

20 1. INTRODUCTION

21

22 In steel fibre reinforced concrete, SFRC, steel fibres and matrix are bonded together through a weak interface, which
23 behaviour is important to understand and accurately model the mechanical behaviour of SFRC, since the properties
24 of this composite are greatly influenced by the interface zone between fibre/matrix and, consequently, by the micro-
25 mechanical fibre reinforcement mechanisms that are mobilised. When these composites are reinforced with low fibre
26 volume ratios, the fibre contribution benefits arise, mainly, not to say almost exclusively, after the crack initiation.

27

28 The post-cracking behaviour of random discontinuous fibre reinforced composites can be predicted by the use of a
29 stress - crack opening displacement relationship, $\sigma-w$. Several authors developed micro-mechanical models to obtain
30 the $\sigma-w$ relationship, since for quasi-brittle materials, the stress-crack opening relationship that simulates the stress
31 transfer between the faces of the crack has a significant impact on the behaviour of a structure after its cracking
32 initiation. In case of FRC, the $\sigma-w$ relationship can be approximated by averaging the contributions of the individual
33 fibres bridging the matrix crack plane, defining for this purpose probability-density functions of the centroidal
34 distance of fibres from the matrix crack plane, and of the orientation angle (Visalvanich and Naaman 1983, Li *et al.*
35 1991, Maalej *et al.* 1995). These models, which are based on an averaging process of all the forces that are carried
36 out by the fibres over a crack plane, can provide the general material composite behaviour with reasonable accuracy
37 by modelling the main mechanisms of a single fibre pullout. However, in general, they do not account for some
38 aspects, such as, fibre bending rupture and matrix spalling at the exit points of inclined fibres.

39

40 Another difficulty on the prediction of the post-cracking behaviour of a FRC in a real structure is that the material
41 behaviour in a test specimen may differ from the behaviour of a real structural element. It is well described in
42 literature that various casting procedures and structural shapes may result in predominant fibre orientation into
43 parallel planes (Stroeven 1986, RILEM TC 162-TDF 2002). In the case of steel fibre reinforced self-compacting
44 concrete, SFRSCC, the predominant fibre orientation can be along the flow itself (in the fresh state) and along the
45 boundary surfaces due to the wall-effect (Stähli *et al.* 2008, Vandewalle *et al.* 2008). A predefined orientation of the
46 steel fibres parallel to the tensile direction in a test specimen may result in overestimating the post-cracking

47 mechanical properties of the steel fibre reinforced concrete, when compared with specimens with equal amount of
48 fibres, however with a random fibre orientation.

49

50 Having in mind the aforementioned aspects and factors that influence and contribute to the post-cracking behaviour
51 of a FRC, approaching the FRC as a continuum material may lead to a rough estimation of the mechanical behaviour
52 of a certain FRC structural element. Even though, material behaviour laws for FRC can be obtained with great
53 accuracy by inverse analysis procedures of test specimens, these laws may not translate the accurate material
54 behaviour within a specific structural element (Cunha 2010). It is feasible to assume FRC as a two-phase material,
55 namely, an unreinforced concrete matrix phase and a fibre phase, with the latter one comprising information about
56 fibre density and orientation depending on where and how the material is applied. Hence, this approach can
57 somehow enhance the numerical simulation of FRC structures, thus excluding the use of biased material behaviour
58 laws, i.e. from a macro-mechanical behaviour point of view, obtained from the inverse analysis of experimental tests.

59

60 Therefore, based on the prior reasoning, in the present work is presented a numerical approach where SFRC is
61 treated as a heterogeneous medium comprised by one homogeneous phase (aggregates and paste), and another one
62 composed by the steel fibres. The fracture process of the cementitious matrix (unreinforced) is modelled with a 3D
63 multi-directional fixed smeared crack model. This unreinforced concrete phase is discretized by solid finite elements.
64 On the other hand, the stress transfer between crack planes due to the fibres bridging active cracks is modelled with
65 3D truss elements. A non-linear behaviour law is assigned to these last elements in order to account the fibre/matrix
66 interface properties. These laws are based upon the micro-mechanical behaviour of the fibres. The random fibre
67 distribution, over the matrix, is simulated with an algorithm based on the Monte Carlo method, providing a realistic
68 distribution of the fibres over a bulk element. The developed algorithm enables to take into account factors that
69 influence the fibre structure, such as: the so-called wall-effect and the high flowability of SFRSCC (Cunha 2009).
70 The geometry, positioning and orientation of the fibres are subsequently inserted in a three dimensional finite
71 element mesh. The cable elements representing the fibres are considered as embedded elements. Within a first stage
72 of the research, as a simplification, the embedded element is modelled with a perfectly bonded formulation. Hence,
73 the bond - slip behaviour is simulated in an indirect fashion from the transformation of a load - slip relationship to a
74 tensile stress - strain relation. Moreover, to the author's knowledge this kind of approach is quite novel, and within

75 numerical models for FRC with the same philosophy only meso-level models using lattice structures are known, e.g.
76 Van Hauwaert and Van Mier (1998), Leite *et al.* (2004) and Bolander (2004).

77

78

79 **2. MATERIALS**

80

81 The materials used in the composition of the SFRSCC, were: cement (C) CEM I 42.5R, limestone filler (LF),
82 superplasticizer (SP) of third generation based on polycarboxilates (Glenium® 77SCC), water (W), three types of
83 aggregates (fine river sand, FS, coarse river sand, CS, and crushed granite 5-12 mm, CA) and DRAMIX® RC-80/60-
84 BN hooked end steel fibres (length, l_f , of 60 mm, diameter, d_f , of 0.75 mm, aspect ratio, l_f/d_f , of 80, and yield stress
85 of 1100 MPa). The method used to define the SFRSCC composition, the mixing procedure and other properties of
86 the SFRSCC in the fresh state can be found elsewhere (Barros *et al.* 2007).

87

88 To study the SFRSCC tensile post-cracking behaviour, two batches with distinct fibre contents (C_f), 30 and 45 kg/m³
89 were used. Table 1 includes the compositions that have best fitted self-compacting requirements for the adopted two
90 fibre contents. Remark that in Table 1, WS is the water necessary to saturate the aggregates, and W/C is the
91 water/cement ratio. The WS parcel was not used to compute the W/C ratio. The fibre pullout tests were performed on
92 the SCC medium with a fibre content of 30 kg/m³.

93

94 For determining the SFRSCC properties in the fresh state, the Abrams cone was used in inverted position (concrete
95 flowed through the small orifice of the cone). A total spread over 700 mm was measured and no sign of segregation
96 was detected as the mixture showed good homogeneity and cohesion.

97

98

99 **3. EXPERIMENTAL RESEARCH**

100

101 **3.1 Fibre pullout tests**

102 **3.1.1 Series of tests, preparation of specimens and SCC properties**

103 The pullout tests were divided into two main groups, according to the geometry type of used fibres: hooked ends and
104 smooth. The influence of the fibre's embedded length, L_b , (10, 20 and 30 mm) and fibre's orientation (0, 30 and 60°)
105 on the pullout response was assessed in both groups. Each series of the smooth fibres comprises three specimens,
106 whereas six specimens compose each series of the hooked fibres. Smooth fibres were obtained by cutting the hooked
107 ends of the RC-80/60-BN fibres with pliers. Code names were given to the test series, which consist on alphanumeric
108 characters separated by underscores. The first character indicates the fibre type (S - smooth; H - hooked), the second
109 string indicates the embedded length in mm (for instance, Lb10 represents a fibre embedded length of 10 mm) and
110 finally the last numeral indicates the angle between the fibre and the fibre pullout load direction, in degrees. Due to
111 technical problems, the series S_Lb10_0 and S_Lb10_30 could not be correctly tested, therefore they are not
112 presented.

113

114 The pullout tests on single steel fibres were performed using cored SCC specimens. A special mould was designed,
115 able to accommodate 81 fibres fixed at its bottom (more details can be found elsewhere, Cunha 2010). This device
116 was used to cast the 81 pullout specimens simultaneously, allowing a correct placement of the fibre and keeping the
117 desired L_b and inclination angle for the fibre. After casting, the SCC slab was cured at a temperature of 20°C and a
118 relative humidity of about 95%. After 30 days, the SCC slab was demolded, and cylindrical cores containing, each
119 one, a single fibre was drilled from the slab. The diameter and height of each specimen was 80 mm.

120

121 The fibre pullout tests were performed at approximately 180 days after concrete casting. The concrete compressive
122 strength was assessed by testing three cubic specimens with an edge length of 150 mm. The average value of the
123 concrete compressive strength, at the age of the fibre pullout tests, was 83.4 MPa with a coefficient of variation of
124 0.9%. This strength in cubes corresponds to an equivalent average strength in cylinders of 69.5 MPa, if adopting an
125 approximate ratio between the strength in cubes and cylinders of 1.2 (Kotsovos 1983, EN 1992-1-1 2004).

126

127 **3.1.2 Test setup**

128 The specimen is mounted in a steel supporting system, see Fig. 1. This frame incorporates a steel system composed
129 by a plate connected to a cylinder that is fixed to the testing machine frame. The cylinder/machine connection allows

130 free rotations of the entire steel frame. A steel ring is coupled to the aforementioned steel system by three steel
131 screws. The protruding end of the steel fibre is fastened to a standard grip that allows a secure hold of the fibre.

132

133 For the measurement of the fibre pullout slip, three LVDT's (linear stroke +/- 5mm) were used. In order to exclude
134 measuring deformations of the testing rig and fibre slip at the grip, the LVDT's were fixed at the upper steel ring and
135 touching the bottom surface of an aluminium plate fastened to the fibre. The plate was fixed to the fibre with two
136 fine screws and was used as a support for this LVDT configuration (Fig. 1). The deformation of the steel frame in
137 which the LVDT's were fixed is marginal, due to its considerable stiffness. Since the three LVDT's were disposed
138 around the test specimen forming an angle of 120° between consecutive LVDT's, the actual slip of the fibre is the
139 average of the three LVDT's readouts. The closed-loop displacement control was performed by the testing machine
140 internal displacement transducer, at a rate of 10 µm/s.

