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# EXPERIMENTAL AND NUMERICAL ANALYSIS OF RC TWO-SPAN SLABS STRENGTHENED WITH NSM CFRP LAMINATES

By Dalfré<sup>1</sup>, G. and Barros<sup>2</sup>, J.A.O.

Synopsis: This work reports the results of an ongoing research program on the use of the near surface mounted (NSM) CFRP laminates for the flexural strengthening of continuous reinforced concrete (RC) slabs. The experimental program is formed by two slab strips of two equal span lengths, and has the main purpose of verifying the possibility of increasing the negative resisting bending moment in 25%, maintaining a relatively high level of moment redistribution. To assess the predictive performance of a FEM-base computer program, the experimental results are compared with the values estimated by the numerical analysis carried out using a FEM-based computer program. The results show that the proposed strengthening technique is able to increase significantly the load carrying capacity of RC slabs. However, the load carrying capacity of the strengthened slabs was limited by the detachment of the strengthened concrete cover layer at the intermediate support. The numerical model predicts with high accuracy the behavior of this type of structures.

**KEYWORDS:** CFRP, Continuous RC slabs strips, Flexural strengthening, NSM, Moment redistribution

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## INTRODUCTION

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60 In general, when a structural Reinforced Concrete (RC) element is strengthened with  
61 fiber reinforced polymer (FRP) systems, its failure mode tends to be more brittle than its  
62 unstrengthened homologous element, due to the intrinsic bond conditions between these  
63 systems and the concrete substrata, as well as the linear-elastic brittle tensile behavior of  
64 FRPs. In case of continuous RC slabs and beams (statically indeterminate structures), the  
65 use of FRP systems to increase their flexural resistance can even compromise the moment  
66 redistribution capacity of these types of elements.

67 Externally Bonded Reinforcement, EBR (ACI 440 2007, FIB 2001), and the Near Surface  
68 Mounted, NSM (Barros and Kotynia 2008; Barros *et al.* 2007) are the most used  
69 techniques for the strengthening of RC elements. However, when compared to EBR, the  
70 NSM technique is especially appropriate to increase the negative bending moments (in  
71 the intermediate supports) of continuous RC slabs since its strengthening process is  
72 simpler and faster to apply than other FRP-based techniques (Barros and Kotynia 2008).  
73 The efficiency of the NSM technique for the flexural (Barros and Fortes 2005; De  
74 Lorenzis *et al.* 2000; Carolin 2003; El-Hacha and Rizkalla 2004; Liu *et al.* 2006; Nordin  
75 2003) and shear (Barros and Dias 2006, Dias and Barros 2008, Dias and Barros 2010;  
76 Anwarul Islam 2009) strengthening of RC members has already been assessed. However,  
77 most of the tests were carried out with simply supported NSM strengthened members.

78 Although many in situ RC elements are of continuous construction, there is a lack of  
79 experimental and theoretical studies in the behavior of statically indeterminate RC  
80 members strengthened with FRP materials. Related to the analysis of the behavior of  
81 continuous elements, the majority of research studies reports the use of EBR technique  
82 (El-Refaie *et al.* 2003; Ashour *et al.* 2004; Grace *et al.* 2004; Akbarzadeh Bengar and  
83 Maghsoudi 2009, Vasseur 2009). Limited information is available in literature dealing  
84 with the behavior of continuous structures strengthened according to the NSM technique  
85 (Liu 2005; Liu *et al.* 2006; Bonaldo 2008). In the present paper the potentialities of the  
86 NSM technique is explored for the increase of the load carrying capacity of two spans  
87 continuous RC slabs. The NSM strengthening configurations applied in the slab strip  
88 were designed to increase in 25% the load carrying capacity of its corresponding  
89 unstrengthened control RC slab. Besides the load carrying capacity of the tested slabs, the  
90 moment redistribution issue is discussed in this paper.

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## EXPERIMENTAL PROGRAM

### Specimen and Test Configuration

The experimental program is composed by the two RC slab strips with the geometry, support and load conditions, reinforcement and strengthening arrangements represented in Figure 1. The steel reinforcement arrangements in the reference slab (with the designation of SL30) were designed for a load of 46.2 kN (10.4 kips), which is the load that introduces a deflection of  $L/480$  ( $L=2800$  mm [110.24 in.] is the span length of the slab) recommended by the ACI 318 (2004), and assuming a moment redistribution of 30%. Furthermore, in the evaluation of these reinforcement arrangements a strain limit of 3.5‰ for the concrete crushing was assumed.

According to the CEB-FIB Model Code (1993), the coefficient of moment redistribution,  $\delta = M_{red}/M_{elas}$ , is defined as the relationship between the moment in the critical section after redistribution ( $M_{red}$ ) and the elastic moment ( $M_{elas}$ ) in the same section calculated according to the theory of elasticity, while  $\eta = (1 - \delta) \cdot 100$  is the moment redistribution percentage. The NSM flexural strengthened slab has the same steel reinforcement arrangement adopted in the reference slab, and a number of CFRP laminates applied in the hogging (intermediate support) and sagging regions (loaded zones) designed in order to increase the load carrying capacity of the reference slab (REF) in 25%.

The design of cross sections subject to flexure was based on stress and strain compatibility, where the maximum strain at extreme concrete compression fiber was assumed equal to 0.0035. In order to increase the load carrying capacity in 25% the strengthening arrangement represented in Figure 1 (c) was adopted. In the hogging region, two  $1.4 \times 20$  mm<sup>2</sup> ( $0.05 \times 0.79$  in.<sup>2</sup>) cross section area CFRP laminates were applied, while in both sagging regions two  $1.4 \times 20$  mm<sup>2</sup> ( $0.05 \times 0.79$  in.<sup>2</sup>) and two  $1.4 \times 10$  mm<sup>2</sup> ( $0.05 \times 0.39$  in.<sup>2</sup>) CFRP laminates were installed. This slab has the designation of SL30s25.

The test with the strengthened slab strip had two phases. In the first phase the slab was loaded up to attain in the loaded sections a deflection corresponding to 50% of the deflection measured in the reference slab when steel reinforcement in the hogging region (H) has attained its yield strain. When attained this deflection level (5.8 mm [0.23 in.]), a temporary reaction system was applied (Figure 2) in order to maintain this deformability during the period necessary to strengthen the slab. To control the maintenance of this deflection, dial gauges were used in order to adjust the temporary reaction system when necessary. Therefore, the strengthening process was applied maintaining the slab with a damage level that can be representative of real slabs requiring structural rehabilitation. After the curing time of the adhesives used to bond the NSM CFRP strips (which in general took about two weeks), the temporary reaction system was removed, while the load was transferred to the slab. This stress transfer process was governed by the criteria of maintaining the deflection level that corresponds to the initiation of the second phase of the test (5.8 mm [0.23 in.]). This second phase ended when the strengthened slab strip has ruptured.

