An MPCC approach on a Stackelberg game in an electric power market: changing the leadership

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An electric power market is formulated as a Stackelberg game where two firms, A and B, produce energy. Two distinct situations, according to the firm who plays the leader role, are analysed. In the first one, the firm A is the leader and the firm B is the follower, and in the second situation the players reverse their roles. In order to select the optimal strategy, the leader uses as knowledge his own perception of the market and anticipates the reactions of the other followers. The main goal of this paper is to understand the behaviour of the various agents that compose the electric power network, such as transmissions capacity, quantities of power generated and demanded, when the leadership changes.

The problem is formulated as a mathematical program with complementarity constraints (MPCC) and reformulated into a nonlinear program (NLP), allowing the use of robust NLP solvers. Computational results using Lancelot, Loqo and Snopt solvers are performed. The numerical experiments show that the firm profit is conditioned by the available information.

Keywords: Electric power; Stackelberg game; leadership; MPCC; NLP solver

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1. Introduction

In the last years, the transformations of the electric power market has been a constant. It becomes a liberalized activity, where planning and operation scheduling are independent activities which are not constrained by centralized procedures.

In Europe, the liberalization process is under way in many countries. The market has been faced with fusions and merges between companies. The directives of European Union for an electric power liberality led up to increasing institutional and physical connections between markets from different countries. Some papers about studies in course, related with German, French and The Netherlands power markets have emerged – see [4,7,18] for more details.

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In a competitive environment, the main goal is to benefit the consumers through price reduction. However, ill effects can occur if the level of concentration in the market grows dangerously and the generator companies combine prices between themselves.

According to [13], there are reasons to consider electric power as a special commodity. In fact, all power travels over the same set of power lines, independently of the firm that generated it; this difference is particularly marked when the networks contain loops and there are transmission capacity limits. Besides, the electricity has unique physical properties, namely Kirchhoff voltage and current laws. As the electricity is difficult to store, and the quantity of power must be instantly adjusted to the demand, the companies that lead the market could easily manipulate the price, changing it to higher values, specially in peak consumption periods.

In order to study the interaction of all market participants and to have a better knowledge of the market conditions, firms and governments need suitable decision-support models. The problem that is presented herein is related with the oligopolist market modeled as a Stakelberg game [12]. In this game theory, there is a noncompetitive situation, where one player (the leader) takes as input his own perception of the market and can anticipate the reactions of the other players, using the information in order to select his optimal strategic. The other players (the followers) do not have the perception of how their decisions have influence in leader decisions. Between followers their behaviour acts like a non-cooperative Nash game, where each one observes the actions of the others and no one can increase their own profit through unilateral decisions. In this paper, the transformations of the electric power network, related to quantities of demanded and generated power in each node when the leader company changes, are analysed.

The Stakelberg game theory was a great motivation for the study of the bilevel optimization problems, because there are many similarities between both [8]. However, solving this kind of problems is a hard task in optimization. But, if the bilevel problem is convex in the second-level [19], this level can be replaced by their own Karush–Kuhn–Tucker (KKT) conditions, and the resulting problem is one-level optimization problem with complementarity constraints.

The organization of this paper is as follows. Section 2 introduces the definition of MPCC and its special features; the nonlinear approach of the previous problem as a way of taking advantages of the efficiently and robust NLP solvers is also presented. Section 3 describes two versions of the electric power market problem and also provides the data of the electric network. The computational experiments and corresponding results are reported in Section 4. Finally, some conclusions and future developments are presented in Section 5.

2. MPCC–NLP approach

The interest on MPCC increased in the last decade due to the subjacent equilibrium concept present in many applications (see [3,5,12] for some applications in the last years).