141

142 **3.1.3 Results**

143 3.1.3.1 Failure modes

144 The totality of both hooked and smooth aligned fibres were completely pulled out. In the case of hooked fibres, after
145 debonding of the fibre/matrix interface, the hooked was fully smoothened. This failure mode was designated as FM1.
146 A similar failure mode, FM2, was observed for some inclined fibres, however, in opposite to aligned fibres, spalling
147 of the matrix at the fibre bending point was observed. Nevertheless, the most common failure mode observed during
148 the pullout tests of inclined hooked fibres was fibre rupture, FM3. Another observed failure mode, FM4, was by
149 matrix spalling. In this case, the fibre was almost fully pulled out from the concrete specimen. However, when the
150 embedded end of the fibre approaches the exit point of the concrete matrix, a portion of concrete near the fibre
151 bending point was detached. This failure mode was only observed for a few fibres with an inclination angle of 60°
152 and a $L_b=10$ mm. Premature fibre or matrix failures were observed, exclusively in pullout specimens with inclined
153 fibres. Moreover, fibre rupture was the predominant failure mode for a 30° inclination angle, whereas for an angle of
154 60° and lower embedded lengths matrix failure was also registered. For hooked inclined fibres, the fibre rupture
155 occurred for stresses lower than the fibre's tensile strength. This can be explained by the fact that inclined fibres are
156 submitted to a mixed tensile - bending mode. Hence, as the inclination angle increases, the bending moment will
157 increase resulting in a decrease of the fibre's tensile rupture stress. The failure modes observed for each series are

158 indicated in Table 2. When more than one failure mode occurred for a specific series, the number of specimens
159 corresponding to each failure type is indicated between parentheses.

160

161 3.1.3.2 Pullout load - slip curves

162 The average pullout load - slip curves for the tested series are depicted in Fig. 2. In general, for both analyzed hooked
163 and smooth aligned fibres, the configuration of the pullout load - slip curve was similar, regardless the fibre
164 embedded length but, as expected, the peak load and the dissipated energy increased with L_b (see Fig. 2a). In smooth
165 fibres, after the peak load was attained a sudden drop was observed, which corresponds to an abrupt increase of
166 damage at the fibre/paste interface (unstable debond). Afterwards, fibre/paste friction was the commanding
167 mechanism of the pullout behaviour. In this part of the post-peak branch, the load decreased with the increase of slip,
168 since the available frictional area decreases, as well as the roughness of the failure surface. On the other hand, the
169 post-peak load decay in hooked fibres was not so abrupt than in smooth fibres, since with the increase of the slip the
170 fibre mechanical anchorage started to become progressively mobilized. At an approximately 4.5 mm slip
171 (corresponding approximately to the smoothened hook length), the pullout process occurs under frictional resistance
172 in similitude to smooth fibres.

173

174 In the case of hooked fibres with a 30° inclination angle, as previously seen, two failure modes occurred, which were
175 reflected into two distinct types of pullout-slip curves. In Fig. 2b, the average curve is represented up to the slip
176 where the fibre rupture took place; therefore the curve averaging was performed only up to a slip correspondent to
177 the peak load. For some specimens, sudden load drops were observed before attaining the peak load. This was a
178 consequence of matrix wedges that have spalled. After each completion of wedge spalling off, a new more stable
179 wedge was formed, and the remaining fibre segment embedded in the matrix was then pulled out. On the other hand,
180 for the post-peak behaviour of smooth fibres with an inclination angle of 30° the load also decreases with the
181 increase of slip. Comparatively to the aligned smooth fibres, the load decay is lesser abrupt, since the influence of the
182 frictional resistance is more significant for inclined fibres.

183

184 A completely distinct behaviour was observed for the series with an inclination angle of 60° (Fig. 2c). As previously
185 seen, the hooked series with the latter inclination angle failed by fibre rupture, with the exception of one specimen

186 (Cunha 2010), whereas in the series of smooth fibres were fully pulled out. As the inclination angle increased, the
187 stresses concentration at the fibre exit point from the matrix increased, therefore the concrete matrix is more prone to
188 cracking and spalling. In terms of pre-peak behaviour, this was reflected in a significant loss of stiffness. Comparing
189 Fig. 2b, and Fig. 2c, it can be perceived that for the series with a 60° angle, cracking and spalling started for a lower
190 load level. Moreover, as a larger portion of concrete was pushed or pulled out, a larger fibre length can be more
191 easily bent. Therefore an additional displacement correspondent to the free deformation of the fibre's protruding part
192 will be added to the measured slip, which promotes the stiffness decrease up to the peak load. Regarding the post-
193 peak in smooth fibres, smoother load decay was observed than in case of smooth fibres aligned at 30° angle, since for
194 a 60° inclination angle the frictional resistance due to the force component perpendicular to the fibre axis is much
195 higher.

196

197 3.1.3.3 Effect of the fibre embedded length

198 In Table 2 are indicated both the average values of the peak pullout load, N_{max} , and the corresponding coefficient of
199 variation, CoV. Generally, the N_{max} increased linearly with the L_b for both hooked and smooth fibres. The series
200 H_Lb_60 was the only exception, since it was observed a decrease on the N_{max} when L_b increased from 20 mm to 30
201 mm. In the case of aligned fibres the influence of L_b was more significant on the smooth fibres, since an increase of
202 more than 100% on N_{max} occurred, increasing L_b from 20 to 30 mm, while smaller increments were registered in the
203 hooked fibres. In fact, for hooked fibres the increment of L_b from 10 to 30 mm provided an increase on the N_{max} of
204 about 20%. These results demonstrate that the pullout response of hooked fibres at given L_b is predominantly
205 influenced by the mobilization and smoothening of the hook, which is in accordance with published findings
206 (Naaman and Najm 1991, Robins *et al.* 2002). For inclined smooth fibres, in resemblance to the aligned hooked
207 fibres, the increase of N_{max} with the increase of L_b was also relatively small, respectively, 17% and 23% for an
208 inclination angle of 30° and 60° (for the comparison between $L_b = 20$ mm and $L_b = 30$ mm). These results point out
209 that, in inclined fibres, the enhanced frictional resistance due to the force component normal to the fibre axis (due to
210 the fibre inclination) plays a more important role on the peak pullout load than the L_b . This is even more relevant on
211 the inclined hooked fibres, since both mechanical deformation of the hook and frictional resistance actuate together.
212 Therefore, for the latter series, the increase of N_{max} with L_b will be smaller than for smooth fibres. Moreover, this

213 increment decreases with the inclination angle (20%, 15% and 7%, respectively for the hooked series with a 0°, 30°
214 and 60° – these values correspond for the comparison between $L_b = 10$ mm and $L_b = 30$ mm).

215

216 The average values of the slip at peak pullout load, s_{peak} , and the corresponding coefficient of variation, CoV, are
217 indicated in Table 2. For both smooth and hooked aligned fibres a slight increase of s_{peak} with L_b was observed,
218 whereas for inclined fibres no clear tendency of the influence of the L_b on the s_{peak} was found. The high values
219 obtained for the CoV hampers the perception of a clear trend between L_b and s_{peak} for the inclined fibres' series.

220

221 A detailed overview of the pullout toughness of the current experimental tests can be found elsewhere (Cunha 2010).
222 Nevertheless, the main conclusion that can be drawn is that, the overall toughness is markedly influenced by the type
223 of failure mode, since fibre fracture significantly reduces the toughness when compared to the cases where fibres
224 underwent a complete pullout.

225

226 3.1.3.4 Effect of the fibre inclination angle

227 In general, the N_{max} increases up to an inclination angle of 30° and then decreases for 60° angle. For both hooked and
228 smooth fibres the highest maximum pullout load was observed for an inclination angle of 30°. However, the increase
229 of the maximum pullout load with the inclination angle was more significant on the smooth fibre series. The series of
230 smooth fibres with a 30° inclination angle had a N_{max} 30% and 125% higher than the aligned smooth fibre series,
231 respectively, for $L_b = 30$ mm and $L_b = 20$ mm. On the other hand, for the hooked fibre series with a 30° inclination
232 angle, the N_{max} is just 7% to 15% higher than aligned hooked fibre series. In spite of the increase of the frictional
233 pullout component with the inclination angle, increasing the angle from 30° to 60° led to a slight decrease on the
234 N_{max} . Remember that for the series of inclined hooked fibres, fibre rupture was the commanding failure mode.
235 Moreover, the average peak pullout load was smaller for the series with a 60° inclination angle than for 30°.

236

237 The slip at peak load, s_{peak} , increased with the inclination angle for both hooked and smooth fibres, especially in the
238 smooth fibres. From 0° to 30° a slight increase on s_{peak} was observed, while a significant increase of s_{peak} was
239 registered from 30° to 60°. In fact, for the series of smooth fibres with 60° inclination angle, the s_{peak} was
240 approximately 5 to 9 times higher than for a 30° angle, whereas for the hooked series it was 1.3 to 2.3 times higher.

241 The significant higher values of s_{peak} for a 60° angle can be ascribed to other additional mechanisms that usually
242 occur for inclined fibres than for aligned fibres. As the fibre inclination angle increases, the stresses concentrated at
243 the fibre bending point also increase. This leads to a more significant portion of concrete that crushes or pushes off at
244 the crack plane. As the volume of concrete that spalls is higher, a larger fibre length is subjected to bending, resulting
245 an additional measured slip due to the fibre deformation. So, for large inclination angles, such as 60° , the slip
246 includes a significant parcel which is due to fibre deformation.