### Measuring Devices

Figure 3 depicts the positioning of the sensors for data acquisition in the tests. To measure the vertical deflection of a slab strip, six linear voltage differential transducers

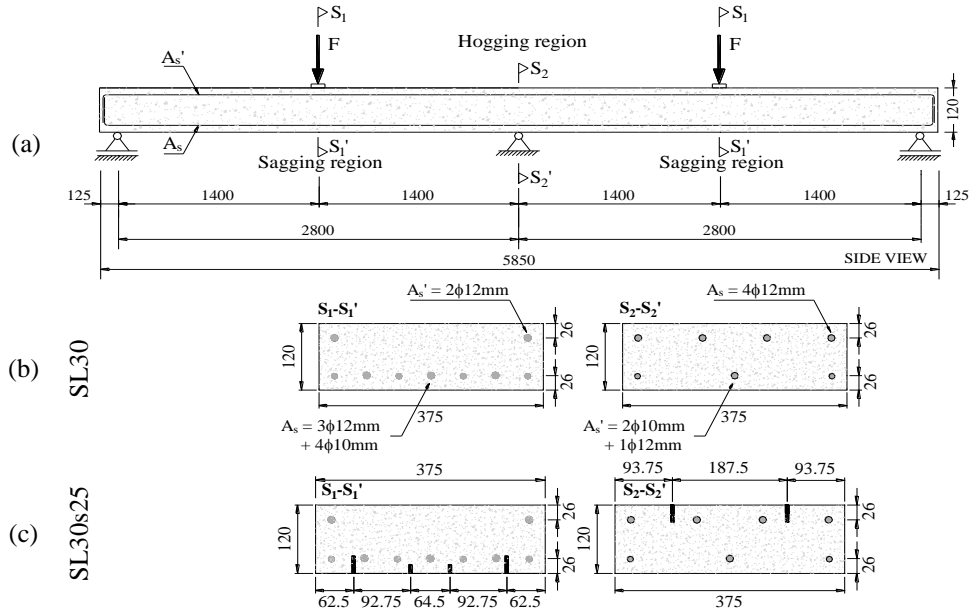
138 (LVDT 82803, LVDT 60541, LVDT 82804, LVDT 19906, LVDT 18897 and LVDT  
139 3468) were supported in a suspension bar (Figures 2 and 3a).  
140 The LVDTs 60541 and 18897, placed at the slab loaded sections, were also used to  
141 control the test at a displacement rate of 10  $\mu\text{m/s}$  up to the deflection of 50 mm (1.97 in.).  
142 After this deflection, the internal LVDTs of the actuators were used to control the test at a  
143 displacement rate of 20  $\mu\text{m/s}$  up to the failure of the slab strip.  
144 The force ( $F_{(522)}$ ) applied at the left span (Figure 3a) was measured using a load cell of  
145  $\pm 200$  kN (44.9 kips) and accuracy of  $\pm 0.03\%$  (designated Ctrl\_1), placed between the  
146 loading steel frame and the actuator of 150 kN (33.7 kips) load capacity and 200 mm (7.9  
147 in.) range. In the right span, the load ( $F_{(123)}$ ) was applied with an actuator of 100 kN (22.5  
148 kips) and 200 mm (7.9 in.) range, and the corresponding force was measured using a load  
149 cell of  $\pm 250$  kN (56.2 kips) and accuracy of  $\pm 0.05\%$  (designated Ctrl\_2). To monitor the  
150 reaction forces, load cells were installed under two supports. One load cell (AEP\_200)  
151 was positioned at the central support (nonadjustable support), placed between the reaction  
152 steel frame (HEB 300 profile) and the slab's support device (Fig. 3a). The other load cell  
153 (MIC\_200) was positioned in-between the reaction steel frame and the apparatus of the  
154 adjustable right support of the slab. These cells have a load capacity of 200 kN (44.9  
155 kips) and accuracy of  $\pm 0.05\%$ .  
156 To monitor the strain variation in the steel bars, concrete and CFRP laminates, the  
157 arrangements of strain gauges (SGs) represented in Figure 3(b-e) were adopted. Eleven  
158 SGs were installed in steel bars, seven of them in steel bars at top surface in the hogging  
159 region (SG1 to SG7) and the other four in steel bars at bottom surface in the sagging  
160 regions (SG8 to SG11, Figure 3b-c). Six SGs were applied at the external concrete  
161 surface in the compression regions (SG12 to SG17, Figure 3d). Finally, three SGs (SG18  
162 to SG20) were bonded along one CFRP laminate in the hogging region and three SGs  
163 (SG21 to SG23 and SG24 to SG26) were installed along one CFRP laminate in both  
164 sagging regions (Figure 3e).

#### 165 166 Material Properties

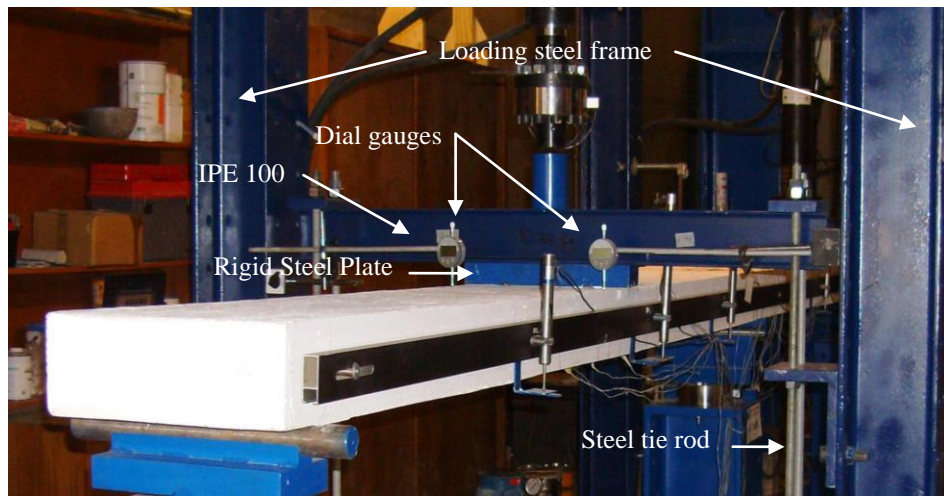
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168 Tables 1 and 2 include values obtained from experimental tests for the characterization of  
169 the main properties of the materials used in the present work. The compressive strength  
170 and the static modulus of elasticity in compression were determined according to NP-  
171 E397 (1993). To characterize the steel bars, uniaxial tensile tests were conducted  
172 according to the standard procedures of ASTM 370 (2002). Unidirectional pultruded  
173 CFRP laminates, supplied by "S&P Clever Reinforcement Ibérica Company" were used  
174 in this study and their tensile behaviour was assessed by performing uniaxial tensile tests  
175 carried out according to ISO 527-1 (1993) and ISO 527-5 (1993) recommendations. Both  
176 CFRP laminates have a width of 1.4 mm (0.05 in.). For the characterization of the tensile  
177 behaviour of the epoxy adhesive, uniaxial tensile tests were performed complying with  
178 the procedures outlined in ISO 527-2. For the adhesive, an elasticity modulus and a  
179 tensile strength of 18.60 GPa (2697 ksi) [11.46%], and 21.12 MPa (3063 psi) [6.06%]  
180 were obtained, respectively, where the values between square brackets correspond to the  
181 coefficient of variation.

