increase. As a result of that price competition increases and prices decrease.

First-period prices Regarding the effect of imperfect information on first-period prices remember that under perfect information $p^1 = t(1 + \frac{\delta}{3})$ while with imperfect information it is equal to $p^{1*} = t(1 + \frac{\delta}{3})\left(\frac{2-\phi}{\phi}\right)$. Again it is straightforward to see that the first period price with perfect information is below first-period equilibrium price under imperfect information as long as $\phi < 1$.

3.6 Welfare analysis

This section investigates the effects of BBPD with targeted advertising on profits, consumer surplus and on total welfare. When price discrimination is permitted, any consumer who observes a message from at least one firm participate in the market. To model the effects of price discrimination, it is enough to consider two groups of buyers: captive consumers and selective consumers, that is those that in fact buy the good. We construct a welfare function taken into account the consumer surplus for all consumers consuming the good less the firms' advertising costs. The gross benefits to consumers are a function of the surplus created by each consumer consuming her ideal product less the average transportation cost incurred by a consumer in the market. To simplify the analysis, throughout this section, it is assumed that $\delta = 1$. Total welfare in period 1 is equal to:

$$w_1^* = (\phi^*)^2 \left(v - 2 \int_0^{1/2} t\theta d\theta \right) + \phi^* (1 - \phi^*) \left(v - 2 \int_0^1 t\theta d\theta \right) - 2A (\phi^*)$$

To simplify our analysis, we can write this equation as:

$$w_1^* = v\phi^* - \frac{1}{4}\phi^* t (4 - 3\phi^*) - 2A (\phi^*)$$

Following the same reasoning, in the period 2, we need to take into account that price discrimination makes some consumers buy inefficiently as some of them buy from the least preferred firm. Second-period equilibrium welfare is equal to:

$$\begin{aligned} w_2^* &= \phi^* (1 - \phi^*) \left(v - 2 \int_0^1 t\theta d\theta \right) + v (\phi^*)^2 + (\phi^*)^2 \left(- \int_0^{\hat{\theta}_A} t\theta d\theta - \int_{\hat{\theta}_A}^{\theta_1} t(1-\theta) d\theta \right) \\ &\quad + (\phi^*)^2 \left(- \int_{\theta_1}^{\hat{\theta}_B} t\theta d\theta - \int_{\hat{\theta}_B}^1 t(1-\theta) d\theta \right) \end{aligned}$$

Substituting the equilibrium values for $\hat{\theta}_A$,$\hat{\theta}_B$and θ_1, we obtain

$$w_2^* = v\phi^* - t \left(\phi^* (1 - \phi^*) + \frac{11 (\phi^*)^2 + 8 (1 - \phi^*)}{36} \right)$$

Overall welfare is equal to $W^* = w_1^* + w_2^*$, which simplifies to

$$W^* = 2v\phi^* - t\left(\phi^*\left(1-\phi^*\right) + \frac{11\left(\phi^*\right)^2 + 8\left(1-\phi^*\right)}{36} + \frac{1}{4}\phi^*\left(4-3\phi^*\right)\right) - 2A\left(\phi^*\right).$$

In equilibrium industry profit under BBPD with advertising (and so with imperfect information) is

$$\Pi_{ind}^* = \frac{34t}{18}\left(\phi^* - 2\right)^2 - 2A\left(\phi^*\right).$$

We can now compute overall consumer surplus using the fact that $ECS^* = W^* - \Pi_{ind}^*$. This simplifies to:

$$ECS^* = 2v\phi^* - \frac{2}{9}t\left(2\phi^{*2} - 26\phi^* + 35\right)$$

Next we compare welfare, consumer surplus and industry profits in different scenarios, for different values of t and a taken into account the restriction imposed between t and a derived in the equilibrium analysis of the no discrimination case which is $a > t$. Social welfare without price discrimination (W^{nd}), with price discrimination with advertising, and thus with imperfect information (W^*) and with price discrimination based on perfect information(W^p) is presented in table 1. Table 2 presents consumer surplus and industry profits for the same scenarios. In order to facilitate the analysis, welfare, consumer surplus and equilibrium industry profit computed assuming that $v = 10$.

Table 2: Social Welfare

t	a	ϕ^{nd}	ϕ^{*}	W^{nd}	W^{*}	W^{P}
1	$a = 1.5$	0.89898	0.68806	1.2122	2.5410	19.444
1	$a = 2$	0.82843	0.63020	1.3726	2.7499	19.444
1	$a = 3$	0.73205	0.55221	1.6077	3.0445	19.444
1	$a = 4$	0.66667	0.5	1.7777	3.25	19.444
2	$a = 2.5$	0.9442	0.72554	2.2301	4.82	18.889
2	$a = 3$	0.89898	0.68806	2.4245	5.0820	18.889
2	$a = 4$	0.82843	0.63020	2.7451	5.4998	18.889
3	$a = 3.5$	0.96148	0.73985	3.2356	7.0827	18.333
3	$a = 4$	0.9282	0.71221	3.4463	7.3686	18.333

Table 3: Consumer Surplus and Industry Profits

t	a	Π^{nd}_{ind}	Π^{*}_{ind}	Π^{P}_{ind}	ECS^{nd}	ECS^{*}	ECS^{P}
1	$a = 1.5$	1.2122	2.5410	1.8889	15.868	9.7485	17.556
1	$a = 2$	1.3726	2.7499	1.8889	13.196	8.2909	17.556
1	$a = 3$	1.6077	3.0445	1.8889	10.765	6.3214	17.556
1	$a = 4$	1.7777	3.25	1.8889	9.1112	5.0	17.556
2	$a = 2.5$	2. 2301	4.82	3.7778	13.323	6.8713	15.111
2	$a = 3$	2. 4245	5.0820	3.7778	11.959	5.7357	15.111
2	$a = 4$	2.7451	5.4998	3.7778	9. 8235	3.9777	15.111
3	$a = 3.5$	3.2356	7.0827	5.6667	11.150	3.5579	12.667
3	$a = 4$	3.4463	7.3686	5.6667	9.9793	2.5795	12.667

Based on our numerical analysis we can establish the following results.