247

248

249 **3.2 Uniaxial tensile tests**

250

251 **3.2.1 Specimens**

252 For each batch eight cylinders with a diameter of 150 mm and 300 mm height were casted. The compressive strength
253 of each SFRSCC batch was assessed by executing compression tests with three cylindrical specimens of 150 mm
254 diameter and 300 mm height. At the date when the tests were performed (approximately at 30 days), the series with
255 30 kg/m^3 of fibres had an average compressive strength of 71.1 MPa with a coefficient of variation, CoV, of 1.9 %,
256 while in the series with 45 kg/m^3 of fibres an average compressive strength of 67.2 MPa was obtained with a CoV of
257 1.4 %.

258

259 **3.2.2 Test setup**

260 The RILEM TC 162-TDF (2001) recommendations for the execution of uniaxial tensile tests with steel fibre
261 reinforced concrete were adopted in this work. According to this document, a notched cylinder of both 150 mm
262 diameter and height should be used. The specimens were sawn out from standard cylinders of 150 mm diameter and
263 300 mm height. Afterwards, a notch along the perimeter with a 15 mm depth and 5 mm thick was swan at mid height
264 of the final test specimen. When the sawing operations were executed the specimens were in its hardened-mature
265 phase. These operations were conducted with care in order to ensure that the notch become perpendicular to the
266 specimen's axis.

267

268 Afterwards, each specimen was ground and carefully cleaned with both compressed air and solvent. The specimen
269 was then directly glued, "in situ", to the loading platens of the testing rig. The selected glue is a high strength epoxy
270 resin, which achieves a tensile strength of about 30 MPa and a bond strength between 4 to 8 MPa (depending on the
271 surface material and treatment characteristics).

272

273 A servo-hydraulic system with a 2000 kN static load carrying capacity with a high stiff frame was used to perform
274 the uniaxial tensile tests, Fig. 3a. A test was performed in closed-loop displacement control using the average signal
275 of three displacement transducers mounted on two steel rings disposed at equal distances along the perimeter of the
276 specimen, Fig. 3b. The adopted gauge length was 35 mm, smaller than the upper limit length of 40 mm suggested by
277 RILEM TC 162-TDF (2001). The following displacement rates were adopted: 5 $\mu\text{m}/\text{min}$ up to a displacement of
278 0.1 mm; 100 $\mu\text{m}/\text{min}$ until the completion of the test, i.e. a 2 mm displacement.

279

280 **3.2.3 Results**

281 3.2.3.1 Stress - displacement curves

282 In all the performed uniaxial tensile tests, the cracking occurred along the notched plane; hence the desired crack
283 localization was assured. The average curve and the envelope of the experimental uniaxial tensile stress - average
284 displacement relationships, σ - δ_{avg} , for a fibre content of 30 and 45 kg/m^3 are presented in Figs. 4 and 5, respectively.
285 Note that δ_{avg} is the average measurement over the gauge length of 35 mm. Hereinafter these series will be
286 designated by Cf30 and Cf45, respectively. A detailed view of the initial part of the experimental response is
287 depicted on the right side of these figures. For both tested series the σ - δ_{avg} response is linear almost up to peak. Only
288 just before the peak load some non-linearity is observed. Once the peak load is attained, the load had a relatively
289 accentuated decrease up to a displacement of about 0.10 mm (see right side of both Figs. 4 and 5). Beyond this
290 displacement value, a plateau or a pseudo-hardening-plastic response has occurred. In general, the post-peak pseudo-
291 hardening is observed in the Cf45 series, but this response was also observed in some specimens of the Cf30 series
292 (Cunha 2010). On the other hand, after the plateau on the Cf30 series, i.e. beyond a displacement of nearby 0.8 mm,
293 the residual stress starts to decrease with sudden strength losses corresponding to the fibre fracture. In fact, during
294 the execution of the tensile tests this was audible by the peculiar sound of fibre fracturing.

295

296 According to Stroeven and Hu (2006) the average orientation angle value of the active fibres crossing a leading crack
297 is 35° (this value was analytically derived), and a similar value (34°) was experimentally observed by Soroushian and
298 Lee (1990). According to the fibre pullout tests carried out, fibres at an inclination angle of 30° with the load
299 direction lead predominantly to fibre rupture when the slip was in the range [0.6-1.0] mm (Fig. 2b, Cunha 2010).
300 These reasons can justify the occurrence of the significant residual stress decay after the displacement of 0.8 mm, in
301 the result of fibres rupture in Cf30 series, where the number of active fibres is relatively reduced for the strength of
302 the matrix.

303

304 Regarding the Cf45 series (Fig. 5), the post-peak pseudo-hardening observed can be ascribed to two reasons: a higher
305 number of active fibres crossing the crack; both the compressive strength and stiffness of the Cf45 series matrix are
306 lower than the Cf30, thus the fibre/matrix bond properties are not so favourable to proportionate fibre rupture. After
307 peak load, the stress starts to decrease until a minimum stress is attained, roughly about 0.1 mm. As micromechanics
308 of hooked ends fibre pullout demonstrate (Cunha 2010), above this displacement the strengthening provided by
309 fibre's hook starts to be the predominant fibre reinforcement mechanism. Since there are more fibres intersecting the
310 crack and due to the lower tensile strength of the concrete, the energy released during the cracking is smaller when
311 compared to the Cf30 series. Moreover, due to the lower tensile strength and matrix stiffness, fibres did not fracture
312 so often as in the Cf30 series. Consequently, beyond a displacement around 0.1 mm a pseudo-hardening phase
313 occurred up to a displacement value of about 1.0 mm.

314

315 In general, the responses exhibit very low scatter in the pre-peak phase. On the other hand, in the post-peak branch
316 the scatter was considerably higher, particularly in Cf30 series, and for a displacement higher than 0.1 mm. Up to
317 0.1 mm the commanding pullout reinforcement mechanism is the chemical bond (Cunha 2010), hence the influence
318 of the fibre dispersion, i.e. fibre spatial distribution is not so important. As the crack width increases, the fibre's
319 hook-ends start to be mobilized; hence the scatter of the post-peak behaviour increases due to the variation of the
320 fibre dispersion between different specimens. Moreover, for the Cf30 series fibre rupture was the predominant fibre
321 failure mode, as it was expected taking into account the results obtained in fibre pullout tests presented in the
322 previous sections.

323

324 3.2.3.2 Stress - crack opening curves

325 In Fig. 6 and 7 are depicted the envelope of the stress-crack width curves obtained in the tests for the Cf30 and Cf45
326 series, respectively. A stress crack width curve ($\sigma-w$) was derived from a stress displacement curve ($\sigma-\delta$) according
327 to the recommendations of RILEM TC 162-TDF (2001). In these figures are also included the average and the
328 characteristic $\sigma-w$ curves. The characteristic $\sigma-w$ curve for the lower bound (L.B.) and upper bound (U.B.) with a
329 confidence level of $k = 95\%$ was obtained from the average curve computed from all tests, $\bar{\sigma}_w(w)$, according to
330 RILEM TC 162-TDF (2001):

$$\sigma_{w,k}(w) = \bar{\sigma}_w(w) \frac{G_{F2mm,k}}{\bar{G}_{F2mm}} \quad (1)$$

331 where \bar{G}_{F2mm} is the average energy dissipated up to a crack width of 2 mm and $G_{F2mm,k}$ is the characteristic energy
332 dissipated for the same crack width. For computing $G_{F2mm,k}$ a t -Student distribution was assumed.

333

334 3.2.3.3 Stress and toughness parameters

335 The average and characteristic values of relevant stress and toughness parameters, as well as the respective
336 coefficients of variation obtained from the performed uniaxial tests are included in Table 3. In this Table σ_{peak} is the
337 maximum stress, while $\sigma_{0.3mm}$, σ_{1mm} and σ_{2mm} are the stress at a crack width value of, respectively, 0.3, 1 and 2 mm.
338 On the other hand G_{F1mm} and G_{F2mm} represent the dissipated energy up to a crack width of, respectively, 1 and 2
339 mm. The characteristic values were obtained for a $k=95\%$ confidence level assuming a t -Student distribution. The
340 number of total fibres, N_f , and effective fibres, N_f^{eff} , counted at the fracture surface are also included in Table 3.
341 Effective fibres were considered all the fibres that had the hook deformed, as well as the fibres that have ruptured. In
342 spite of some researchers do not consider the ruptured fibres as "effective", in the authors' opinion they should be
343 considered, since they are able of transferring forces between the crack surfaces up to reasonable crack width. The
344 "fully effectiveness" of this type of fibres can be questionable, but it is feasible to admit that they are "partially
345 effective". For the sake of simplicity, let's assume that the fibre slip when it is being pulled out is approximately
346 equal to the crack width. Then, as it can be observed in Cunha (2009), in the pullout behaviour of inclined fibres,

347 depending on the inclination angle and L_b , a fibre that fails by rupture can sustain forces up to a slip, i.e. crack width
348 that varies from 0.7 mm to 4 mm.

349

350 From the analysis of Table 3 it is verified that, in general, the stress and toughness parameters increased with the
351 fibre content, as it would be expected. The only exception was the peak stress which was nearby 10 % lower for the
352 series with a content of fibres of 45 kg/m^3 . Note that this decrease is not due to the content of fibre, even though it
353 could be indirectly appointed to it, since the aggregates, cement and additions contents for each series are distinct in
354 order to attain self-compactability requirements. Moreover, the peak stress cannot be regarded as the tensile strength,
355 but as estimation, since due to the notch introduced in specimen, apart the fact that a stress concentration exists at the
356 notch tip, cracking does not start at the weakest material point (Van Mier and Van Vliet, 2002).

357 Regarding the post-cracking stress parameters, a significant increase with the increase of fibre content was observed.