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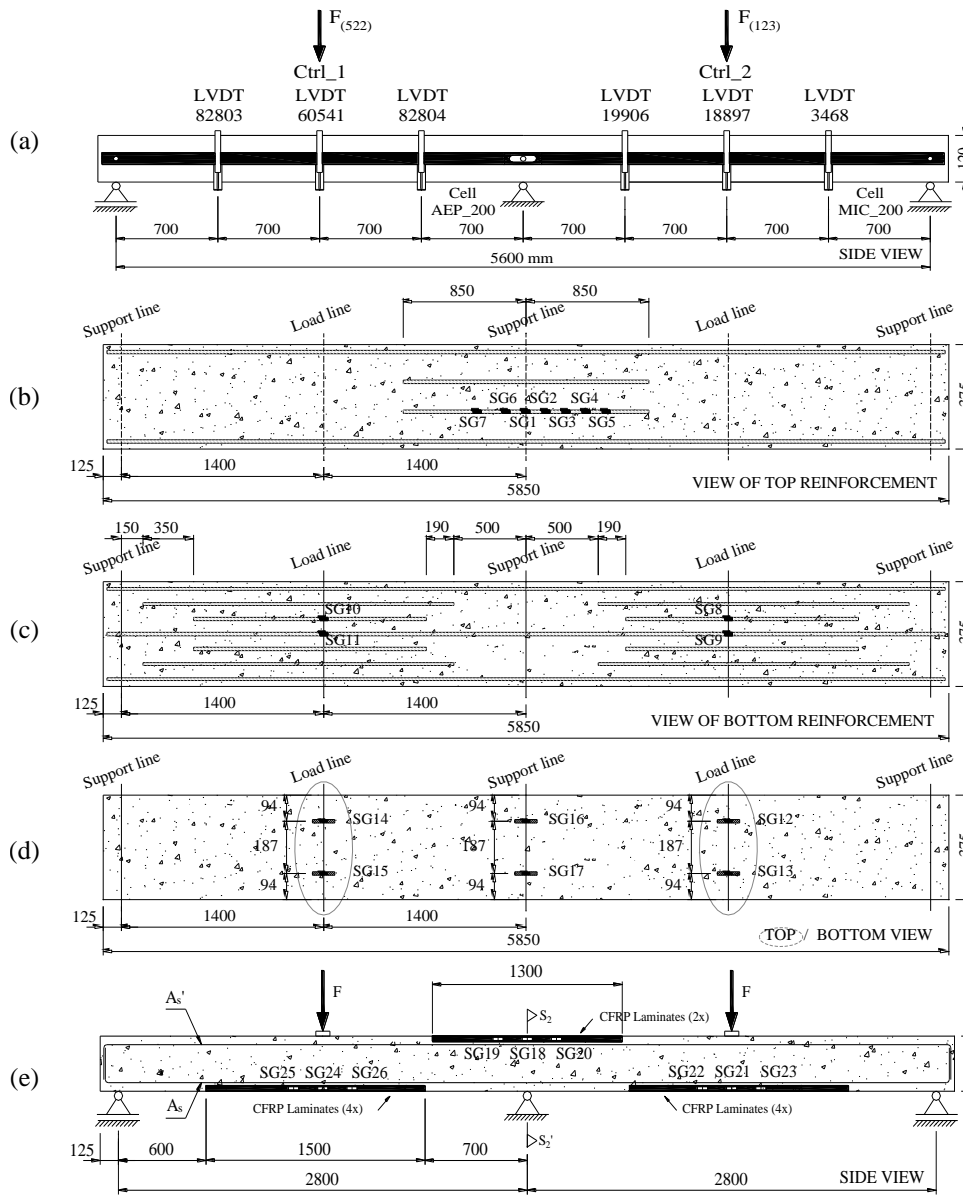
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185 Figure 1 — Slab strips: (a) test configurations, (b and c) specimens cross-sectional dimensions  
 186 of sagging (S1-S1') and hogging regions (S2-S2'). All dimensions are in mm (1 mm = 0.04 in.).  
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189  
 190 Figure 2 — Apparatus to sustain and control the mid-span displacement level applied in  
 191 the slab strips to be strengthened.  
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194 Figure 3 — Arrangement of displacement transducers and strain gauges: (a)  
 195 displacement transducers; layout of strain gauges at steel bars at hogging (b) and sagging  
 196 (c) region; (d) strain gauges at concrete slab surfaces, (e) layout of strain gauges at CFRP  
 197 laminates for SL30s25 (all dimensions are in mm – 1 mm = 0.04 in).

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Table 1 — Characteristics of plain concrete.

Slab strip	Property	
	$f_{cm}$ (MPa), [psi]	$E_c$ (GPa), [ksi]
SL30	(30.10), [4365] {1.08}	(31.52), [4570] {0.86}
SL30s25	(32.59), [4726] {1.15}	(30.62), [4441] {2.42}

{ value } = Standard deviation

202

203

Table 2 — Summary of the properties of steel reinforcement and CFRP laminates.

Steel bar diameter ( $\phi$ s)	Steel reinforcement				CFRP laminate height	CFRP Laminate		
	Modulus of Elasticity (GPa) [ksi]	Yield stress (0.2 %) <sup>a</sup> (MPa) [psi]	Strain at yield stress <sup>b</sup>	Tensile strength (MPa) [psi]		Ultimate tensile stress (MPa) [ksi]	Ultimate tensile strain (%)	Modulus of Elasticity (GPa) [ksi]
10 mm (0.39 in.)	(178.24) [25851] {2.48%}	(446.95) [64824] {3.25%}	0.0027 {0.45%}	(575.95) [83534] {0.34%}	10 mm (0.39 in.)	(2867.63) [415914] {3.07%}	17.67 {3.04%}	(159.304) [23105] {3.15%}
12 mm (0.47 in.)	(198.36) [28769] {2.77%}	(442.47) [64174] {2.87%}	0.0024 {0.19%}	(539.88) [78302] {1.84%}	20 mm (0.79 in.)	(2782.86) [403619] {2.73%}	17.76 {3.13%}	(156.69) [22725] {0.73%}

204 <sup>a</sup>Yield stress determined by the “Offset Method”, according to ASTM 370 (2002)205 <sup>b</sup>Strain at yield point, for the 0.2 % offset stress

206 { value } Coefficient of Variation (COV) = (Standard deviation/Average) x 100

207

208 Strengthening system

209

210 The first step of the NSM strengthening process consisted in opening the slits for the  
 211 installation of the CFRP laminates, by using a conventional diamond saw cut machine.  
 212 The slits had a width that varied between 4.5 mm (0.17 in.) and 4.6 mm (0.18 in.) and a  
 213 depth of 15 mm (0.59 in.) or 27 mm (1.06 in.), depending on the depth of the cross  
 214 section of the used CFRP laminate, 10 mm (0.39 in.) or 20 mm (0.79 in.), respectively. In  
 215 order to eliminate the dust resultant from the sawing process, the slits were cleaned using  
 216 compressed air before bonding the laminates to the concrete into the slits. The CFRP  
 217 laminates were cleaned with acetone to remove any possible dirt. Finally, the slits were  
 218 filled with the epoxy adhesive using a spatula, and the CFRP laminates were introduced  
 219 into the slits.

220

221 Main results of the experimental program

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223 The applied loads ( $F_{(522)}$  or  $F_{(123)}$ ) versus deflection curves of the tested slab strips are  
 224 presented in Figures 4 to 6. Additionally, Table 3 presents the main results obtained

225 experimentally. In this Table,  $\bar{F}_{\max}$  is the average load ( $\bar{F}_{\max} = (F_{(522)} + F_{(123)})/2$ ),  
 226  $R_{L, \bar{F}_{\max}}$  is the load registered at the load cell (MIC\_200) and  $\Delta\bar{F}_{\max}/\bar{F}_{\max}^{REF}$  is the increase  
 227 in terms of load carrying capacity provided by the strengthening technique at  $\bar{F}_{\max}$ .  
 228 Figure 6 shows that the adopted NSM strengthening configuration conducted to a  
 229 significant increase of the load carrying during the second phase of the test loading process.  
 230 Four phases occurred during each test in the following sequence: a) the uncracked elastic  
 231 response; b) crack propagation in the hogging and sagging regions with steel bars in  
 232 elastic stage; c) yielding of the steel reinforcement at the hogging region and crack  
 233 propagation in the sagging regions with steel bars in elastic stage; d) yielding of the steel  
 234 reinforcement at the hogging and sagging regions.  
 235 As expected, the unstrengthened control slab strip behaved in a perfectly plastic manner in  
 236 the post-yielding phase (after the formation of plastic hinges at hogging and sagging  
 237 regions), whereas the strengthened slab strips exhibited continuous hardening up to  
 238 failure. The failure mechanism of the reference slab was governed by flexure failure  
 239 mode, i.e. by yielding of internal reinforcements, with extensive cracking in the tension  
 240 flange, followed by concrete crushing in compression parts.  
 241 The SL30s25 failed by the detachment of the top concrete cover that includes the  
 242 laminates in the hogging region (Figure 7 a2). This slab strip had four CFRP laminates  
 243 mounted in the tension face of the slab over the sagging: two of  $1.4 \times 20 \text{ mm}^2$  ( $0.05 \times 0.79$   
 244  $\text{in.}^2$ ) cross section area and two of  $1.4 \times 10 \text{ mm}^2$  ( $0.05 \times 0.39 \text{ in.}^2$ ). Additionally, two CFRP  
 245 laminates of  $1.4 \times 10 \text{ mm}^2$  ( $0.05 \times 0.39 \text{ in.}^2$ ) cross section area were placed in hogging  
 246 regions. As already mentioned, in the first phase of the test, the strengthened slab strips  
 247 were loaded up to a deflection of 5.80 mm (0.23 in.), which corresponds to a  $\bar{F} = 17 \text{ kN}$   
 248 (3.8 kips). Flexural cracks were first observed at a  $\bar{F}$  of about 6 kN (1.3 Kips).  
 249 Upon further loading, several flexural cracks formed over the hogging region of both  
 250 slabs, as shown in Figure 7. The number of flexural cracks has increased with the load,  
 251 and herringbone cracks formed in the concrete surrounding the CFRP laminates at  
 252 hogging region.