DEFINITION 2.1 (MPCC Problem) *MPCC is defined as*

$$\begin{aligned} \min_{z} \quad & F(z) \\ s.t. \quad & c_i(z) = 0, \ \ i \in E, \\ & c_i(z) \geq 0, \ \ i \in I, \\ & 0 \leq z_1 \perp z_2 \geq 0, \end{aligned} \tag{1}$$

where $z = (z_0, z_1, z_2)$, with the control variable $z_0 \in \mathbb{R}^n$ and the state variables $z_1, z_2 \in \mathbb{R}^p$; F is the objective function, c_i, $i \in E \cup I$ are the equality and inequality constraints, respectively. The sets E and I are the disjoint finite sets of indices. The objective function F and the constraints c_i, $i \in E \cup I$ are assumed twice continuously differentiable. The constraints related to complementarity are defined with the operator $\perp$ and demand that the product of the two nonnegative quantities must be zero, i.e. $z_{1i}z_{2i} = 0$, $i \in \{1, \ldots, p\}$.

The MPCC problem is nonsmooth mostly due to the complementarity constraints. The optimal conditions are complex and very difficult to verify. Besides, the feasible set of MPCC is ill-posed since the constraint qualifications – namely the Mangasarian Fromovitz (MFCQ) and the Linear Independent (LICQ) – which are commonly assumed to prove convergence of standard nonlinear programming do not hold at any feasible point [8,15]. This implies mostly that the multiplier set is unbounded, the active constraint normals are linearly dependent and the linearizations of the MPCC can become inconsistent arbitrarily close to a solution.

The violation of constraint qualifications has led to a number of specific algorithms for MPCCs. In spite of being specially designed to address MPCC problems they do not represent a real solution, since these algorithms still need rather strong assumptions to ensure convergence. On the other side, they also require significative computational effort when compared with nonlinear solvers in the market. The search of new techniques and algorithms in order to solve real problems with large dimension is still an intensive research area.

Recently, some authors proposed to solve MPCC problems by reformulating into an equivalent NLP problem [11]. This formulation allows to take advantage of certain NLP algorithms' features in order to obtain rapid local convergence. Reliability and robustness of NLP solvers can also be acceded by using these problems for numerical testing allowing also to see its performance under some specific problems irregularities.

An MPCC defined in Equation (1) can be reformulated as an equivalent NLP problem of the following form:

DEFINITION 2.2 (NLP formulation of the MPCC problem)

$$\begin{aligned} \min_{z} \quad & F(z) \\ s.t. \quad & c_i(z) = 0, \ \ i \in E, \\ & c_i(z) \geq 0, \ \ i \in I, \\ & z_1 \geq 0, \\ & z_2 \geq 0, \\ & z_1^{\mathrm{T}} z_2 \leq 0. \end{aligned} \tag{2}$$

Recall that the complementarity constraint was replaced by a nonlinear inequality, relaxing the problem. The transformation from an MPCC problem into an NLP problem allows to use some standard NLP solvers.

One can easily show that the reformulated problem has the same properties as the previous one, including the constraint qualifications violation and second-order conditions. However, in the last few years, some studies show that strong stationarity is equivalent to the KKT conditions of the MPCC-NLP problem [16,17]. This fact has advantages because strong stationarity is a useful and practical computation characterization, since it is relatively easy to find a stationarity point in an NLP solver, under reasonable assumptions.

3. The electric power market problem

The problem described in this paper is based on the model proposed in [6]. It is a competitive power market, formulated as an oligopolistic equilibrium model.

There are a number of generator firms, each owing a given number of units. These make an hourly bid to an Independent System Operator (ISO). The ISO, taking into consideration the network, solves a social welfare maximization problem, announces a dispatch for each bidder and possibly charges distinct prices at each node. It decides how much power to buy from generators and how much power to distribute to consumers and what prices to charge. All these decisions are made with the optimal power flow in mind.

The leader generator first decides and takes as input all the perceptions and information that it could have about the market (including predictable bids of the other firms, demand and supply functions) and maximizes its profit inside a set of spatial price equilibrium constraints and Kirchhoff's voltage and current laws. The followers' units make their own decisions taking into account the leader decision.

3.1 *Formulation*

The notation used in the mathematical formulation are as follows.