358 A small increase of the fibre content of 15 kg/m^3 provided high increments on the values of the residual stresses,
359 having varied from 2 to 7 times depending on the crack width value. Such increase on the residual stresses is not
360 simply justified by the higher number of fibres crossing the crack surface for the Cf45 series. It must be noticed that
361 there are other factors that have contributed for this fibre reinforcement effectiveness. One of these factors is the
362 predominant fibre failure mode that, as already mentioned, was fibre rupture in the Cf30 series, while in Cf45 series
363 was the fibre pullout.

364

365 In what concerns to the dissipated energy (G_F), in the Cf45 series a significant increase (2 to 2.6 times) up to both
366 deflections of 1 and 2 mm was observed. In general, the CoV values obtained for the G_F parameters were
367 considerably smaller for the Cf45 series. At a first glance, these values may seem relatively high. The magnitude of
368 such values is, however, smaller than the ones reported by other authors with the same test procedure and specimen's
369 dimensions for conventional fibre reinforced concrete (Barragan 2002, Läofgren *et al.* 2008).

370

371 In Fig. 8 is depicted the relationship between the total number of fibres, N_f , and the number of effective fibres, N_f^{eff} ,
372 at the fracture surface obtained for all the tested specimens of both Cf30 and Cf45 series. Since the uniaxial tensile
373 test specimens (with 150 mm height) were obtained from distinct parts of a standard cylinder with 300 mm height,

374 additionally, the specimens obtained from the bottom and upper part of the standard cylinder are distinguished in
375 Fig. 8. This process enables an indirect assessment of an eventual influence of the material specific weight (gravity)
376 on the fibre distribution. Fig. 8 shows a linear relationship between the N_f and N_f^{eff} . Moreover, a tendency for an
377 eventual higher concentration of fibres in the bottom part of the specimen was not found. Dupont (2003) has also
378 found a linear relationship between the total number of fibres and the number of effective fibres for small fibre
379 contents. However, for higher fibre contents this relationship becomes nonlinear, i.e. the N_f^{eff} increment becomes
380 smaller with the further increase of N_f . In fact, due to the group effect, N_f^{eff} may decrease as the fibre spacing
381 decreases due to the higher probability of mutual influence of adjacent fibres (Naaman and Shah 1976).

382

383 3.2.3.4 Influence of the number of effective fibres on the post-cracking parameters

384 Throughout Figs. 9a to 9f are depicted the relationships between N_f^{eff} and the aforementioned post-cracking
385 parameters obtained from the uniaxial tensile tests.

386

387 Regarding the peak stress, σ_{peak} , no significant relation was observed with the N_f^{eff} increase (Fig. 9a). In spite of
388 σ_{peak} for the Cf45 series have been, in general, smaller than for Cf30 series, due to the reasons already pointed out,
389 there was no significant relation between σ_{peak} and the provenience of specimen, i.e. from lower or upper part of a
390 standard cylinder. This suggests that there was no significant segregation of the matrix skeleton and paste in the
391 casted cylinders. On the other hand, for the residual stresses at a crack width of 0.3 mm, $\sigma_{0.3mm}$, and 1 mm, σ_{1mm} , a
392 linear relationship between these residual stresses and the N_f^{eff} is quite evident (Figs. 9b and 9c). This was expected,
393 since the residual stress sustained by the crack is intimately related to number of mobilized fibres. Concerning the
394 Cf45 series, the residual stresses for the "top specimens" exhibited a lower scatter when compared to the values from
395 the "bottom specimens". Due to technical problems occurred during the test program, the number of specimens from
396 the bottom part (5) was different of the specimens obtained from the top part of the standard cylinder (3), therefore
397 no conclusive elations can be withdraw in this subject.

398

399 The relation between the residual stress at a crack opening of 2 mm, σ_{2mm} , and the N_f^{eff} is represented in Fig. 9d.
400 When compared to the $\sigma_{0.3mm} - N_f^{eff}$ and $\sigma_{1mm} - N_f^{eff}$ relationships the overall trend is distinct. First of all, there are
401 two clear distinct trends for the $\sigma_{2mm} - N_f^{eff}$ relationship. For the Cf30 series, the increase of σ_{2mm} with N_f^{eff} is
402 marginal and can be assumed null. On the other hand, for the Cf45 series a linear increase of σ_{2mm} with N_f^{eff} is quite
403 visible, in spite of a higher scatter than those register for the $\sigma_{0.3mm} - N_f^{eff}$ and $\sigma_{1mm} - N_f^{eff}$ relationships. Moreover, a
404 clear jump on the σ_{2mm} value from the series Cf30 to Cf45 is visible, even though the small difference of N_f^{eff}
405 between the Cf30 series' specimen with the highest N_f^{eff} and the Cf45 series with the lowest N_f^{eff} . This considerable
406 jump on σ_{2mm} value from the Cf30 to Cf45 specimens is not ascribed to the increase of effective fibres, hence this
407 jump comprises differences on the fibre micro-mechanical behaviour between the two series with distinct fibre
408 content, as previously stated. Since for the series Cf30, in general, the fibres ruptured before a 2 mm crack width, in
409 case of Cf45 series, in general, the fibres were fully pulled-out enabling a higher crack bridging stress transfer
410 effectiveness.
411
412 Finally, in both series and regardless the extracting location of the specimen, the dissipated energy has shown a linear
413 increase with N_f^{eff} , for both considered crack width limits (Fig. 9e and 9f). Nevertheless, there are two aspects that
414 should be emphasized. For the energy dissipated up to a 2 mm crack width, G_{F2mm} , there is also a jump on the
415 G_{F2mm} value from the Cf30 to Cf45 series for the same reasons pointed for the $\sigma_{2mm} - N_f^{eff}$ relationship. The other
416 aspect is that the increment rate of both G_{F1mm} and G_{F2mm} with N_f^{eff} for the Cf45 series was slightly smaller than for
417 Cf30 series due to the higher stiffness of the Cf30 matrix.

418
419
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423

424 4. NUMERICAL SIMULATION

425

426 4.1 Numerical model

427 In steel fibre reinforced cementitious composites (SFRC), steel fibres and matrix are bonded together through a weak
428 interface, which behaviour is important to understand the mechanical behaviour of SFRC, since properties of the
429 composite are greatly influenced by this interface zone (Pereira 2006). Taking this into account and that the fibre
430 contribution for the post-cracking behaviour of a composite is significantly higher than the unreinforced matrix
431 contribution, it was settled to model the SFRC as a two phase material. In the developed model, SFRC is treated as a
432 heterogeneous medium comprised by one homogeneous phase (aggregates and paste), and another one composed by
433 the steel fibres. The fracture process of the cementitious matrix (unreinforced) is modelled with a multi-directional
434 three-dimensional smeared crack model. The formulation of this crack model can be found elsewhere (Ventura *et al.*
435 2008). On the other hand, the stress transfer between crack planes due to the fibres bridging an active crack is
436 modelled with embedded cable elements.

437

438 The random fibre distribution in the matrix is simulated with an algorithm based on the Monte Carlo method,
439 providing a realistic distribution of the fibres in a certain specimen. In the present work only one mesh was used in
440 the numerical simulations of the experimental tests. In Cunha (2010) can be found detailed statistical information of
441 several "virtual meshes" generated with the Monte-Carlo procedure for both cylinders and prismatic specimens. The
442 geometry, positioning and orientation of the fibres are subsequently inserted in a three dimensional finite element
443 mesh. This approach was adopted, mainly, due to the following reasons: 1) a homogenization of the reinforcements
444 (fibres) crossing a certain solid element is difficult due to the random nature of the fibre distribution; 2) the discrete
445 modelling of the reinforcements as cable elements located along the solid element nodes leads to a higher
446 computational cost due to an unnecessary concrete mesh refinement. Moreover, this mesh refinement could lead to
447 numerical errors caused by distorted elements for comprising the fibre distribution.

448

449 The contribution of the steel fibres crossing a solid volume is given by:

$$K^{rc} = K^c + \sum_{i=1}^{n_f} K_i^f \quad (2)$$

450 where K^{rc} , K^c and K_i^f are, respectively, the stiffness matrix of the reinforced solid element (plain concrete + fibre
451 reinforcement contribution), the stiffness matrix of plain concrete and the stiffness matrix of the i^{th} embedded fibre;
452 n_f is the total number of fibres crossing the "mother" element.

453

454 A tri-linear stress-strain ($\sigma_f - \varepsilon_f$) diagram was used for modelling the fibres' bond - slip behaviour. This relationship
455 was obtained by a manual fitting procedure of the fibre pullout tests average curves obtained for three distinct
456 inclinations angles, α , (0° , 30° and 60°). In Fig. 10 is depicted the procedure adopted to obtain the $\sigma_f - \varepsilon_f$, where ε_f , s
457 and l_b are, respectively, the embedded cable strain, the steel fibre's slip and the crack band-width ($l_b=5$ mm, same as
458 the height of the finite element where the crack is forced to appear); σ is stress computed from the pullout force, P ,
459 divided by the fibre's cross sectional area, A_f . Afterwards the completion of the $\sigma_f - \varepsilon_f$ diagram's last branch, the
460 stress transfer stops abruptly. The crack band-width is the concept implicit in the 3D smeared crack model in order to
461 assure independence of the results from the mesh refinement (Bazant and Oh 1983). The $\sigma_f - \varepsilon_f$ law assigned to each
462 embedded cable depends on the inclination angle, θ , between the cable and the normal vector of the active crack
463 surface, \hat{n} (see Fig. 11).