## 253 254 NUMERICAL SIMULATION

255  
 256 For the prediction of the behaviour of RC continuous slabs strengthened with NSM  
 257 laminate arrangements capable of increasing the load carrying capacity and assuring high  
 258 level of moment redistribution for this type of structure, a computer program, based on  
 259 the finite element method (FEM), was used.

### 260 261 Constitutive laws

262  
 263 According to the present model, a concrete slab is considered a plane shell formulated  
 264 under the Reissner-Mindlin theory (Barros 1995). In order to simulate the progressive  
 265 damage induced by concrete cracking and concrete compression nonlinear behavior, the  
 266 thickness of a shell element was discretized in 20 layers that were considered in a state of  
 267 plane stress.  
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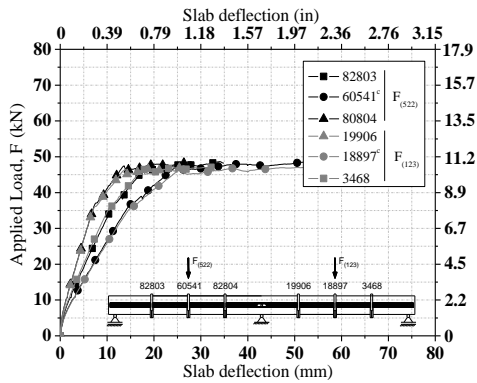


Figure 4 — Load-deflection curves of SL30.

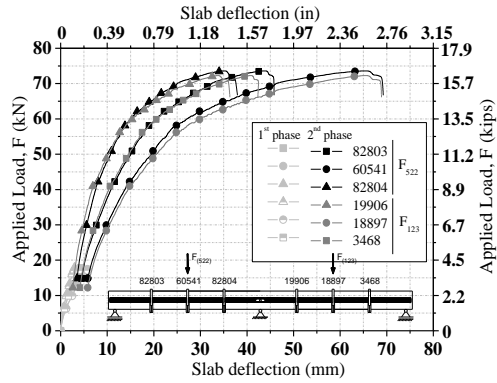


Figure 5 — Load-deflection curves of SL30s25.

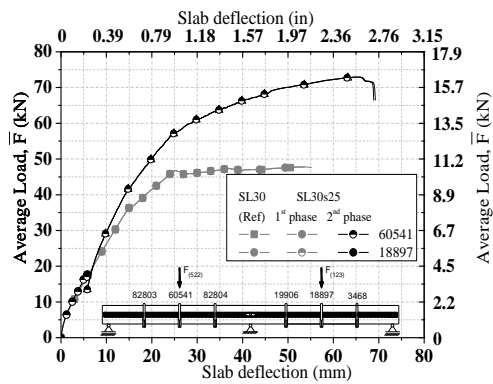
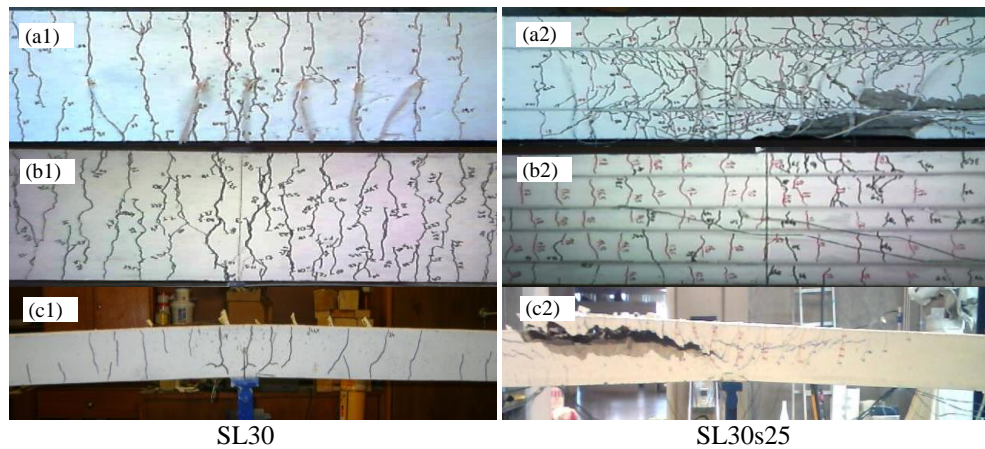


Figure 6 — Load-midspan deflection.

Table 3 — Main results of the experimental program.

Slab strips ID	$\bar{F}_{max}$ , kN (kips)	$R_{L,\bar{F}_{max}}$ , kN (kips)	$\frac{\Delta \bar{F}_{max}}{\bar{F}_{max}^{REF}}$ (%)
SL30	47.85 (10.7)	16.43 (3.7)	-----
SL30s25	72.96 (16.4)	26.19 (5.9)	52.47

269



SL30

SL30s25

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271  
272

Figure 7 — Crack patterns: plant view at hogging (a1-a2) and sagging regions (b1-b2); lateral view (c1-c2) at hogging region.

273 The incremental strain vector derived from the incremental nodal displacements obtained  
 274 under the framework of a nonlinear FEM analysis is decomposed in an incremental crack  
 275 strain vector,  $\Delta \underline{\varepsilon}^{cr}$ , and an incremental strain vector of the concrete between cracks,  
 276  $\Delta \underline{\varepsilon}^{co}$ . This last vector is decomposed in an elastic reversible part,  $\Delta \underline{\varepsilon}^e$ , and an  
 277 irreversible or plastic part,  $\Delta \underline{\varepsilon}^p$ , resulting

$$\Delta \underline{\varepsilon} = \Delta \underline{\varepsilon}^{cr} + \Delta \underline{\varepsilon}^{co} = \Delta \underline{\varepsilon}^{cr} + \Delta \underline{\varepsilon}^e + \Delta \underline{\varepsilon}^p \quad (1)$$

278  
 279 The incremental stress vector can be computed from the incremental elastic strain vector,  
 $\Delta \underline{\sigma} = \underline{D}^{co} \Delta \underline{\varepsilon}^{co}$  (2)

280 where  $\underline{D}^{co}$  is the concrete tangent constitutive matrix,

$$\underline{D}^{co} = \begin{bmatrix} \underline{D}_{mb}^{co} & \phi \\ \phi & \underline{D}_s^{co} \end{bmatrix} \quad (3)$$

281 with  $\underline{D}_{mb}^{co}$  and  $\underline{D}_s^{co}$  being the in-plane and the out-of-plane shear stiffness matrices,  
 282 respectively. In the present model, concrete behavior is assumed linear elastic in terms of  
 283 out-of-plane shear. Therefore, the concrete nonlinear behaviour is only considered in the  
 284  $\underline{D}_{mb}^{co}$  constitutive matrix.

