

A MPCC approach on a Stackelberg game in an electric power market: changing the leadership

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Abstract

An electric power market is studied as a Stackelberg game where two firms, A and B, produce energy. It is analyzed two distinct situations, according to the firm who plays the leader role: the first one, when the firm A is the leader and the other firm is the follower, and the second that is the reverse of the players roles. The main goal is to understand the behavior of the various agents that compose the electric power network, such as transmissions capacity, quantities of power generated and demanded, when changing leadership.

The problem is formulated as a Mathematical Program with Complementarity Constraints (MPCC) and reformulated into a Nonlinear Program (NLP), allowing the use of robust NLP solvers. Numerical results are presented and some final considerations are carried out.

*Key words: Electric power, Stackelberg game, leadership, MPCC, NLP solver.
MSC 2000: 90C30, 90C33, 90C99, 91C99*

1 Introduction

In the last years, the transformations of the electric power market has been a constant. It becomes a liberalized activity, where planning and operation scheduling are independent activities which are not constrained by centralized procedures.

In Europe, the liberalization process is under way in many countries. The market has been faced with fusions and merges between companies. The directives of European Union for an electric power liberality led up to increasing institutional and physical connections between markets from different countries. Some papers about studies in course, related with German, French and The Netherlands power markets have emerged - see [4], [7] and [19] for more details.

In a competition environment the main goal is to benefit the consumers through price reduction. Although, ill effects can occur if the level of concentration in the market grows dangerously and the generator companies combine prices between themselves.

According to [14], there are reasons to consider electric power as a special commodity: all power travels over the same set of power lines, independently of the firm that generated it; this difference is particularly marked when the networks contains loops and there are transmission capacity limits; also electricity has unique physical properties, namely Kirchhoff voltage and current laws. As the electricity is difficult to store, and the quantity of power must be instantly adjusted to the demand, the companies that lead the market could easily manipulate the price, changing it to higher values, specially in peak consumption periods.

In order to study the interaction of all market participants and to have a better knowledge of the market conditions, firms and governments need suitable decision-support models. The problem that is presented herein is related with the oligopolist market modeled as a Stakelberg game [13]. In this game theory, there is a non competitive situation, where one player - namely leader - takes as input his own perception of the market and can anticipate the reactions of the other players, using the information in order to select his optimal strategic. The other players - namely followers - do not have the perception how their decisions have influence in leader decisions. Between followers their behavior act like a non-cooperative Nash game, where each one observes the actions of the others and no one can increase their own profit through unilateral decisions.

The Stakelberg game theory was a great motivation for the study of the bilevel optimization problems, because there are many similarities between both [8]. However, solving this kind of problems is a hard task in optimization. But, if the bilevel problem is convex in the second level [20], this level can be replaced by their own Karush-Kuhn-Tucker (KKT) conditions, and the resulting problem is one level optimization problem with complementarity constraints. In this paper, it is studied the transformations of the electric power network, related to quantities of demanded and generated power in each node, when the leader company changes.

The organization of this paper is as follows: section 2 introduces the definition of Mathematical Program with Complementarity Constraints and it special features; it is presented the nonlinear approach of the previous problem as a way of taking advantages of the efficiently and robust NLP solvers. In section 3, we present two versions of the electric power problem and we also provide the data specifying the network used in the computational experiments. The numerical results are shown in section 4 and finally we conclude in the last section.

2 MPCC-NLP approach

The interest on Mathematical Program with Complementarity Constraints (MPCC) increased in the last decade due to the subjacent equilibrium concept present in many applications (see [3, 5, 13] for some applications in the last years).

Definition 2.1 *MPCC Problem*

Mathematical Program with Complementarity Constraints (MPCC) is defined as:

$$\begin{aligned}
 \min_z \quad & F(z) \\
 \text{s.t.} \quad & c_i(z) = 0, i \in E, \\
 & c_i(z) \geq 0, i \in I, \\
 & 0 \leq z_1 \perp z_2 \geq 0,
 \end{aligned} \tag{1}$$

where $z = (z_0, z_1, z_2)$, with the control variable $z_0 \in \mathbb{R}^n$ and the state variables $z_1, z_2 \in \mathbb{R}^p$; F is the objective function, $c_i, i \in E \cup I$ are the equality and inequality constraints, respectively. The sets E and I are the disjoint finite sets of indices. The objective function F and the constraints $c_i, i \in E \cup I$ are assumed twice continuously differentiable. The constraints related to complementarity are defined with the operator \perp and demand that the product of the two nonnegative quantities must be zero, *i.e.*, $z_{1i}z_{2i} = 0, i \in \{1, \dots, p\}$.