Proposition 10 *As long as $a > t$, when we move from no discrimination to BBPD with advertising:*

(i) industry profits increase;

(ii) consumer surplus falls down, and

(i) social welfare increases.

Proposition 11 *As long as $a \in R \backslash \left\{ \frac{t(\delta+3)}{6} \right\}$ when we move from BBPD with perfect information to BBPD with imperfect information:*

(i) industry profits increase;

(ii) consumer surplus falls down, and

(i) social welfare decreases.

Proposition 9 and 10 suggest that it is important to investigate the economic and welfare effects of price discrimination in markets with imperfect informed consumers. While industry profits decrease with BBPD in markets with perfect informed consumers, the same might not occur under imperfect informed consumers. The simple model developed shows that advertising might help firms to introduce imperfect information into the market which may act to soften price competition. As a result of that BBPD with advertising might boost industry at the expense of consumer surplus.

Chapter 4

Conclusions

The economics literature on oligopoly price discrimination by purchase history is relatively new and has focused mostly on markets with perfectly informed consumers. With the exception of Esteves (2009a) the possibility of firms being able to use advertising as a way to transmit relevant information to otherwise uninformed consumers has not been considered. This dissertation has taken a step in investigating the impact of Behaviour-Based Price Discrimination in markets where through the firms' advertising decisions consumers become imperfectly informed. That is, this thesis has provided a first study about the dynamic effects of customer poaching in horizontally product differentiated markets. We developed a two-period Hotelling model with BBPD and informative advertising. In the second period firms can recognize their old and the rival previous consumers and they may send them targeted ads with different prices.

A relevant theme of this dissertation was the investigation of the effects of BBPD on firm's advertising and pricing strategies and thus on profits and welfare. It was shown that being price discrimination permitted profits are higher under imperfect informed consumers than under fully informed consumers. More interesting are the results obtained when we move from the benchmark case where firm advertise but price discrimination is not permitted to the case with advertising and BBPD. We show that BBPD boosts industry profit and welfare at the expense of consumer surplus. Thus, BBPD doesn't necessarily lead to a prisioners' dilemma situation. This suggests that in fact a good economic understanding of the effects of BBPD should take into account different forms of market competition.

Appendix

This appendix collects the proofs that were omitted from the text.

Proof of Proposition 1: In period 1 firms simultaneously chooses prices and advertising levels. Each firm goal is to solve the following maximization problem:

$$\max_{p_i,\phi_i}\left\{(1+\delta)\left[p_i\phi_i\left(1-\phi_j\right)+p_i\phi_i\phi_j\left(\frac{p_j-p_i+t}{2t}\right)\right]-A\left(\phi_i\right)\right\}.$$

From the above problem we get the FOC with respect to p_i given by equation (2.3) which is:

$$p_i=\frac{1}{2\phi_j}\left(2t-t\phi_j+\phi_j p_j\right).$$

As we are looking for a symmetric equilibrium it must be true that $p_i=p_j=p^{nd}$ thus

$$\begin{aligned} p^{nd} &= \frac{1}{2\phi_j}\left(2t-t\phi_j+\phi_j p^{nd}\right) \\ p^{nd} &= \frac{t\left(2-\phi_j\right)}{\phi_j}. \end{aligned}$$

The FOC with respect to ϕ_i defined in equation (2.4) is:

$$\phi_i=\frac{1}{a}p_i\left(\delta+1\right)\left(1-\phi_j+\phi_j\left(\frac{p_j-p_i+t}{2t}\right)\right).$$

Because the game is symmetric, we are looking for a symmetric equilibrium $\left(\phi_i=\phi_j=\phi^{nd}\right.$ and $\left.p_i=p_j=p^{nd}\right)$. The above equation simplifies to:

$$\frac{t\left(\delta+1\right)}{2a}=\left(\frac{\phi^{nd}}{2-\phi^{nd}}\right)^2.$$

Solving this equation in order ϕ^{nd} we have:

$$\frac{\phi^{nd}}{2-\phi^{nd}} = \pm\sqrt{\frac{t(\delta+1)}{2a}}$$

$$\phi^{nd} = \left(2-\phi^{nd}\right)\sqrt{\frac{t(\delta+1)}{2a}} \vee \phi^{nd} = -\left(2-\phi^{nd}\right)\sqrt{\frac{t(\delta+1)}{2a}}$$

$$\phi^{nd} = 2\sqrt{\frac{t(\delta+1)}{2a}} - \phi^{nd}\sqrt{\frac{t(\delta+1)}{2a}} \vee \phi^{nd} = \phi^{nd}\sqrt{\frac{t(\delta+1)}{2a}} - 2\sqrt{\frac{t(\delta+1)}{2a}}$$

$$\phi^{nd}\left(1+\sqrt{\frac{t(\delta+1)}{2a}}\right) = 2\sqrt{\frac{t(\delta+1)}{2a}} \vee \phi^{nd}\left(1-\sqrt{\frac{t(\delta+1)}{2a}}\right) = -2\sqrt{\frac{t(\delta+1)}{2a}}$$

$$\phi^{nd} = \frac{2\sqrt{\frac{t(\delta+1)}{2a}}}{1+\sqrt{\frac{t(\delta+1)}{2a}}} \vee \phi^{nd} = \frac{-2\sqrt{\frac{t(\delta+1)}{2a}}}{1-\sqrt{\frac{t(\delta+1)}{2a}}}$$

$$\phi^{nd} = \frac{2}{1+\sqrt{\frac{2a}{t(\delta+1)}}} \vee \phi^{nd} = \frac{2}{1-\sqrt{\frac{2a}{t(\delta+1)}}}.$$

Since we obtain two solutions, it is important to impose that $0 < \phi^{nd} \leq 1$. So, we must verify in which circumstances the previous condition is valid. Starting with the first solution, it is obvious that $\phi^{nd} > 0$. It is also straightforward to see that $\phi^{nd} \leq 1$, iff $a \geq \frac{t(\delta+1)}{2}$.