Indices:

i node in the network
ij arc from node i to node j
m number of Kirchhoff voltage loops in the network

Sets:

$\mathcal{N}$ set of all nodes
$\mathcal{A}$ set of all arcs
$\mathcal{S}_f$ set of generator nodes under control of leader firm f
$\mathcal{P}$ set of all generators nodes
$\mathcal{D}$ set of all demand nodes
$\mathcal{L}$ set of Kirchhoff's voltage loops m
$\mathcal{L}_m$ set of ordered arcs (clockwise) associated with loop m

Recall that a node can be simultaneously a generator and a consumer, so $\mathcal{P}$ and $\mathcal{D}$ are not necessarily disjoint and their union could be a proper subset of $\mathcal{N}$. The uniqueness of the net flow on each arc is ensured by the Kirchhoff's laws in the linearized DC models and, consequently, the number of (independent) loops are $\#\mathcal{A} - \#\mathcal{N} + 1$ (where $\#X$ is the set X cardinality).

Parameters:

a_i, b_i intercept and slope of supply function (marginal cost) for the generator at node $i \in \mathcal{P}$
c_i, d_i intercept and slope of demand function for consumer at node $i \in \mathcal{D}$
$\overline{\alpha}_i$ upper bound of the bid for the unit at node $i \in \mathcal{S}_f$
$\overline{Q}_{S_i}$ upper bound of production capacity for the unit at node $i \in \mathcal{P}$
$\overline{T}_{ij}$ maximum transmission capacity on arc $ij \in \mathcal{A}$
r_{ij} reactance on arc $ij \in \mathcal{A}$
s_{ijm} ± 1 corresponding to the orientation of the arc $ij \in \mathcal{A}$ in loop $m \in \mathcal{L}$ ($+1$ if ij has the same orientation as the loop m)

First-level decision variable

α bid for the unit at node $i \in \mathcal{P}$

In this model, it is assumed that the generator firms can only manipulate α (the intercept in the bid function) and not the slope b, due to market and optimization assumptions.

Let α_i be fixed for the competitive firms (i.e. α_i fixed $\forall i \in P \backslash S_f$) and variables for the leader firms (i.e. α_i variable $\forall i \in S_f$).

Primal variables in the second-level

Q_{S_i} vector defined by quantity of power generated by the unit at node i ($Q_{S_i} = a_i + b_i Q_{S_i}$ if $i \in \mathcal{P}$ and $Q_{S_i} = 0$ if $i \notin \mathcal{P}$)

Q_{D_i} quantity of power demanded at node i ($Q_{D_i} = c_i - d_i Q_{D_i}$ if $i \in \mathcal{D}$ and $Q_{D_i} = 0$ if $i \notin \mathcal{D}$)

T_{ij} matrix defined by MW transmitted from node i to node j

Dual variables in the second-level

λ_i marginal cost at node i

μ_i marginal value of generation capacity for unit at node i

θ_{ij} marginal value of transmission capacity on arc ij

γ_m shadow price for Kirchhoff voltage law for loop m

Next, the second-level convex quadratic problem is defined. The objective function is related to the maximization of social welfare:

$$\max \sum_{i \in \mathcal{D}} \left(c_i Q_{D_i} - \frac{1}{2} d_i Q_{D_i}^2 \right) - \sum_{i \in \mathcal{P}} \left(\alpha_i Q_{S_i} + \frac{1}{2} b_i Q_{S_i}^2 \right). \tag{3}$$

This function relates the maximization profits (firms) to the the maximization of the utility of the product (consumers).

The following constraints report to a spatial price equilibrium plus a constraint due to Kirchhoff voltage law.

