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"Price Discrimination with Private and Imperfect Information"

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Price Discrimination with Private and Imperfect Information*

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Abstract

    This paper investigates the competitive and welfare effects of information accuracy improvements in markets where firms can price discriminate after observing a private and noisy signal about a consumer's brand preference. I show that firms charge more to customers they believe have a brand preference for them, and that this price has an inverted-U shaped relationship with the signal's accuracy. In contrast, the price charged after a disloyal signal has been observed falls as the signal's accuracy rises. While industry profit and overall welfare fall monotonically as price discrimination is based on increasingly more accurate information, the reverse happens to consumer surplus. The model is also extended to a public information setting. For any level of the signal's accuracy, moving from public to private information boosts industry profit and welfare at the expense of consumer surplus.

    JEL Code: D43, D80, L13, L40

    Keywords: Competitive Price Discrimination, Customer Recognition, Imperfect Information.

1   Introduction

"Dynamic pricing is a new version of an old practice: price discrimination. It uses a potential buyer's electronic fingerprint—his record of previous purchases, his address, maybe the other sites he has visited—to size up how likely he is to balk if the price is high. If the consumer looks price-sensitive, he gets a bargain; if he doesn't he pays a premium."

Paul Krugman (2000)

    The increasing use of the Internet and the development of more sophisticated methods for acquiring, storing and analysing consumer information have dramatically improved the capability of sellers to predict the consumers types or preferences and to set prices accordingly. However, it is frequently the case that due to insufficient information or inaccurate statistical inferences firms may not be able to perfectly predict (recognise) the tastes of individual consumers. When

a firm has access to inaccurate information some consumers can be wrongly recognised and be offered the wrong intended price.[1,2] On the other hand, perhaps because each firm purchases customer data from a different marketing company, firms may possess *private* information about consumers, implying that they don't know for sure how a given consumer is classified by rival firms for price discrimination purposes.

With few exceptions,[3] the vast majority of previous literature on competitive price discrimination with customer recognition has mainly considered situations in which information is perfect and freely available to all firms or it is not available to any firm. A good understanding of the profit, consumer surplus and welfare implications of price discrimination with customer recognition needs to be founded on a good understanding of the informational context in which it is implemented. This paper is a step in this direction. It explores the following questions. How does profit, consumer surplus and overall welfare change as firms have access to more accurate information about consumers? Are consumers better off when firms possess private information about them or rather when firms have public information?

The paper adds to the literature on pricing with customer recognition a theoretical model that encompasses situations where firms are uncertain about (i) the consumer preferences and (ii) the information the rivals possess. One way of dealing with this possibility is by introducing some randomness in the accuracy of a firm's private information. The paper addresses, in section 2, a Hotelling model where two firms A and B sell their products directly to consumers whose loyalty degree towards the right-hand firm (firm A) is indexed by their location along an interval. Each firm's private imperfect information comes from the observation of a *noisy* signal that is not seen by the rival firm, which informs with only some level of accuracy whether a customer favours brand A or rather brand B. Each firm has imperfect private information about whether each consumer has a preference for it or for its rival, but no information about the extent of this preference. One important implication of this assumption is that markets are no longer completely separate as some consumers will be misrecognised and will receive the wrong intended price.[4]

Section 3 looks at the equilibrium price behaviour of firms when price discrimination is based on private and imperfect information. As in the extant literature (e.g. Thisse and Vives (1988), Chen (1997) and Fudenberg and Tirole (2000)) it is shown that firms will always set a higher price to a consumer recognised as loyal than to a consumer they believe have a brand

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[1] In the telecommunications industry, for instance, AT&T once mailed three rebate checks to a marketing executive of MCI to switch phone services (Chen, et al. (2001), p.24). Credit card issuers occasionally send targeting offers to the deceased, to children, and even family pets (Chen et al.(2001), footnote 2).

[2] In the Internet, user cookies are misleading. A cookie is unique to a computer, not a person. The information stored in a cookie typically aggregates behavioral clues from multiple users of different gender, age, and interests. So if firms use cookies to predict a consumer taste they may not be able to perfectly predict each consumer type.

[3] Basically Chen et al. (2001) and Liu and Serfes (2004). See the discussion of these papers in the subsequent related literature.

[4] The model addressed in this paper fits well pricing policies that will be possible through the mobile wireless technologies. Particularly relevant will be the possibility of firms setting their prices according to the geographic location of consumers—i.e., to set different prices to consumers with different location-based information. In general, a typical cell phone that is turned on sends out signals every ten minutes and identifies the location of the nearest cell tower. These signals can then be used to determine a cell phone user's location so that even without the precise auto-location technologies (e.g. GPS), a user's location can be predicted fairly precisely. Thus, after receiving a signal more or less accurate about each individual's location, a firm might be able to price discriminate between consumers located near its store and consumers located closer to a rival's store. To read more on mobile wireless technologies and consumer uses see for instance www.ftc.gov/bcp/reports/wirelesssummary.pdf

preference for the rival's product (Proposition 2). In section 4, I explore the competitive effects of information improvements on the equilibrium outcomes. Regarding the effects on prices, we will see that the price for customers recognised as loyal has an inverted-U relationship with the signal's accuracy and may be above (below) the non-discrimination level for low (high) levels of the signal's accuracy. In contrast, the price charged to a customer recognised as disloyal is always below the non-discrimination level, and lower the more accurate is the signal (Corollary 1).

The welfare analysis is presented in section 5. Here, it is shown that, when firms have information to price discriminate, consumer welfare is always above the non-discrimination level and increases monotonically as the accuracy of the private signal rises (Corollary 4). In contrast, industry profit and overall welfare are always below the non-discrimination levels and fall monotonically as firms possess increasingly more accurate information (Proposition 3).

The remainder of the paper deals with public and imperfect information. Section 6 extends the model of section 2 to the case where both firms observe a public noisy signal about each consumer's brand preference. The signal is "public" in the sense that its actual realization is common knowledge of both firms. Section 7 compares the equilibrium outcomes with private and public information. Here it is shown that firms price more aggressively when both have the *same* piece of information. So moving from private to public information is good for consumers, but bad for industry profit and overall welfare (Proposition 6).

The advances in information technologies have provided unprecedented opportunities for the collection and sharing of information about consumers. The way this information is used has increasingly attracted the attention of regulators, policy makers and privacy advocates. This paper offers some criterion to assess *how* overall and consumer welfare evolve as price discrimination is based on more precise information as well as on private rather than on public information. It suggests that any advice to a regulatory authority should take into account whether the target is welfare or solely consumer surplus. It highlights that when personal information about consumers is used for price discrimination purposes, any policy restricting the use and/or the access to more accurate information about consumers would act in favour of industry profits and overall welfare but against consumer welfare.[5]

Related literature   This paper is related to the literature on price discrimination in imperfectly competitive markets,[6] mainly to the literature on price discrimination based on customer recognition.[7] Most of this literature has assumed that firms can *perfectly* (i) recognise each individual consumer's tastes and offer them personalized prices (e.g. Thisse and Vives (1988)) or (ii) can group consumers into two different segments, their own (loyal) and the rival's customers and price accordingly (e.g. Chen (1997), Villas-Boas (1999), Fudenberg and Tirole (2000), Shaffer and Zhang (2000)). In either case all firms offer personalized prices/price discriminate on the basis of the *same* piece of information and with *no uncertainty* about consumer types.

Thisse and Vives (1988) show that when firms have the same piece of information about each individual consumer (i.e. each firm observes a public fully accurate signal of each consumer's brand preference), and firms base their prices on this observed signal (i.e. firms set personalized

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[5]It is important to stress that this paper looks only on the use of information about consumers for price discrimination purposes. Obviously, the discussion on consumer privacy issues is far more complex than the present analysis.

[6]See Armstrong (2006) and Stole (2007).

[7]See Fudenberg and Villas-Boas (2007) for a comprehensive review and Esteves (2009b).

prices), then each consumer is a completely separate market to be contested. As a result, they show that price discrimination may intensify competition leading all prices and profits to fall compared to the non-information (no discrimination) case.[8] Thus, it can be said that the model proposed in this paper is a natural way of thinking of situations where firms set personalized prices. It extends the Thisse and Vives's model in the sense that even though a firm cannot observe the true brand preference of each individual consumer, it is able to observe a *noisy* private signal of a consumer's brand preference, and thereby to set its prices accordingly.

Other papers have extended the Thisse and Vives's analysis to frameworks where, although firms are unable to observe the brand preference of individual consumers, they are able to recognise them only as their own customers or as the rival's customers. These models have assumed that by observing the consumers' past purchasing decisions, firms obtain a public and perfectly accurate signal of whether a consumer prefers its brand or the rival's one and price differently to loyal and the rival's customers. However, because information is fully precise and public there is neither misrecognition of consumers nor uncertainty about the rival's information. In these papers information may disclose exogenous switching costs (e.g. Chen (1997) and Taylor (2003)), or exogenous brand preferences (e.g. Villas-Boas (1999), Fudenberg and Tirole (2000) and Shaffer and Zhang (2000)). These models assume that firms observe the *same* piece of information and thereby each consumer is classified into the right segment. For instance, in Fudenberg and Tirole (2000) after a firm has observed whether or not a consumer bought its product previously, it is able, in period 2, to segment the market into old (loyal) customers and rival's (disloyal) customers. Once one firm recognises a customer as a loyal one it must be the case that the competitor recognises that customer as a disloyal one. Hence, customer recognition (and market segmentation) is based on public information, implying that firms are not uncertain about the information the rival is using for setting its discriminatory prices. As each firm tries to poach each other's customers, a common finding is that price discrimination acts to intensify competition leading all segment prices to fall as well as profits (Chen (1997), Villas-Boas (1999), Fudenberg and Tirole (2000), Taylor (2003)).[9]

This paper is most closely related to Chen et al. (2001) who investigate the profit and welfare effects of price discrimination when firms cannot perfectly recognise (target) a captive customer from a switcher.[10] Firms compete only for switchers. When targetability is not perfect it may happen that a captive consumer receives the price tailored to a switcher and vice-versa. In this way, a low level of targetability tends to soften price competition for switchers as firms try to avoid that some captive consumers receive a very low price. For this reason they show that profits may increase with improvements in targetability when it departs from a low enough level. As firms' targetability is sufficiently high further improvements in targetability

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[8] In Thisse and Vives all prices fall when consumer brand preferences are uniformly distributed.

[9] Villas-Boas (1999) analyses an in infinite-horizon model with overlapping generations of consumers, where although firms are able to recognise old customers, they are not able to distinguish a first-time customer in the market from one that bought the rival's product previously. In this case, each firm has private information concerning its old customers. That is, firms know with certainty whether a customer is an old one or not, but when they face a new customer they are uncertain about whether that customer is a first-time customer to the market or a rival's previous customer. Even in this context, Villas-Boas shows that each firm's temptation to try to attract the rival's previous customers makes both firms cut prices in relation to the situation where price discrimination cannot occur. Hence he also finds that price discrimination leads to lower equilibrium prices and profits for all competing firms.

[10] They define targetability as a firm's ability to predict the preferences and purchase behaviours of individual consumers.

will intensify competition and lead to a prisoner's dilemma. The present paper complements Chen et al. (2001) analysis by proposing a different framework for the consumers brand tastes. While in Chen et al. (2001) there are three distinct group of consumers, captive to each firm and switchers and firms compete only for switchers, here there is a continuum of consumer loyalties. In doing so we will see that improvements in information accuracy will produce different profit and welfare outcomes. I elaborate more on these differences in section 5.

Finally, another relevant paper is Liu and Serfes (2004) on price discrimination and information quality improvements.[11] Using the Hotelling model they propose a unifying framework that has the two-groups case (i.e., the Fudenberg and Tirole model) and the personalized case (i.e., Thisse and Vives model) as special cases. Their modeling approach allows for the investigation of the profit and welfare effects of price discrimination as information quality improves. The quality of information in their paper is measured by the ability of firms to increasingly identify more groups of consumers in the Hotelling line, being the more informative case the one where each firm knows the location of each individual consumer. Nevertheless, in their analysis given the number of identified groups (quality of information) there is no uncertainty about who each consumer is and about the rival's information. As in the present paper they find that moving from no discrimination (no information) to discrimination is bad for industry profit and good for consumer surplus. However, conditional on the availability of information they find that equilibrium profits have a U-relationship with respect to information improvements and consumer surplus have an inverse U-shape as a function of the information quality. The lack of uncertainty in their model explains the different outcomes obtained in their model and in the present paper.

This paper complements Chen et. (2001) and Liu and Serfes (2004) and so puts forward that a good economic understanding of the profit and welfare effects of information improvements do depend on the way the information improvement is modelled, on what is learned about consumer demand and on the nature of preferences.

2   The model

There are two firms A and B who sell competing brands of a good produced, without loss of generality, at zero marginal cost.[12] The total number of consumers in the market is normalized to one. Each consumer wishes to buy a single unit either from firm A or B and he is willing to pay at most v. The reservation value v is sufficiently high so that nobody stays out of the market. Consumers are heterogeneous with respect to the brand loyalty degree. Following Raju, et al. (1990) the degree of brand loyalty can be defined as the minimum difference between the prices of two competing brands necessary to induce a consumer to buy his less preferred brand. A consumer's brand loyalty towards brand A is represented by a parameter l uniformly distributed on the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$, with density 1 and it is assumed that $l \leq v$. All else equal, a consumer with $0 < l \leq \frac{1}{2}$ prefers brand A, while a consumer with $-\frac{1}{2} \leq l < 0$ prefers brand B.[13] A type l consumer who buys his favorite brand, say brand i, enjoys a net surplus equal to $v - p_i$; if he buys his less preferred brand, say j, his net surplus is $v - p_j - |l|$, where $i, j = A, B$.