285 For linear elastic uncracked concrete,  $\underline{D}_{mb}^{co}$  is designated by  $\underline{D}_{mb}^{eco}$ , which is defined  
 286 elsewhere (Barros and Figueiras 2001). For the case of cracked concrete with concrete  
 287 between cracks exhibiting an elasto-plastic behavior,  $\underline{D}_{mb}^{co}$  of (3) is replaced by  
 288  $\underline{D}_{mb}^{epcrco}$  (Sena-Cruz *et al.* 2004):

$$\underline{D}_{mb}^{co} \Rightarrow \underline{D}_{mb}^{epcrco} = \underline{D}_{mb}^{epco} - \underline{D}_{mb}^{epco} \left[ \underline{T}^{cr} \right] \left( \hat{\underline{D}}^{cr} + \underline{T}^{cr} \underline{D}_{mb}^{epco} \left[ \underline{T}^{cr} \right]^T \right)^{-1} \underline{T}^{cr} \underline{D}_{mb}^{epco} \quad (4)$$

289 where

$$\underline{D}_{mb}^{epco} = \underline{H} - \frac{\underline{H} \frac{\partial f}{\partial \underline{\sigma}} \left( \frac{\partial f}{\partial \underline{\sigma}} \right)^T \underline{H}}{h + \left( \frac{\partial f}{\partial \underline{\sigma}} \right)^T \underline{H} \left( \frac{\partial f}{\partial \underline{\sigma}} \right)} \quad (5)$$

290 and

$$\underline{H} = \left( \left[ \underline{D}_{mb}^{eco} \right]^{-1} + h_c \Delta \lambda \frac{\partial^2 f}{\partial \underline{\sigma}^2} \right)^{-1} \quad (6)$$

291 where  $\partial f / \partial \underline{\sigma}$  is the flow vector,  $h_c$  is a scalar function that depends on the hydrostatic  
 292 pressure,  $\underline{T}^{cr}$  is a transformation matrix that depends on the direction of the cracks  
 293 formed at a sampling point (Sena-Cruz *et al.* 2004), and  $\hat{\underline{D}}^{cr}$  is the constitutive matrix of  
 294 the set of cracks. In case of one crack per each sampling point,

$$\underline{\hat{D}}^{cr} = \underline{D}^{cr} = \begin{bmatrix} D_I^{cr} & 0 \\ 0 & D_{II}^{cr} \end{bmatrix} \quad (7)$$

295 where  $D_I^{cr}$  and  $D_{II}^{cr}$  are the softening modulus of the fracture modes I and II of the  
 296 smeared cracks, respectively.  $D_I^{cr}$  is characterized by the stress at crack initiation,  
 297  $\sigma_{n,1}^{cr}$  (see Figure 8), the fracture energy,  $G_f$ , the shape of the softening law and the crack  
 298 band width,  $l_b$ .

299 In smeared crack models the fracture zone is distributed over  $l_b$ , which must depend on  
 300 the finite element geometric characteristics in order to assure that the results of the FEM  
 301 analysis are not dependent on the finite element mesh (Bazant and Oh 1983). The fracture  
 302 mode II modulus,  $D_{II}^{cr}$ , of (7) is obtained from (Barros 1995):

$$D_{II}^{cr} = \frac{\left(1 - \frac{\varepsilon_n^{cr}}{\varepsilon_{n,u}^{cr}}\right)^{p_1}}{1 - \left(1 - \frac{\varepsilon_n^{cr}}{\varepsilon_{n,u}^{cr}}\right)^{p_1}} G_c \quad (8)$$

303 where  $G_c$  is the concrete elastic shear modulus and  $p_1$  an integer parameter that can  
 304 obtain distinct values in order to simulate different levels of concrete shear stiffness  
 305 degradation (Barros 1995). In case of cracked concrete with concrete between cracks in  
 306 linear and elastic state,  $\underline{D}_{mb}^{co}$  is still obtained from (4) replacing  $\underline{D}_{mb}^{epco}$  by  $\underline{D}_{mb}^{eco}$ .

307

### 308 Steel constitutive law

309 For modelling the behaviour of the steel bars, the stress-strain relationship represented in  
 310 Figure 9 was adopted (Sena-Cruz 2004). The curve (under compressive or tensile  
 311 loading) is defined by the points  $PT1 = (\varepsilon_{sy}, \sigma_{sy})$ ,  $PT2 = (\varepsilon_{sh}, \sigma_{sh})$  and  $PT3 = (\varepsilon_{su}, \sigma_{su})$ ,  
 312 and a parameter  $p$  that defines the shape of the last branch of the curve. Unloading and  
 313 reloading linear branches with slope  $E_s$  are assumed in the present approach.

### 314 FRP constitutive law

315

316 A linear elastic stress-strain relationship was adopted to simulate the behaviour of NSM  
 317 CFRP laminates applied in the RC slabs.

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319

## 319 SIMULATION OF THE TESTS

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### 321 Materials properties and finite element mesh

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323 Tables 4 and 5 include the values of the parameters adopted for the characterization of the  
 324 constitutive models for the concrete and steel, respectively.

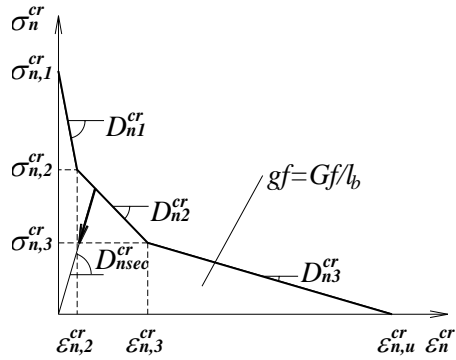


Figure 8 — Tri-linear tensile-softening diagram (Sena-Cruz 2004).

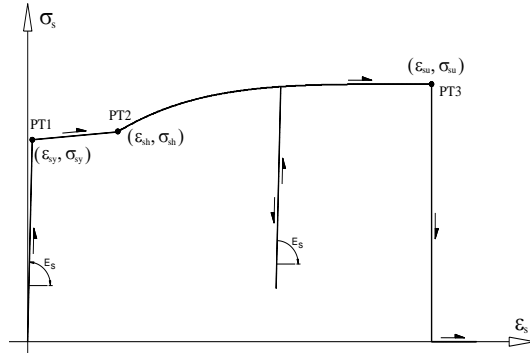


Figure 9 — Uniaxial constitutive model for the steel bars (Sena-Cruz 2004).

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The CFRP laminates were assumed as an isotropic material with an elasticity modulus of 156 GPa and null value for the Poisson's coefficient, since the consideration of their real anisotropic properties have marginal influence in terms of their contribution for the behaviour of NSM strengthened RC slabs.

Due to the structural symmetry, only half of the slab was considered in the numerical simulations. Figure 10 shows the eight node finite element mesh adopted to discretize the half part of the slab. The support conditions are also represented in this figure. The slab thickness was discretized in 20 layers.

### Results and discussion

Figures 11 to 14 represent relevant results of the simulations corresponding to the slabs of the SL30 series. The figures show that the numerical model is able to capture with good accuracy the behaviour of the constituent materials of this structural system during the loading process of the tested slabs.