Look now to the second solution, i.e., $\phi^{nd} = \frac{2}{1-\sqrt{\frac{2a}{t(\delta+1)}}}$. We observe that $\phi^{nd} > 0$ iff $a < \frac{t(\delta+1)}{2}$ and $\phi^{nd} \leq 1$ iff $a \geq \frac{t(\delta+1)}{2}$ which is impossible. Thus, this solution doesn't satisfy the condition $0 < \phi^{nd} \leq 1$. Therefore, the only valid solution is $\phi^{nd} = \frac{2}{1+\sqrt{\frac{2a}{t(\delta+1)}}}$ as long as $a \geq \frac{t(\delta+1)}{2}$. Replacing ϕ^{nd} by $\frac{2}{1+\sqrt{\frac{2a}{t(\delta+1)}}}$ into the equation $p^{nd} = \frac{t\left(2-\phi^{nd}\right)}{\phi^{nd}}$ we obtain $p^{nd} = \sqrt{\frac{2at}{(\delta+1)}}$.$\blacksquare$

Proof of Corollary 1: Using the expression for overall profit with no discrimination and the quadratic advertising cost function, we have that:

$$\Pi^{nd} = (1+\delta)\, p^{nd}\phi^{nd}\left(\frac{2-\phi^{nd}}{2}\right) - \frac{a\left(\phi^{nd}\right)^2}{2}.$$

Using now the equilibrium solutions for p^{nd} and ϕ^{nd} we obtain:

$$\begin{aligned}
\Pi^{nd} &= \frac{2(1+\delta)\sqrt{\frac{2at}{(\delta+1)}}}{2\left(1+\sqrt{\frac{2a}{t(1+\delta)}}\right)}\left(2-\frac{2}{1+\sqrt{\frac{2a}{t(1+\delta)}}}\right) \\
&\quad -\frac{a}{2}\left(\frac{2}{1+\sqrt{\frac{2a}{t(1+\delta)}}}\right)^2 \\
&= \frac{(1+\delta)\sqrt{\frac{2at}{(\delta+1)}}}{1+\sqrt{\frac{2a}{t(1+\delta)}}}\left(\frac{2+2\sqrt{\frac{2a}{t(1+\delta)}}-2}{1+\sqrt{\frac{2a}{t(1+\delta)}}}\right) \\
&\quad -\frac{a}{2}\left(\frac{2}{1+\sqrt{\frac{2a}{t(1+\delta)}}}\right)^2 \\
&= \frac{(1+\delta)\sqrt{\frac{2at}{(\delta+1)}}}{1+\sqrt{\frac{2a}{t(1+\delta)}}}\left(\frac{2\sqrt{\frac{2a}{t(1+\delta)}}}{1+\sqrt{\frac{2a}{t(1+\delta)}}}\right)-\frac{a}{2}\left(\frac{2}{1+\sqrt{\frac{2a}{t(1+\delta)}}}\right)^2 \\
&= \frac{2(1+\delta)\sqrt{\frac{4a^2t}{t(1+\delta)^2}}}{\left(1+\sqrt{\frac{2a}{t(1+\delta)}}\right)^2}-\frac{a}{2}\left(\frac{2}{1+\sqrt{\frac{2a}{t(1+\delta)}}}\right)^2 \\
&= \frac{4(1+\delta)\sqrt{\frac{a^2}{(1+\delta)^2}}}{\left(1+\sqrt{\frac{2a}{t(1+\delta)}}\right)^2}-\frac{a}{2}\left(\frac{2}{1+\sqrt{\frac{2a}{t(1+\delta)}}}\right)^2 \\
&= \frac{4a}{\left(1+\sqrt{\frac{2a}{t(1+\delta)}}\right)^2}-\frac{2a}{\left(1+\sqrt{\frac{2a}{t(1+\delta)}}\right)^2} \\
&= \frac{2a}{\left(1+\sqrt{\frac{2a}{t(1+\delta)}}\right)^2}. \blacksquare
\end{aligned}$$

Proof of Proposition 4: We consider that firms discount future profits by a factor δ. Consider first the case of firm A. A consumer located at θ is indifferent between buying product A or B if $\theta = \frac{p_B^1 - p_A^1 + t}{2t}$. Its overall profit can be written as follow:

$$\Pi_A = \pi_A^1 + \delta \pi_A^2$$

where

$$\begin{aligned}\pi_A^1 &= p_A^1 \phi_A \left[(1-\phi_B) + \phi_B \theta\right] - A(\phi_A) \\ &= \frac{p_A \left(6t\phi_A + 2t\delta\phi_B - 3t\phi_A\phi_B - 3\phi_A\phi_B p_A + 3\phi_A\phi_B p_B - t\delta\phi_A\phi_B\right)}{6t + 2t\delta} - A(\phi_A)\end{aligned}$$

and

$$\begin{aligned}\pi_A^2 &= \frac{2}{9}t\phi_B - \frac{4}{3}t\phi_A + \frac{5}{9}t\phi_A\phi_B + \frac{8}{9}t\phi_A\theta^* - \frac{8}{9}t\phi_B\theta^* \\ &\quad + \frac{8}{9}t\frac{\phi_A}{\phi_B} + \frac{2}{9}\frac{t}{\phi_A}\phi_B + \frac{10}{9}t\phi_A\phi_B(\theta^*)^2 - \frac{10}{9}t\phi_A\phi_B\theta^*.\end{aligned}$$

As a remark notice that we are assuming the the first period cutoff is equal to

$$\theta^* = \frac{t\delta\left(2\phi_B - 2\phi_A + \phi_A\phi_B\right) + 3\phi_A\phi_B\left(t - p_A^1 + p_B^1\right)}{2t\phi_A\phi_B(\delta+3)}.$$