The MPCC problem is nonsmooth mostly due to the complementarity constraints. The optimal conditions are complex and very difficult to verify. Besides, the feasible set of MPCC is ill-posed since the constraint qualifications – namely the Mangasarian Fromovitz (MFCQ) and the Linear Independent (LICQ) – which are commonly assumed to prove convergence of standard nonlinear programming do not hold at any feasible point [8, 16]. This implies mostly that the multiplier set is unbounded, the active constraint normals are linearly dependent and the linearizations of the MPCC can become inconsistent arbitrarily close to a solution.

The violation of constraint qualifications has led to a number of specific algorithms for MPCCs. In spite of being specially designed to address MPCC problems they do not represent a real solution, since by these algorithms still need rather strong assumptions to ensure convergence. On the other side they also require significant computational effort when compared with nonlinear solvers in the market. The search of new techniques and algorithms in order to solve real problems with large dimension is still an intensive research area.

Recently, some authors proposed to solve MPCC problems by reformulating into an equivalent NLP problem [12]. This formulation allows to take advantage of certain NLP algorithms features in order to obtain rapid local convergence. Reliability and robustness of NLP solvers can also be acceded by using these problems for numerical testing allowing also to see its performance under some specific problems irregularities.

A MPCC defined in (1) can be reformulated as an equivalent NLP problem of the following form:

Definition 2.2 *NLP formulation of the MPCC problem*

$$\begin{aligned}
 \min_z \quad & F(z) \\
 \text{s.t.} \quad & c_i(z) = 0, i \in E, \\
 & c_i(z) \geq 0, i \in I, \\
 & z_1 \geq 0, \\
 & z_2 \geq 0, \\
 & z_1^T z_2 \leq 0.
 \end{aligned} \tag{2}$$

Recall that the complementarity constraint was replaced by a nonlinear inequality, relaxing the problem. The transformation from a MPCC problem into a NLP problem allows the use of standard NLP solvers.

One can easily show that the reformulated problem has the same properties as the previous one, including the constraint qualifications violation and second-order conditions. However, in the last few years, some studies show that strong stationarity is equivalent to the KKT conditions of the MPCC-NLP problem [17, 18]. This fact has advantages because strong stationarity is a useful and practical computation characterization, since it is relatively easy to find a stationarity point in a NLP solver, under reasonable assumptions.

3 The electric power market problem

The problem described in this paper is based on the model proposed in [6]. It is a competitive power market, formulated as an oligopolistic equilibrium model.

There are a number of generator firms, each owning a given number of units. These make an hourly bid to an Independent System Operator (ISO). The ISO, taking in consideration the network, solves a social welfare maximization problem, announces a dispatch for each bidder and possibly distinct prices at each node. It decides how much power to buy from generators and how much power to distribute to consumers and what prices to charge. All these decisions are made with the optimal power flow in mind.

The leader generator first decides and takes as input all the perceptions and information that it could have about the market (including predictable bids of the other firms, demand and supply functions) and maximizes its profit inside a set of spatial price equilibrium constraints and Kirchhoff's voltage and current laws. The followers units make their own decisions taking into account the leader decision.

3.1 Formulation

The notation used in the mathematical formulation follows.

Indices:

- i node in the network
- ij arc from node i to node j
- m number of Kirchhoff voltage loops in the network

Sets:

- \mathcal{N} set of all nodes
- \mathcal{A} set of all arcs
- \mathcal{S}_f set of generator nodes under control of leader firm f
- \mathcal{P} set of all generators nodes
- \mathcal{D} set of all demand nodes
- \mathcal{L} set of Kirchhoff' voltage loops m
- \mathcal{L}_m set of ordered arcs (clockwise) associated with loop m

Recall that, a node can be, simultaneously a generator and a consumer, so \mathcal{P} and \mathcal{D} are not necessarily disjoint and their union could be a proper subset of \mathcal{N} . The

uniqueness of the net flow on each arc is ensured by the Kirchhoff's laws in the linearized DC models and, consequently, the number of (independent) loops are $\#\mathcal{A} - \#\mathcal{N} + 1$ (where $\#X$ is the set X cardinality).