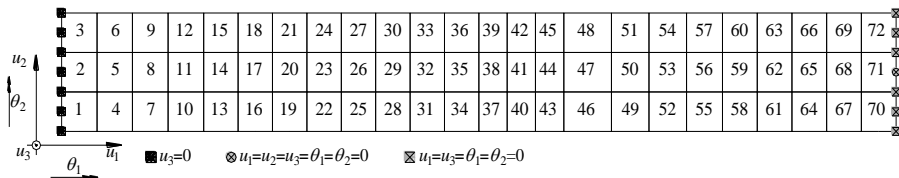
Table 6 resumes the results obtained numerically for two scenarios: when a plastic hinge formed at the hogging region (superscript H); when a plastic hinge formed at the sagging regions (superscript S). In this Table,  $F_y^H$  and  $F_y^S$  are the loads at the formation of the plastic

hinge at hogging and sagging regions, respectively,  $u_y^H$  and  $u_y^S$  are the average deflection for

$F_y^H$  and  $F_y^S$ , respectively,  $\varepsilon_c^H$  and  $\varepsilon_c^S$  are the maximum concrete strains registered at H and

S regions,  $\varepsilon_s^H$  and  $\varepsilon_s^S$  are the maximum strains in steel bars at H and S regions, respectively,

and, finally,  $\varepsilon_f^H$  and  $\varepsilon_f^S$  are the maximum strains in the CFRP laminates at H and S regions.



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Figure 10 — Finite element mesh adopted to discretize the half part of a RC slab.

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Table 4 — Values of the parameters of the concrete constitutive model.

Poisson's ratio ( $\nu_c$ )	0.15
Initial Young's modulus ( $E_c$ )	29.83 GPa (4326 ksi)
Compressive strength ( $f_c$ )	28.40 MPa (4119 psi)
Strain at peak compression stress	$\varepsilon_{c,1} = 1.98 \times 10^{-3}$
Parameter defining the initial yield surface (Sena-Cruz 2004)	$\alpha_0 = 0.4$
Tri-linear tension softening/stiffening diagram <sup>(1)</sup>	$f_{ct} = 1.50$ MPa (217 psi) $G_f = 0.052$ N/mm (0.30 lb/in.) $\xi_1 = 0.015$ ; $\alpha_1 = 0.6$ $\xi_2 = 0.2$ ; $\alpha_2 = 0.25$
Parameter defining the mode I fracture energy available to the new crack (Barros 1995)	$n = 2$
Shear retention factor ( $p_f$ factor of Equation (8))	$p_1 = 2$
Crack band-width	Square root of the area of Gauss integration point
Threshold angle (Barros 1995)	$\alpha_{th} = 30^\circ$
Maximum number of cracks per integration point	2

351 <sup>(1)</sup>  $f_{ct} = \sigma_{n,1}^{cr}$ ;  $\xi_1 = \varepsilon_{n,2}^{cr} / \varepsilon_{n,u}^{cr}$ ;  $\alpha_1 = \sigma_{n,2}^{cr} / \sigma_{n,1}^{cr}$ ;  $\xi_2 = \varepsilon_{n,3}^{cr} / \varepsilon_{n,u}^{cr}$ ;  $\alpha_2 = \sigma_{n,3}^{cr} / \sigma_{n,1}^{cr}$  (see Figure 8)

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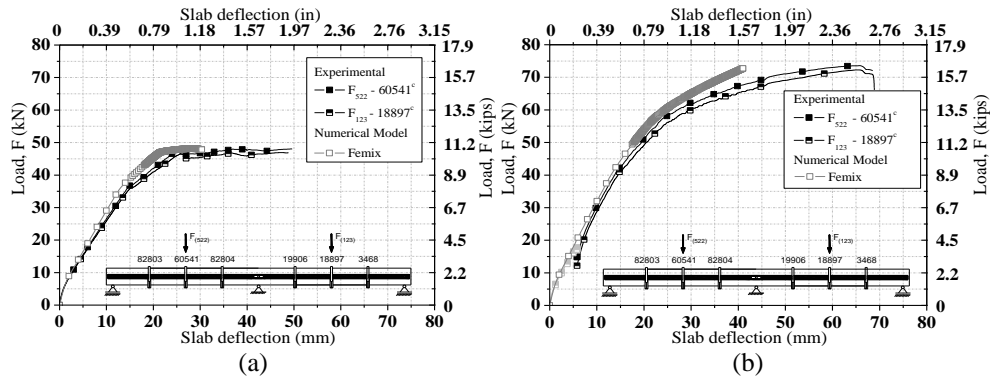
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Table 5 — Values of the parameters of the steel constitutive model (see Figure 9).

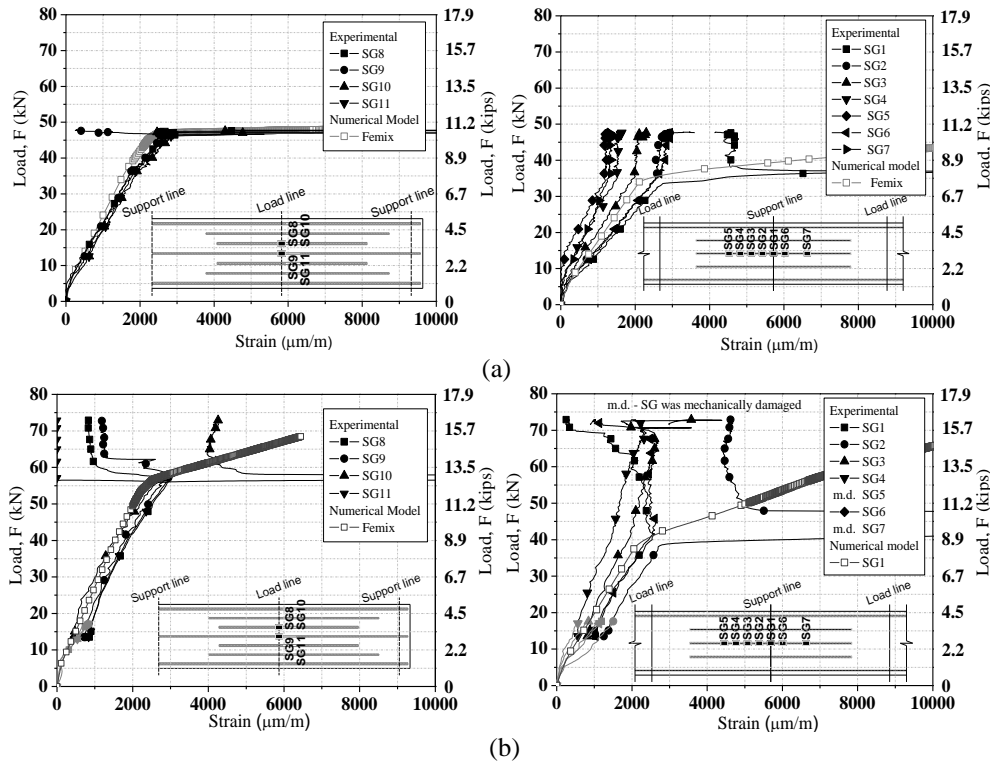
Steel bar diameter	PT1 $\left( \begin{array}{c} \varepsilon_{sy} [-] \\ \sigma_{sy} (MPa), [psi] \end{array} \right)$	PT2 $\left( \begin{array}{c} \varepsilon_{sh} [-] \\ \sigma_{sh} (MPa), [psi] \end{array} \right)$	PT3 $\left( \begin{array}{c} \varepsilon_{su} [-] \\ \sigma_{su} (MPa), [psi] \end{array} \right)$	$E_s$ (GPa) [ksi]
8 mm (0.31 in.)	$2.50 \times 10^{-3}$ (421.00), [61060]	$4.42 \times 10^{-2}$ (526.25), [76326]	$8.85 \times 10^{-2}$ (555.72), [80600]	(200.80) [29123]
10 mm (0.39 in.)	$2.50 \times 10^{-3}$ (446.00), [64686]	$3.07 \times 10^{-2}$ (446.00), [64686]	$1.31 \times 10^{-1}$ (557.50), [80858]	(178.24) [25851]
12 mm (0.47 in.)	$2.50 \times 10^{-3}$ (445.00), [64541]	$3.05 \times 10^{-2}$ (445.00), [64541]	$1.02 \times 10^{-1}$ (547.35), [79386]	(198.36) [28769]