The derivate of overall profit with respect to p_A is $\frac{d\pi_A^1}{dp_A} + \frac{d\pi_A^2}{dp_A} = 0$ which can be writen as:

$$\frac{d\pi_A^1}{dp_A} + \frac{d\pi_A^2}{d\theta^*}\frac{d\theta^*}{dp_A} = 0$$

where

$$\begin{aligned}\frac{d\pi_A^1}{dp_A} &= \frac{6t\phi_A + 2t\delta\phi_B - 3t\phi_A\phi_B - 6\phi_A\phi_B p_A + 3\phi_A\phi_B p_B - t\delta\phi_A\phi_B}{2t(\delta+3)} \\ &= \frac{3t\phi_A + t\delta\phi_B - \frac{3}{2}t\phi_A\phi_B - 3\phi_A\phi_B p_A + \frac{3}{2}\phi_A\phi_B p_B - \frac{1}{2}t\delta\phi_A\phi_B}{t(\delta+3)}\end{aligned}$$

$$\frac{d\pi_A^2}{d\theta^*} = \frac{8}{9}t\phi_A - \frac{8}{9}t\phi_B - \frac{10}{9}t\phi_A\phi_B + \frac{20}{9}t\theta^*\phi_A\phi_B$$

$$\frac{d\theta^*}{dp_A} = -\frac{3}{6t+2t\delta}.$$

Using the previous expression we have that:

$$\begin{aligned}\frac{d\pi_A^2}{d\theta^*}\frac{d\theta^*}{dp_A} &= \left(\frac{8}{9}t\phi_A - \frac{8}{9}t\phi_B - \frac{10}{9}t\phi_A\phi_B + \frac{20}{9}t\theta^*\phi_A\phi_B\right)\left(-\frac{3}{6t+2t\delta}\right) \\ &= -\frac{4\phi_A - 4\phi_B - 5\phi_A\phi_B + 10\phi_A\phi_B\theta^*}{3\delta+9} \\ &= -\frac{4t\phi_A - 4t\phi_B - 2t\delta\phi_A + 2t\delta\phi_B - 5\phi_A\phi_B p_A + 5\phi_A\phi_B p_B}{t\left(\delta+3\right)^2}.\end{aligned}$$

Therefore, from $\frac{d\pi_A^1}{dp_A} + \frac{d\pi_A^2}{d\theta^*}\frac{d\theta^*}{dp_A} = 0$ it follows that:

$$\begin{aligned}0 &= \frac{1}{3t+t\delta}\left(3t\phi_A + t\delta\phi_B - \frac{3}{2}t\phi_A\phi_B - 3\phi_A\phi_B p_A + \frac{3}{2}\phi_A\phi_B p_B - \frac{1}{2}t\delta\phi_A\phi_B\right) \\ &\quad -\frac{1}{t\left(\delta+3\right)^2}\left(4t\phi_A - 4t\phi_B - 2t\delta\phi_A + 2t\delta\phi_B - 5\phi_A\phi_B p_A + 5\phi_A\phi_B p_B\right)\end{aligned}$$

and so, the best-response price function of firm A with respect to p_B is:

$$p_A = \frac{(1+\delta)\,t\left(10\phi_A + 2\delta\phi_B\right) + t\phi_B\left(8 - 9\phi_A\right) - t\delta\phi_A\phi_B(\delta+6) + \phi_A\phi_B p_B\left(3\delta - 1\right)}{\phi_A\phi_B\left(8+6\delta\right)}.$$

Looking now at the FOC with respect to ϕ_A given by $\frac{\partial\Pi_A}{\partial\phi_A} = 0$ we have:

$$\begin{aligned}\frac{d\pi_A^1}{d\phi_A} + \frac{d\pi_A^2}{d\phi_A} &= 0 \text{ which simplifies to} \\ \frac{d\pi_A^1}{d\phi_A} + \frac{d\pi_A^2}{d\theta^*}\frac{d\theta^*}{d\phi_A} &= 0.\end{aligned}$$

Using the fact that

$$\begin{aligned}\frac{d\pi_A^1}{d\phi_A} &= -\frac{p_A\left(3t\phi_B - 6t + 3\phi_B p_A - 3\phi_B p_B + t\delta\phi_B\right)}{6t+2t\delta} - a\phi_A \\ &= -\frac{p_A\left(3\phi_B\left(t + p_A - p_B\right) - 6t + t\delta\phi_B\right)}{6t+2t\delta} - a\phi_A,\end{aligned}$$

$$\frac{d\theta^*}{d\phi_A} = -\frac{\delta}{\phi_A^2\left(\delta+3\right)}$$

and

$$\begin{aligned}\frac{d\pi_A^2}{d\theta^*}\frac{d\theta^*}{d\phi_A} &= \left(\frac{8}{9}t\phi_A - \frac{8}{9}t\phi_B - \frac{10}{9}t\phi_A\phi_B + \frac{20}{9}t\theta^*\phi_A\phi_B\right)\left(-\frac{\delta}{\phi_A^2(\delta+3)}\right) \\ &= \frac{2\delta}{3\phi_A^2(\delta+3)^2}\left(2t(2-\delta)(\phi_B-\phi_A)+5\phi_A\phi_B(p_A-p_B)\right).\end{aligned}$$

Therefore, from $\frac{\partial \Pi_A}{\partial \phi_A} = 0$ we get:

$$\begin{aligned}a\phi_A &= \frac{2\delta\left(2t(2-\delta)(\phi_B-\phi_A)+5\phi_A\phi_B(p_A-p_B)\right)}{3\phi_A^2(\delta+3)^2} \\ &\quad -\frac{p_A\left(3\phi_B(t+p_A-p_B)+t(\delta\phi_B-6)\right)}{2t(\delta+3)}.\end{aligned}$$

from which we obtain firm A's best-response advertising function given ϕ_j.

$$\begin{aligned}a\phi_A &= \frac{2\delta\left(2t(\phi_A-\phi_B)(\delta-2)+5\phi_A\phi_B(p_A-p_B)\right)}{3\phi_A^2(\delta+3)^2} \\ &\quad -\frac{p_A\left(-6t+\phi_B(3t+3p_A-3p_B+t\delta)\right)}{2t(\delta+3)}.\end{aligned}$$

Symmetric expressions hold for firm B's best-response functions:

$$p_B = \frac{(1+\delta)t(10\phi_B+2\delta\phi_A)+t\phi_A(8-9\phi_B)-t\delta\phi_A\phi_B(\delta+6)+\phi_A\phi_B p_A(3\delta-1)}{\phi_A\phi_B(8+6\delta)}$$

and

$$\begin{aligned}a\phi_B &= \frac{2\delta\left(-2t(\phi_A-\phi_B)(\delta-2)-5\phi_A\phi_B(p_A-p_B)\right)}{3\phi_B^2(\delta+3)^2} \\ &\quad -\frac{p_B\left(-6t+\phi_A(3t-3p_A+3p_B+t\delta)\right)}{2t(\delta+3)}.\end{aligned}$$

