

## EMBEDDING THE DIRECT ALGORITHM IN A PENALTY APPROACH FOR SOLVING ENGINEERING DESIGN PROBLEMS

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### ABSTRACT

In this paper we investigate the performance of DIRECT algorithm when solving constrained engineering design problems. For this purpose, the hyperbolic penalty approach is employed and the algorithm is modified in order to preserve feasibility of solutions. The algorithm is illustrated on six well-known engineering problems with promising results. Comparisons with other global optimization solvers are reported and discussed.

**Keywords:** global optimization, constrained optimization, DIRECT algorithm, hyperbolic penalty function, engineering design problems.

### 1. INTRODUCTION

Many engineering applications, such as structural optimization, engineering design, VLSI design, economics, allocation and location problems (Floudas and Pardalos 1987), involve difficult optimization problems that must be solved efficiently and effectively. Due to the nature of these applications, the solutions usually need to be constrained in specific parts of the search space that are delimited by linear and/or nonlinear constraints.

Hence, the kind of problems to be addressed in this paper are

$$\begin{aligned} & \text{minimize} && f(x) \\ & \text{subject to} && g_i(x) \leq 0 \\ & && x \in \Omega \end{aligned} \tag{1}$$

where  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  and  $g_i : \mathbb{R}^n \rightarrow \mathbb{R}^m$ , for  $i = 1, \dots, m$ , are nonlinear functions and  $\Omega = \{x \in \mathbb{R}^n : l \leq x \leq u\}$  is a closed set. We assume that the objective function  $f$  is nonconvex and may possess many local minima in the feasible region. This class of global optimization problems arises frequently in engineering applications.

Different deterministic as well as stochastic algorithms have been developed for tackling such problems. Deterministic approaches such as Feasible Direction and Generalized Gradient Descent make strong assumptions on the continuity and differentiability of the objective function (Floudas and Pardalos 1987). Therefore their

applicability is limited since these characteristics are rarely met in problems that arise in real life applications. On the other hand, stochastic optimization algorithms such as Genetic Algorithms, Evolution Strategies, Particle Swarm Optimization and Electromagnetism-like mechanism do not make such assumptions and they have been successfully applied for tackling constrained optimization problems during the past few years (Hu et al. 2003, Coello 2002, Parsopoulos and Vrahatis 2002, Rocha and Fernandes 2009).

A well-known approach for solving this kind of constrained optimization problems is based on penalty functions (Coello 2000). The penalty techniques transform the constrained problem into an unconstrained problem by penalizing  $f$  when constraints are violated and then minimizing the penalty function using methods for unconstrained problems.

This paper presents a numerical study of a penalty approach, based on the hyperbolic penalty function, where the unconstrained problems are solved by a global deterministic algorithm, the DIRECT algorithm.

In Section 2, the penalty approach is described, as well as the proposed DIRECT algorithm. Following, the numerical results when applying the proposed method are presented to some engineering design problems, described in the literature. Finally some conclusions and final remarks are reported and discussed in Section 4.

## 2. THE PENALTY APPROACH

Penalty functions methods have made themselves among the most common methods for solving constrained optimization problems by their advantages of simplicity and easy to be implemented. Hence, in order to solve problem (1) a penalty approach will be used that converts a constrained optimization problem into an unconstrained one by adding a penalty term to the objective function of infeasible solutions so that they will be penalized for violating the constraints. Although penalty functions are very simple and easy to implement they often require several parameters that are usually problem dependent and chosen by priori knowledge by users. Too large parameters lead to heavy selective pressure that would cause the algorithm hard to converge to satisfactory solutions not mention to the global optimum. However too small parameters make the search too broad and hard to find a feasible solution. Different penalty functions have been suggested and can also be classified based on the way the penalties are added: death, static, dynamic, annealing, adaptive (Coello 2002).