Parameters:

- a_i, b_i intercept and slope of supply function (marginal cost) for the generator at node $i \in \mathcal{P}$
- c_i, d_i intercept and slope of demand function for consumer at node $i \in \mathcal{D}$
- $\bar{\alpha}_i$ upper bound of the bid for the unit at node $i \in \mathcal{S}_f$
- \bar{Q}_{S_i} upper bound of production capacity for the unit at node $i \in \mathcal{P}$
- \bar{T}_{ij} maximum transmission capacity on arc $ij \in \mathcal{A}$
- r_{ij} reactance on arc $ij \in \mathcal{A}$
- s_{ijm} ± 1 corresponding to the orientation of the arc $ij \in \mathcal{A}$ in loop $m \in \mathcal{L}$ (+1 if ij has the same orientation as the loop m)

First-Level decision variable

- α bid for the unit at node $i \in \mathcal{P}$

In this model, it is assumed that the generator firms can only manipulate α (the intercept in the bid function) and not the slope b , due to market and optimization assumptions.

Let α_i be fixed for the competitive firms (*i.e.*, α_i fixed $\forall i \in \mathcal{P} \setminus \mathcal{S}_f$) and variables for the leader firms (*i.e.*, α_i variable $\forall i \in \mathcal{S}_f$).

Primal variables in the second-level

- Q_{S_i} vector defined by quantity of power generated by the unit at node i
($Q_{S_i} = a_i + b_i Q_{S_i}$ if $i \in \mathcal{P}$ and $Q_{S_i} = 0$ if $i \notin \mathcal{P}$)
- Q_{D_i} quantity of power demanded at node i
($Q_{D_i} = c_i - d_i Q_{D_i}$ if $i \in \mathcal{D}$ and $Q_{D_i} = 0$ if $i \notin \mathcal{D}$)
- T_{ij} matrix defined by MW transmitted from node i to node j

Dual variables in the second-level

- λ_i marginal cost at node i
- μ_i marginal value of generation capacity for unit at node i
- θ_{ij} marginal value of transmission capacity on arc ij
- γ_m shadow price for Kirchhoff voltage law for loop m

Let Δ be the matrix with the information about the pair (node,arc) in the electric network:

$$\Delta_{il} = \begin{cases} 1, & \text{if } l = ij \in \mathcal{A} \text{ for some } j \in \mathcal{N} \\ -1, & \text{if } l = ji \in \mathcal{A} \text{ for some } j \in \mathcal{N} \\ 0, & \text{other values} \end{cases} \quad (3)$$

Let R be the matrix (arc, cycle) related with the reactance coefficients:

$$R_{ij,m} = \begin{cases} s_{ijm} r_{ij}, & \text{if } ij \in \mathcal{L}_m \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

The notation $diag(w)$ represents the diagonal matrix whose diagonal entries are the components of the vector w .

It is provided in [9] that due to the convexity of the second level problem, for each vector α , there exists a unique globally optimal solution $(Q_D(\alpha), Q_S(\alpha), T(\alpha))$.

Replacing the second level of problem by its KKT optimality conditions leads to a MPCC optimization problem. The maximization of the leader firm profit in the electric power market is described by the following MPCC problem:

$$\begin{aligned}
\max \quad & \Pi_f(\lambda, Q_D, Q_S, T, \theta, \lambda, \mu) \equiv \sum_{i \in D} (c_i Q_{D_i} - d_i Q_{D_i}^2) \\
& - \sum_{i \in \mathcal{S}_f} \left(a_i Q_{S_i} + \frac{b_i}{2} Q_{S_i}^2 \right) - \sum_{ij \in A} \theta_{ij} \bar{T}_{ij} \\
& - \sum_{i \in P \setminus \mathcal{S}_f} (\mu_i \bar{Q}_{S_i} + a_i Q_{S_i} + b_i Q_{S_i}^2) \\
\text{s.t.} \quad & 0 \leq \alpha_i \leq \bar{\alpha}_i, \forall i \in \mathcal{S}_f \\
& 0 \leq \bar{Q}_S - Q_S \quad \perp \quad \mu \geq 0 \\
& 0 \leq Q_S \quad \perp \quad -\lambda + \mu + \alpha + \text{diag}(b)Q_S \geq 0 \\
& 0 \leq Q_D \quad \perp \quad \lambda - c + \text{diag}(d)Q_D \geq 0 \\
& 0 \leq \theta \quad \perp \quad \bar{T} - T \geq 0 \\
& 0 \leq T \quad \perp \quad \Delta^T \lambda + \theta + R\gamma \geq 0 \\
& Q_D - Q_S + \Delta T = 0 \\
& R^T T = 0 \\
& \lambda \text{ free} \\
& \gamma \text{ free}
\end{aligned} \tag{5}$$

3.2 Data

The electric power network includes a circuit with 30 nodes. Six are nodes with generators – 3 for the leader and the 3 for the follower – and the remaining 21 are demand nodes. Connecting the nodes there are 41 arcs and 12 loops. Figure 1 shows a network scheme with additional information.

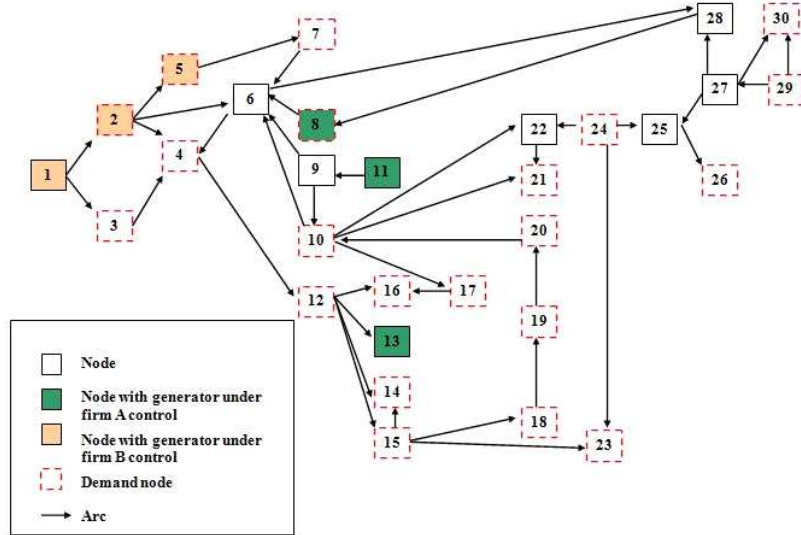


Figure 1: Electric network scheme

The data related with production, demand, transmission values are based on [14]. The generator cost function, reactance and upper bounds for supply and transmission flows values are also given. As a safety measure of the network the upper bounds values for the transmission capacity are 60% of the values assumed in [14].

It is studied two distinct situations. The first one (case A), assumes firm A as leader firm B as follower. The second case (B), the firms change their role by considering firm B as the leader and firm A as the follower. To solve the both dominant firm situations it is assumed that the bids for the units of the follower company are equals to their marginal costs, which means $\alpha_i = a$, $i \in P \setminus S_f$, where a is a constant.

The demand curve for each costumer node is determined by $P_i = 40 - d_i Q_{D_i}$ where d_i is chosen so that $P_i = \$30/MWh$ when Q_{D_i} equals the value assumed in [1].

The code of these cases are in AMPL language and can be found in the MacMPEC [11] with the name *monteiro.mod* and *monteiroB.mod*. Each problem has 136 variables, 201 constraints where 62 of them are complementarity constraints.

The MPCC-NLP approach was used to solve the problem, meaning that all complementarity constraints were reformulated as nonlinear constraints according the definition (2).

4 Computational results

Three nonlinear solvers were used in order to solve both problem cases. These solvers provide a sample for available nonlinear optimization software, implementing different techniques.

Lancelot [10] is a standard Fortran 77 package for large scale nonlinear optimization, developed by Conn, Gould and Toint. The software uses an augmented Lagrangian approach and combines a trust region approach adapted to handle the bound constraints.

Loqo [2] was developed by Vanderbei and is a software for solving smooth constrained optimization problems. It is based on an infeasible primal-dual interior point method applied to a sequence of quadratic approximations. It uses line search to induce global convergence and an exact Hessian matrix.