Since we are looking for a symmetric equilibrium it must be the case that $p_A = p_B = p$ and $\phi_A = \phi_B = \phi^*$. Evaluating both firms' best resonse functions in the symmetric conditions it is easy to find the first period price and advertising equilibrium given by

$$\begin{aligned} p &= \frac{1}{3\phi^*}\left(6t + 2t\delta - 3t\phi^* - t\delta\phi^*\right) \\ &= t(1 + \frac{\delta}{3})\left(\frac{2 - \phi^*}{\phi^*}\right) \end{aligned}$$

and

$$\phi^* = \frac{t\left(6 + \delta\right) - \sqrt{t^2\delta^2 + 72at}}{t\left(3 + \delta\right) - 6a} \vee \phi^* = \frac{t\left(6 + \delta\right) + \sqrt{t^2\delta^2 + 72at}}{t\left(3 + \delta\right) - 6a}.$$

Remember that the equilibrium level of advertising must satisfy the condition $\phi^* \in]0, 1]$. As we have two possible solutions we go now to prove that the solution $\phi^* = \frac{t(6+\delta) - \sqrt{t^2\delta^2 + 72at}}{t(3+\delta) - 6a}$ satisfies the conditions $\phi^* > 0$ and $\phi^* \leq 1$.

Look first at the condition $\phi^* > 0$. Next we study in which circumstances the expression $\frac{t(6+\delta) - \sqrt{t^2\delta^2 + 72at}}{t(3+\delta) - 6a}$ is positive. We have to take into account the sign of the numerator and denominator simultaneously. The simplest process is to draw up a framework where we study the sign of the numerator and denominator separately and then the sign of the whole expression. Note that $t > 0$. The next computations show that $a = \frac{t}{6}\left(\delta + 3\right)$ is a zero of both the numerator and denominator.

$$\begin{aligned} 6t + t\delta - \sqrt{t^2\delta^2 + 72at} &= 0 \\ a &= -\frac{1}{72t}\left(t^2\delta^2 - (6t + t\delta)^2\right) \\ a &= \frac{t}{6}\left(\delta + 3\right) \end{aligned}$$

$$\begin{aligned} -6a + 3t + t\delta &= 0 \\ a &= \frac{1}{2}t + \frac{1}{6}t\delta \\ a &= \frac{t}{6}\left(\delta + 3\right) \end{aligned}$$

The next table studies the sign of the expression $\frac{t(6+\delta) - \sqrt{t^2\delta^2 + 72at}}{t(3+\delta) - 6a}$ for values of $t > 0$.

a	0		$\frac{t}{6}(\delta+3)$	$+\infty$
$6t+t\delta-\sqrt{t^2\delta^2+72at}$	$n.d.$	$+$	0	$-$
$-6a+3t+t\delta$	$n.d.$	$+$	0	$-$
$\frac{6t+t\delta-\sqrt{t^2\delta^2+72at}}{-6a+3t+t\delta}$	$n.d.$	$+$	$n.d.$	$+$

From the table it is easy to see that follows that $\frac{t(6+\delta)-\sqrt{t^2\delta^2+72at}}{t(3+\delta)-6a}$ is positive iff $a \in \mathbb{R}/\left\{\frac{t}{6}(\delta+3)\right\}$. Looking now at the condition $\phi^* \leq 1$, we have to solve the inequality $\frac{t(6+\delta)-\sqrt{t^2\delta^2+72at}}{t(3+\delta)-6a} \leq 1$. To solve this inequality, one can rewrite the previous inequality as $\frac{t(6+\delta)-\sqrt{t^2\delta^2+72at}}{t(3+\delta)-6a} - 1 \leq 0$, which simplifies to:

$$\frac{6a+3t-\sqrt{t^2\delta^2+72at}}{t(3+\delta)-6a} \leq 0.$$

Then, we find zeros of the numerator and the denominator. From

$$6a+3t-\sqrt{t^2\delta^2+72at} = 0$$

we have

$$a = \frac{t}{6}(3-\delta) \vee a = \frac{t}{6}(\delta+3).$$

From

$$-6a+3t+t\delta = 0$$

we obtain

$$a = \frac{t}{6}(\delta+3).$$

Regarding the computation of the numerator's zeros we need to take into account that as we have an irrational equation, some caution is needed with the solutions obtained. This is due to the square. Therefore we need to assess whether, in fact, the two solutions obtained are solutions of the equation given. From the evaluation of $6a+3t-\sqrt{t^2\delta^2+72at}$ at the solution $a = \frac{t}{6}(3-\delta)$ we obtain $-2t(\delta-6)$ which is different from zero. Thus, $a = \frac{t}{6}(3-\delta)$ is not solution of the equation.

Likewise, from the evaluation of $6a+3t-\sqrt{t^2\delta^2+72at}$ at the solution $a=\frac{t}{6}(3+\delta)$, we find that $6a+3t-\sqrt{t^2\delta^2+72at}=3t-t(\delta+6)+t(\delta+3)$, which is in fact equal to zero. Thus $a=\frac{t}{6}(3+\delta)$ is a solution of the equation. Therefore this equation has only one solution given by $a=\frac{t}{6}(3+\delta)$.

Knowing the values that override the numerator and denominator, we move on to build below a table of signs.

a	0		$\frac{t}{6}(\delta+3)$	$+\infty$
$6a+3t-\sqrt{t^2\delta^2+72at}$	$n.d.$	$-$	0	$+$
$-6a+3t+t\delta$	$n.d.$	$+$	0	$-$
$\frac{6a+3t-\sqrt{t^2\delta^2+72at}}{-6a+3t+t\delta}$	$n.d.$	$-$	$n.d.$	$-$

From this table it follows that for any $a\in\mathbb{R}/\left\{\frac{t}{6}(\delta+3)\right\}$ it is always true that $\phi^*\leq 1$.