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[11] For a paper on information quality improvements and personalized products (rather than personalized prices) see Bernhardt, Liu and Serfes (2007).

[12] The assumption of zero marginal costs can be relaxed without altering the basic nature of the results derived throughout the model.

[13] Shaffer and Zhang (2002) and Liu and Serfes (2006) model the distribution of consumer brand preferences in a similar way. However, they allow for asymmetric loyalties towards firms as $l_A > l_B > 0$.

Each consumer has private information about his loyalty degree. Firms are not able to observe the degree of loyalty of individual customers. However, it is assumed that each firm has consumer-specific private information—acquired either from specialized information vendors or from the firm's databases that record the customers' individual purchase histories—that enables it to predict (with some probability) whether a customer prefers its brand or rather the rival's one. Suppose that for each consumer firm i observes a private noisy signal, s_i, that informs whether that customer is loyal to brand A ($s_i = \alpha$) or loyal to brand B ($s_i = \beta$). The signal gives no information about the extent of this preference. After a noisy signal has been observed for a particular consumer each firm only knows that he is a loyal or a disloyal consumer with some probability. This means that consumer recognition is imperfect, and so market segmentation for the purpose of price discrimination will be also imperfect. A more accurate signal is associated with better information which is then reflected in a higher ability of firms to classify correctly potential customers.

When price discrimination is permitted, a firm pricing strategy consists of choosing a price to a customer it believes is likely to prefer its good and choosing a different price to a customer perceived as one that favours the rival's product.

In short, conditional on a given consumer true type, suppose that firm A and B observe an independent private noisy signal $s_i \in \{\alpha, \beta\}$, $i = A, B$, about that consumer's brand preference. While α informs that the consumer prefers brand A, β informs that the consumer favours brand B. It is common knowledge that the probability of each signal conditional on each consumer's loyalty degree l is given by:

$$q(l) = \Pr(s_i = \alpha \mid l), \tag{1}$$

$$1 - q(l) = \Pr(s_i = \beta \mid l). \tag{2}$$

Assume further that $q(l)$ is increasing in l, meaning that the greater is the degree of loyalty of a particular consumer, the higher is the probability of a firm observing a loyal signal. For instance, when say firm A's signal is based on data on consumers' past purchasing behaviour, it is more likely that firm A observes a loyal signal for a consumer that bought its product many times in the past than for a customer that bought few times or did not buy at all from A in the past. For the sake of simplicity consider that

$$q(l) = \frac{1}{2} + bl, \qquad 0 \leqq b \leqq 1, \tag{3}$$

where b measures the signal's accuracy. When $b = 0$ the signal has no informational content and firms have no way to distinguish customers. In contrast, the signal discloses increasingly more accurate information as b approaches 1, thereby allowing firms to better recognise customers. For intermediate values of b some consumers are incorrectly classified by firms, i.e. some consumers loyal to brand A are *misrecognised* as loyal to brand B and vice-versa.

Updating beliefs After receiving a signal for a given customer firms update their own beliefs over this customer's true loyalty degree and form beliefs about the rival's signal for the same customer. The density function of l, denoted $f(l)$ (which in this case is equal to 1) and $q(l)$ are common knowledge of both firms and form the basis of their prior and posterior beliefs. Hence, after receiving signal $s_i \in \{\alpha, \beta\}$, using Bayes rule, each firm's posterior belief about a

customer's loyalty degree is given by the conditional density function $h_{s_i}(l)$ where[14]

$$h_\alpha(l) = \Pr(l \mid s_i = \alpha) = \frac{\Pr(s_i = \alpha \mid l) f(l)}{\Pr(s_i = \alpha)} = \frac{q(l)}{\Pr(s_i = \alpha)}, \tag{4}$$

$$h_\beta(l) = \Pr(l \mid s_i = \beta) = \frac{\Pr(s_i = \beta \mid l) f(l)}{\Pr(s_i = \beta)} = \frac{[1 - q(l)]}{\Pr(s_i = \beta)}, \tag{5}$$

and

$$\Pr(s_i = \alpha) = \int_{-\frac{1}{2}}^{\frac{1}{2}} q(l) f(l) dl = \frac{1}{2}, \tag{6}$$

$$\Pr(s_i = \beta) = \int_{-\frac{1}{2}}^{\frac{1}{2}} [1 - q(l)] f(l) dl = \frac{1}{2}. \tag{7}$$

Given signal $s_i = k$, firm i believes that firm j's signal is $s_j = \alpha$ with probability $\rho_k = \Pr(s_j = \alpha \mid s_i = k)$. Therefore,

$$\rho_\alpha = \Pr(s_j = \alpha \mid s_i = \alpha) = \frac{\Pr(s_j = \alpha, s_i = \alpha)}{\Pr(s_i = \alpha)} = 2\lambda_{\alpha\alpha} \tag{8}$$

$$\rho_\beta = \Pr(s_j = \alpha \mid s_i = \beta) = \frac{\Pr(s_j = \alpha, s_i = \beta)}{\Pr(s_i = \beta)} = 2\lambda_{\beta\alpha} \tag{9}$$

where

$$\lambda_{kr} = \Pr(s_i = k, s_j = r); \qquad k, r = \{\alpha, \beta\}.$$

Conditional on a consumer loyalty degree l, the signals observed by firms are independently distributed. That is

$$\Pr(s_i = k, s_j = r \mid l) = \Pr(s_i = k \mid l) \Pr(s_j = r \mid l).$$

Thus,

$$\lambda_{kr} = \int_{-\frac{1}{2}}^{\frac{1}{2}} \Pr(s_i = k, s_j = r \mid l) f(l) dl. \tag{10}$$

and so,

$$\lambda_{\alpha\alpha} = \Pr(s_i = \alpha, s_j = \alpha) = \int_{-\frac{1}{2}}^{\frac{1}{2}} [q(l)]^2 f(l) dl = \frac{1}{4} + \frac{b^2}{12}, \tag{11}$$

$$\lambda_{\beta\beta} = \Pr(s_i = \beta, s_j = \beta) = \int_{-\frac{1}{2}}^{\frac{1}{2}} [1 - q(l)]^2 f(l) dl = \frac{1}{4} + \frac{b^2}{12}, \tag{12}$$

$$\lambda_{\alpha\beta} = \Pr(s_i = \alpha, s_j = \beta) = \int_{-\frac{1}{2}}^{\frac{1}{2}} q(l) [1 - q(l)] f(l) dl = \frac{1}{4} - \frac{b^2}{12}. \tag{13}$$

Since both firms observe signal α with equal probability it follows that $\lambda_{\beta\alpha} = \lambda_{\alpha\beta}$.

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[14] Notice that $h_{s_i}(l)$ satisfies the property $\int_{-\frac{1}{2}}^{\frac{1}{2}} h_{s_i}(l) dl = 1$.

Lemma 1. *For any level of the signal's accuracy it follows that $\rho_\alpha \geqslant \rho_\beta$.*

Proof. See the Appendix.

In words, when firm A observes signal α for a particular consumer it believes that it is more likely that the rival observes the same type of signal for that consumer. The more accurate is the signal the higher is the likelihood that firm B has observed signal α given that firm A has observed signal α. (Note that $\rho_\alpha = \frac{1}{2} + \frac{b^2}{6}$.) Hence, more accurate signals reduce each firm's uncertainty about the rival's private information.

Firms also form beliefs about the loyalty degree of a given consumer after signals s_i and s_j have been observed. This is given by the conditional density function $g_{kr}(l)$ where,[15]

$$g_{kr}(l) = \Pr\left(l \mid s_i = k, s_j = r\right) = \frac{\Pr\left(s_i = k \mid l\right)\Pr\left(s_i = r \mid l\right)f\left(l\right)}{\Pr\left(s_i = k, s_j = r\right)}. \tag{14}$$

Thus,

$$g_{\alpha\alpha}(l) = \Pr\left(l \mid s_i = \alpha, s_j = \alpha\right) = \frac{[q(l)]^2}{\lambda_{\alpha\alpha}},$$

$$g_{\alpha\beta}(l) = g_{\beta\alpha}(l) = \Pr\left(l \mid s_i = \alpha, s_j = \beta\right) = \frac{q(l)\left[1 - q\left(l\right)\right]}{\lambda_{\alpha\beta}},$$

$$g_{\beta\beta}\left(l\right) = \Pr\left(l \mid s_i = \beta, s_j = \beta\right) = \frac{\left[1 - q\left(l\right)\right]^2}{\lambda_{\beta\beta}}.$$

Finally,

$$G_{kr}(x) = \Pr\left(l < x \mid s_i = k, s_j = r\right) = \int_{-\frac{1}{2}}^{x} g_{kr}(l)dl. \tag{15}$$

3   Equilibrium Analysis

Consider first the benchmark case where price discrimination cannot occur either because it is illegal or because the signal has no informational content (i.e. $b = 0$).

Non-discrimination benchmark case: Here the setup is analogous to a standard symmetric Hotelling model, playing the loyalty parameter l the same role as the transportation cost. If firms cannot price discriminate in the symmetric equilibrium they will set the non-discrimination price $p_N = \frac{1}{2}$. With non discrimination, equilibrium profit per firm is $\pi_N = \frac{1}{4}$, consumer surplus is $CS_N = v - \frac{1}{2}$, and total welfare is $W_N = 2\pi_N + CS_N = v$.

Consider now the case where price discrimination is permitted and firms price differently to consumers they believe are likely to prefer their product and to consumers they believe have a preference for the rival's product. Formally, upon observing signal $s_i \in \{\alpha, \beta\}$, firm A chooses $p_{s_A}^A \in \left\{p_\alpha^A, p_\beta^A\right\}$ and firm B chooses $p_{s_B}^B \in \left\{p_\alpha^B, p_\beta^B\right\}$. For the sake of simplicity, assume $s_A = k$

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[15]Notice that $g_{kr}(l)$ satisfies the property $\int_{-\frac{1}{2}}^{\frac{1}{2}} g_{kr}(l)dl = 1$.

and $s_B = r$, where $k, r = \{\alpha, \beta\}$. I will use the Bayesian Nash Equilibrium (BNE) as the solution concept. Equilibrium prices are obtained solving the ensuing maximization problems:

$$\underset{p_{s_A}^A}{Max} E\left[\pi^A \mid s_A = k\right] \quad \text{and} \quad \underset{p_{s_B}^B}{Max} E\left[\pi^B \mid s_B = r\right],$$

where

$$E\left(\pi^A \mid s_A = \alpha\right) = p_\alpha^A \sum_{r \in \{\alpha, \beta\}} \Pr\left(p_\alpha^A < p_r^B + l \mid s_A = \alpha, s_B = r\right) \Pr\left(s_B = r \mid s_A = \alpha\right)$$

$$= p_\alpha^A \left\{\rho_\alpha \left[1 - G_{\alpha\alpha}\left(p_\alpha^A - p_\alpha^B\right)\right] + (1 - \rho_\alpha)\left[1 - G_{\alpha\beta}\left(p_\alpha^A - p_\beta^B\right)\right]\right\}, \tag{16}$$

and,

$$E\left(\pi^A \mid s_A = \beta\right) = p_\beta^A \sum_{r \in \{\alpha, \beta\}} \Pr\left(p_\beta^A < p_r^B + l \mid s_A = \beta, s_B = r\right) \Pr\left(s_B = r \mid s_A = \beta\right)$$

$$= p_\beta^A \left\{\rho_\beta \left[1 - G_{\beta\alpha}(p_\beta^A - p_\alpha^B)\right] + (1 - \rho_\beta)\left[1 - G_{\beta\beta}(p_\beta^A - p_\beta^B)\right]\right\}. \tag{17}$$

Symmetric expressions hold for firm B. Considering, for instance, the perspective of firm A, from the first-order conditions it follows that:

$$p_\alpha^A = \frac{\rho_\alpha \left[1 - G_{\alpha\alpha}\left(p_\alpha^A - p_\alpha^B\right)\right] + (1 - \rho_\alpha)\left[1 - G_{\alpha\beta}\left(p_\alpha^A - p_\beta^B\right)\right]}{\rho_\alpha g_{\alpha\alpha}\left(p_\alpha^A - p_\alpha^B\right) + (1 - \rho_\alpha) g_{\alpha\beta}\left(p_\alpha^A - p_\beta^B\right)} \tag{18}$$

and

$$p_\beta^A = \frac{\rho_\beta \left[1 - G_{\beta\alpha}\left(p_\beta^A - p_\alpha^B\right)\right] + (1 - \rho_\beta)\left[1 - G_{\beta\beta}\left(p_\beta^A - p_\beta^B\right)\right]}{\left[\rho_\beta g_{\beta\alpha}\left(p_\beta^A - p_\alpha^B\right) + (1 - \rho_\beta) g_{\beta\beta}\left(p_\beta^A - p_\beta^B\right)\right]}. \tag{19}$$

Similar expressions hold for firm B.[16]

Since the game is symmetric one must look at an equilibrium solution where $p_\alpha^A = p_\beta^B$ and $p_\beta^A = p_\alpha^B$. In doing so, the above system is reduced to two equations with two unknowns. Denoting as p_L the price charged to customers recognised as loyal and as p_D the price offered to customers recognised as disloyal it follows that $p_L = p_\alpha^A = p_\beta^B$ and $p_D = p_\beta^A = p_\alpha^B$. Since G is a cubic function the model does not allow for closed-form solutions. However, imposing the condition $y = p_L - p_D$ gives the answer implicitly.