464

465

466 4.2 Simulations

467

468 The model performance is appraised by simulating the uniaxial tensile tests already presented in this work. In Figs.
469 12a and 12b are depicted, respectively, the mesh used exclusively for the concrete matrix phase and the mesh of the
470 3D embedded truss elements for the case of the self-compacting concrete reinforced with 30 Kg/m^3 of steel fibres
471 (Cf30 series). In the present mesh are used Lagrangian 8-noded solid elements for modelling the plain concrete
472 contribution. Since the specimen had a notch at its mid-height, all the non-linear behaviour was localized at the notch
473 region, thus a $2 \times 2 \times 1$ Gauss-Legendre integration scheme is used (1 integration point in the loading direction). The
474 remaining solid elements are modelled with linear elastic behaviour, and a $2 \times 2 \times 2$ Gauss-Legendre integration

475 scheme is adopted. The Cornelissen *et al.* (1986) softening law was used for modelling the post-cracking behaviour
476 of SCC. The material properties of the plain concrete matrix used in the simulations are included in Table 4. These
477 values were obtained taking into account the strength class registered for the Cf30 and Cf45 series.

478

479 On the other hand, the steel fibres are modelled with 3D embedded elements with two integration points (Gauss-
480 Legendre). For all the embedded elements is assumed a nonlinear behaviour. From the fibre pullout tests were
481 ascertained three distinct $\sigma_f - \varepsilon_f$ laws corresponding, respectively, to the studied fibre pullout inclination angles, α
482 (0° , 30° and 60°). Due to the impossibility of having a $\sigma-s$ law for every possible inclination angle, the $\sigma-s$ laws
483 obtained from the pullout tests with an angle α of 0° , 30° and 60° was assigned, respectively, to the embedded cables
484 with an orientation towards the active crack surface (θ) ranging from $[0^\circ, 15^\circ]$, $[15^\circ, 45^\circ]$ and $[45^\circ, 75^\circ]$. The strain in
485 an embedded element, ε_f , is obtained from the displacement field of its "mother" element, and the corresponding slip
486 is determined multiplying ε_f by the crack band width. The relationships used for each α , are given in Table 5.

487

488 In Figs. 13 and 14 are depicted the numerical simulation of the uniaxial tensile tests of the Cf30 and Cf45 series,
489 respectively. Regarding the Cf30 series a good agreement with the experimental response was obtained. On the other
490 hand, for the Cf45 series two numerical simulations were carried out. The first one adopting the $\sigma-s$ laws obtained
491 from the fibre pullout tests (in which fibre rupture was observed for a $\alpha=30^\circ$ and 60°). Up to a crack width of
492 approximately 1.0 mm a relatively good agreement with the experimental response is observed. Significant stress
493 decay is, however, observed after this crack width limit due, mainly, to the rupture of the embedded cables with an
494 inclination θ between 15° and 45° . Note that, the embedded cables with an inclination θ between 45° and 75° have
495 also rupture, but only for crack widths higher than 2 mm. Since for the uniaxial tensile tests of Cf45 series, fibre
496 rupture did not occur so often, due to both a less resistant matrix and the reduction of the average fibre orientation
497 angle towards the crack plane, another simulation was carried out assuming that the embedded cables with an
498 inclination θ between 15° and 45° did not rupture (see Table 5). With this approach the quality of the simulation was
499 improved (Fig. 14).

500

501

502 5. CONCLUSIONS

503

504 In the present work the tensile behaviour of self-compacting concrete (SCC) reinforced with two distinct hooked end
505 steel fibre contents (30 and 45 kg/m³) was characterized by performing displacement controlled tensile tests. These
506 results were discussed based upon the micro-mechanical behaviour of the steel fibres used in the composition. The
507 fibres' micro-mechanical behaviour was assessed by means of single fibre pullout tests.

508

509 The fibre orientation (0°, 30° and 60°) and fibre embedded length (10, 20 and 30 mm) influence on the fibre pullout
510 behaviour was studied. In general, two main pullout failures modes were observed during the pullout tests. The
511 complete fibre pullout was observed for aligned hooked fibres, aligned smooth fibres and inclined smooth fibres,
512 whereas for inclined hooked fibres the observed principal failure mode was fibre rupture. In general, the maximum
513 pullout load had an almost linear increase with the L_b for both hooked and smooth fibres. Regarding the effect of the
514 fibre orientation angle, the maximum pullout load increased up to an inclination angle of 30° and then decreased for a
515 60° inclination angle. For both smooth and hooked aligned fibres a slight increase of the slip at peak load with the
516 fibre L_b was observed. On the other hand, regarding the influence of the fibre orientation angle, a slight increase on
517 the slip at peak load was observed for a 30° angle, whereas for a 60° angle the slip at peak stress increased
518 considerably. This significant increase for a 60° angle can be ascribed to other additional mechanisms that usually
519 occur on the pullout of inclined fibres in opposite to aligned fibres. Those phenomena add to the measured slip a
520 supplementary displacement corresponding to the fibre deformation in consequence of matrix spalling around fibre
521 exit point.

522

523 From the stress-displacement curves obtained in the uniaxial tensile tests the stress-crack opening relationships were
524 derived, and residual stress and energy dissipation parameters, able of indicating the effectiveness of fibre
525 reinforcement mechanisms, were determined. The relatively high compactness of the matrix system of these
526 SFRSCC, the number of effective fibres bridging the fracture surface and the results obtained in the fibre pullout
527 tests were all taken into account to interpret the post-cracking tensile behaviour of the tested composites. In general,
528 a linear relationship between the post-cracking parameters and the effective number of fibres was observed. A strong
529 dependency on the type of fibre failure mode and the stiffness of the matrix and number of effective fibres was

530 detected, which justified the occurrence of a pseudo-hardening branch in the softening phase of the SFRSCC with
531 the highest fibre content (Cf45 series), as well as the significant residual strength decay occurred in the SFRSCC
532 with the lowest fibre content (Cf30). In fact, in Cf45 series the predominant failure mode was fibre pullout, while in
533 Cf30 series fibre rupture was the predominant failure mode.

534

535 Taking the results obtained in fibre pullout tests and considering SFRSCC as a two phase material modelled by a 3D
536 multi-directional smeared crack model for the concrete and considering the fibres as discrete steel embedded short
537 cables distributed in the matrix according to a Monte Carlo method, the carried out tensile test were simulated with
538 good accuracy. With a realistic approximation of the actual fibre distribution and with the knowledge of the micro-
539 mechanical behaviour of the fibres, it was possible to predict the macro-mechanical behaviour of laboratory's
540 specimens.

541

542

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544

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641 **TABLES AND FIGURES**

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650

Table 1 – Compositions for 1 m³ of SFRSCC.

Cement Type	Limestone	Water	Water to saturate	Super	Fine	Coarse	Crushed	Steel Fibres	W/C
I 42.5R	Filler		aggregates	plasticizer	Sand	Sand	Calcareous		
(C)	(LF)	(W)	(WS)	(SP)	(FS)	(CS)	(CA)	(Cf)	
[kg]	[kg]	[dm ³]	[dm ³]	[dm ³]	[kg]	[kg]	[kg]	[kg]	
359.4	312.2	96.9	64.7	6.9	108.2	709.4	665.2	30	0.29
401.7	344.2	117.3	65.4	7.6	178.3	668.1	668.1	45	0.31

Table 2 – Overview of the experimental pullout results.

Series	Failure mode	N_{max} [N]	CoV [%]	s_{peak} [mm]	CoV [%]
S_Lb10_60	<i>FM4</i>	154.2	43.8	3.34	45.6
S_Lb20_0	<i>FM1</i>	77.4	2.0	0.12	11.8
S_Lb20_30	<i>FM2</i>	173.5	18.2	0.19	7.4
S_Lb20_60	<i>FM2</i>	172.8	8.7	2.02	2.0
S_Lb30_0	<i>FM1</i>	155.2	9.7	0.25	14.4
S_Lb30_30	<i>FM2</i>	203.7	13.8	0.38	32.3
S_Lb30_60	<i>FM2</i>	189.4	15.0	2.17	11.8
H_Lb10_0	<i>FM1</i>	321.8	5.6	0.59	8.7
H_Lb10_30	<i>FM2(2)*</i> , <i>FM3(4)</i>	360.9	13.9	0.94	11.4
H_Lb10_60	<i>FM3(5), FM4(1)</i>	342.0	2.3	2.40	20.8
H_Lb20_0	<i>FM1</i>	347.8	2.8	0.65	9.4
H_Lb20_30	<i>FM2(2), FM3(4)</i>	400.1	4.9	1.00	9.7
H_Lb20_60	<i>FM3</i>	335.2	3.0	2.33	15.1
H_Lb30_0	<i>FM1</i>	388.2	1.6	0.69	11.0
H_Lb30_30	<i>FM3</i>	416.0	3.4	0.80	19.3
H_Lb30_60	<i>FM3</i>	365.1	2.5	2.64	23.2

FM1 - Fibre pullout; *FM2* – Fibre pullout with matrix spalling; *FM3*- Fibre rupture; *FM4* – Matrix spalling

* (i) – *i* is the number of specimens with this type of failure mode

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Table 3 – Stress and toughness parameters obtained from the uniaxial tensile tests.

Cf (kg/m ³)		N_f	N_f^{eff}	σ_{peak}	$\sigma_{0.3mm}$	σ_{1mm}	σ_{2mm}	G_{F1mm}	G_{F2mm}
		[-]	[-]	[MPa]	[MPa]	[MPa]	[MPa]	[N/mm]	[N/mm]
30	Avg.	27	19	3.392	0.649	0.520	0.184	0.685	1.007
	CoV	30.8%	28.8%	13.0%	36.6%	45.1%	22.8%	32.3%	33.7%
	k95%	20	14	3.024	0.450	0.324	0.250	0.500	0.724
45	Avg.	67	39	3.019	1.219	1.466	1.342	1.342	2.645
	CoV	16.1%	16.5%	9.9%	18.0%	12.7%	13.6%	13.6%	13.1%
	k95%	58	34	2.768	1.036	1.310	1.189	1.189	2.356

Table 4 – Plain concrete properties used in the numerical simulations.