From the results of the two previous tables, we conclude that $0<\phi^*\leq 1$ for any $a\in\mathbb{R}/\left\{\frac{t}{6}(\delta+3)\right\}$. Regarding the second solution for ϕ^* given by $\phi^*=\frac{t(6+\delta)+\sqrt{t^2\delta^2+72at}}{t(3+\delta)-6a}$, the condition $\phi^*>0$ implies that $a<\frac{t}{6}(\delta+3)$ and likewise to garante $\phi^*\leq 1$ it must be the case that $a>\frac{t}{6}(\delta+3)$, which is impossible. Thus, the only valid solution is $\phi^*=\frac{t(6+\delta)-\sqrt{t^2\delta^2+72at}}{t(3+\delta)-6a}$.

Now we prove (ii). Notice that $\theta^*=\frac{t\delta(2\phi_B-2\phi_A+\phi_A\phi_B)+3\phi_A\phi_B\left(t-p_A^1+p_B^1\right)}{2t\phi_A\phi_B(\delta+3)}$. In a symmetric it must be true that $\theta^*\left(p_A^1,p_B^1\right)=\frac{1}{2}$. Remember that the second period equilibrium prices are given by:

$$
\begin{aligned}
p_A^O &= \frac{t\left[4+\phi_B\left(2\theta^*-3\right)\right]}{3\phi_B}\\
p_B^O &= \frac{t\left[4+\phi_A\left(-2\theta^*-1\right)\right]}{3\phi_A}\\
p_A^R &= \frac{t\left[2+\phi_A\left(1-4\theta^*\right)\right]}{3\phi_A}\\
p_B^R &= \frac{t\left[2+\phi_B\left(4\theta^*-3\right)\right]}{3\phi_B}.
\end{aligned}
$$

Replacing in last conditions θ^* by $\frac{1}{2}$and ϕ^*into ϕ_A and ϕ_B, and taking into account that $p_A^O = p_B^O = p^O$ $p_A^R = p_B^R = p^R$ we obtain that the second-period equilibrium prices are

$$\begin{aligned} p^O &= \frac{2t}{3}\left(\frac{2-\phi^*}{\phi^*}\right) \\ p^R &= \frac{t}{3}\left(\frac{2-\phi^*}{\phi^*}\right). \end{aligned}$$

This completes the proof. ■

Proof of Corollary 2: Using Proposition 4 we can now compute firm A overall profit with BBPD. Remember that $\Pi_A = \pi_A^1 + \delta\pi_A^2 - A(\phi^*)$. Evaluating $\pi_A^1 = p_A\phi_A[(1-\phi_B)+\phi_B\theta] - A(\phi_A)$ at p^* and ϕ^* we have that:

$$\begin{aligned} \pi_A^1 &= p^*\phi^*\left((1-\phi^*)+\frac{1}{2}\phi^*\right) - A(\phi^*) \\ &= \frac{1}{2}\phi^*\left(p^*(2-\phi^*)\right) - A(\phi^*) \end{aligned}$$

Doing the same to π_A^2 we obtain:

$$\begin{aligned} \pi_A^2 &= \frac{2}{9}t\phi^* - \frac{4}{3}t\phi^* + \frac{5}{9}t(\phi^*)^2 + \frac{8}{18}t\phi^* - \frac{8}{18}t\phi^* + \frac{8}{9}t + \frac{2}{9}t + \frac{10}{36}t(\phi^*)^2 - \frac{10}{18}t(\phi^*)^2 \\ &= \frac{5}{18}t(\phi^*-2)^2 \end{aligned}$$

In sum, the overall objective function is

$$\Pi_A = \frac{1}{2}\phi^*\left(p^*(2-\phi^*)\right) + \frac{5\delta}{18}t(\phi^*-2)^2 - A(\phi^*)$$

Thus, each firm overall profit with BBPD with advertising equals

$$\Pi^* = \frac{t}{18}(\phi^*-2)^2(8\delta+9) - A(\phi^*) \blacksquare$$

Proof of Proposition 5: In order to compare $\phi^* = \frac{t(6+\delta)-\sqrt{t^2\delta^2+72at}}{t(3+\delta)-6a}$ and $\phi^{nd} = \frac{2}{1+\sqrt{\frac{2a}{t(1+\delta)}}}$ we can start with the hypothesis that $\phi^* < \phi^{nd}$, and investigate whether this condition is true or false. For simplify let us consider $\delta = 1$. Then from $\phi^* < \phi^{nd}$ we obtain:

$$\frac{\sqrt{t^2+72at}-7t}{6a-4t} < \frac{2}{\sqrt{a/t}+1} \text{ which simplifies to}$$

$$\frac{\left(\sqrt{t^2+72at}-7t\right)\left(\sqrt{a/t}+1\right)-2\left(6a-4t\right)}{\left(6a-4t\right)\left(\sqrt{a/t}+1\right)} < 0$$

Determining the roots of the numerator we obtain

$$\left(\sqrt{t^2+72at}-7t\right)\left(\sqrt{a/t}+1\right)-2\left(6a-4t\right)=0$$

$$a=\frac{2}{3}t$$

Doing the same for the denominator:

$$\left(6a-4t\right)\left(\sqrt{a/t}+1\right)=0$$

$$a=\frac{2}{3}t$$

After verifying that $t = \frac{3}{2}a$ is in fact a solution of both expressions we can write the following table:

a	0		$\frac{2}{3}t$	$+\infty$
$\left(\sqrt{t^2+72at}-7t\right)\left(\sqrt{a/t}+1\right)-2\left(6a-4t\right)$	$n.d.$	$-$	0	$+$
$\left(6a-4t\right)\left(\sqrt{a/t}+1\right)$	$n.d.$	$+$	0	$-$
$\frac{\left(\sqrt{t^2+72at}-7t\right)\left(\sqrt{a/t}+1\right)-2(6a-4t)}{(6a-4t)\left(\sqrt{a/t}+1\right)}$	$n.d.$	$-$	$n.d.$	$-$

We find therefore find that $\phi^{nd} > \phi^*$ as long as $a \in R\backslash\left\{\frac{2t}{3}\right\}$. As ϕ^{nd} is defined as long as $a \geq t$ the previous condition is always true. This completes the proof that

$\phi^{nd} > \phi^{*}$.■

Proof of Proposition 6: Now we look at the behaviour of both second period prices as we move from no discrimination to discrimination. Notice that from proposition 4 we have $p^{O} = \frac{2t}{3}\left(\frac{2-\phi^{*}}{\phi^{*}}\right)$ and $p^{R} = \frac{t}{3}\left(\frac{2-\phi^{*}}{\phi^{*}}\right)$. From proposition 1 we have $p^{nd} = t\left(\frac{2-\phi^{nd}}{\phi^{nd}}\right)$. In order to compare second-period prices with price discrimination with second-period prices with no discrimination we solve the following inequalities $p^{O} < p^{nd}$ and $p^{R} < p^{nd}$.

