

**Curriculum reorganization and Project based learning:
a teaching experiment of Geometry in the 6th grade**

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Introduction

This study intends to analyze the implications that an unconventional approach of Mathematics, in context of classroom, will have in 6 grade pupils. This approach consists of a reorganization of part of the mathematics curriculum, more concretely of the Thematic Unit - Geometry, through the methodology of Project Based Learning.

The curricular reorganization was based on some principles concerning the teaching and learning of Mathematics, namely that: Mathematics must contribute for the development of individuals who can understand and use mathematics; learning is considered a process of active construction of knowledge; and activities of more open character, as Projects, present some potentialities concerning the teaching and learning of Mathematics.

In the scope of this problematic the following research questions had been defined:

- what kind of task should be presented to pupils, in order to constitute a challenge, and provide the opportunity to construct mathematical knowledge?
- How to organize the activity in such a way that the geometry contents from 6th grade relate one another and that it develops pupils' knowledge of concepts and procedures, mathematical communication and reasoning?
- In which way does this approach influence the knowledge of concepts and procedures, the ability of mathematical communication and reasoning?

Theoretical Framework

When the performance of Portuguese pupils started to be compared with that of pupils from other countries (SIAEP - Second International Assessment of Educational Progress, in 1990/91 with children between 9 and 13 years old; TIMSS - Third International Mathematics and Science Study, applied in 1995 to pupils from 7th and 8th grade and PISA - Programme for International Student Assessment, in 2003, for 15 years old students) the education of Mathematics started to deserve greater attention and concern. These studies had shown some of the fragilities of the education system, in Portugal.

Throughout the last decades, several studies gave various recommendations, in order to improve the pupils' learning and, consequently, to diminish the poor results achieved in external evaluations. Many of these recommendations include the necessity of a more effective curricular development on the part of teachers and changes in the teaching and learning of mathematics.

In Portugal, as well as in other countries, the creation of the curriculum was for some decades in charge of a group of specialists and the teachers had only “to put in practice” the guidelines that others determined. This led to a huge distance between the curriculum prescribed in official documents and the one that was implemented in the classroom.

In 2000, the National Council of Teachers of Mathematics (NCTM) published *Principles & Standards for School Mathematics*, where it outlines six important principles for school Mathematics of 21st century. According to this organization, the curriculum must be coherent and effective. The mathematics curriculum needs to introduce ideas in such a way that they build on one another allowing students to perceive the relationships among important mathematical ideas and build connections. The curriculum must also “*focus on important mathematics, that is worth the time and attention of students and that will prepare them for continued study and for solving problems in a variety of school, home, and work settings*” as well as challenge them to learn more and with more depth.

Project Based Learning (PBL) is a pedagogical methodology little used in the teaching and learning of the mathematics but it presents some potentialities. According to Leite, Malpique and Ribeiro (1989), Cromley (2000), Rey et al (2005), PBL:

- allows pupils to develop their learning from experience and personal involvement in activities that meet their interests, promoting autonomy and responsibility;
- is a way to incorporate many features of effective learning: problem solving, cooperative work;
- gives rise to multiple perspectives to solve a problem: the problem and the solutions “relate with meaning”;
- promotes the acquisition of complex competences;
- familiarizes students with the interpretation of new situations.

Thus, PLB may be seen as a methodology that develops complex mathematical skills, as students construct their knowledge in response to problematic situations in addition to providing meaningful application of mathematics to concrete daily life situations.

Research methodology

Our work, besides describing and analyzing the situations arising from an educational practice, aims at understand phenomena that might happen, establish relations between them, formulate hypotheses for the situations encountered and, where possible, contribute to changes in educational practices (Bogdan and Biklen, 1994). Therefore, we chose a qualitative methodology, action research.

The target population was a 6th grade class of twenty-four students (average age of eleven years old). The class was divided into groups of three elements, each with different performances in math. Data collection was based on the following methods: observation, audio recording, written documents (prepared by students and by researcher) and interviews.

The investigation, which was developed over four weeks, has gone through three distinct phases: in phase 1, prior to the beginning of the project, a diagnostic test was given to assess students knowledge in relation to what is are planned to be investigated. The 2nd phase corresponded to the development of the project and data collection and finally, the third phase, occurred after the completion of the project.

In the latter, we applied two evaluation tests, one to each group and another to each student, in order to observe possible changes in relation to their knowledge.

The learning experience (the project)

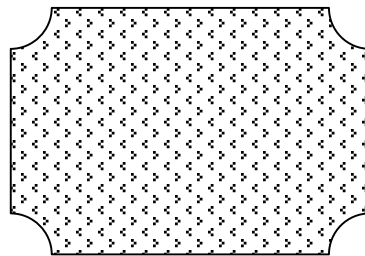
The learning experience under investigation, was the construction of a scale model of a quarter of a city that students would have to devise and build.

The task proposal, presented to students in the form of a worksheet, had some clear guidelines about the intervention area, as in reality.

Task: Building a Scale Model of an area of a city.

The figure below is the plant from an area of a city that needs intervention: Construction of buildings, gardens, ...