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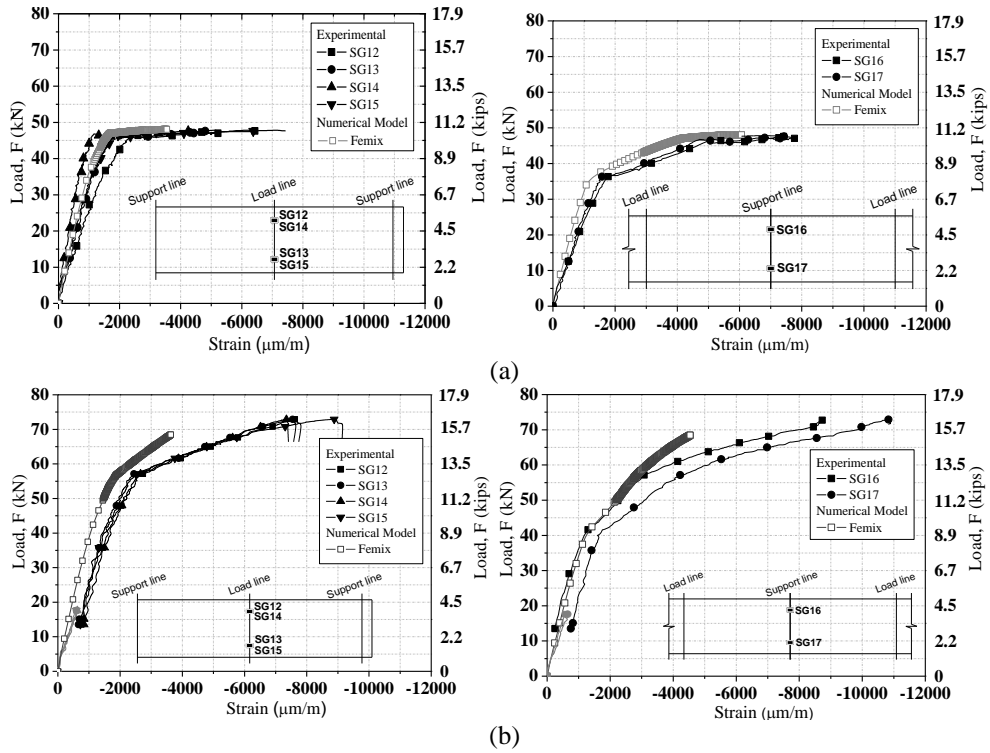
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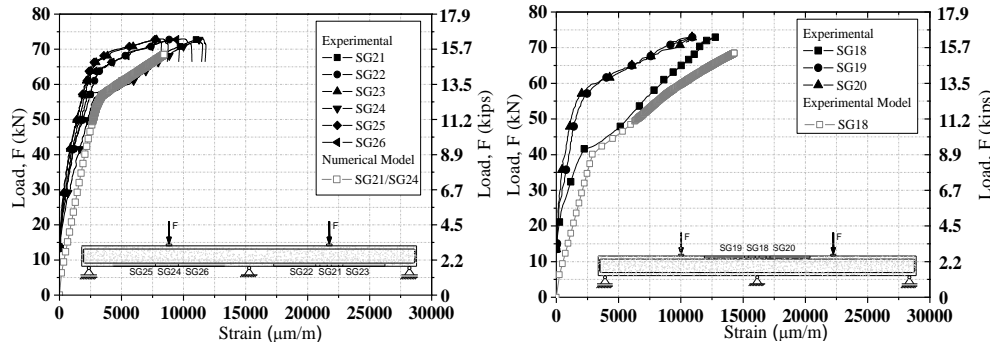
356 Figure 11 — Force-loaded section deflection relationship: (a) SL30 and (b) SL30s25.  
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358 Figure 12 — Force-strain relationships in steel: (a) SL30 and (b) SL30s25.



359 Figure 13 — Force –strain relationships in concrete: (a) SL30 and (b) SL30s25.  
 360



361 Figure 14 — Force –strain relationships in CFRP laminates of SL30s25.  
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363 Table 7 presents the relevant results when the maximum concrete compressive strain  
 364 attained 3.5 ‰ (symbols with subscript “cu”) in the hogging and sagging regions  
 365 ( $\epsilon_{c,max} = 3.5\text{‰}$ , which is assumed the concrete crushing strain).

366 In this Table, *IR* represents the increase in terms of load carrying capacity provided by  
 367 the strengthening technique, calculated according to the following equation:

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Table 6 — Numerical results at the formation of the hinges

Hinge at hogging region (H)			Hinge at sagging region (S)		
Slab strip ID	SL30	SL30s25	Slab strip ID	SL30	SL30s25
$F_y^H$ (kN) [kips]	(36.23) [8.1]	(40.12) [9.0]	$F_y^S$ (kN) [kips]	(45.16) [10.2]	(53.72) [12.1]
$u_y^H$ (mm) [in]	(13.00) [0.51]	(13.00) [0.51]	$u_y^S$ (mm) [in]	(19.79) [0.78]	(19.88) [0.78]
$\varepsilon_c^H$ (‰)	-1.21	-1.23	$\varepsilon_c^H$ (‰)	-3.51	-2.52
$\varepsilon_c^S$ (‰)	-1.00	-1.05	$\varepsilon_c^S$ (‰)	-1.50	-1.66
$\varepsilon_s^S$ (‰)	1.60	1.54	$\varepsilon_s^S$ (‰)	2.23	2.23
$\varepsilon_s^H$ (‰)	2.40	2.30	$\varepsilon_s^H$ (‰)	11.94	6.05
$\varepsilon_f^H$ (‰)	-----	2.90	$\varepsilon_f^H$ (‰)	-----	7.74
$\varepsilon_f^S$ (‰)	-----	2.05	$\varepsilon_f^S$ (‰)	-----	3.00

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Table 7 — Experimental results at concrete crushing

Concrete crushing initiation at hogging region ( $\varepsilon_{cu}^H = 3.5\text{‰}$ )			Concrete crushing initiation at sagging regions ( $\varepsilon_{cu}^S = 3.5\text{‰}$ )		
Slab strip ID	SL30	SL30s25	Slab strip ID	SL30	SL30s25
$F_{cu}^H$ (kN) [kips]	(45.15) [10.1]	(62.34) [14.0]	$F_{cu}^S$ (kN) [kips]	(48.04) [10.8]	(67.67) [15.2]
$u_{cu}^H$ (mm) [in]	(19.77) [0.78]	(26.83) [1.06]	$u_{cu}^S$ (mm) [in]	(27.10) [1.07]	(33.48) [1.32]
$\varepsilon_{c,max}^S$ (‰)	-1.50	-2.67	$\varepsilon_{c,max}^H$ (‰)	-6.01	-4.39
$\varepsilon_{s,max}^S$ (‰)	2.23	4.37	$\varepsilon_{s,max}^S$ (‰)	8.81	6.17
$\varepsilon_{s,max}^H$ (‰)	11.93	8.75	$\varepsilon_{s,max}^H$ (‰)	20.54	10.85
$\varepsilon_{f,max}^H$ (‰)	-----	11.17	$\varepsilon_{f,max}^H$ (‰)	-----	13.86
$\varepsilon_{f,max}^S$ (‰)	-----	5.76	$\varepsilon_{f,max}^S$ (‰)	-----	8.07
$\eta$ (%)	22.75	18.82	$\eta$ (%)	26.88	19.37
IR (%)		38.07	IR (%)		40.86

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$$IR = \frac{F_{cu}^{CFRP} - F_{cu}^{REF}}{F_{cu}^{CFRP}} 100 \quad (9)$$

380 where  $F_{cu}^{CFRP}$  and  $F_{cu}^{REF}$  are the load of the strengthened and reference slabs, respectively.