From $p^{O} < p^{nd}$ we obtain:

$$\frac{2t}{3}\left(\frac{2-\phi^{*}}{\phi^{*}}\right) < t\left(\frac{2-\phi^{nd}}{\phi^{nd}}\right)$$

We already know from proposition 5 that $\phi^{nd} > \phi^{*}$. To simplify the computations we consider $\phi^{*} = x$ e $\phi^{nd} = x + \varepsilon$, where $\varepsilon \longrightarrow 0$. Therefore, we can rewrite the last inequality in the following way

$$\begin{aligned}
\frac{2t}{3}\left(\frac{2-x}{x}\right) &< t\left(\frac{2-x-\varepsilon}{x+\varepsilon}\right) \\
\frac{2}{3}\left(\frac{2-x}{x}\right) &< \left(\frac{2-x-\varepsilon}{x+\varepsilon}\right) \\
\frac{4}{3x} - \frac{2}{3} - \left(\frac{2}{x+\varepsilon} - \frac{\varepsilon}{x+\varepsilon} - \frac{x}{x+\varepsilon}\right) &< 0 \\
\frac{1}{3x\left(x+\varepsilon\right)}\left(4\varepsilon - 2x + x\varepsilon + x^{2}\right) &< 0
\end{aligned}$$

As $\varepsilon \longrightarrow 0$ the previous expression simplifies to

$$\frac{(x-2)}{3x} < 0$$

Since $x \in]0,1]$ the previous expression is always negative. This proves that $p^{O} < p^{nd}$.

Similarly, we can prove that $p^R < p^{nd}$. We obtain that

$$\begin{aligned} \frac{t}{3}\left(\frac{2-\phi^*}{\phi^*}\right) &< t\left(\frac{2-\phi^{nd}}{\phi^{nd}}\right) \\ \frac{t}{3}\left(\frac{2-x}{x}\right) &< t\left(\frac{2-x-\varepsilon}{x+\varepsilon}\right) \\ \frac{t}{3}\left(\frac{2-x}{x}\right) - t\left(\frac{2-x-\varepsilon}{x+\varepsilon}\right) &< 0 \end{aligned}$$

As $\varepsilon \longrightarrow 0$ the previous expression simplifies to

$$\left(-\frac{2t}{3}\right)\left(\frac{2-x}{x}\right) < 0$$

As $\left(\frac{2-x}{x}\right)$ is always positive and $\left(-\frac{2t}{3}\right)$ is always negative the previous inequality is always true. This completes the proof that $p^R < p^{nd}$. ■

Proof of Proposition 7: Here we prove that $p_1 > p_1^{nd}$. Using the fact that $p^1 = t(1+\frac{\delta}{3})\left(\frac{2-\phi^*}{\phi^*}\right)$ and $p_1^{nd} = t\left(\frac{2-\phi^{nd}}{\phi^{nd}}\right)$ consider as an hypothesis that $p^1 > p_1^{nd}$. From $p_1 - p_1^{nd} > 0$ we have

$$t(1+\frac{\delta}{3})\left(\frac{2-\phi^*}{\phi^*}\right) - t\left(\frac{2-\phi^{nd}}{\phi^{nd}}\right) > 0$$

Since $\phi^{nd} > \phi^*$, consider that $\phi^* = x$ e $\phi^{nd} = x+\varepsilon$, where $\varepsilon \longrightarrow 0$. Therefore, we can rewrite the last inequality as follows:

$$t(1+\frac{\delta}{3})\left(\frac{2-x}{x}\right) - t\left(\frac{2-x-\varepsilon}{x+\varepsilon}\right) > 0$$

As $\varepsilon \longrightarrow 0$ the previous expression simplifies to

$$\frac{t\delta}{3}\left(\frac{2-x}{x}\right) > 0$$

As $\left(\frac{2-x}{x}\right)$ and $\frac{t\delta}{3}$ are always positive the previous inequality is always true. This proves that $p^1 > p_1^{nd}$ for any $\phi \in]0,1]$.■

Proof of Proposition 8: Look first on first-period equilibrium profits with and without discrimination.

Using the expression of π_1^* and the equilibrium level of advertising with discrimination we have:

$$
\begin{aligned}
\pi_1^* &= p\phi^* \left[(1-\phi^*) + \phi^*\theta\right] - A(\phi^*) \\
&= p\phi^* \left((1-\phi^*) + \frac{1}{2}\phi^*\right) - A(\phi^*) \\
&= t(1+\frac{\delta}{3})\left(\frac{2-\phi^*}{\phi^*}\right)\phi^* \left((1-\phi^*) + \frac{1}{2}\phi^*\right) - A(\phi) \\
&= \frac{t}{2}(\phi^*-2)^2\left(\frac{1}{3}\delta+1\right) - A(\phi^*).
\end{aligned}
$$

Doing the same for the no-discrimination case:

$$
\begin{aligned}
\pi_1^{nd} &= \frac{1}{2}p\phi\left(2-\phi^{nd}\right) - A\left(\phi^{nd}\right) \\
&= \frac{t}{2}\left(2-\phi^{nd}\right)^2 - A\left(\phi^{nd}\right)
\end{aligned}
$$

Consider for instance as an hypothesis that $\pi_1^* < \pi_1^{nd}$. It follows that:

$$
\frac{t}{2}(\phi^*-2)^2\left(\frac{1}{3}\delta+1\right) - A(\phi^*) < \frac{t}{2}\left(2-\phi^{nd}\right)^2 - A\left(\phi^{nd}\right) \tag{4.1}
$$