Property	Series	
	Cf30	Cf45
Density	$\rho = 2.4 \times 10^6 \text{ N/mm}^3$	
Poisson ratio	$\nu = 0.2$	
Initial Young modulus	$41.3 \times 10^3 \text{ N/mm}^2$	$40.6 \times 10^3 \text{ N/mm}^2$
Compressive strength	71.1 N/mm^2	67.2 N/mm^2
Tensile strength	4.6 N/mm^2	4.5 N/mm^2
Fracture energy	0.117 N/mm	0.114 N/mm
Crack band-width	$l_b = 5 \text{ mm}$ (equal to the element height at the notch)	
Threshold angle	30°	

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Table 5 – Tri-linear stress-strain diagrams used for modelling the fibres' bond - slip behaviour.

α [°]	Failure mode	$\sigma_{f,1}$ [MPa]	$\sigma_{f,2}$ [MPa]	$\sigma_{f,3}$ [MPa]	$\varepsilon_{f,1}$ [-]	$\varepsilon_{f,2}$ [-]	$\varepsilon_{f,3}$ [-]
0	Pullout	588	803	441	0.030	0.090	0.500
30	Rupture	453	679	905	0.016	0.050	0.200
	Pullout	588	803	441	0.030	0.090	0.500
60	Rupture	283	362	656	0.020	0.16	0.400

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Figure 2 – Average pullout load-slip curves for a fibre inclination angle: (a) 0 degrees, (b) 30 degrees and (c) 60 degrees.

Figure 3 – Uniaxial tensile test setup: a) general view and b) positioning of displacement transducers (not scaled).

Figure 4 – Uniaxial tensile stress – displacement relationship, σ – δ_{avg} , for 30 kg/m³.

Figure 5 – Uniaxial tensile stress – displacement relationship, σ – δ_{avg} , for 45 kg/m³.

Figure 6 – Uniaxial stress - crack width relationship for 30 kg/m³.

Figure 7 – Uniaxial stress - crack width relationship for 45 kg/m³.

Figure 8 – Relationship between the total number of fibres and the number of effective fibres ant the crack surface.

Figure 9 – Relationships between the number of effective fibres and the post-cracking parameters: (a) peak stress, (b), (c) and (d) stress at a 0.3, 1 and 2 mm crack width, respectively; (e) and (f) dissipated energy up to 1 and 2 mm crack width, respectively.

Figure 10 – Determination of the embedded cable's stress - strain diagram based on the experimental pullout force-slip relationship.

Figure 11 – Three-dimensional scheme of the embedded cable intersecting an active crack (n is the vector normal to the crack plane).

Figure 12 – Three-dimensional finite element mesh: (a) concrete phase and (b) fibre phase (Cf30 series).

Figure 13 – Numerical simulation of the Cf30 series' uniaxial tensile tests.

Figure 14 – Numerical simulation of the Cf45 series' uniaxial tensile tests.

Cunha, V.M.C.F.; Barros, J.A.O.; Sena Cruz, J.M. (2011) "An integrated approach for modelling the tensile behaviour of steel fibre reinforced self-compacting concrete." *Cement and Concrete Research*, 41, 64–76.

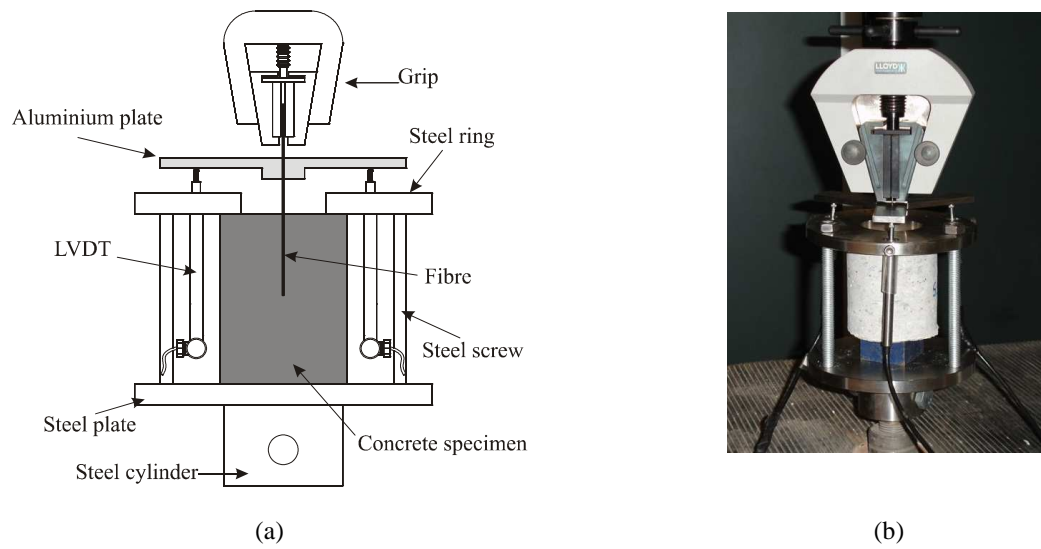
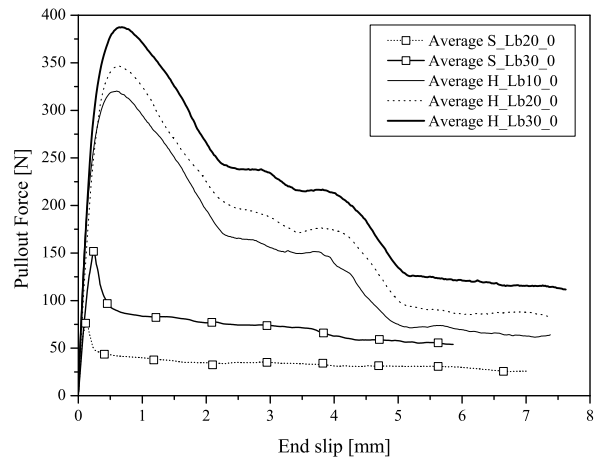
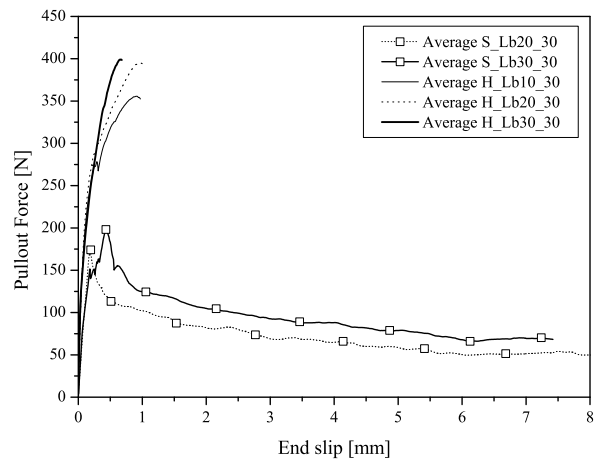


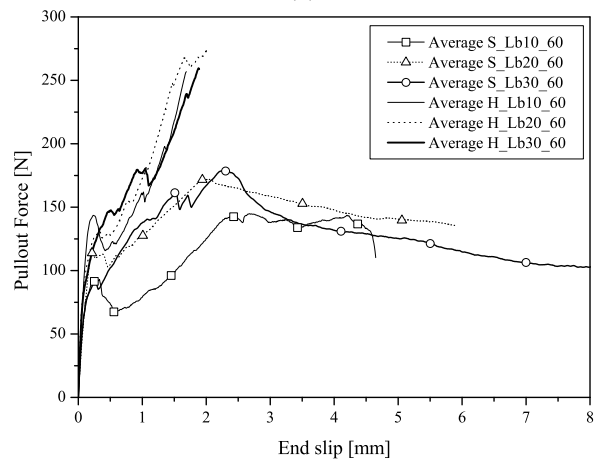
Fig. 1 – Set-up of the single fibre pullout test: (a) schematic representation; b) photo.



(a)



(b)



(c)

Fig. 2 – Average pullout load-slip curves for a fibre inclination angle: (a) 0 degrees, (b) 30 degrees and (c) 60 degrees.

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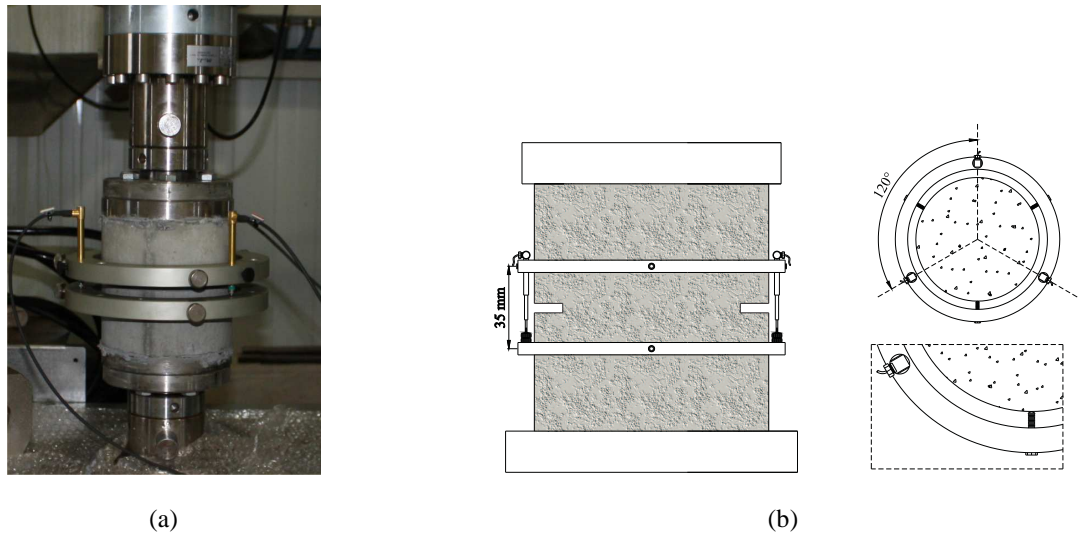


Fig. 3 – Uniaxial tensile test setup: a) general view and b) positioning of displacement transducers (not scaled).