Proposition 1. *When firms price discriminate on the basis of private and imperfect information about consumer brand preferences, the symmetric BNE in prices is given by*

$$p_L = \frac{\frac{1}{4} + \frac{1}{8}b - \frac{1}{4}y - \frac{1}{2}by^2 - \frac{1}{3}b^2y^3}{\frac{1}{2} + by + b^2y^2}, \tag{20}$$

and

$$p_D = \frac{\frac{1}{4} - \frac{1}{8}b + \frac{1}{4}y + \frac{1}{2}by^2 + \frac{1}{3}b^2y^3}{\frac{1}{2} + by + b^2y^2}. \tag{21}$$

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[16] Second-order conditions are also satisfied as can be seen in the proof of Proposition 1 in the Appendix provided.

Proof. See the Appendix.

The symmetric interior solution presented in Proposition 1 is the only possible equilibrium solution. Despite the cubic equation in y, there is only one real root for all values of $b \in [0,1]$, thus the symmetric equilibrium derived is unique.

Proposition 2. *(i) When the signal has no informational content (i.e. $b = 0$), price discrimination is unfeasible and equilibrium prices are $p_L = p_D = \frac{1}{2}$.*
(ii) For any informative signal (i.e. $b > 0$), $p_L > p_D$.
(iii) The more informative is the signal the greater is the difference between the two discriminatory prices, i.e. $p_L - p_D$ increases with b.

Using (20) and (21) part (i) is easily obtained. To prove (ii) and (iii) Figure 1 plots $y = p_L - p_D$ as an implicit function of b, showing that for any level of the signal's accuracy $y > 0$.[17]

Proposition 2 claims that firms will charge more to customers they believe have a brand preference for their product than to customers that, from their perspective, are more price-sensitive, which are those customers recognised as disloyal. Furthermore, it predicts that the gap between loyal and disloyal prices will be greater as the accuracy of information improves.
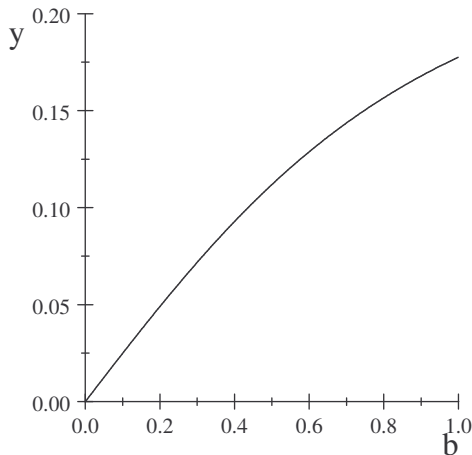

Figure 1: y as a function of b

When the signal's precision is low firms are more uncertain about each consumer's loyalty degree and about the rival's private information. This acts to soften price competition in the market leading to a smaller gap between loyal and disloyal prices. In contrast, when information's accuracy improves it is less likely that firms receive the wrong signal for a given consumer and it is more likely that both firms observe the same signal (for instance, signal α). Note that the probability of a customer be in fact loyal say to firm A, after signal α has been observed, is given by $h_\alpha(l)$ (see equation (4)) which is increasing in b. Note also that the probability of both firms observing the same signal, i.e. λ_{kk}, given by equation (11), is increasing in b, meaning that as information's accuracy increases it is also more likely that both firms classify customers in the

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[17]More precisely, y is defined implicitly as follows: $y + 2by^2 + \frac{5}{3}b^2y^3 - \frac{1}{4}b = 0$.

same way. As information quality improves, consumers can be recognised more accurately by both firms which now compete more aggressively. The gap between prices increases.

4   Competitive effects of information improvements

This section investigates the competitive effects of price discrimination as the firms' ability to recognise customers more accurately gradually improves.

4.1   Prices

Using the equilibrium solutions presented in (20) and (21) we obtain the following result.

Corollary 1. *When $b > 0$ then:*
(i) the price charged to a customer recognised as disloyal is always below the non-discrimination price, i.e. $p_D < p_N$.
(ii) The price charged to a customer recognised as loyal is above the non-discrimination price when the signal's accuracy is not too high, and below the non-discrimination price when the signal's precision is high.
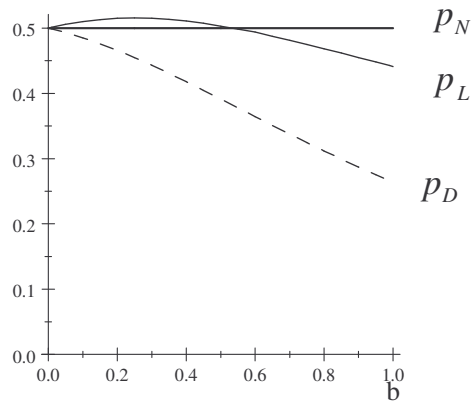

Figure 2: Relation between prices and b

In models of competitive price discrimination with perfect information, there are mainly two effects at work. There is the *"surplus extraction effect"* through which price discrimination allows a firm to extract greater surplus from those consumers willing to pay more for its product. There is also the *"business stealing effect"* as the ability to set discriminatory prices gives firms an incentive to reduce the price to disloyal consumers as away to entice them to switch. Apart from these two effects, when price discrimination is based on imperfect information (imperfect recognition), there is another effect at work, namely the *"misrecognition effect"*.[18] When firms rely on imperfect information for price discrimination purposes they may classify consumers incorrectly and consequently offer them the wrong intended price.

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[18]In Chen et al. (2001) there is a similar effect which they designate as the "mistargeting effect".

Consider the following example. Suppose that a given consumer prefers brand A. Under imperfect and private information four events are relevant: (i) both firms observe signal α and the consumer is correctly recognised by both firms; (ii) while firm A observes signal α, firm B observes signal β and the consumer is misrecognised by firm B; (iii) firm A observes signal β and firm B observes signal α, in which case the consumer is misrecognised by firm A, and (iv) both firms observe the wrong signal, that is signal β, and the consumer is misrecognised by both firms.[19] Figure 2 shows that the price to a consumer recognised as disloyal is always below its non-discrimination counterpart and decreases as information's quality improves. When information precision is low, misrecognition of consumers is more likely and a firm has less incentives to decrease its price to a consumer that generates a disloyal signal, because there is a good chance that the consumer turns out to be a loyal consumer. However, as information's accuracy increases firms are more certain about who each consumer is. Thus, as the private signal's precision rises the "misrecognition effect" becomes smaller leading the "business stealing effect" to become larger. In this way, the price to a consumer recognised as disloyal falls as information's precision increases.

An interesting finding of the paper is that the price charged to customers recognised as loyal has an inverted U-shaped relationship with the signal's accuracy. Figure 2 shows that p_L is above its non-discrimination counterpart when the signal's accuracy is (approximately) below 0.5, the reverse happens for $b > 0.5$.[20] In order to explain this non-monotonic relationship we must have in mind that there have to be opposing forces at work, with a balance between the forces that changes as the parameter b increases. Apart from strategic reasoning, when a firm receives a loyal signal for a given consumer it has an incentive to raise its price as a way of appropriating greater surplus from that consumer. When information is imperfect the expected surplus that a firm can extract after a loyal signal has been observed depends on the probability that a correct signal of a consumer type has been observed. Because misrecognition decreases with increases in b the "surplus extraction effect" becomes larger as the quality of information improves. Consider now the strategic interaction between firms. Suppose that firm A receives a loyal signal (i.e., signal α) for a particular consumer. As the signal's precision raises firm A is more certain about the true type of that consumer and so it has more incentives to raise the price to a customer recognised as loyal. When the signal's precision departs from a low level of precision, firm A acknowledges that if firm B receives as well signal α, it prices less aggressively because the consumer may turn out to be its own. However, as the quality of information becomes increasingly more precise, although firm A is more certain about the consumer type, it knows that the same happens to the rival firm. The "misrecognition effect" becomes smaller, firms compete more aggressively for each consumer leading the "business stealing effect" to become increasingly larger.

Summing up, for sufficiently low levels of the private signal's accuracy (i.e. $b \lessapprox 0.25$) the misrecognition effect acts to soften price competition and reinforces the surplus extraction effect. Since increases in b increase the informational content of the signal and competition is

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[19]It is important to stress that with private information, misrecognition of consumers can occur either when both firms misrecognise (i.e. when both receive the wrong signal) or when only one of them misrecognises (i.e. when both receive different signals). We will see that, with public information, misrecognition of consumers will only occur when both firms observe the wrong signal.

[20]Numerical analysis shows that p_L reaches its maximum value approximately at 0.5154 for $b \cong 0.25$ and is equal to the non-discrimination level for $b \cong 0.5$. When $b = 1$ the price charged to a perceived loyal customer falls to 0.44 (lower than the non-discrimination level).

not so intense, the price to a consumer recognised as loyal first increases with improvements in information's accuracy. However, as information becomes increasingly more precise, the "misrecognition effect" becomes weaker and weaker, firms compete more aggressively and the "business stealing effect" becomes larger. This explains why the price to a customer recognised as loyal is a decreasing function of the quality of information for $b \gtrapprox 0.25$. Nevertheless, when $0.25 \lessapprox b \lessapprox 0.5$, the "misrecognition effect" allows the "surplus extraction effect" to dominate the "business stealing effect". In contrast, when information becomes increasingly more and more accurate (i.e. $b \gtrapprox 0.5$) the role of the "misrecognition effect" becomes so small that the "business stealing effect" dominates the "surplus extraction effect", thus the price to a loyal signal falls below its non-discrimination counterpart.

Note that in Chen et al. (2001) because firms only compete for switchers the price to customers recognised as captive (loyal) is always above the non-discrimination level and is increasingly higher as firms become increasingly able to distinguish a captive from a switcher.

Since in the present paper each consumer is a market to be contested, a very accurate signal *intensifies* price competition explaining why both prices are below the non-discrimination level. This confirms a common finding in the previous literature. When each firm's strong market is the rival's weak market (i.e., in the terminology of Corts (1998) there is best response asymmetry), price discrimination acts to intensify competition leading all segment prices to fall (e.g. Thisse and Vives (1988), Chen (1997) and Fudenberg and Tirole (2000)).

4.2 Probability of winning a customer

When say *Amazon* infers that a particular customer is a loyal one it may be interested in determining what is the probability of winning that customer with a price tailored to a loyal consumer, taking into account that say *Barnes&Noble* also offers that customer a price based on its beliefs about that customer's loyalty degree. This section investigates how this probability evolves as information becomes more accurate.

Price discrimination is allowed Due to symmetry let γ_L and γ_D denote the probability of a firm winning a customer with a loyal and a disloyal signal. Using, for instance, the perspective of firm A,

$$
\begin{aligned}
\gamma_L &= \Pr\left(\text{firm } A \text{ wins customer} \mid s_A = \alpha\right) \\
&= \sum_{r \in \{\alpha, \beta\}} \Pr\left(p_\alpha^A < p_r^B + l \mid s_A = \alpha, s_B = r\right) \Pr\left(s_B = r \mid s_A = \alpha\right) \\
&= \left[1 - G_{\alpha\alpha}(y)\right] \rho_\alpha + \left[1 - G_{\alpha\beta}(0)\right]\left(1 - \rho_\alpha\right)
\end{aligned}
$$

and,

$$
\begin{aligned}
\gamma_D &= \Pr\left(\text{firm } A \text{ wins customer} \mid s_A = \beta\right) \\
&= \sum_{r \in \{\alpha, \beta\}} \Pr\left(p_\alpha^A < p_r^B + l \mid s_A = \alpha, s_B = r\right) \Pr\left(s_B = r \mid s_A = \alpha\right) \\
&= \left[1 - G_{\beta\alpha}(0)\right] \rho_\beta + \left[1 - G_{\beta\beta}(-y)\right]\left(1 - \rho_\beta\right).
\end{aligned}
$$

After some algebra we obtain:

$$
\gamma_L = \frac{1}{2} + \frac{1}{4}b - \frac{1}{2}y - by^2 - \frac{2}{3}b^2y^3 = \frac{1}{2} + b^2y^3 + by^2 + \frac{1}{2}y \tag{22}
$$

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and

$$\gamma_D = \frac{1}{2} - \frac{1}{4}b + \frac{1}{2}y + by^2 + \frac{2}{3}b^2y^3 = \frac{1}{2} - b^2y^3 - by^2 - \frac{1}{2}y \qquad (23)$$

Corollary 2. *The greater is the accuracy of each firm's private signal the greater is the probability of a firm winning a customer with a loyal price and the lower is the probability of a firm winning a customer with a disloyal price.*

It is straightforward to verify that for any $b > 0$ it immediately follows that $y > 0$ and so $\gamma_L > \gamma_D$. Further, while γ_L is strictly increasing in b, γ_D is strictly decreasing in b.[21] As expected when the signal has non informational content it follows that $\gamma_L = \gamma_D = 0.5$. As the quality of information improves, the greater is the likelihood of firms facing in fact a loyal customer after observing a loyal signal and, the higher is the probability of firms winning a customer with a loyal signal. The reverse happens for the probability of winning a customer with a disloyal price.