Intervention Area



Scale 1/1000.

The planning and building must meet the following conditions:

- The intervention must have an urbanized area and also green spaces;
- The urbanized area may not exceed three fifths of the total area of intervention;
- The urbanized area is formed by both "private space" and "public space";
- In the intervention area a cylindrical fountain should be built;
- The total volume of public buildings should not exceed $21\,000\text{ m}^3$;
- In the "private space" area, the buildings can not be over three floors.

While the construction of "scale models" is not a novel task (it even appears in some textbooks), this type of task never comes up with this character of a possible approach to geometry in the manner in which it is proposed, that is, as a resource for teaching and learning geometry through PBL.

Although the final product has been formulated and imposed from the outset - building a scale model - its achievement did not guarantee, however, that all contents of geometry would be discussed. For this reason, it was necessary to introduce certain conditions in the realization of the model as to ensure some of the "new learning" that was intended for the school year: the area and perimeter of the circle, the volume of the cylinder and scale. Scale, even though is not a geometric content, was taken into account, since it implied a certain rigor in building the model and was an opportune situation to first address this content.

The initial obstacles the students encountered were deciding where to start and the direction that the project would take. Little by little, the first questions related to the project began to emerge, which constituted the starting point for research. However, this situation was not similar in all groups. Some have more easily identified issues / problems.

Helia: What is a scale?

Sandra: We can put all the private houses in one area and public buildings in another?

Walter: And how do we know the area?

Initially, neither group was able to identify the different partial problems that they have to deal with in order to construct the model. The development of this project proceeded around the successive partial problems and their solutions.

One of the first problems was how to determine the intervention area. One group identified the quarter of circles as being the missing figures in the "corners" of the rectangle and determined the area of the figure by the difference between the areas of the rectangle and the circle.

Another group, however, opted to determine the area dividing the land into five parts (supposedly the same) using schemes. The schemes became increasingly refined until the final version appears, as shown below:

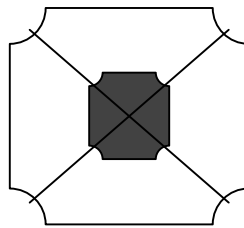


Fig. 1 – Final Scheme presented by one group

Since they considered this division as correct, it was necessary to confront them in order to determine the area of each of the five pieces found and to check whether they all had the same area. Only at that moment they decide to abandon this strategy and use the one that other groups had already used (to determine the area of the figure the difference between the area of the rectangle and the circle and calculate $3/5$ of that area).

The dynamics and work done by each group were diverse: some more organized and anxious to comply with the conditions of the proposed activity, others more concerned with aesthetic issues of the model and yet another with the written record of everything they were doing.

Throughout the project the students needed to mobilize diverse knowledge, attitudes and abilities leading to the development of skills.

For example, during the construction of plans for "buildings" the students had to relate two-dimensional representations to the objects in three dimensions (spatial reasoning).

In another situation, a group focused on the area of the rectangle but had difficulties in determining the correct operation to calculate the area. This shows that the students had associated the concept of area to an operation.

Ricardo: What's on this side?

Jessica: 310

Ricardo: And how much is 310×310 ?

Jessica: $310 + 310$ gives 620.

Ricardo: It times, it is not more!

Noah: It's more, it is not times.

Ricardo: To find the area is times.

Jessica: Stop! I am not understanding anything I'm doing!

The use of schemas, mental and written calculation, measurement, using the ruler and square in tracing lines, the need to estimate area measurements of different spaces, height of buildings and construction of networks for different solids, were the most frequently observed procedures. The mostly mobilized concepts were scale and area although all groups have approached others such as the perimeter of the circle and volume of some geometric solids.

The lack of rigor in mathematical language was common practice in almost all groups especially in the initial phase of the project. This lack of accuracy has been gradually fading away and becoming progressively more accurate.

Much of the communication observed in several groups reports to metacognitive behaviours: planning the activity, analysis and regulation of work carried out, confirming the study by Artzt and Curcio (1998). Although all the skills under study were observed in all groups through the different attitudes and behaviors, mathematical communication was most evident during the development of the project.

From the moment pupils started the investigation until its fulfilment, the project was viewed in different ways by different groups, providing a wealth of information about how it was perceived, developed and presented at the end of the activity.

Some Results

Concerning the task presented to students

In this proposed task we intended to promote learning in an autonomous and active way. Although initially students revealed a great dependence of the teacher, with the progress of the project the autonomy of the groups was clearly increasing. The groups engaged, very actively, in developing their project, providing ideas, suggesting and implementing strategies for solving the problems identified. It was also noticeable the creative capacity of some elements of some groups when presenting ideas and strategies for solving some situations.

It was also clear from interviews with students, that this activity attracted considerable interest, besides having contributed to a more practical view of mathematics and in interpreting a variety of real-life situations. It should be noted that throughout the development of the project the students had to determine areas and volumes of buildings with different shapes, defining public and private buildings, building scale models of buildings, estimating areas of different types of buildings, among others,

which constituted a real challenge, thus promoting the development of understanding of certain concepts and mathematical processes.