Here, the solution of problem (1) is obtained by solving a sequence of subproblems, whose objective function is given by the hyperbolic penalty method (Xavier 2001)

$$P(x, \lambda, \tau) = f(x) + \sum_{i=1}^m (\lambda g_i(x) + \sqrt{\lambda^2 g_i(x)^2 + \tau^2}) \quad (2)$$

where  $\lambda, \tau \geq 0$  and  $\lambda \rightarrow \infty, \tau \rightarrow 0$ . The sequence of subproblems is obtained by controlling the two parameters in two different phases of the optimization process. In the first phase, the initial parameter  $\lambda$  increases, thus causing a reduction in the penalty to the points outside the feasible region while at the same time there is a

reduction in the penalty for the points inside the feasible region. This phase continues until a feasible point is obtained. From this point on,  $\lambda$  remains constant and the values of  $\tau$  decrease sequentially.

In this context, the subproblems to be solved in each  $k$  iteration are given by

$$\underset{x \in \Omega}{\text{minimize}} P(x, \lambda^k, \tau^k) \quad (3)$$

and are approximately solved using the DIRECT algorithm that is a deterministic global optimization method. DIRECT algorithm from Jones et al. (1993), is an acronym for DIviding RECTangles, and it is a deterministic sampling global optimization method designed for finding the global minima for bound constrained non-smooth problems, where no derivative information is needed. It is guaranteed to converge to the global optimal function value, if the objective function is continuous or at least continuous in the neighborhood of a global optimum. The most important advantage of DIRECT stems from its approach to balancing local and global search - the simple idea of not sampling just one point per iteration, but rather sampling several points using all possible weightings of local versus global search. This approach leads to an algorithm with no tuning parameters, making the algorithm easy-to-use and robust.

The first step in the DIRECT algorithm is to transform the search space to be the unit hypercube. The function is then sampled at the center-point of this cube. Computing the function value at the center-point instead of doing it at the vertices is an advantage when dealing with problems in higher dimensions. The hypercube is then divided into smaller hyperrectangles whose centerpoints are also sampled. Instead of using a Lipschitz constant when determining the rectangles to sample next, DIRECT identifies a set of potentially optimal rectangles in each iteration. All potentially optimal rectangles are further divided into smaller rectangles whose center-points are sampled. See (Finkel 2003; Finkel and Kelly 2004) for more details.

The hyperbolic penalty method for constrained optimization embedding the DIRECT algorithm is described in the Algorithm 1. Step 4 refers to the solution of the subproblem (3) parametrized by the penalty parameters using the DIRECT algorithm. The herein used measure of constraint violation is given by

$$\text{viol} = \sum_{i=1}^m \max\{0, g_i(x)\}. \quad (4)$$

### 3. NUMERICAL RESULTS

Problems of practical interest are important for assessing the effectiveness of any algorithm. Thus, to evaluate the performance of the embedded DIRECT algorithm in the hyperbolic penalty function method a set of 6 benchmark engineering problems is used.

Following there is a summary of the characteristics of the engineering design problems selected, where all of them have simple bounds and inequality constraints (Costa and Fernandes 2008, Lee and Geem 2005, Ray and Liew 2002, Rocha and Fernandes 2009).

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**Algorithm 1** Hyperbolic Penalty + DIRECT Algorithm