Price discrimination is illegal Again symmetry allows us to ease notation and to denote as γ_L^N and γ_D^N the probability of each firm winning a customer after observing a loyal and a disloyal signal, respectively, when discrimination is not allowed. Using (22) and (23) and the fact that $y = 0$ one gets that:

$$\gamma_L^N = \frac{1}{2} + \frac{1}{4}b \qquad (24)$$

and

$$\gamma_D^N = \frac{1}{2} - \frac{1}{4}b. \qquad (25)$$

With non discrimination, firms set the price $\frac{1}{2}$ regardless of observing signal α or β. Therefore, the probability of winning a customer with a loyal signal only depends on b and due to the model assumptions the greater is b the greater is that probability. The reverse happens for the probability of winning a consumer with a disloyal signal. Obviously the probability of a firm winning a customer with the non-discriminatory price is always equal to $\frac{1}{2}$. From the comparison between equations (22) and (24) and using the fact $y > 0$, it is easy to see that $\gamma_L^N > \gamma_L$ while $\gamma_D^N < \gamma_D$. Under non-discrimination consumers have no incentive to swap brands, thus $\gamma_L^N > \gamma_L$. Conversely, because each firm tries to attract those customers who prefer the rival's product by offering them a lower price it is obvious that the probability of winning a customer after observing a disloyal signal should be higher than its no-discrimination counterpart.

4.3 Expected number of *Inefficient Shoppers*

When firms are allowed to price discriminate, some consumers might have an incentive to swap to their less preferred brand. It is straightforward to see that the expected number of *inefficient shoppers* (EIS) is equal to:[22]

$$EIS = \frac{1}{2}y + by^2 + \frac{2}{3}b^2y^3. \qquad (26)$$

---

[21] It is easy to check that as $\frac{dy}{db} > 0$ then $\frac{d\gamma_L}{db} > 0$ and $\frac{d\gamma_D}{db} < 0$ for any $b \in \left]0, 1\right]$.

[22] Because the model abstracts from any previous competition, it is more convenient to adopt the term inefficient shoppers rather than the usual "switchers" which is more indicate to situations in which consumers choose one brand in one period and a different one in the other period.

The number of customers who buy inefficiently in equilibrium under price discrimination depends on the informativeness of the signal and on the difference between prices, that is y. As expected, when discrimination is not allowed *every* consumer buys his most preferred brand, thereby implying that no consumer buys inefficiently. In contrast, when price discrimination is permitted, more accurate information gives rise to more inefficient shopping in equilibrium. (When $b > 0$ it follows that $\frac{dEIS}{db} > 0$.) We have seen that as information's precision improves the difference between p_L and p_D is higher. As a result of that, those customers with a smaller loyalty degree— i.e. those located in the middle—will have more incentives to buy the wrong brand. When the signal reaches its maximum level of accuracy, i.e. when $b = 1$, approximately 12.5% of customers buy inefficiently, meaning that firms can attract some consumers that prefer the rival's brand.

4.4 Profits

This section investigates how profits respond to information's accuracy improvements. Let $E\pi_L$ and $E\pi_D$ represent, respectively, each firm expected profit from a loyal and a disloyal signal.

$$E\pi_L = p_L \left(\frac{1}{2} + \frac{1}{4}b - \frac{1}{2}y - by^2 - \frac{2}{3}b^2y^3 \right), \tag{27}$$

$$E\pi_D = p_D \left(\frac{1}{2} - \frac{1}{4}b + \frac{1}{2}y + by^2 + \frac{2}{3}b^2y^3 \right). \tag{28}$$

Given that $\Pr(s_i = \alpha) = \Pr(s_i = \beta) = \frac{1}{2}$ each firm expected aggregate profit, denoted by $E\Pi$ is equal to

$$E\Pi = \frac{1}{2} \left(E\pi_L + E\pi_D \right). \tag{29}$$

Corollary 3. *(i) The profit from a loyal signal exhibits an inverted U-shaped relationship with the private signal's accuracy. Conversely, the profit from a disloyal signal falls monotonically as the signal's accuracy improves.*
(ii) Expected profit with private information is always below the non-discrimination profit and falls monotonically as the accuracy of the private signal rises.
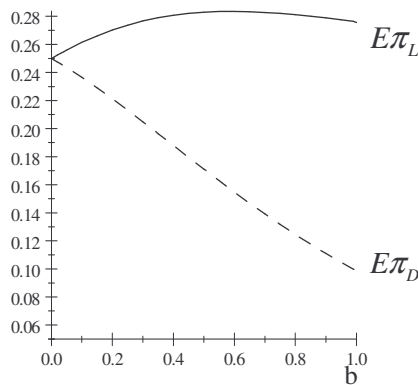

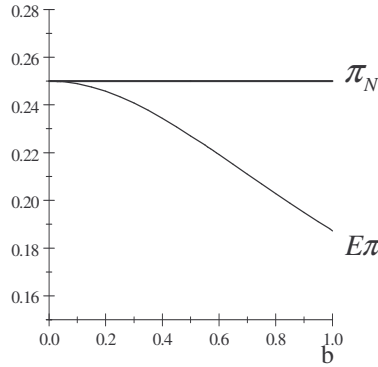Figure 3: Expected equilibrium profits

Figure 4: Expected aggregated profit

Figure 3 illustrates the firm's profit conditional on a loyal and a disloyal signal and the precision of the private signal. Obviously, the relationship between the profit from customers recognised as loyal (disloyal) and the accuracy of the private information is strongly related to the relationship between the price conditional on a loyal (disloyal) signal and the accuracy of the signal. We have seen before that when a firm's signal is not very accurate—i.e. b approximately below 0.5—a firm is able to extract more surplus from customers recognised as loyal which clearly is good for profits. However, when the signal becomes increasingly more accurate firms compete more aggressively which leads to lower prices and profits. Similarly, because the price charged to customers perceived as disloyal always falls as the accuracy of information improves, the same happens to profits from perceived disloyal customers.

Look next at expected aggregate profit as firms rely on more accurate information. Figure 4 shows that equilibrium expected aggregate profit (and so industry profit) *falls monotonically* as the accuracy of the private signal rises and it is always below the non-discrimination level. This is true even when b is low and firms can increase the price to consumers recognised as loyal. Although firms can charge higher prices to consumers recognised as loyal when the signal is not too precise, the probability of winning a customer with a loyal price is smaller. Thus, the expected surplus extraction benefit is not enough to overcome the reduction in profits from a lower price being charged to a consumer recognised as disloyal. Because more accurate information gives rise to more aggressive pricing, the model shows that firm and industry profit increase when price discrimination is based on highly inaccurate information.

It is further worth stressing that this paper's findings are also different from the predictions in Chen, et al. (2001). In their model there are only three types of consumers, each firm has a captive segment and they only compete for switchers. They find that equilibrium profits have an inverse U-shaped relation with information accuracy. They show that both firms might benefit from price discrimination based on more accurate information (which they designate as targetability) when the level of information precision is low. Specifically, they show that when firms depart from low levels of targetability, profits will increase as targetability improves. The intuition for their result is as follows. When targetability is not too high, improvements in it allow firms to extract more surplus from their captive customers as they are increasingly able to identify them. At the same time, because targetability remains at a relatively low level, firms can not fully separate switchers from captive customers which clearly softens price competition

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for the switchers. Thus, while in their model competing firms may all benefit from more accurate information, here this is never the case.

Liu and Serfes (2004) also find that equilibrium profits are always below the nondiscrimination counterparts. However, in contrast to our results, in their model equilibrium profits exhibit a U-shape as a function of information accuracy. They find that when information is of low quality firms segment the Hotelling line into a small number of intervals and so the competition effect dominates the surplus extraction effect. In contrast, as the partition becomes more refined firms are more certain about the location of each consumer and the surplus extraction effect becomes the dominant one. Profits increase.

Corollary 3 proves that as in the previous literature firms face a prisoner's dilemma. However, in this paper the information technology available for each firm acts as a restriction to more aggressive price discrimination, which of course is good for profits. Notice that when the information technology is fully uninformative there is a credible kind of commitment to uniform pricing, which benefits all competing firms. Thus, the current analysis and earlier work put forward that competing firms could all benefit from regulatory policies protecting consumer privacy, which would limit the firms' ability to recognise customers and thereby to set discriminatory prices in a more aggressive competitive context.

5   Welfare analysis

This section investigates the welfare effects of price discrimination based on more accurate information. As usual expected overall welfare is defined as the sum of expected industry profit and expected consumer surplus (ECS). Hence, $EW = 2E\Pi + ECS$.

Lemma 2.   *Expected consumer surplus is given by:*

$$ECS = v - p_L\gamma_L - p_D\gamma_D - \frac{1}{4}y^2 - \frac{2}{3}by^3 - \frac{1}{2}b^2y^4. \tag{30}$$

Proof.   See the Appendix.

As expected, when $b = 0$, ECS is equal to consumer surplus with non-discrimination, that is $v - p_N$.

Corollary 4. (Consumer welfare)
(i) As a whole consumer surplus with discrimination is above the non-discrimination level, and it increases monotonically as the accuracy of the private signal increases.
(ii) Each individual consumer is increasingly better off as price discrimination is based on more accurate information.

In order to evaluate the welfare effects of improvements in the private signal's accuracy, I assume, without any loss of generality, that $v = 2$. Figure 5 shows expected consumer surplus as the accuracy of the private signal rises. Thus, it confirms part (i) of corollary 4. Likewise Figure 6 confirms part (ii). It shows expected consumer surplus for each loyalty degree and for a fixed level of the signal's accuracy.
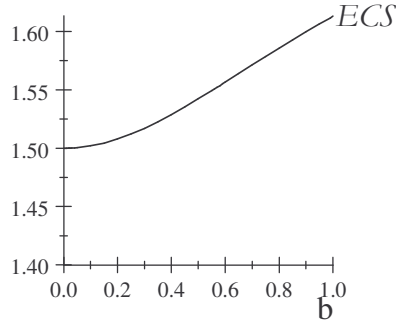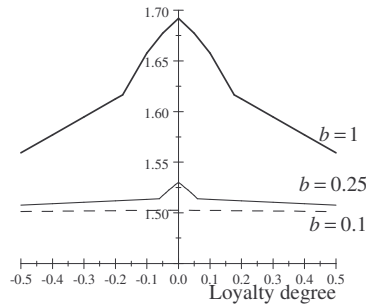
Figure 5: Expected consumer surplus

Figure 6: ECS per consumer

When discrimination is not allowed (or when $b = 0$) expected consumer surplus per consumer is constant, regardless the loyalty degree l. When $v = 2$ expected consumer surplus per individual consumer equals 1.5. Although ECS remains approximately at the same level for tiny increases in b, when the signal becomes increasingly informative we observe that ECS increases with b. We have seen that as the signal becomes more informative price competition on a segment basis is more intense and thereupon prices fall. As a result, consumers can buy at better deals.

Despite consumers as a whole are unequivocally better off with price discrimination, we have seen that for not too accurate signals ($b \lesssim 0.5$) some consumers are expected to face a higher price than under non-discrimination. At a first glance this could suggest that not all consumers would benefit when price discrimination is based on low quality information. Nevertheless, Figure 5 shows that even in the range where loyal customers are expected to pay a higher price with discrimination, expected consumer surplus is above its non-discrimination counterpart. Notice that p_L reaches its maximum value when b is approximately equal to 0.25 and even in this case ECS for the most loyal customers i.e., with $l = \left\{-\frac{1}{2}, \frac{1}{2}\right\}$ is higher than ECS under non-discrimination. The intuition is as follows. Although for small levels of the signal's accuracy loyal customers are expected to pay higher prices, it is also true that with some positive probability they will be misrecognised (i.e. they will be classified as a disloyal consumer) and so they will have a chance to buy the product at the lowest price p_D. Though a firm tends to charge higher prices to perceived loyal customers when b is low, it is also true that it is more likely that the firm mistakenly recognises a true loyal customer as a disloyal when b

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is low. As a result, expected consumer surplus is always above the non-discrimination level and *increases monotonically* with information improvements.

Look now at the expected overall welfare. Using (29) and (30) it ensues that:

$$
\begin{aligned}
EW &= p_L\gamma_L + p_D\gamma_D + \left(v - p_L\gamma_L - p_D\gamma_D - \frac{1}{4}y^2 - \frac{2}{3}by^3 - \frac{1}{2}b^2y^4\right) \qquad (31) \\
&= v - \left(\frac{1}{4}y^2 + \frac{2}{3}by^3 + \frac{1}{2}b^2y^4\right).
\end{aligned}
$$

Thus, we may establish the following proposition.