The development of this project proceeded around the successive partial problems and their solutions. This sequence, problem - solution was constant and very visible throughout the project as if it were a result of working in a spiral in which, to reach the end of the project, one would have to solve a series of situations related to the above. The following diagram may illustrate the work done by students throughout the project:

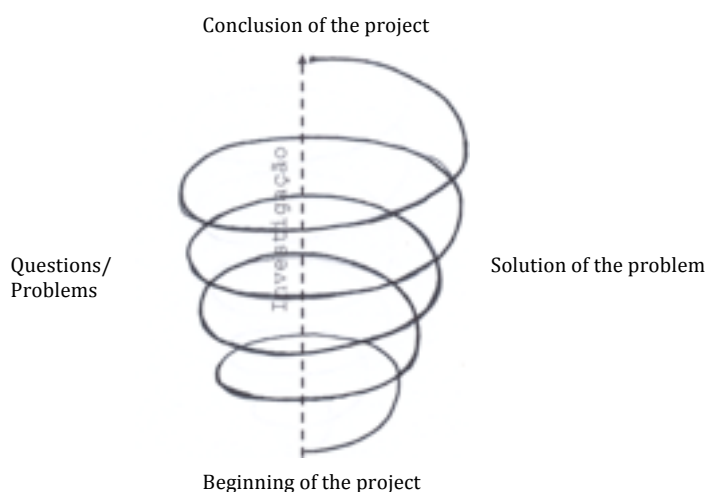


Fig. 2 - Project Development

This form of work seems to contradict some views of PBL as going through distinct phases. In this case, they do not precede in a linear manner as refer, for example, Castro and Ricardo (2003). The planning of tasks, research, information organization and analysis were phases that presented this sequence for each particular problem that arose. The sum of the various solutions allowed the construction and finalization of the model.

The fact that the activity has been developed in small group enabled the learning of cognitive and metacognitive strategies through sharing and exchanging ideas, as refers Peterson (1992). While there have been some difficulties in working together, there were several times when students had to explain, justify, discuss strategies, ask questions, examine and reflect on what they observed and performed.

Through the exploration of several different situations (modeling, analysis, discussion and reflection on the findings), we observed the construction of knowledge that was directly or indirectly related to the task. So the knowledge was not transmitted but constructed by the learner, according to their know-how and experiences.

Despite the margin of freedom that is associated with PBL, the final product presented by the three study groups was not as distinct as it could be. It should be noted that the shapes of the buildings were, in most cases, square prisms with heights that varied only between one and three floors.

As for the geometric contents that were part of the planning and supposedly could be addressed (as, for example, symmetries, the area of the parallelogram and classification of quadrilaterals), they weren't.

For this reason an activity, such as PBL, does not always develop in the "direction" that the teacher initially supposed. The teacher's role becomes sensitive in these situations because on the one hand, he should not interfere in the "path" that the groups defined according to their interests but on the other hand, letting the pupils follow their own can lead to a work that does not go according to the teachers' objectives. Knowing when and how to "interfere" is a difficult and delicate matter for a teacher in activities that are more open, as is the case of projects and mathematical investigations (as mentioned by Abrantes, 1994).

Concerning the influence of PBL on students

The analysis of data collected in this study shows that PLB influenced students regarding:

i) The knowledge of concepts and procedures

Of the various concepts covered by the groups (proportion, area, perimeter, volume and scale), the concepts of proportion and scale were the most relevant by the number of times they were mobilized throughout the project and, in the case of scale, by the results evidenced in the tests.

The development of PBL also contributed to the mobilization of different procedures as the use of diagrams, the handling of various instruments, the construction of three-dimensional models from their representations in two dimensions, the calculation and estimation of various measures. However, calculation was the most experienced procedure for all groups.

ii) Their ability to communicate

This activity facilitated communication as it required students to think about strategies to be adopted and the results obtained. The communication was the mathematical competence that stood out in all study groups. This is consistent with the views of Greenes and Schulman (1996) who reported that student involvement in research activities or projects calls for increased communication. It was also noted that the most observed behaviours related to mathematical communication reports the metacognitive behaviours associated mainly to the phase of analyzing situations and planning activities.

iii) Their reasoning

The fact that the activity was developed in groups and the need to solve various problems/situations, facilitated the development of mathematical reasoning, as students were encouraged to question each other ideas, to explain and justify their views or procedures. There was an improved performance in the evaluation tests.

It was found that in all groups, the use of procedures led to the development of spatial reasoning. In fact, when they developed strategies to calculate area by decomposition and related three-dimensional objects with their two-dimensional representations they were developing spatial reasoning.

Final Thoughts

Although data obtained from this study does not allow generalizations, there are however some aspects that confirm some of the advantages presented by several authors, for Project Based Learning and should be taken into account in future teaching practices.

This study shows that it is possible to approach math differently from the expositive and so called traditional way and that this approach motivates and encourages pupils to learn mathematics, even those pupils who believe that they are not able to succeed in math. It also shows that success can be more comprehensive, according to what is proposed and how it is proposed. This activity attracted considerable interest and commitment showing pupils an pleasurable, educational and enriching aspect of the discipline.

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