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1. **Given**  $\lambda^1 = 20$ ,  $\tau^1 = 10$ ,  $\epsilon_{\max} = 10^{-6}$ ,  $\lambda_{\max} = 10^{12}$ ,  $\tau_{\min} = 10^{-12}$ ,  $\gamma_\lambda = 2$ ,  
( $\gamma_\lambda > 1$ ),  $\gamma_\tau = 0.01$ , ( $0 < \gamma_\tau < 1$ ),  $\eta^* = 10^{-6}$ ,  $k_{\max} = 20$
  2. **Set**  $k = 1$
  3. **While** ( $\text{viol} > \epsilon_{\max}$  OR  $|f^* - f(x^k)| > 10^{-4}|f^*|$ ) AND  $k < k_{\max}$  **do**
  4.     Find an approximate minimizer  $x^k$  to the subproblem (3) using DIRECT
  5.     **If**  $\text{viol} > \epsilon_{\max}$  **then**  $\lambda^{k+1} = \min\{\lambda_{\max}, \gamma_\lambda \lambda^k\}$  and  $\tau^{k+1} = \tau^k$
  6.     **Else**  $\tau^{k+1} = \max\{\tau_{\min}, \gamma_\tau \tau^k\}$  and  $\lambda^{k+1} = \lambda^k$
  7.     **Set**  $k = k + 1$
  8. **End while**
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1. Design of a tension/compression spring where the weight of a tension/ compression spring is minimized, subject to constraints on the minimum deflection, shear stress, surge frequency, limits on outside diameter and on design variables. The problem has 3 design variables and 4 inequality constraints.
2. Design of a speed reducer where the objective in this problem is to minimize the total weight of a speed reducer, subject to constraints on bending stress of the gear teeth, surface stress, transverse deflections of the shafts and stresses in the shafts. It has 7 design variables and 11 inequality constraints.
3. Design of a disc brake where the objective is to minimize both the mass of the brake and the stopping time. The problem has 4 design variables and 6 inequality constraints.
4. Design of a tubular column where the objective is to minimize the total cost of the material and construction of a tubular column. The problem has 2 design variables and 2 inequality constraints.
5. Design of three-bar truss where the objective is to minimize the volume of a 3-bar truss structure, subject to stress constraints. The problem has 2 design variables that represent cross-sectional areas of two bars (the third bar is equal to the first bar), and 3 inequality constraints.
6. Design of four-bar truss structure where the objective is to minimize the volume and the displacement of a 4-bar truss structure, subject to stress constraints. The problem has 4 design variables that represent cross-sectional areas and 1 inequality constraint.

The results reported in Table 1 include the number of iterations (Nit), the number of objective function evaluations (Nfe) and the optimal solution found (Fopt) with the hyperbolic penalty method as described in Algorithm 1. The stopping criteria was the specified in Step 3 of the algorithm. We remark that we also stopped the algorithm when the penalty parameters stabilized and the solution could not improve, since DIRECT is a deterministic algorithm. The conditions used to stop the DIRECT algorithm (in Step 4 of Algorithm 1) were the maximum number of iterations and the

maximum objective function evaluations set to 10000. To assess the performance of the hyperbolic penalty method Table 1 also reports the results of the penalty approach with the quadratic penalty function, when applied to the same engineering problems.

Table 1: Comparison results of the penalty approach (in terms of  $f_{opt}$ ,  $N_{fe}$  and  $N_{it}$ ).

		spring	speed	brake	tubular	3-bar	4-bar
$f^*$		0.0126	2994.4999	0.1274	26.5313	263.896	1400
quadratic penalty function	$f_{opt}$	0.0280	3040.4774	0.1329	26.7402	263.8975	1400.0532
	$N_{fe}$	40115	120375	29163	3619	100155	10056
	$N_{it}$	4	12	2	11	10	1
hyperbolic penalty function	$f_{opt}$	0.0156	2995.9333	0.1274	26.5329	263.9665	1400.0542
	$N_{fe}$	8117	90156	40099	30070	50068	30090
	$N_{it}$	8	9	7	3	3	3

The results reported in Table 1 show that, in general, the hyperbolic penalty function performs better when compared with the quadratic penalty function. The optimal solutions found are more closer to the optimal solution known in the literature ( $f^*$ ). We remark, for only three problems the hyperbolic function is more expensive in terms of objective function evaluations than quadratic function, but in these cases the optimal solution found is better.

#### 4. CONCLUSIONS

This paper presents the hyperbolic penalty method to solve constrained engineering design problems. The unconstrained problems that arrive from the penalty approach are solved by a deterministic global optimization method, named DIRECT. To assess the performance of the herein proposed method a set of six well-known engineering problems is solved. A comparison with a quadratic penalty function is reported and show promising results.

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