Proposition 3. *Price discrimination is always good for consumers although bad for profits and overall welfare. Further, consumers are increasingly better off, while firms and welfare are increasingly worse off as price discrimination is based on more accurate private signals.*

Equation (31) shows that welfare is equal to v minus the disutility incurred by those consumers who buy inefficiently. As y increases monotonically as the private signal rises, the higher is b the higher is $\left(\frac{1}{4}y^2 + \frac{2}{3}by^3 + \frac{1}{2}b^2y^4\right)$, so the lower is welfare. Because with non-discrimination aggregate welfare equals v, it is clear-cut that with discrimination expected welfare is always below the non-discrimination level. As firms become increasingly able to recognise customers, and to segment them more accurately, the stronger is the damage of price discrimination on welfare. In fact, welfare reaches its minimum value when the signal's accuracy reaches its maximum level. The reason is that the expected number of consumers who buy inefficiently in equilibrium reaches its maximum value when $b = 1$. Because in the present model there is no role for price discrimination to increase aggregate output, variations in welfare are uniquely explained by the "disutility" supported by those consumers who do not buy the most preferred brand. In this paper, information improvements give rise to an increasing number of inefficient shoppers which clearly is not good for welfare.[23]

At this stage it is useful to discuss the main differences between our predictions and the predictions in Liu and Serfes (2004) and Chen, et al. (2001).

As in the first part of corollary 4, Liu and Serfes (2004) also predict that consumer surplus is always above its no discrimination counterpart. However, while here consumer surplus increases monotonically as the quality of information increases, they predict that consumer surplus exhibits an inverse U-relationship with the information quality, meaning that moderate information quality is the most beneficial informational context for consumers. After a certain level of information quality (the peak) further increases in quality lead some consumers to start paying higher prices and consumer welfare decreases.

In Chen et al. (2001) better information is the worst scenario for a captive customer who is expected to pay the monopoly price and the best scenario for switchers. As in the present framework, in Chen et al., firms observe a noisy signal about each consumer type (captive or switcher) and lacks the kind of certainty present in Liu and Serfes. For low levels of accuracy

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[23]This result is in contrast with that achieved in Thisse and Vives (1988). The reason is that in their model price discrimination based on perfect information gives rise to no welfare loss since no consumer actually switches in equilibrium. Here because firms can only segment consumers into loyal or disloyal (as in Fudenberg and Tirole (2000)) there are always some consumers who buy inefficiently. Therefore, here price discrimination based on better information increases the welfare loss.

firms are not certain about who each consumer is, they have less incentives to charge a customer recognised as a switcher a low price because the consumer may turn out to be a captive. As the signal becomes more accurate firms can better distinguish a captive from a switcher. Consequently, as firms become increasingly able to protect their base of captive customers from price reductions, they compete more aggressively for switchers.

Here we have a continuum of consumers, firms have no captive customers which means that each consumer is a market to be contested. As information accuracy improves each firm is more certain about who each consumer is, that is, each firm can better distinguish its weak and its strong market. Firms price more aggressively at the benefit of all consumers.

The present analysis highlights that as information accuracy improves industry profit and welfare fall monotonically, while consumer surplus increases monotonically. This suggests that any advice to a regulatory authority should take into account whether the target is welfare or solely consumer surplus. In those markets that could be reasonably well represented by the features of the current model, restrictions protecting consumer privacy and limiting the efficacy of price discrimination would benefit industry profits and overall welfare but at the expense of consumer welfare.

In sum, taking into account our findings and the findings in Chen et al. (2001) and Serfes and Liu (2004) it follows that a good economic understanding of the profit and welfare effects of information improvements do depend on the way the information improvement is modelled, on what is learned about consumer demand and on the nature of preferences.

6   Price discrimination with public information

This section extends the previous model of imperfect information to the case where both firms observe a noisy public signal about each consumer's brand preference. The signal is "public" in the sense that its actual realization is common knowledge of both firms. This means that although firm A observes s_A and firm B observes s_B each firm knows the signal of each other. Alternatively, this public information case could also fit a situation where both firms observe two public signals about a particular consumer's brand loyalty rather than one.[24]

To ease notation I denote by s the public signal observed by firms where $s = (s_A = k, s_B = r)$ and $(k, r) = \{(\alpha, \alpha), (\beta, \beta), (\alpha, \beta), (\beta, \alpha)\}$. Again, suppose that α informs that the consumer is loyal to firm A while β informs that the consumer is loyal to firm B. In this scenario firms may observe a *biased* signal in favour of one firm, that is two signals informing that the consumer is either loyal to firm A or loyal to firm B (i.e. (α, α) or (β, β)); or, alternatively, firms may observe a *non-biased* signal meaning that one of the signals reveals that the consumer prefers A, while the other reveals that the consumer prefers B (i.e. (α, β) or (β, α)). Since now signals are public firms only need to update their beliefs about each consumer's brand loyalty degree after observing signal s. Thus, using our previous computations $g_{rk} = \Pr(l \mid s = (r, k))$ is now the density function of l conditional on the public signal observed by firms.

6.1   Equilibrium analysis

In the public information framework, after the public signal has been observed, each firm may classify a particular consumer into three different segments. The consumer can be recognised as

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[24]It can fit as well the case where a firm exogenously shares its customer information with its competitor. Perhaps because both firms purchase customer data from the same marketing company.

(i) a strong loyal customer, (ii) a strong disloyal customer and (iii) a non-biased consumer. Under price discrimination each firm tailors a different price for each different type of signal. Thus, if price discrimination is permitted, firm i chooses simultaneously $p_s^i \in \left\{ p_{\alpha\alpha}^i, p_{\beta\beta}^i, p_{\alpha\beta}^i, p_{\beta\alpha}^i \right\}$, $i = A, B$. These prices are the solution to the following problem

$$\underset{p_s^i \geq 0}{Max} E\left[\pi^i \mid s\right],\tag{32}$$

where, for instance for firm A, one gets

$$
\begin{aligned}
E\left[\pi^A \mid s = (\alpha, \alpha)\right] &= p_{\alpha\alpha}^A \Pr\left(p_{\alpha\alpha}^A < p_{\alpha\alpha}^B + l \mid s = (\alpha, \alpha)\right) \\
&= p_{\alpha\alpha}^A \left[1 - G_{\alpha\alpha}\left(p_{\alpha\alpha}^A - p_{\alpha\alpha}^B\right)\right],
\end{aligned}
$$

$$
\begin{aligned}
E\left[\pi^A \mid s = (\beta, \beta)\right] &= p_{\beta\beta}^A \Pr\left(p_{\beta\beta}^A < p_{\beta\beta}^B + l \mid s = (\beta, \beta)\right) \\
&= p_{\beta\beta}^A \left[1 - G_{\beta\beta}\left(p_{\beta\beta}^A - p_{\beta\beta}^B\right)\right],
\end{aligned}
$$

$$
\begin{aligned}
E\left[\pi^A \mid s = (\alpha, \beta)\right] &= p_{\alpha\beta}^A \Pr\left(p_{\alpha\beta}^A < p_{\alpha\beta}^B + l \mid s = (\alpha, \beta)\right) \\
&= p_{\alpha\beta}^A \left[1 - G_{\alpha\beta}\left(p_{\beta\beta}^A - p_{\beta\beta}^B\right)\right],
\end{aligned}
$$

$$
\begin{aligned}
E\left[\pi^A \mid s = (\beta, \alpha)\right] &= p_{\beta\alpha}^A \Pr\left(p_{\beta\alpha}^A < p_{\beta\alpha}^B + l \mid s = (\beta, \alpha)\right) \\
&= p_{\beta\alpha}^A \left[1 - G_{\alpha\beta}\left(p_{\beta\alpha}^A - p_{\beta\alpha}^B\right)\right].
\end{aligned}
$$

Symmetric expressions hold for firm B. Due to symmetry, in a Bayesian Nash equilibrium it follows that $p_{\alpha\alpha}^A = p_{\beta\beta}^B$, $p_{\beta\beta}^A = p_{\alpha\alpha}^B$ and $p_{\alpha\beta}^A = p_{\alpha\beta}^B = p_{\beta\alpha}^A = p_{\beta\alpha}^B$. Following the same reasoning as before let p_L^{pub} and p_D^{pub} be, respectively, the equilibrium prices firms charge after observing a double loyal and a double disloyal signal, in the public information context. Likewise let p_{NB}^{pub} be the price firms charge after observing a non-biased signal. Again, $p_L^{pub} = p_{\alpha\alpha}^A = p_{\beta\beta}^B$ and $p_D^{pub} = p_{\beta\beta}^A = p_{\alpha\alpha}^B$. After some algebra it is possible to establish the following propositions.

Proposition 4. *When information is public, the BNE in prices is given by*

$$p_L^{pub} = \frac{\frac{1}{8} + \frac{1}{8}b + \frac{1}{24}b^2 - \frac{1}{4}z - \frac{1}{2}bz^2 - \frac{1}{3}b^2z^3}{\left(\frac{1}{2} + bz\right)^2},\tag{33}$$

$$p_D^{pub} = \frac{\frac{1}{8} - \frac{1}{8}b + \frac{1}{24}b^2 + \frac{1}{4}z + \frac{1}{2}bz^2 + \frac{1}{3}b^2z^3}{\left(\frac{1}{2} + bz\right)^2},\tag{34}$$

where $z = p_L^{pub} - p_D^{pub}$; and

$$p_{NB}^{pub} = \frac{1}{2} - \frac{1}{6}b^2.\tag{35}$$

Proof. See the Appendix.

Proposition 5. *(i) When the public signal has no informational content (i.e. $b = 0$), $p_L^{pub} = p_D^{pub} = p_{NB}^{pub} = \frac{1}{2}$.*

(ii) For any informative public signal, the price charged to customers perceived as loyal is always higher than the price charged to customers perceived as disloyal. That is, $p_L^{pub} > p_D^{pub}$.

(iii) As the public signal becomes increasingly more precise, the higher is the difference between p_L^{pub} and p_D^{pub}.

(iv) The more accurate is the public signal, the lower is the price charged to non-biased consumers.

(v) For any level of the public signal's informativeness, $p_L^{pub} > p_{NB}^{pub} > p_D^{pub}$.

Part (i) can be easily proved using (33), (34) and (35) and making $b = 0$. The same happens to part (iv) if we look at (35). Part (iii) is proved by plotting $z = p_L^{pub} - p_D^{pub}$ as an implicit function of b, where $b \in [0, 1]$. This is done in Figure 7. The reader can observe that z is positive and monotonically increases with b. Intuition suggests that when firms are more certain about the loyalty of a particular consumer they have more incentives to charge that consumer a higher price. The same reasoning applies when firms observe a double disloyal signal. Obviously when firms observe a non-biased signal it is equally likely that the consumer can turn out to be a truly loyal or disloyal. So, the price charged to a consumer with that type of signal should be below p_L^{pub} but above p_D^{pub}.

Under public information, expected equilibrium profits conditional on each signal are equal to:

$$E\pi_L^{pub} = \frac{1}{\lambda_{\alpha\alpha}} \left(p_L^{pub} \right) \left(\frac{1}{8} + \frac{1}{8}b + \frac{1}{24}b^2 - \frac{1}{4}z - \frac{1}{2}bz^2 - \frac{1}{3}b^2 z^3 \right), \tag{36}$$

$$E\pi_D^{pub} = \frac{1}{\lambda_{\beta\beta}} \left(p_D^{pub} \right) \left(\frac{1}{8} - \frac{1}{8}b + \frac{1}{24}b^2 + \frac{1}{4}z + \frac{1}{2}bz^2 + \frac{1}{3}b^2 z^3 \right), \tag{37}$$

$$E\pi_{NB}^{pub} = \frac{2}{\lambda_{\alpha\beta}} \left(p_{NB}^{pub} \right) \left(\frac{1}{8} - \frac{1}{24}b^2 \right) = p_{NB}^{pub} \left(\frac{1}{4} - \frac{1}{12}b^2 \right). \tag{38}$$

Due to symmetry each firm's expected aggregated profit under public information denoted $E\Pi^{pub}$ equals:

$$E\Pi^{pub} = E\pi_L^{pub} \Pr(\text{double loyal}) + E\pi_D^{pub} \Pr(\text{double disloyal}) + E\pi_{NB}^{pub} \Pr(\text{non-biased}).$$

7 Private versus public information: a comparative analysis

This section aims to examine whether firms, consumers (as a whole) and welfare benefit from price discrimination based on public rather than on private information. In this regard, in what follows I compare equilibrium prices, profits, consumer surplus and welfare attained with private and public information.

7.1 Equilibrium prices

Figure 7 plots the difference between the highest and the lowest price under private information—namely, $y = p_L - p_D$—and under public information—namely, $z = p_L^{pub} - p_D^{pub}$. If price dispersion is measured by the range of prices, it can be said that the level of price dispersion is greater
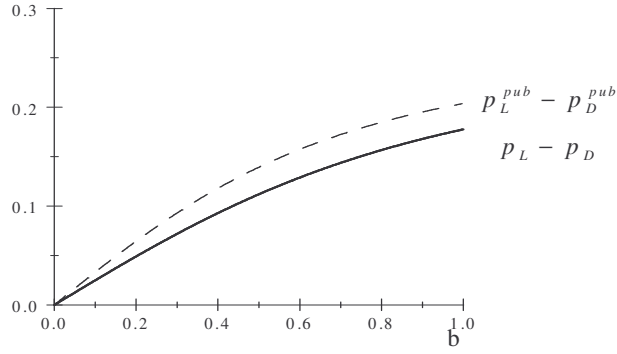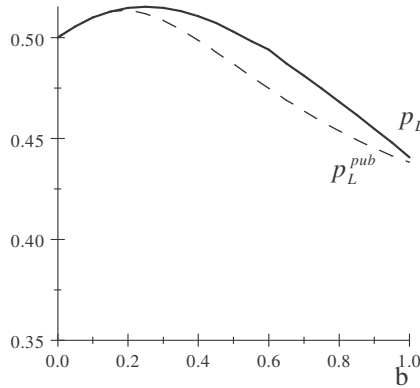
Figure 7: Comparison between y and z

Figure 8: Price to loyal customers

with public than with private information, greater as firms rely on more accurate information about consumer brand preferences.