- Nonnegative demand variables:

$$Q_{D_i} \geq 0, \quad i \in \mathcal{D}. \tag{4}$$

- Lower and upper bounds for transmission variables:

$$0 \leq T_{ij} \leq \overline{T}_{ij}, \quad ij \in \mathcal{A}. \tag{5}$$

- Minimum and maximum capacity of production:

$$0 \leq Q_{S_i} \leq \overline{Q}_{S_i}, \quad i \in \mathcal{P}. \tag{6}$$

- Conservation constraints:

$$Q_{D_i} - Q_{S_i} + \sum_{j:ij \in \mathcal{A}} T_{ij} - \sum_{j:ji \in \mathcal{A}} \overline{T}_{ij} = 0. \tag{7}$$

- Kirchhoff voltage law:

$$\sum_{ij \in \mathcal{L}_m} s_{ijm} r_{ij} T_{ij} = 0. \tag{8}$$

If in Equation (6), by economic reasons, the minimum production level could not be zero, it is possible to change the lower bound and still use the same model.

The description of the first level of the electric power is complete by taking into account that for the follower firms the bids are already fixed. The determination of the dominant company profit consists of finding a bid vector $\alpha^f \equiv (\alpha_i: \ i \in \mathcal{S}_f)$, a vector of supplies Q_S, a vector of demands Q_D and a vector of transmission capacities T, by solving the following maximization problem.

$$\begin{aligned} \max \quad & \pi_f(\lambda, Q_S) \equiv \sum_{i \in \mathcal{S}_f} \left(\lambda_i Q_{S_i} - a_i Q_{S_i} - \frac{b_i}{2} Q_{S_i}^2 \right) \\ \text{s.t.} \quad & 0 \le \alpha_i \le \overline{\alpha}_i, \quad \forall i \in \mathcal{S}_f, \end{aligned} \tag{9}$$

where Q_S, Q_D and T, for each value of α, are the solution of the second-level problem (3)–(8).

Due to the convexity of the second-level problem, the following property is assured (for more details see [12]):

For each vector α there exists a unique globally optimal solution of the quadratic program (3)–(8), denoted $(Q_D(\alpha), Q_S(\alpha), T(\alpha))$; furthermore, this solution is a piecewise linear function in α.

In order to solve this bilevel problem, the ISO's lower-level optimization problem is replaced by its stationarity conditions giving rise to a system of complementarity constraints.

The maximization of the leader firm profit in the electric power market is described by the following MPCC problem:

$$\begin{aligned} \max \quad & \pi_f(\lambda, Q_S) \equiv \sum_{i \in \mathcal{S}_f} \left(\lambda_i Q_{S_i} - a_i Q_{S_i} - \frac{b_i}{2} Q_{S_i}^2 \right) \\ \text{s.t.} \quad & 0 \le \alpha_i \le \overline{\alpha}_i, \quad \forall i \in \mathcal{S}_f, \\ & 0 \le \overline{Q}_S - Q_S \perp \mu \ge 0, \\ & 0 \le Q_S \perp -\lambda + \mu + \alpha + \operatorname{diag}(b) Q_S \ge 0, \\ & 0 \le Q_D \perp \lambda - c + \operatorname{diag}(d) Q_D \ge 0, \\ & 0 \le \theta \perp \overline{T} - T \ge 0, \\ & 0 \le T \perp \Delta^{\mathrm{T}} \lambda + \theta + R\gamma \ge 0, \\ & Q_D - Q_S + \Delta T = 0, \\ & R^{\mathrm{T}} T = 0, \\ & \lambda \text{ free}, \\ & \gamma \text{ free}, \end{aligned} \tag{10}$$

where Δ is the matrix with the information about the pair (node, arc) in the electric network:

$$\Delta_{il} = \begin{cases} 1 & \text{if}\, l = ij \ \in \mathcal{A} \ \text{ for some } j \in \mathcal{N}, \\ -1 & \text{if}\, l = ji \in \mathcal{A} \ \text{ for some } j \in \mathcal{N}, \\ 0 & \text{other values} \end{cases} \tag{11}$$

and R is the matrix (arc, cycle) related with the reactance coefficients:

$$R_{ij,m} = \begin{cases} s_{ijm} r_{ij} & \text{if}\, ij \in \mathcal{L}_m, \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

The notation $\operatorname{diag}(w)$ represents the diagonal matrix whose diagonal entries are the components of the vector w.