381 From the analysis of the results included in Tables 6 and 7 and represented in Figures 11  
382 to 14 it can be outlined the following:

383 - after crack initiation, which occurred for a load of about 6 kN (1.3 kips), the slab  
384 stiffness decreased significantly, but the elasto-cracked stiffness was almost maintained  
385 up to the formation of the plastic hinge at the intermediate support, at a load level of  
386 about 36 kN (8.1 kips) and 40 kN (8.9 kips) for the reference and SL30s25 slabs,  
387 respectively.

388 - For a compressive strain of 3.5 ‰, the increase of the load carrying capacity provided  
389 by the strengthening system was of about 39 %. This value reveals that the aimed  
390 increase in terms of slab's load carrying capacity was attained.

391 - Up to the formation of the plastic hinges the strains in the laminates ranged from 2.05‰  
392 to 7.74‰, which justifies the relative low contribution of the laminates for the load  
393 carrying capacity up to this load level. In fact, the force-deflection relationship evinces  
394 that, up to the formation of the plastic hinge at the intermediate support, CFRP strips did  
395 not contribute significantly for the slabs' load carrying capacity. However, at concrete  
396 crushing at the sagging regions, the maximum strain in the CFRP laminates varied  
397 between 8.07‰ and 13.86‰, which is 45 to 78 % of the CFRP laminate ultimate strain.

398 - The deflection at  $F_y^S$ ,  $u_y^S$ , was not significantly affected by the presence of the CFRP  
399 laminates.

400 - The contribution of the CFRP laminates for the slab's maximum load carrying capacity  
401 was limited due to the occurrence of concrete crushing, and the premature failure mode  
402 by the detachment of the concrete cover layer that includes the laminates at the hogging  
403 region.

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#### 405 MOMENT REDISTRIBUTION ANALYSIS

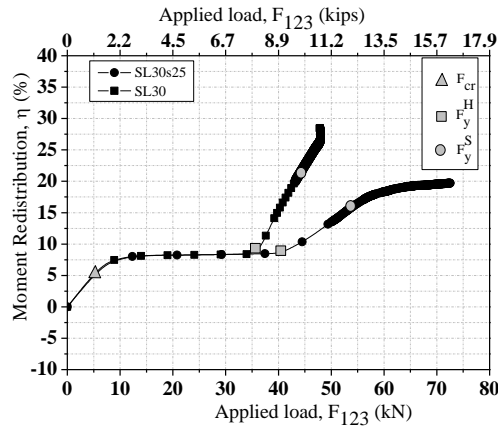
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407 The percentages of moment redistribution obtained numerically for the slab strips are  
408 shown in Figure 15. The SL30 slab strip exhibited a moment redistribution rate of about  
409 8.84 % at the yielding of steel reinforcement at the central support section. At the  
410 yielding of reinforcement at the sagging region, the moment redistribution increased to  
411 about 22.76 %. For a compressive strain of 3.5 ‰ at H and S moment redistribution rates  
412 of about 22.74% and 26.88% were obtained, respectively.

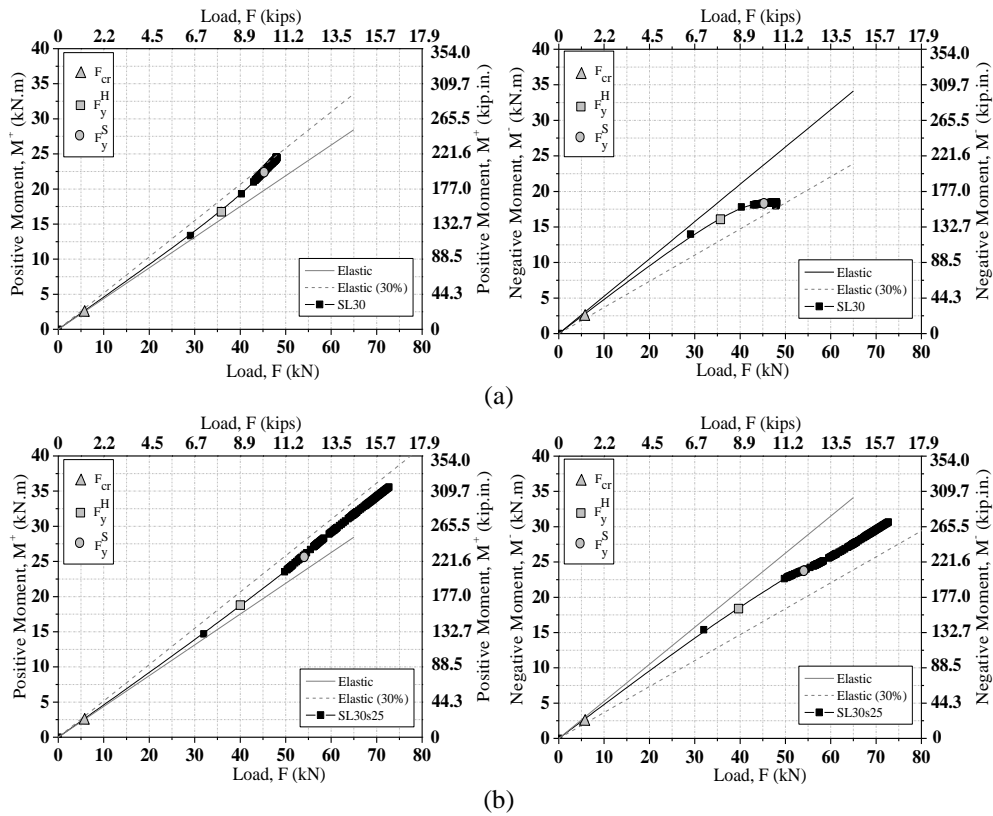
413 Concerning to SL30s25 slab strip, a moment redistribution of 8.49 % was obtained when the  
414 steel reinforcement yields at the hogging region. Afterwards, a moment redistribution of 15.76 %  
415 was obtained at the yielding of steel reinforcement at the sagging region. Finally, for a  
416 compressive strain of 3.5 ‰ at H and S, respectively, moment redistribution rates of about  
417 18.22% and 19.37% were obtained. Figure 16 shows the variation of the negative (M) and  
418 positive (M<sup>+</sup>) moments with the increase of the applied load. The tendency of the M-F  
419 relationship to approximate to the elastic relationship when a 30% of moment redistribution was  
420 assumed indicates that the moment redistribution mechanism was formed.

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423 Figure 15 — Moment redistribution-applied load relationship obtained numerically for  
 424 the slabs.  
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426 Figure 16 — Bending moment -applied load relationship: (a) SL30 and (b) SL30s25.  
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## CONCLUSIONS

This work deals with the use of the near surface mounted (NSM) CFRP laminates for the flexural strengthening of continuous reinforced concrete (RC) slabs not only in terms of load carrying capacity, but also in the context of moment redistribution capacity. The strengthening procedures adopted in the laboratory tests followed, as much as possible, the real strengthening practice for this type of interventions.

The obtained results show that the proposed technique is able to increase the load carrying capacity of RC slabs and preserves relevant levels of moment redistribution. However, the load carrying capacity of the strengthened slab was limited by the detachment of the strengthened concrete cover layer at the intermediate support.

For validation purposes, a computer program, based on the finite element method (FEM), was used. Using the obtained experimental results, the capability of the FEM-based computer program to predict with high accuracy the behaviour of this type of structures up to its collapse was highlighted.

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