Figure 8 illustrates the relationship between the price charged to customers recognised as loyal under private information (p_L) and under public information (p_L^{pub}).

Like in the private information setting, under public information there is a non-monotonic relationship between the quality of information and the price charged to a consumer generating a loyal signal. Note however that with public information misrecognition of consumers only occurs when both firms observe a wrong signal. For sufficiently low levels of information's precision, as the probability of misrecognition decreases the "surplus extraction effect" increases, allowing firms to charge more to consumers recognised as loyal. (This explains why the price to a loyal consumer increases as the quality of information improves when $b \lesssim 0.2$.) However, as information becomes more and more accurate, firms are more likely to receive a correct signal, the role of the "misrecognition effect" is increasingly smaller leading firms to compete more aggressively in prices. The "business stealing effect" becomes larger explaining why the price to a customer recognised as loyal is a decreasing function of the quality of information when $b \gtrsim 0.2$. In this way, as the signal is increasingly more precise (i.e. when $b \gtrsim 0.4$) the "business stealing effect" dominates the "surplus extraction effect" and the price to a loyal consumer is
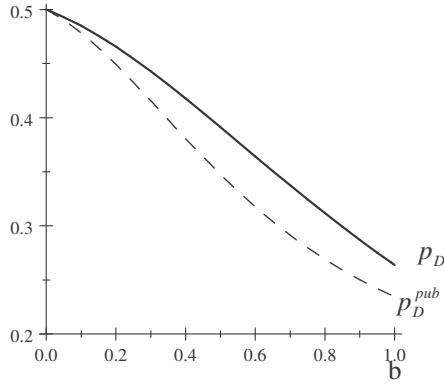
23

Figure 9: Price to disloyal customers

below its non-discrimination counterpart.

Figure 9 illustrates the relationship between the price to customers recognised as disloyal under private information (p_D) and under public information (p_D^{pub}).

Numerical analysis allows us to establish the following corollary.

Corollary 5. *(i) Moving from private to public information reduces the price to customers perceived as disloyal.*

(ii) When the signal's accuracy is not too low (i.e. $b \gtrsim 0.2$) the price to a customer perceived as loyal is higher under private than under public information.

This result claims that for sufficiently accurate signals (i.e. $b \gtrsim 0.2$) the prices to loyal and disloyal consumers are lower under public than under private information. Although the current analysis offers no clear cut results with respect to the comparison between the price to a loyal consumer in both information regimes when the signal's accuracy is too low, it shows that as information's accuracy becomes increasingly higher consumers perceived as loyal are expected to pay higher prices under private than under public information. Numerical analysis shows that while under private information the price to a loyal consumer reaches the no-discrimination level for $b \cong 0.5$ under public information this happens for $b \cong 0.4$. Similarly, we find that whilst under private information the price to a loyal consumer reaches its maximum value approximately at 0.5154 for $b \cong 0.25$ under public information the price to a loyal consumer reaches its maximum value approximately at 0.5138 for $b \cong 0.2$.

In the private information setting firms are uncertain about each consumer's type and also about the rival's private information. The latter type of uncertainty disappears in the public information framework. Whilst under private information a given consumer may be recognised differently by both firms, under public information this is no longer the case because firms always classify consumers in the same way. As a consequence, at least for sufficiently accurate signals, firms will compete more aggressively when they have access to same piece of information, resulting in lower prices under public information.
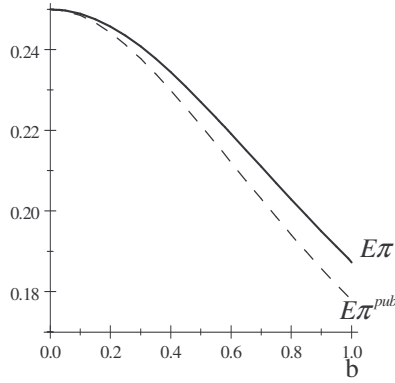
24

Figure 10: Equilibrium Profits

7.2 Expected profits

Since firms will play more aggressively when they have access to the same piece of information about consumers the next result ensues.

Corollary 6. *Firms are better off when price discrimination is based on private information rather than on public information.*

Figure 10 shows that expected profit with public information is always below its private information counterpart. In both cases firms are worse off with information than with no information (no discrimination).

The model suggests that firms are better off when they possess private information about brand preferences for price discrimination purposes. This may be the result of firms purchasing customer data from different marketing companies or buying different information packages from the same marketing company. In some way this suggests that under symmetry firms would have no incentives to share their information with their competitors. Liu and Serfes (2006) develop a two-period model with two firms who produce horizontally and vertically differentiated products to investigate in which circumstances will firms have an incentive to share their information about customers. In their model the information is about the consumers brand preferences and allows the firm who possesses it to price discriminate. They show when firms are symmetric, in a purely horizontally differentiation model neither firm has incentives to share its information with its rival. Only when firms are sufficiently asymmetric will firms share their information in equilibrium. Thus the present analysis suggests that any regulatory policy restricting firms to disclose private information about their customers to rival firms would benefit firms' profitability at the expense of consumer welfare.

7.3 Expected number of *inefficient shoppers*

Lemma 3 The expected number of *inefficient shoppers* under public information denoted EIS^{pub} is:

$$EIS^{pub} = \left(\gamma_L^{N,pub} - \gamma_L^{pub}\right) = \frac{1}{2}z + bz^2 + \frac{2}{3}b^2z^3. \tag{39}$$

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Proof. See the Appendix.

Corollary 7. *The number of customers buying the wrong brand is higher under public than under private information and increases monotonically with information accuracy improvements.*

Using (26) and (39) and the fact that when $b > 0$, $z, y > 0$ and $z > y$ it follows that

$$EIS^{pub} - EIS = \frac{1}{2}(z - y) + b\left(z^2 - y^2\right) + \frac{2}{3}b^2\left(z^3 - y^3\right) > 0.$$

Because price discrimination leads to more intense competition and to lower prices to consumers recognised as disloyal in the public information case, it is obvious that those consumers that are not extremely loyal to one of the brands (i.e. those located in the middle) will have more incentives to buy the wrong brand under public information. On the other hand, the groups of consumers that generate inefficiency in both information frameworks are those who generate two identical signals as in this case they will receive a different price from the two firms. (Notice that consumers who generate different signals will always buy efficiently under public and under private information because they will receive the same price from the two firms.) So it is clear that more consumers buy inefficiency when firms observe the same piece of information.

7.4 Welfare

Lemma 4. Expected consumer surplus with public information denoted ECS^{pub} equals:

$$ECS^{pub} = v - p_{NB}^{pub}\gamma_{NB}^{pub} - p_L^{pub}\gamma_L^{pub} - p_D^{pub}\gamma_D^{pub} - \frac{1}{4}z^2 - \frac{2}{3}bz^3 - \frac{1}{2}b^2z^4.$$

Proof. See the Appendix.

Lemma 5. Expected overall welfare with public information denoted EW^{pub} is equal to:

$$EW^{pub} = v - EIS^{pub} = v - \frac{1}{4}z^2 - \frac{2}{3}bz^3 - \frac{1}{2}b^2z^4. \tag{40}$$

Proposition 6. *For any level of the signal's accuracy, moving from public to private information boosts industry profit and overall welfare and lowers consumer surplus.*

Proof. See the Appendix.

Figure 11 plots expected consumer surplus with private (ECS) and with public information (ECS^{pub}), for the case where $v = 2$.

To summarize, the present analysis offers some criterion to assess *how* welfare evolves as price discrimination is based on more precise information as well as on private rather than on public information. It predicts that consumer welfare increases monotonically with information improvements and is higher under public than under private information. The reverse happens to industry profit. Paradoxically, it predicts that overall welfare is unambiguously greater when price discrimination is based on private and imperfect information. Obviously, we need to be extremely careful in drawing any public policy in models with unit demand assumptions. If aggregate output increased with either more accurate information or when moving from private to public information, then the effects of such changes would need to be taken into account.
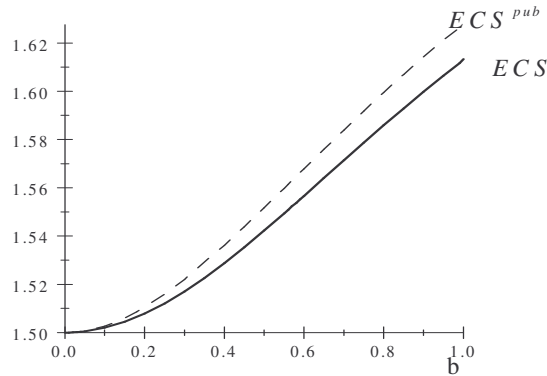
Figure 11: Expected Consumer Surplus

8   Conclusions

This paper has tried to provide a more complete picture of the prices, profits and welfare effects of price discrimination as information technologies gradually improve the firms' ability to recognise the consumers' types. The main contribution was to extend the previous literature by allowing firms to price discriminate on the basis of imperfect and private information. We have seen that while imperfect information tends to lead firms to misrecognise customers, private information gives rise to some uncertainty about the rival's information for price discrimination. Besides a customer may be wrongly recognised by some firm, he can also be recognised in a different way by the competing firms.

In the private information setting, it was shown that customers recognised as loyal pay *always* a higher price than those recognised as disloyal. This result is in consonance with the practice of charging more to old than to first-time customers. It was also shown that customers recognised as loyal are expected to pay prices above the non-discrimination level when information is not too accurate while the reverse happens when information becomes more precise. By contrast, it was found that customers recognised as disloyal are expected to pay lower prices under price discrimination, and increasingly lower prices as the accuracy of the private signal rises.

By proposing a different way of modeling information improvements, it was shown that the equilibrium outcomes may differ from those in Chen et al. (2001) and Liu and Serfes (2004). It was shown that the availability of the private signal (and so price discrimination) benefits consumers, but is bad for industry profit and welfare. More importantly, we found that consumer welfare increases monotonically as the accuracy of the private signal rises, while the reverse happens to industry profit and overall welfare. The model suggests that any public policy protecting consumer privacy, by restricting firms to recognise customers more accurately, would benefit all competing firms at the expense of consumer welfare.

This paper has also analysed price discrimination with public and imperfect information. Extending the model in this direction has proved to be helpful to understand whether or not firms would have an incentive to share their private information with their rivals. It was shown that as a whole consumers are better off with public than with private information. In contrast, industry profit and welfare fall when moving from private to public information.

Taking into account the findings in Chen et al. (2001) and Serfes and Liu (2004) this paper

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highlighted that a good economic understanding of the profit and welfare effects of price discrimination with information improvements do depend on the way the information improvement is modelled, on what is learned about consumer demand, and on the nature of preferences.

In light of the above, this paper has tried to contribute to the ongoing debate on the economic implications of price discrimination with customer recognition. Notwithstanding the model addressed in this paper is far from covering all complex aspects of real markets, it has tried to offer a closer approximation of reality where the quality of consumer-specific information that firms have been using to implement their pricing strategies is increasingly improving thanks to advances in information technologies.

Appendix

This appendix collects the proofs and computations that were omitted from the text.