The Snopt, developed by Gill, Murray and Saunders, is a software package for solving large-scale linear and nonlinear programs. The functions used should be smooth but not necessary convex and it is specially effective for problems whose functions and gradients are expensive to evaluate.

The NEOS Server [15] platform was used to interface with the selected solvers. NEOS (*Network Enabled Optimization System*) is an optimization service that is available through the Internet. It is a large set of software packages considered as the state of the art in optimization.

The numerical results using NLP solvers are presented in Table 1 where the objective function and the first level variables are shown. Note that in case A, the bids of the nodes 1, 2 and 5 are fixed. It is assumed that firm A, that is owner of the nodes 8, 11 and 13, knows the nodes bids that are under control of firm B. Conversely, when firm roles changes in case B.

	Solver	Profit function (π_f)	Bid (α^f) for each generator node					
			1	2	5	8	11	13
Case A	LANCELOT	37.53	20	17.50	10	35.83	40	29.80
	LOQO	37.53	20	17.50	10	35.83	36.09	20
	SNOPT	37.53	20	17.50	10	35.83	39.99	0
Case B	LANCELOT	827.86	29.56	28.30	13.80	32.50	30	30
	LOQO	823.40	29.46	28.30	13.80	32.50	30	30
	SNOPT	827.86	29.56	28.30	13.80	32.50	30	30

Table 1: Objective function and bid results

Although it has been reached an identical value for all solvers for the objective function, in each case, the same did not happen for the bid variable, which take us to believe for the existence of the several local optima.

The other variable values are also different. In Figure 4, it is shown the electric power transmitted between nodes over the power lines. The arcs that are emphasized are the optimal electric power flow. It is possible to see that the only visual difference between the two schemes are that in case B, it is transmitted electric power for the node 5 to the node 7. In these figures, for visual issues, it is only visible the differences in the nodes for the Lancelot solver. For more details about the behavior of the other solvers it is possible consult Tables 2 and 3 in appendix.

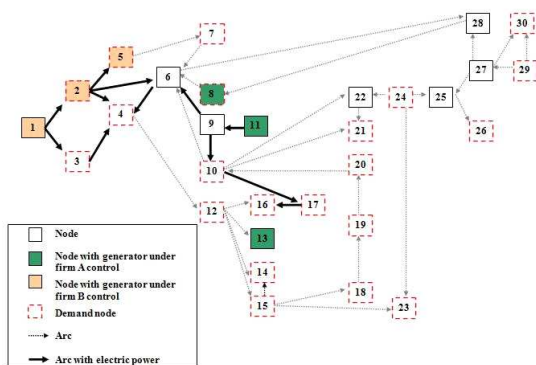


Figure 2: Optimal electric power flow in case A

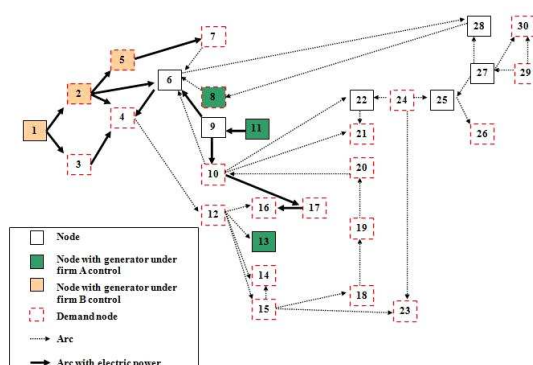


Figure 3: Optimal electric power flow in case B

In spite of the images seem quite similar, the values for generated and demanded electric power in each node is different, as Figures 4 and 5 expose for the Lancelot solver.

There are some demand nodes that practically do not receive electric power. This may be explained for two reasons: economical ones because it is possible that the transportation of the energy for these places are too expensive and by the existence of large demander nodes close to the generator units that absorb all the power produced.