As we already prove on proposition 5 that it is always true that $\phi^{nd} > \phi^*$ and $\frac{\partial A(\phi)}{\partial \phi} > 0$ then it is always true that $A(\phi^*) < A\left(\phi^{nd}\right)$. Thus it is always true that

$$
\frac{t}{2}(\phi^*-2)^2\left(\frac{1}{3}\delta+1\right) - A(\phi^*) < \frac{t}{2}\left(2-\phi^{nd}\right)^2 - A(\phi^*).
$$

Then from the previous equation it follows that

$$
\frac{t}{2}(\phi^*-2)^2\left(\frac{1}{3}\delta+1\right) < \frac{t}{2}\left(2-\phi^{nd}\right)^2.
$$

To simplify consider $\phi^* = x$ and $\phi^{nd} = x + \varepsilon$, where $\varepsilon \longrightarrow 0$.

$$
\begin{aligned}
\frac{t}{2}(x-2)^2\left(\frac{1}{3}\delta+1\right) &< \frac{t}{2}(2-x-\varepsilon)^2 \\
\frac{t}{2}(x-2)^2\left(\frac{1}{3}\delta+1\right)-\frac{t}{2}(2-x-\varepsilon)^2 &< 0 \\
\frac{1}{6}t\left(\delta x^2-6x\varepsilon-4\delta x-3\varepsilon^2+12\varepsilon+4\delta\right) &< 0 \\
\delta x^2-6x\varepsilon-4\delta x-3\varepsilon^2+12\varepsilon+4\delta &< 0
\end{aligned}
$$

As ε is close to zero, it follows that

$$
\begin{aligned}
\delta x^2-4\delta x+4\delta &< 0 \\
\delta (x-2)^2 &< 0
\end{aligned}
$$

As $\delta > 0$ the previous inequality is always false. Thus, it is always true that $\pi_1^* > \pi_1^{nd}$.

Look now on second-period profits We know that

$$\pi_2^* = \frac{5}{18}t(2-\phi^*)^2$$

and

$$\pi_2^{nd} = \frac{t}{2}\left(2-\phi^{nd}\right)^2$$

Assuming as an hypothesis that $\pi_2^* > \pi_2^{nd}$:

$$\frac{5}{18}t(2-\phi^*)^2 > \frac{t}{2}\left(2-\phi^{nd}\right)^2$$

Take into account that $\phi^* < \phi^{nd}$. Using the fact that $\phi^* = x$ and $\phi^{nd} = x+\varepsilon$, where $\varepsilon \longrightarrow 0$, we can write the inequality as follows:

$$\begin{aligned}
\frac{5}{18}t\left(2-x\right)^2 &> \frac{t}{2}\left(2-x-\varepsilon\right)^2 \\
\frac{5}{18}t\left(2-x\right)^2-\frac{t}{2}\left(2-x-\varepsilon\right)^2 &> 0 \\
-\frac{1}{18}t\left(4x^2+18x\varepsilon-16x+9\varepsilon^2-36\varepsilon+16\right) &> 0 \\
4x^2-16x+18x\varepsilon+9\varepsilon^2-36\varepsilon+16 &< 0
\end{aligned}$$

As $\varepsilon \longrightarrow 0$, we have that

$$\begin{aligned}
4x^2-16x+16 &< 0 \\
4\left(x-2\right)^2 &< 0
\end{aligned}$$

It is evident that the inequality is always false. Thus, it is always true that $\pi_2^* < \pi_2^{nd}$.

■

Proof of Proposition 9: From corollary 2 we know that for any $\delta \in \left]0,1\right]$, overall equilibrium profit under BBPD with advertising is equal to $\Pi = \frac{t}{18}\left(\phi^*-2\right)^2\left(8\delta+9\right)-A(\phi^*)$ while overall equilibrium profit without BBPD is given by $\Pi^{nd} = \frac{t}{2}\left(2-\phi^{nd}\right)^2\left(1+\delta\right)-A\left(\phi^{nd}\right)$.

Assume as an hypothesis that $\Pi < \Pi^{nd}$. Thus:

$$\frac{t}{18}\left(\phi^*-2\right)^2\left(8\delta+9\right)-A(\phi^*) < \frac{t}{2}\left(2-\phi^{nd}\right)^2\left(1+\delta\right)-A\left(\phi^{nd}\right)$$

As we already prove it is always true that $\phi^{nd} > \phi^*$ and $\frac{\partial A(\phi)}{\partial \phi} > 0$ then it is always true that $A\left(\phi^*\right) < A\left(\phi^{nd}\right)$. Thus it is always true that:

$$\frac{t}{18}\left(\phi^*-2\right)^2\left(8\delta+9\right)-A(\phi^*) < \frac{t}{2}\left(2-\phi^{nd}\right)^2\left(1+\delta\right)-A\left(\phi^*\right)$$

Therefore,

$$\frac{t}{18}\left(\phi^*-2\right)^2\left(8\delta+9\right) < \frac{t}{2}\left(2-\phi^{nd}\right)^2\left(1+\delta\right)$$

Assuming that $\phi^{*}=x$ e $\phi^{nd}=x+\varepsilon$, where $\varepsilon\longrightarrow 0$,we have

$$\frac{t}{18}\left(x-2\right)^{2}\left(8\delta+9\right)-\left(\frac{t}{2}\left(2-x-\varepsilon\right)^{2}\left(1+\delta\right)\right)<0\Leftrightarrow$$

$$-\frac{1}{18}t\left(4\delta-36\varepsilon-4x\delta+18x\varepsilon+9\varepsilon^{2}-36\delta\varepsilon+x^{2}\delta+9\delta\varepsilon^{2}+18x\delta\varepsilon\right)<0$$

$$\begin{aligned}-\frac{1}{18}t\left(4\delta-4x\delta+x^{2}\delta\right) &< 0\\ -\frac{1}{18}t\delta\left(x-2\right)^{2} &< 0\end{aligned}$$

which for $t>0$ and $\delta>0$ is always false. Therefore, the condition $\Pi>\Pi^{nd}$ is always true. ■

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