Proof of Lemma 1. If $\rho_\alpha \geqslant \rho_\beta$ one must observe that:

$$2\lambda_{\alpha\alpha} \geq 2\lambda_{\beta\alpha} \text{ or } \frac{1}{4} + \frac{b^2}{12} \geq \frac{1}{4} - \frac{b^2}{12}$$

which is true $\forall b \in [0,1]$. *Q.E.D.*

Proof of Proposition 1. From the first-order conditions for both firms we obtain:

$$p_\alpha^A = \frac{\rho_\alpha \left[1 - G_{\alpha\alpha}\left(p_\alpha^A - p_\alpha^B\right)\right] + (1-\rho_\alpha)\left[1 - G_{\alpha\beta}\left(p_\alpha^A - p_\beta^B\right)\right]}{\rho_\alpha g_{\alpha\alpha}\left(p_\alpha^A - p_\alpha^B\right) + (1-\rho_\alpha) g_{\alpha\beta}\left(p_\alpha^A - p_\beta^B\right)}, \tag{41}$$

$$p_\beta^A = \frac{\rho_\beta \left[1 - G_{\beta\alpha}\left(p_\beta^A - p_\alpha^B\right)\right] + (1-\rho_\beta)\left[1 - G_{\beta\beta}\left(p_\beta^A - p_\beta^B\right)\right]}{\left[\rho_\beta g_{\beta\alpha}\left(p_\beta^A - p_\alpha^B\right) + (1-\rho_\beta) g_{\beta\beta}\left(p_\beta^A - p_\beta^B\right)\right]}. \tag{42}$$

$$p_\alpha^B = \frac{\rho_\alpha G_{\alpha\alpha}\left(p_\alpha^A - p_\alpha^B\right) + (1-\rho_\alpha) G_{\beta\alpha}\left(p_\beta^A - p_\alpha^B\right)}{\rho_\alpha g_{\alpha\alpha}\left(p_\alpha^A - p_\alpha^B\right) + (1-\rho_\alpha) g_{\beta\alpha}\left(p_\beta^A - p_\alpha^B\right)} \tag{43}$$

and

$$p_\beta^B = \frac{\rho_\beta G_{\alpha\beta}\left(p_\alpha^A - p_\beta^B\right) + (1-\rho_\beta) G_{\beta\beta}\left(p_\beta^A - p_\beta^B\right)}{\rho_\beta g_{\alpha\beta}\left(p_\alpha^A - p_\beta^B\right) + (1-\rho_\beta) g_{\beta\beta}\left(p_\beta^A - p_\beta^B\right)}. \tag{44}$$

Considering for instance the perspective of firm A, second-order partial derivatives with respect to both prices are

$$
\begin{aligned}
\frac{\partial^2 E\left(\pi^A \mid s_A = \alpha\right)}{\partial p_\alpha^{A2}} &= -2\rho_\alpha g_{\alpha a}\left(p_\alpha^A - p_\alpha^B\right) - 2\left(1-\rho_\alpha\right) g_{\alpha\beta}\left(p_\alpha^A - p_\beta^B\right) \\
&\quad -p_\alpha^A \rho_\alpha g_{\alpha\alpha}'\left(p_\alpha^A - p_\alpha^B\right) - p_\alpha^A\left(1-\rho_\alpha\right) g_{\alpha\beta}'\left(p_\alpha^A - p_\beta^B\right),
\end{aligned}
$$

$$\frac{\partial^2 E\left(\pi^A \mid s_A = \beta\right)}{\partial p_\beta^{A2}} = -2\rho_\beta g_{\beta\alpha}\left(p_\beta^A - p_\alpha^B\right) - 2\left(1 - \rho_\beta\right) g_{\beta\beta}\left(p_\beta^A - p_\beta^B\right)$$

$$-p_\beta^A \rho_\beta g_{\beta\alpha}'\left(p_\beta^A - p_\alpha^B\right) - p_\beta^A \left(1 - \rho_\beta\right) g_{\beta\beta}'\left(p_\beta^A - p_\beta^B\right)$$

and,

$$\frac{\partial^2 E\left(\pi^A \mid s_A = \alpha\right)}{\partial p_\alpha^A \partial p_\beta^A} = 0.$$

Symmetric conditions hold for firm B. Due to symmetry we are looking for an equilibrium where $p_\alpha^A = p_\beta^B = p_L$ and $p_\beta^A = p_\alpha^B = p_D$. Thus, making $y = p_L - p_D$, and using without loss of generality the perspective of firm A, one gets:

$$p_L = p_A^\alpha = \frac{\rho_\alpha\left[1 - G_{\alpha\alpha}\left(y\right)\right] + \left(1 - \rho_\alpha\right)\left[1 - G_{\alpha\beta}\left(0\right)\right]}{\rho_\alpha g_{\alpha\alpha}\left(y\right) + \left(1 - \rho_\alpha\right) g_{\alpha\beta}\left(0\right)}$$

$$p_D = p_\beta^A = \frac{\rho_\beta\left[1 - G_{\beta\alpha}\left(0\right)\right] + \left(1 - \rho_\beta\right)\left[1 - G_{\beta\beta}\left(-y\right)\right]}{\left[\rho_\beta g_{\beta\alpha}\left(0\right) + \left(1 - \rho_\beta\right) g_{\beta\beta}\left(-y\right)\right]}.$$

Then second-order partial derivatives for firm A are then equal to:

$$\frac{\partial^2 E\left(\pi^A \mid s_A = \alpha\right)}{\partial p_\alpha^{A2}} = -2\rho_\alpha g_{\alpha a}\left(y\right) - 2\left(1 - \rho_\alpha\right) g_{\alpha\beta}\left(0\right) - p_L \rho_\alpha g_{\alpha\alpha}'\left(y\right) - p_L\left(1 - \rho_\alpha\right) g_{\alpha\beta}'\left(0\right)$$

and

$$\frac{\partial^2 E\left(\pi^A \mid s_A = \beta\right)}{\partial p_\beta^{A2}} = -2\rho_\beta g_{\beta\alpha}\left(0\right) - 2\left(1 - \rho_\beta\right) g_{\beta\beta}\left(-y\right) - p_D \rho_\beta g_{\beta\alpha}'\left(0\right) - p_D\left(1 - \rho_\beta\right) g_{\beta\beta}'\left(-y\right).$$

Since,

$$1 - G_{\alpha\alpha}\left(y\right) = \frac{1}{\lambda_{\alpha\alpha}} \int_y^{\frac{1}{2}} \left(\frac{1}{2} + bl\right)^2 dl$$

$$= \frac{1}{\lambda_{\alpha\alpha}} \left(\frac{1}{8} + \frac{1}{8}b + \frac{1}{24}b^2 - \frac{1}{4}y - \frac{1}{2}by^2 - \frac{1}{3}b^2 y^3\right),$$

$$1 - G_{\alpha\beta}\left(0\right) = \frac{1}{\lambda_{\alpha\beta}} \int_0^{\frac{1}{2}} \left(\frac{1}{2} + bl\right)\left(\frac{1}{2} - bl\right) dl$$

$$= \frac{1}{\lambda_{\alpha\beta}} \left(\frac{1}{8} - \frac{1}{24}b^2\right),$$

$$g_{\alpha\alpha}\left(y\right) = \frac{1}{\lambda_{\alpha\alpha}} \left(\frac{1}{2} + by\right)^2,$$

$$g_{\alpha\beta}\left(0\right) = \frac{1}{\lambda_{\alpha\beta}} \left(\frac{1}{4}\right),$$

$$1 - G_{\beta\beta}(-y) = \frac{1}{\lambda_{\beta\beta}} \int_{-y}^{\frac{1}{2}} \left(\frac{1}{2} - bl\right)^2 dl$$

$$= \frac{1}{\lambda_{\beta\beta}} \left(\frac{1}{8} - \frac{1}{8}b + \frac{1}{24}b^2 + \frac{1}{4}y + \frac{1}{2}by^2 + \frac{1}{3}b^2y^3\right),$$

$$1 - G_{\beta\alpha}(0) = \frac{1}{\lambda_{\beta\alpha}} \int_0^{\frac{1}{2}} \left(\frac{1}{4} - b^2 l^2\right) dl$$

$$= \frac{1}{\lambda_{\beta\alpha}} \left(\frac{1}{8} - \frac{1}{24}b^2\right),$$

$$g_{\beta\alpha}(0) = \frac{1}{\lambda_{\beta\alpha}} \left(\frac{1}{4}\right),$$

and

$$g_{\beta\beta}(-y) = \frac{1}{\lambda_{\beta\beta}} \left(\frac{1}{2} + by\right)^2.$$

It is now easy to check that second-order conditions for a maximum are satisfied. Using the expressions derived above and the fact that

$$g'_{\alpha\alpha}(y) = \frac{2b}{\lambda_{\alpha\alpha}} \left(\frac{1}{2} + by\right)$$

$$g'_{\alpha\beta}(0) = g'_{\beta\alpha}(0) = 0$$

$$g'_{\beta\beta}(-y) = -\frac{2b}{\lambda_{\beta\beta}} \left(\frac{1}{2} + by\right)$$

it is straightforward to observe that:

$$\frac{\partial^2 E\left(\pi^A \mid s_A = \alpha\right)}{\partial p_\alpha^{A2}} = -2\rho_\alpha g_{\alpha\alpha}(y) - 2(1 - \rho_\alpha) g_{\alpha\beta}(0) - p_L \rho_\alpha g'_{\alpha\alpha}(y)$$

$$= -4\lambda_{\alpha\alpha} \frac{1}{\lambda_{\alpha\alpha}} \left(\frac{1}{2} + by\right)^2 - 4(\lambda_{\alpha\beta}) \frac{1}{\lambda_{\alpha\beta}} \left(\frac{1}{4}\right) - p_L 2\lambda_{\alpha\alpha} \frac{2b}{\lambda_{\alpha\alpha}} \left(\frac{1}{2} + by\right)$$

$$= -4\left(\frac{1}{2} + by\right)^2 - 1 - 4bp_L \left(\frac{1}{2} + by\right) < 0.$$

and

$$\frac{\partial^2 E\left(\pi^A \mid s_A = \beta\right)}{\partial p_\beta^{A2}} = -2\rho_\beta g_{\beta\alpha}(0) - 2(1 - \rho_\beta) g_{\beta\beta}(-y) - p_D(1 - \rho_\beta) g'_{\beta\beta}(-y)$$

$$= -4\lambda_{\beta\alpha} \frac{1}{\lambda_{\beta\alpha}} \left(\frac{1}{4}\right) - 4\lambda_{\beta\beta} \frac{1}{\lambda_{\beta\beta}} \left(\frac{1}{2} + by\right)^2 - p_D(2\lambda_{\beta\beta}) \left(-\frac{2b}{\lambda_{\beta\beta}} \left(\frac{1}{2} + by\right)\right)$$

$$= -1 - 4\left(\frac{1}{2} + by\right)^2 + 4bp_D \left(\frac{1}{2} + by\right)$$

$$= -1 - 4\left(\frac{1}{2} + by\right) \left(\frac{1}{2} + b(y - p_D)\right).$$

Despite the fact that $b(y - p_D) < 0$ we find that for $0 \le b \le 1$, $0.41 \le \frac{1}{2} + b(y - p_D) \le 0.5$ thus $\frac{\partial^2 E(\pi^A | s_A = \beta)}{\partial p_\beta^{A2}} < 0$. In sum, since $\frac{\partial^2 E(\pi^A | s_A = \alpha)}{\partial p_\alpha^{A2}} < 0$ and $\left(\frac{\partial^2 E(\pi^A | s_A = \alpha)}{\partial p_\alpha^{A2}}\right)\left(\frac{\partial^2 E(\pi^A | s_A = \beta)}{\partial p_\beta^{A2}}\right) - \left(\frac{\partial^2 E(\pi^A | s_A = \alpha)}{\partial p_\alpha^A \partial p_\beta^A}\right)^2 > 0$ it follows that second-order conditions for a maximum are as well satisfied.

After some algebra, we find that the prices firms set after observing a loyal and a disloyal signal are, respectively, as follows:

$$p_L = \frac{\frac{1}{4} + \frac{1}{8}b - \frac{1}{4}y - \frac{1}{2}by^2 - \frac{1}{3}b^2y^3}{\frac{1}{2} + by + b^2y^2} \tag{45}$$

$$p_D = \frac{\frac{1}{4} - \frac{1}{8}b + \frac{1}{4}y + \frac{1}{2}by^2 + \frac{1}{3}b^2y^3}{\frac{1}{2} + by + b^2y^2}. \tag{46}$$

Q.E.D.

Proof of Lemma 2. For each individual consumer with brand loyalty parameter $l \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ each firm observes a binary signal. Representing by $(s_A = i, s_B = j)$ the signal observed by the two firms for a particular consumer, where $i, j \in \{\alpha, \beta\}$, we may analyse four possible scenarios with respective equilibrium prices:

$(s_A = i, s_B = j)$	$\left(p_i^A, p_j^B\right)$	$\Pr(s_A = i, s_B = j \mid l)$
$(s_A = \alpha, s_B = \alpha)$	(p_L, p_D)	$\Pr(s_A = \alpha, s_B = \alpha \mid l) = (q(l))^2$
$(s_A = \alpha, s_B = \beta)$	(p_L, p_L)	$\Pr(s_A = \alpha, s_B = \beta \mid l) = q(l)(1 - q(l))$
$(s_A = \beta, s_B = \alpha)$	(p_D, p_D)	$\Pr(s_A = \beta, s_B = \alpha \mid l) = (1 - q(l))q(l)$
$(s_A = \beta, s_B = \beta)$	(p_D, p_L)	$\Pr(s_A = \beta, s_B = \beta \mid l) = (1 - q(l))^2$

Therefore,

$$
\begin{aligned}
ECS =\ & (v - p_L) \int_{-\frac{1}{2}}^{\frac{1}{2}} (1 - q(l))\, q(l) f(l) dl + (v - p_D) \int_{-\frac{1}{2}}^{\frac{1}{2}} (1 - q(l))\, q(l) f(l) dl \\
& + \int_0^{\frac{1}{2}} \max \{v - p_L, v - p_D - l\} (q(l))^2 f(l) dl \\
& + (v - p_D) \int_{-\frac{1}{2}}^0 (q(l))^2 f(l) dl \\
& \int_{-\frac{1}{2}}^0 \max \{v - p_L, v - p_D + l\} (1 - q(l))^2 f(l) dl \\
& + (v - p_D) \int_0^{\frac{1}{2}} (1 - q(l))^2 f(l) dl.
\end{aligned}
$$

After some algebra and using the fact that $y = p_L - p_D$, one finds that

$$
\begin{aligned}
ECS &= (v - p_L) \left(\int_{-\frac{1}{2}}^{\frac{1}{2}} \left(\frac{1}{2} - bl\right) \left(\frac{1}{2} + bl\right) dl + \int_{y}^{\frac{1}{2}} \left(\frac{1}{2} + bl\right)^2 dl + \int_{-\frac{1}{2}}^{-y} \left(\frac{1}{2} - bl\right)^2 dl \right) \\
&\quad + (v - p_D) \left(\int_{-\frac{1}{2}}^{\frac{1}{2}} \left(\frac{1}{2} - bl\right) \left(\frac{1}{2} + bl\right) dl + \int_{-\frac{1}{2}}^{y} \left(\frac{1}{2} + bl\right)^2 dl + \int_{-y}^{\frac{1}{2}} \left(\frac{1}{2} - bl\right)^2 dl \right) \\
&\quad - \int_{0}^{y} l \left(\frac{1}{2} + bl\right)^2 dl + \int_{-y}^{0} l \left(\frac{1}{2} - bl\right)^2 dl,
\end{aligned}
$$

from which we get

$$
\begin{aligned}
ECS &= v - p_L \left(\frac{1}{2} + \frac{1}{4}b - \frac{1}{2}y - by^2 - \frac{2}{3}b^2 y^3 \right) - p_D \left(\frac{1}{2} - \frac{1}{4}b + \frac{1}{2}y + by^2 + \frac{2}{3}b^2 y^3 \right) \\
&\quad - \frac{1}{4}y^2 - \frac{2}{3}by^3 - \frac{1}{2}b^2 y^4.
\end{aligned}
$$

Q.E.D.