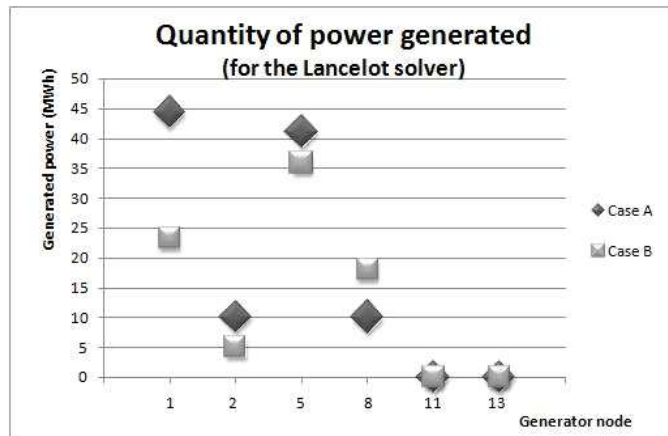


Figure 4: Generated power with Lancelot solver

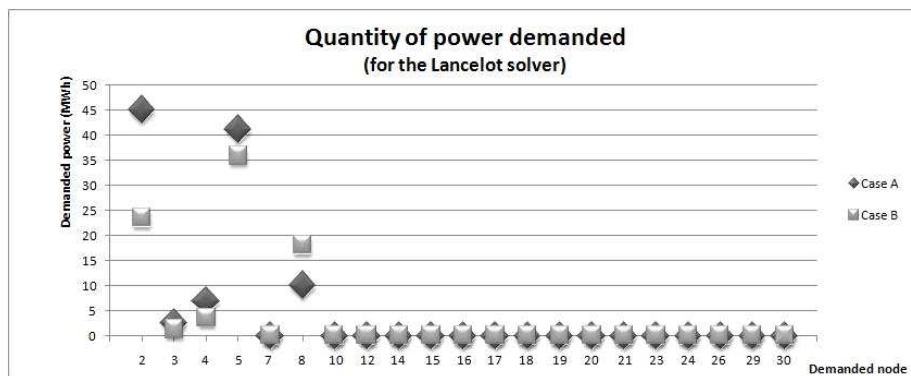


Figure 5: Demanded power with Lancelot solver

5 Conclusions and further work

Due to the economics and politics importance, the electric power market has been a target of many studies. In this paper it has been shown that the MPCC-NLP approach is a reliable and robust approach to solve real problems, providing a powerful tool for decision makers. These tools can indeed provide an advance to producers in presence of information about the market conditions.

The power of information is a decisive factor in order to obtain the dominance of the market. The behavior of the various agents that compose the electric power network, such as transmissions capacity, quantities of power generated and demanded, are different when there are distinct perspectives of the market.

As future work, we will study of this problem as a Nash model, where both firms compete at the same level with the same market information.

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A Generated and demanded power for the two cases

Demanded nodes	Case A			Case B		
	LANCELOT	LOQO	SNOPT	LANCELOT	LOQO	SNOPT
1	44.30	44.30	44.30	23.28	28.58	23.28
2	10.90	10.90	10.90	5.11	0	5.11
5	41.04	41.04	41.04	35.84	35.84	35.84
8	10.01	10.01	10.01	18.01	18.01	18.01
11	1.30e-05	1.60e-05	0	1.61e-07	1.61e-07	0
13	0	0	0	0	0	0

Table 2: Generated power

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Demanded nodes	Case A			Case B		
	LANCELOT	LOQO	SNOPT	LANCELOT	LOQO	SNOPT
2	44.98	44.98	44.98	23.43	22.51	23.43
3	2.55	2.55	2.55	1.34	1.57	1.34
4	6.87	6.87	6.87	3.61	4.50	3.61
5	41.04	41.04	41.04	35.84	35.84	35.84
7	0	0	0	6.29e-09	0	0
8	10.01	10.01	10.01	18.01	18.01	18.01
10	0	0	0	0	6.68e-08	0
12	0	0	0	0	0	0
14	0	0	0	0	0	0
15	0	0	0	0	0	0
16	1.32e-05	0	1.32e-05	4.77e-08	0	0
17	1.32e-05	0	0	0	3.50e-09	0
18	0	0	0	0	0	0
19	0	0	0	0	0	0
20	0	0	0	0	0	0
21	0	0	0	0	0	0
23	0	0	0	0	0	0
24	0	0	0	0	0	0
26	0	0	0	0	0	0
29	0	0	0	0	0	0
30	0	0	0	0	0	0

Table 3: Demanded power