The objective function of Equation (10) is neither convex nor concave due to the bilinear term $\lambda_i Q_{S_i} - a_i Q_{S_i}$. For computational reasons, and exploiting the constraints of the problem, the objective function can be reformulated as

$$\max \Pi_f(\lambda, Q_D, Q_S, T, \theta, \lambda, \mu) \equiv \sum_{i \in D}(c_i Q_{D_i} - d_i Q_{D_i}^2), \\ - \sum_{i \in S_f} \left(a_i Q_{S_i} + \frac{b_i}{2} Q_{S_i}^2 \right) - \sum_{ij \in A} \theta ij \overline{T}_{ij} \\ - \sum_{i \in P \setminus S_f} (\mu_i \overline{Q}_{S_i} + a_i Q_{S_i} + b_i Q_{S_i}^2). \tag{13}$$

3.2 *Data*

The electric power network includes a circuit with 30 nodes. Six are nodes with generators – 3 for the leader and the 3 for the follower – and the remaining 21 are demand nodes. Connecting the nodes there are 41 arcs and 12 loops.

The data related with production, demand, transmission values are based on [13]. The generator cost function, reactance and upper bounds for supply and transmission flows values are also given. As a safety measure of the network, the upper bounds values for the transmission capacity are 60% of the values assumed in [13].

We studied two distinct situations. The first one (case A) assumes firm A as leader and firm B as follower. In the second case (B), the firms change their role by considering firm B as the leader and firm A as the follower. To solve both the dominant firm situations, it is assumed that the bids for the units of the follower company are equal to their marginal costs, which means $\alpha_i = a$, $i \in P \setminus S_f$, where a is a constant.

The demand curve for each consumer node is determined by $P_i = 40 - d_i Q_{D_i}$ where d_i is chosen so that $P_i = \$30/MWh$ when Q_{D_i} equals the value assumed in [1]. The code of these cases are in AMPL language and can be found in the MacMPEC library [10] with the name *monteiro.mod* and *monteiroB.mod*. Each problem has 136 variables, 201 constraints where 62 of them are complementarity constraints.

The MPCC-NLP approach was used to solve the problem, meaning that all complementarity constraints were reformulated as nonlinear constraints according the definition (2).

4. Computational results

Three nonlinear solvers implementing different techniques were used in order to solve both cases.

Lancelot [9] is a standard Fortran 77 package for large-scale nonlinear optimization, developed by Conn, Gould and Toint. The software uses an augmented Lagrangian approach and combines a trust region approach adapted to handle the bound constraints.

Loqo [2] was developed by Vanderbei and is a software for solving smooth constrained optimization problems. It is based on an infeasible primal–dual interior point method applied to a sequence of quadratic approximations. It uses line search to induce global convergence and an exact Hessian matrix.

The Snopt, developed by Gill, Murray and Saunders, is a software package for solving large-scale linear and NLPs. The functions used should be smooth but not necessarily convex and it is specially effective for problems whose functions and gradients are expensive to evaluate.

The NEOS Server [14] platform was used to interface with the selected solvers. NEOS (*network enabled optimization system*) is an optimization service that is available through the Internet. It has a large set of software packages and it is considered as the state of the art in optimization.

As the numerical results using the three solvers are very similar, it is only reported the ones obtained by Lancelot. Table 1 shows the objective function, the first level variables values and computational time.

Note that in case A, the bids of the nodes 1, 2 and 5 are fixed. It is assumed that firm A, that is owner of the nodes 8, 11 and 13, knows the nodes' bids that are under control of firm B. Conversely, when firm roles changes in case B.

Analysing the above information, it is possible to verify that when the firm takes the follower's role - and as consequence looses the advantage of anticipated knowledge of the market – its profit decreases.