Proof of Proposition 4. Given that firms may observe four different types of signals, upon observing signal $s = (i, j)$, each firm chooses simultaneously a different price for the signal observed. Hence, firm i chooses $p_s^i \in \left\{ p_{\alpha\alpha}^i, p_{\beta\beta}^i, p_{\alpha\beta}^i, p_{\beta\alpha}^i \right\}$, $i = A, B$. These prices are the solution to the following problem

$$
\underset{p_s^i \geq 0}{Max} E \left[\pi^i \mid s \right], \tag{47}
$$

Considering for example, the perspective of firm A, first-order conditions for firm A are:

$$
\frac{\partial E \left(\pi^A \mid s = (\alpha, \alpha) \right)}{\partial p_{\alpha\alpha}^A} = \left[1 - G_{\alpha\alpha} \left(p_{\alpha\alpha}^A - p_{\alpha\alpha}^B \right) \right] - \left(p_{\alpha\alpha}^A \right) g_{\alpha\alpha} \left(p_{\alpha\alpha}^A - p_{\alpha\alpha}^B \right) = 0,
$$

$$
\frac{\partial E \left(\pi^A \mid s = (\beta, \beta) \right)}{\partial p_{\beta\beta}^A} = \left[1 - G_{\beta\beta} \left(p_{\beta\beta}^A - p_{\beta\beta}^B \right) \right] - \left(p_{\beta\beta}^A \right) g_{\beta\beta} \left(p_{\beta\beta}^A - p_{\beta\beta}^B \right) = 0,
$$

$$
\frac{\partial E \left(\pi^A \mid s = (\alpha, \beta) \right)}{\partial p_{\alpha\beta}^A} = \left[1 - G_{\alpha\beta} \left(p_{\alpha\beta}^A - p_{\alpha\beta}^B \right) \right] - \left(p_{\alpha\beta}^A \right) g_{\alpha\beta} \left(p_{\alpha\beta}^A - p_{\alpha\beta}^B \right) = 0,
$$

$$
\frac{\partial E \left(\pi^A \mid s = (\beta, \alpha) \right)}{\partial p_{\beta\alpha}^A} = \left[1 - G_{\beta\alpha} \left(p_{\beta\alpha}^A - p_{\beta\alpha}^B \right) \right] - \left(p_{\beta\alpha}^A \right) g_{\beta\alpha} \left(p_{\beta\alpha}^A - p_{\beta\alpha}^B \right) = 0.
$$

Second-order conditions are as well satisfied. Due to symmetry we are looking for an equilibrium where $p_{\alpha\alpha}^A = p_{\beta\beta}^B$, $p_{\beta\beta}^A = p_{\alpha\alpha}^B$ and $p_{\alpha\beta}^A = p_{\alpha\beta}^B = p_{\beta\alpha}^A = p_{\beta\alpha}^B$. Making $z = p_{\alpha\alpha}^A - p_{\alpha\alpha}^B$ we have

$$
\left[1 - G_{\alpha\alpha} \left(z \right) \right] - \left(p_{\alpha\alpha}^A \right) g_{\alpha\alpha} \left(z \right) = 0
$$

from which it follows:

$$
p_{\alpha\alpha}^A = \frac{1 - G_{\alpha\alpha} \left(z \right)}{g_{\alpha\alpha} \left(z \right)}. \tag{48}
$$

We also have

$$
\left[1 - G_{\beta\beta} \left(-z \right) \right] - \left(p_{\beta\beta}^A \right) g_{\beta\beta} \left(-z \right) = 0
$$

from which we obtain

$$p_{\beta\beta}^A = \frac{1 - G_{\beta\beta}(-z)}{g_{\beta\beta}(-z)}. \tag{49}$$

Finally, from

$$[1 - G_{\alpha\beta}(0)] - \left(p_{\alpha\beta}^A\right) g_{\alpha\beta}(0) = 0$$

we get

$$p_{\alpha\beta}^A = p_{\beta\alpha}^A = \frac{1 - G_{\alpha\beta}(0)}{g_{\alpha\beta}(0)}. \tag{50}$$

As before,

$$1 - G_{\alpha\alpha}(z) = \frac{1}{\lambda_{\alpha\alpha}} \left(\frac{1}{8} + \frac{1}{8}b + \frac{1}{24}b^2 - \frac{1}{4}z - \frac{1}{2}bz^2 - \frac{1}{3}b^2 z^3 \right),$$

$$1 - G_{\alpha\beta}(0) = \frac{1}{\lambda_{\alpha\beta}} \left(\frac{1}{8} - \frac{1}{24}b^2 \right),$$

$$g_{\alpha\alpha}(z) = \frac{1}{\lambda_{\alpha\alpha}} \left(\left(\frac{1}{2} + bz \right)^2 \right),$$

$$g_{\alpha\beta}(0) = \frac{1}{\lambda_{\alpha\beta}} \left(\frac{1}{4} \right),$$

$$1 - G_{\beta\beta}(-z) = \frac{1}{\lambda_{\beta\beta}} \left(\frac{1}{8} - \frac{1}{8}b + \frac{1}{24}b^2 + \frac{1}{4}z + \frac{1}{2}bz^2 + \frac{1}{3}b^2 z^3 \right),$$

$$1 - G_{\beta\alpha}(0) = \frac{1}{\lambda_{\beta\alpha}} \left(\frac{1}{8} - \frac{1}{24}b^2 \right),$$

$$g_{\beta\alpha}(0) = \frac{1}{\lambda_{\beta\alpha}} \left(\frac{1}{4} \right),$$

and

$$g_{\beta\beta}(-z) = \frac{1}{\lambda_{\beta\beta}} \left(\left(\frac{1}{2} + bz \right)^2 \right),$$

Rearranging and using the fact $p_L^{pub} = p_{\alpha\alpha}^A = p_{\beta\beta}^B$ and $p_D^{pub} = p_{\beta\beta}^A = p_{\alpha\alpha}^B$ we find that:

$$p_L^{pub} = \frac{\frac{1}{8} + \frac{1}{8}b + \frac{1}{24}b^2 - \frac{1}{4}z - \frac{1}{2}bz^2 - \frac{1}{3}b^2 z^3}{\left(\frac{1}{2} + bz \right)^2}, \tag{51}$$

$$p_D^{pub} = \frac{\frac{1}{8} - \frac{1}{8}b + \frac{1}{24}b^2 + \frac{1}{4}z + \frac{1}{2}bz^2 + \frac{1}{3}b^2 z^3}{\left(\frac{1}{2} + bz \right)^2}, \tag{52}$$

$$p_{NB}^{pub} = \frac{1}{2} - \frac{1}{6}b^2. \tag{53}$$

Q.E.D.

33

Proof of Lemma 3. Due to the normalization of consumers to unit, the expected number of consumers that buy the product at the highest price under public information (p_L^{pub}) is given by,

$$
\begin{aligned}
\gamma_L^{pub} &= \Pr(A \text{ wins} \mid s = (\alpha, \alpha)) \Pr(s = (\alpha, \alpha)) + \Pr(B \text{ wins} \mid s = (\beta, \beta)) \Pr((s = (\beta, \beta)) \\
&= \frac{1}{4} + \frac{1}{4}b + \frac{1}{12}b^2 - \frac{1}{2}z - bz^2 - \frac{2}{3}b^2 z^3.
\end{aligned}
$$

Similarly, the expected number of consumers that buy the product at the lowest price (p_D^{pub}) is given by

$$
\begin{aligned}
\gamma_D^{pub} &= \Pr(A \text{ wins} \mid s = (\beta, \beta)) \Pr(s = (\beta, \beta)) + \Pr(B \text{ wins} \mid s = (\alpha, \alpha)) \Pr(s = (\alpha, \alpha)) \\
&= \frac{1}{4} - \frac{1}{4}b + \frac{1}{12}b^2 + \frac{1}{2}z + bz^2 + \frac{2}{3}b^2 z^3,
\end{aligned}
$$

and the number of consumers that pay the non-biased price equals

$$
\begin{aligned}
\gamma_{NB}^{pub} &= \Pr(A \text{ wins} \mid s = (\alpha, \beta)) \Pr(s = (\alpha, \beta)) \\
+ \Pr(A \text{ wins} & \mid s = (\beta, \alpha)) \Pr(s = (\beta, \alpha)) + \Pr(B \text{ wins} \mid s = (\alpha, \beta)) \Pr(s = (\alpha, \beta)) \\
+ \Pr(B \text{ wins} & \mid s = (\beta, \alpha)) \Pr(s = (\beta, \alpha)) = \frac{1}{2} - \frac{1}{6}b^2.
\end{aligned}
$$

Under non-discrimination the expected number of consumers that pay p_L^{pub}, p_D^{pub} and p_{NB}^{pub} is

$$
\gamma_L^{N,pub} = \frac{1}{4} + \frac{1}{4}b + \frac{1}{12}b^2,
$$

$$
\gamma_D^{N,pub} = \frac{1}{4} - \frac{1}{4}b + \frac{1}{12}b^2,
$$

$$
\gamma_{NB}^{N,pub} = \frac{1}{2} - \frac{1}{6}b^2.
$$

Thus, the expected number of *inefficient shoppers* under public information denoted by EIS^{pub} is given by:

$$
EIS^{pub} = \left(\gamma_L^{N,pub} - \gamma_L^{pub} \right) = \frac{1}{2}z + bz^2 + \frac{2}{3}b^2 z^3. \ Q.E.D. \tag{54}
$$

Proof of Lemma 4. When the signal observed by firms to each customer is public we may analyse the following scenarios:

$s = (i, j)$	$\left(p_i^A, p_j^B \right)$	$\Pr(s = (i,j) \mid l)$
$s = (\alpha, \alpha)$	$\left(p_L^{pub}, p_D^{pub} \right)$	$\Pr(s = (\alpha, \alpha) \mid l) = (q(l))^2$
$s = (\alpha, \beta)$	$\left(p_{NB}^{pub}, p_{NB}^{pub} \right)$	$\Pr(s = (\alpha, \beta) \mid l) = q(l)(1 - q(l))$
$s = (\beta, \beta)$	$\left(p_D^{pub}, p_L^{pub} \right)$	$\Pr(s = (\beta, \alpha) \mid l) = (1 - q(l))^2$
$s = (\beta, \alpha)$	$\left(p_{NB}^{pub}, p_{NB}^{pub} \right)$	$\Pr(s = (\beta, \beta) \mid l) = (1 - q(l))q(l)$

Therefore, expected consumer surplus under public information, denoted by ECS^{pub} is

$$
\begin{aligned}
ECS \;=\; & \left(v - p_{NB}^{pub}\right) \int_{-\frac{1}{2}}^{\frac{1}{2}} \left(1 - q(l)\right) q(l) f(l) dl + \int_{0}^{\frac{1}{2}} \max\left(v - p_{L}^{pub}, v - p_{D}^{pub} - l\right) \left(q(l)\right)^2 f(l) dl \\
& + \left(v - p_{D}^{pub}\right) \int_{-\frac{1}{2}}^{0} \left(q(l)\right)^2 f(l) dl + \int_{-\frac{1}{2}}^{0} \max\left(v - p_{L}^{pub}, v - p_{D}^{pub} + l\right) \left(1 - q(l)\right)^2 f(l) dl \\
& + \left(v - p_{D}^{pub}\right) \int_{0}^{\frac{1}{2}} \left(1 - q(l)\right)^2 f(l) dl.
\end{aligned}
$$

Using previous computations and using the fact that $z = p_{L}^{pub} - p_{D}^{pub}$, after some algebra:

$$
ECS^{pub} = v - p_{NB}^{pub}\gamma_{NB}^{pub} - p_{L}^{pub}\gamma_{L}^{pub} - p_{D}^{pub}\gamma_{D}^{pub} - \frac{1}{4}z^2 - \frac{2}{3}bz^3 - \frac{1}{2}b^2 z^4. \tag{55}
$$

Q.E.D.

Proof of Proposition 6. We prove analytically that $EW^{pub} < EW$. Using (31) and (40) and the fact that $z > y$ it follows that

$$
EW^{pub} - EW = -\frac{1}{4}\left(z^2 - y^2\right) - \frac{2}{3}b\left(z^3 - y^3\right) - \frac{1}{2}b^2\left(z^4 - y^4\right) < 0.
$$

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