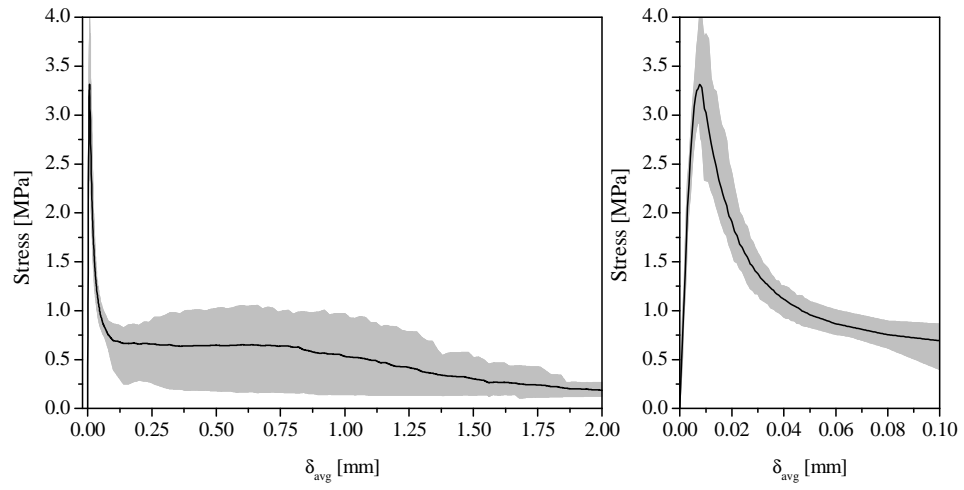


Fig. 4 – Uniaxial tensile stress – displacement relationship, σ – δ_{avg} , for 30 kg/m³.

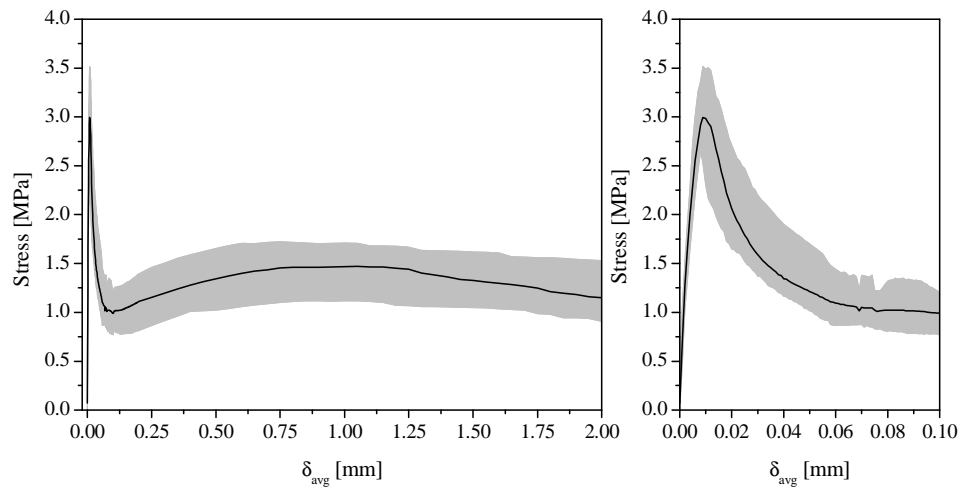


Fig. 5 – Uniaxial tensile stress – displacement relationship, σ – δ_{avg} , for 45 kg/m³.

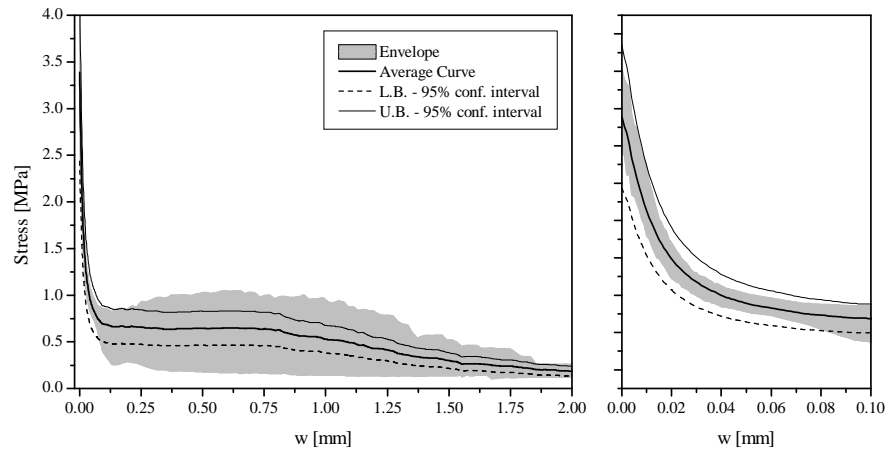


Fig. 6 – Uniaxial stress - crack width relationship for 30 kg/m³.

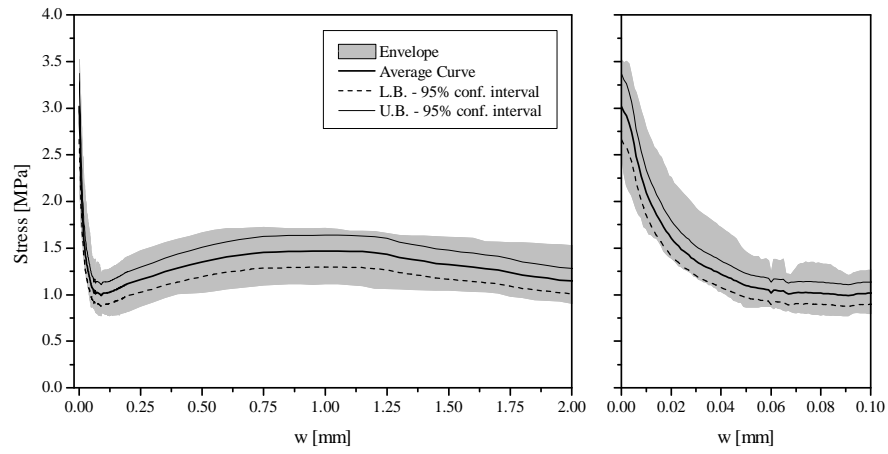


Fig. 7 – Uniaxial stress - crack width relationship for 45 kg/m³.

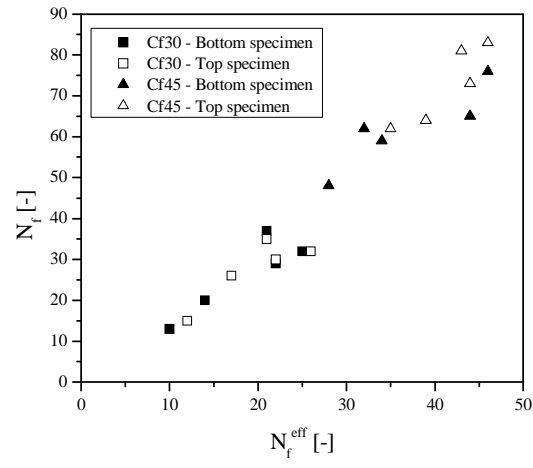


Fig. 8 – Relationship between the total number of fibres and the number of effective fibres ant the crack surface.