The other variable values, such as power transmissions and demand values, are also different. Figures 1 and 2 show the electric power transmitted between nodes over the power lines in the two cases. The arcs that are emphasized are the optimal electric power flow. It is possible to see that the only difference between the two schemes are that in case B, it is transmitted electric power for the node 5 to the node 7. For more details about the behaviour of the variables, consult Tables A1 and A2 in the Appendix.

In spite of the images seeming quite similar, the values for generated and demanded electric power in each node are different, as Figures 3 and 4 expose for the Lancelot solver.

There are some demand nodes with the electric power close to zero. This may be explained for two reasons: economical ones because it is possible that the transportation of the energy to these

Table 1. Objective function and bid results.

	Profit function		Bid (α^f) for each generator node						
	(π_A)	(π_B)	1	2	5	8	11	13	Time (s)
Case A	37.53	572.05	20	17.50	10	35.83	40	29.80	5.56
Case B	13.58	827.86	29.56	28.30	13.80	32.50	30	30	11.10

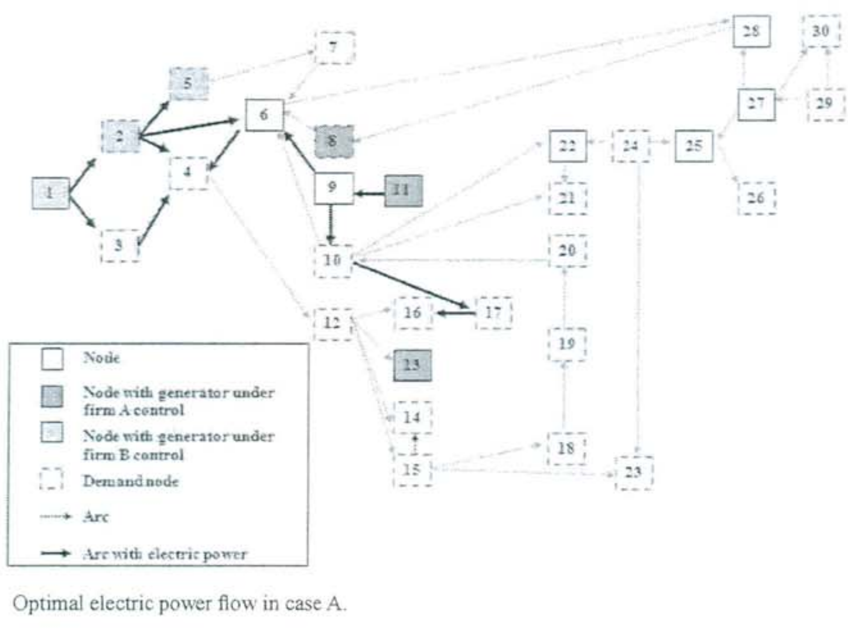

Figure 1. Optimal electric power flow in case A.

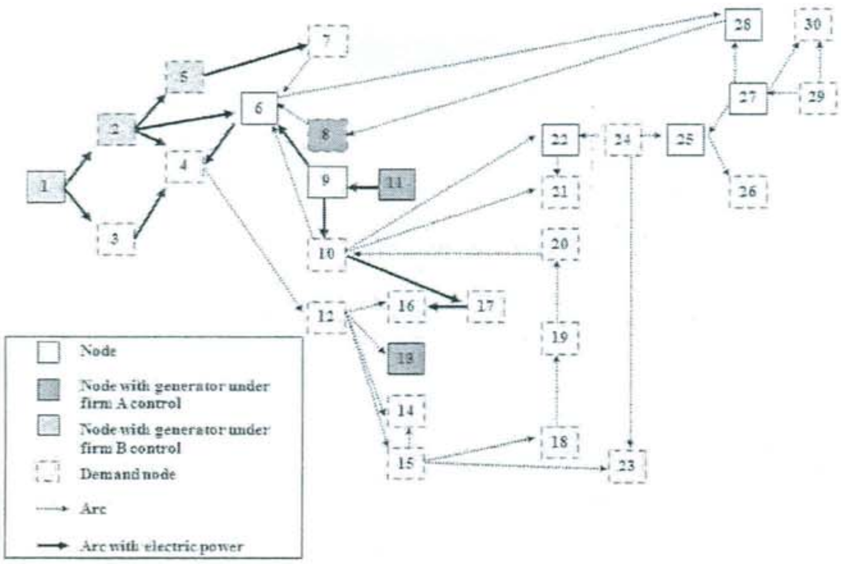

Figure 2. Optimal electric power flow in case B.

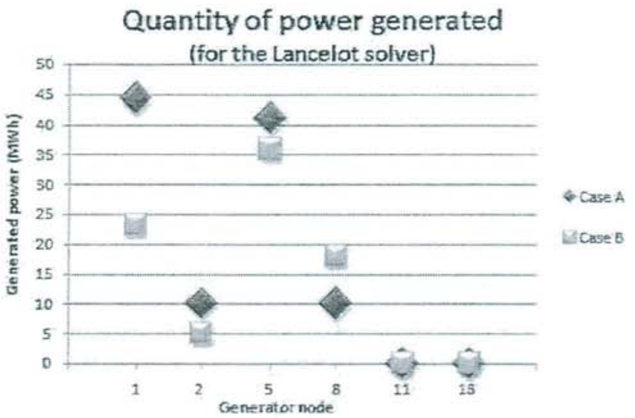

Figure 3. Generated power with Lancelot solver.

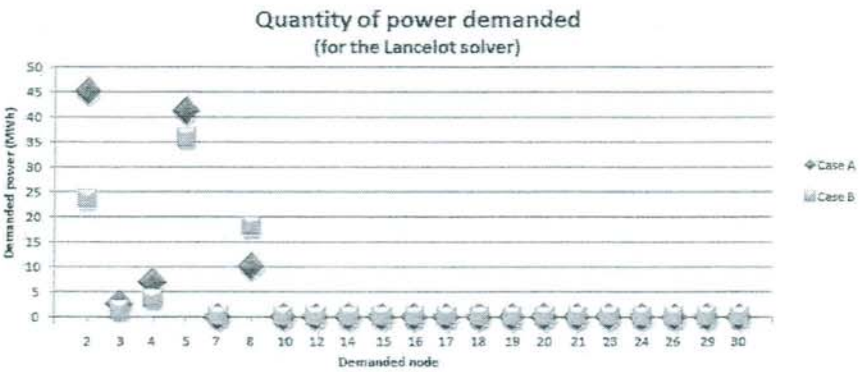

Figure 4. Demanded power with Lancelot solver.

places are too expensive and by the existence of large demand nodes, close to the generator units, that absorb all the power produced.

5. Conclusions and future work

A long and challenging struggle is expected, as reform is an ongoing process requiring detailed design of new models. In this paper an electric power market modeled as a Stackelberg game is presented. The information that a leader takes as input before all the other companies is an important element to obtain larger profits.

It is also possible to conclude that the MPCC-NLP approach is a reliable and robust approach to solve real problems. This approach can indeed provide an advance to producers in presence of information about the market conditions.

As future work, this electric power market will be studied as a Nash model, where both firms compete at the same level with simultaneous moves and with the same market information. The main goal will be to compare the hierarchical Stackelberg game and the symmetric Nash game, and analyse the importance of the information in the market.

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Appendix A. Generated and demanded power for the two cases

Table A1. Generated power.

Demanded nodes	Case A	Case B
1	44.30	23.28
2	10.90	5.11
5	41.04	35.84
8	10.01	18.01
11	1.30e-05	1.61e-07
13	0	0

Table A2. Demanded power.

Demanded nodes	Case A	Case B
2	44.98	23.43
3	2.55	1.34
4	6.87	3.61
5	41.04	35.84
7	0	6.29e-09
8	10.01	18.01
10	0	0
12	0	0
14	0	0
15	0	0
16	1.32e-05	4.77e-08
17	1.32e-05	0
18	0	0
19	0	0
20	0	0
21	0	0
23	0	0
24	0	0
26	0	0
29	0	0
30	0	0