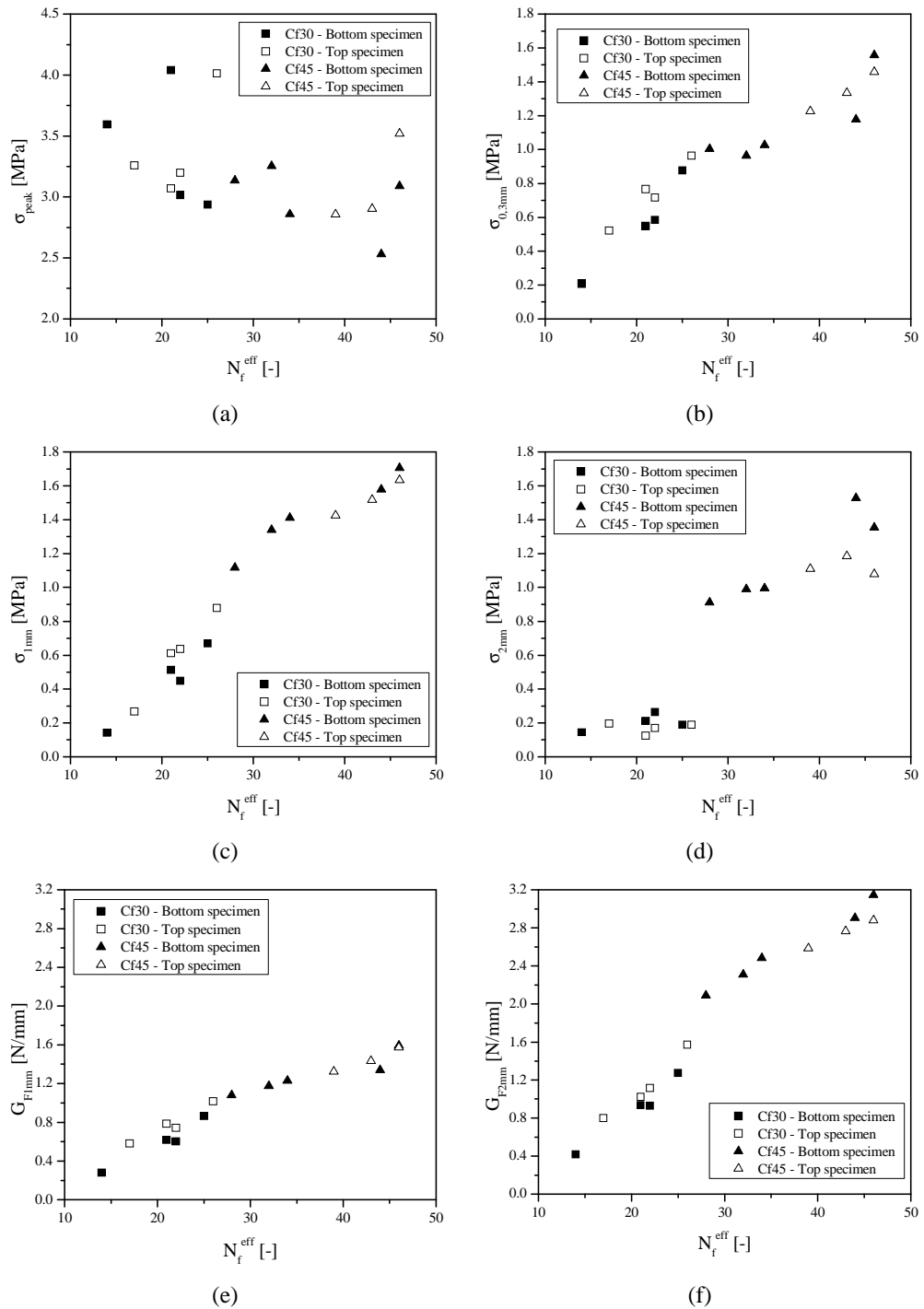


Fig. 9 – Relationships between the number of effective fibres and the post-cracking parameters: (a) peak stress, (b), (c) and (d) stress at a 0.3, 1 and 2 mm crack width, respectively; (e) and (f) dissipated energy up to 1 and 2 mm crack width, respectively.

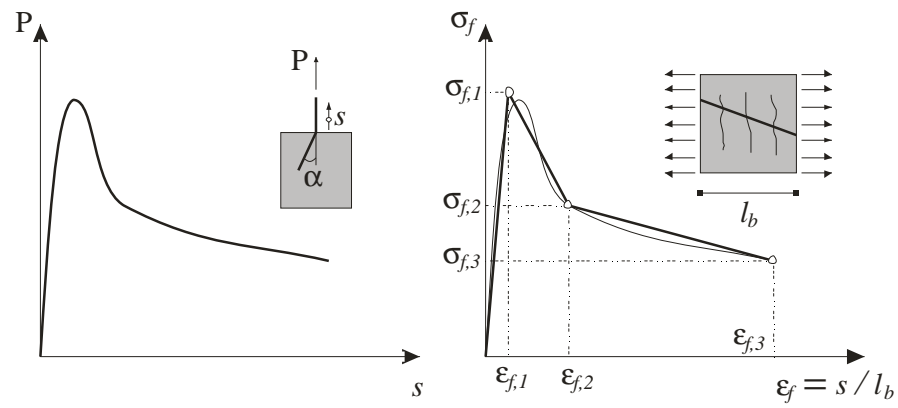


Fig. 10: Determination of the embedded cable's stress - strain diagram based on the experimental pullout force-slip relationship.

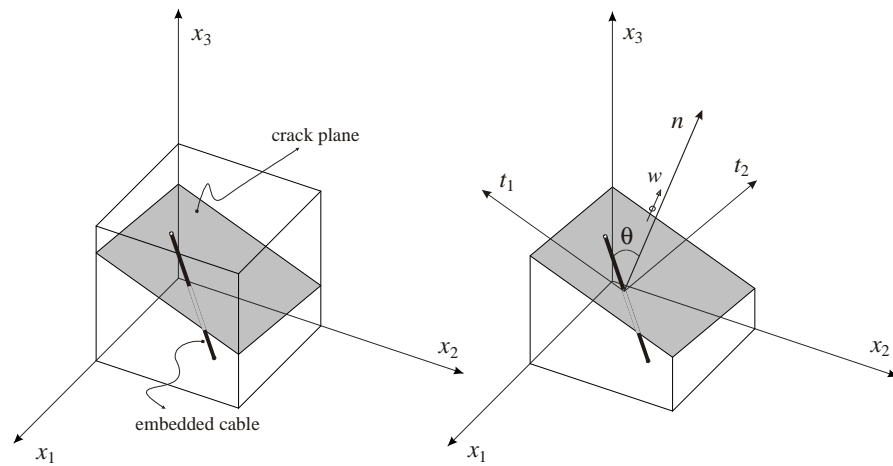


Fig. 11 – Three-dimensional scheme of the embedded cable intersecting an active crack (n is the vector normal to the crack plane).

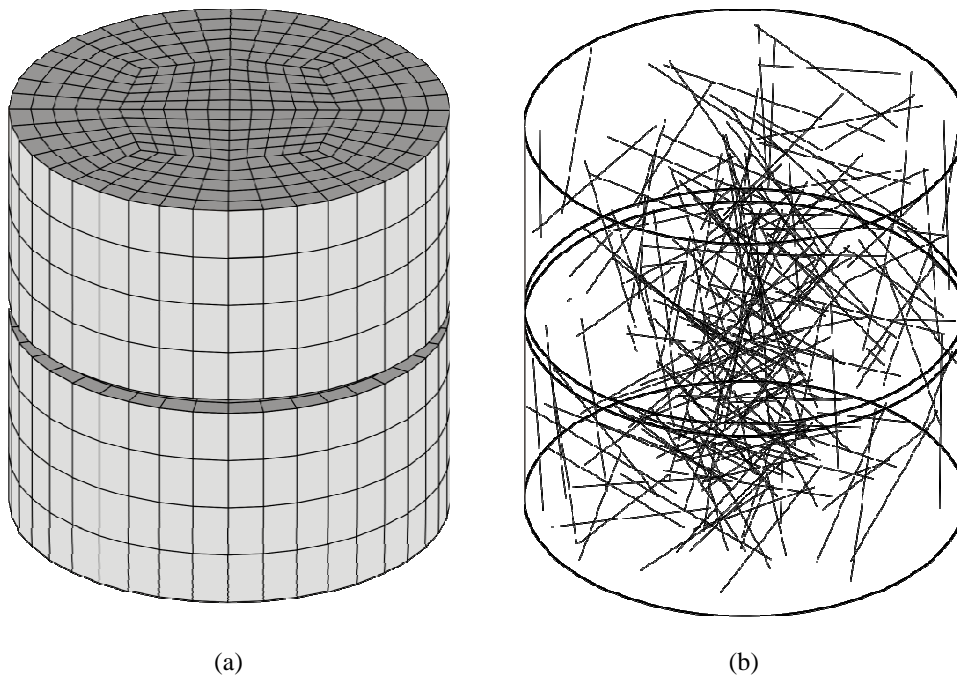


Fig. 12 – Three-dimensional finite element mesh: (a) concrete phase and (b) fibre phase (Cf30 series).

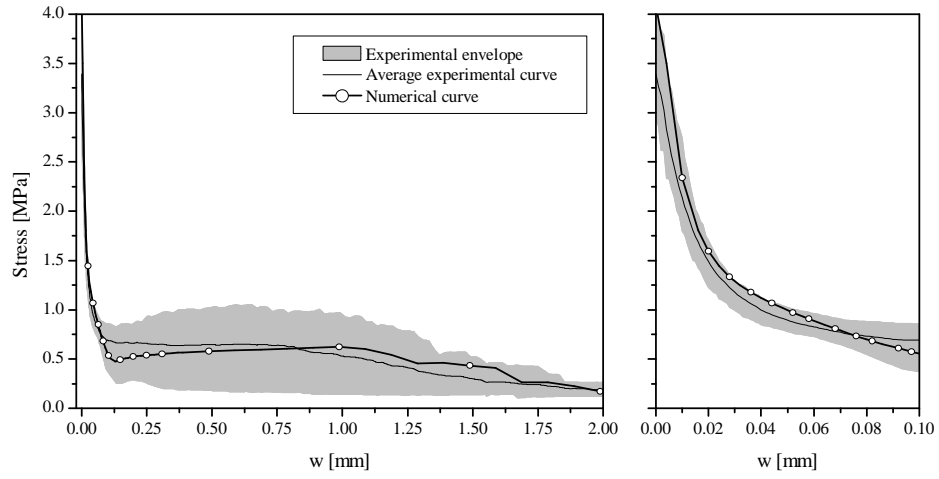


Fig. 13 – Numerical simulation of the Cf30 series' uniaxial tensile tests.

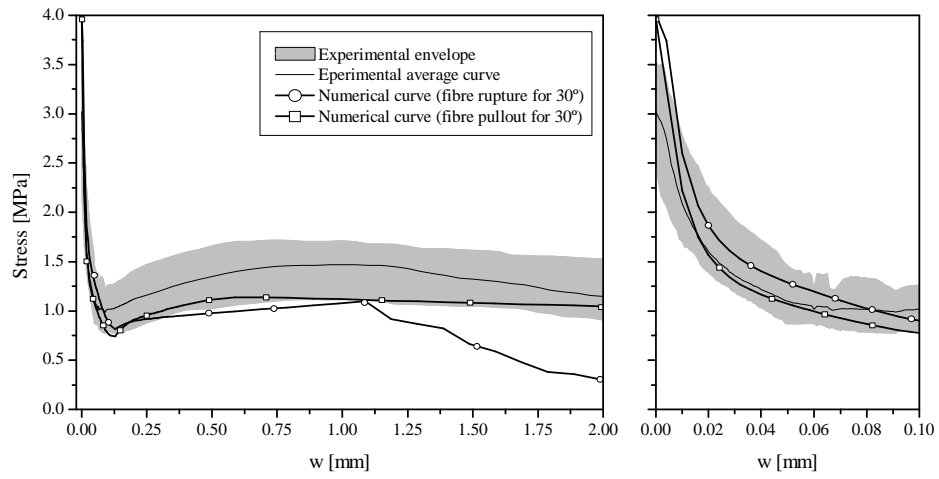


Fig. 14 – Numerical simulation of the Cf45 series' uniaxial tensile tests.